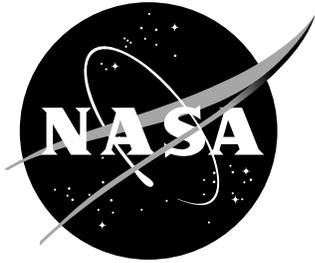


NASA/TM-2017-219612



# A Comparison of Three Random Number Generators for Aircraft Dynamic Modeling Applications

*Jared A. Grauer  
Langley Research Center, Hampton, Virginia*

---

May 2017

## The NASA STI Program Office . . . in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

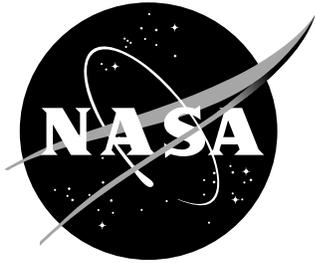
- **TECHNICAL PUBLICATION.** Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA counterpart of peer-reviewed formal professional papers, but having less stringent limitations on manuscript length and extent of graphic presentations.
- **TECHNICAL MEMORANDUM.** Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.
- **CONTRACTOR REPORT.** Scientific and technical findings by NASA-sponsored contractors and grantees.
- **CONFERENCE PUBLICATION.** Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or co-sponsored by NASA.
- **SPECIAL PUBLICATION.** Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.
- **TECHNICAL TRANSLATION.** English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office's diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results . . . even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at <http://www.sti.nasa.gov>
- E-mail your question via the Internet to [help@sti.nasa.gov](mailto:help@sti.nasa.gov)
- Fax your question to the NASA STI Help Desk at (301) 621-0134
- Phone the NASA STI Help Desk at (301) 621-0390
- Write to:  
NASA STI Help Desk  
NASA Center for AeroSpace Information  
7115 Standard Drive  
Hanover, MD 21076-1320

NASA/TM-2017-219612



# A Comparison of Three Random Number Generators for Aircraft Dynamic Modeling Applications

*Jared A. Grauer*  
*Langley Research Center, Hampton, Virginia*

National Aeronautics and  
Space Administration

Langley Research Center  
Hampton, Virginia 23681-2199

---

May 2017

The use of trademarks or names of manufacturers in this report is for accurate reporting and does not constitute an official endorsement, either expressed or implied, of such products or manufacturers by the National Aeronautics and Space Administration.

---

Available from:

NASA Center for Aerospace Information (CASI)  
7115 Standard Drive  
Hanover, MD 21076-1320  
(301) 621-0390

National Technical Information Service (NTIS)  
5285 Port Royal Road  
Springfield, VA 22161-2171  
(703) 605-6000

## Abstract

Three random number generators, which produce Gaussian white noise sequences, were compared to assess their suitability in aircraft dynamic modeling applications. The first generator considered was the MATLAB<sup>®</sup> implementation of the Mersenne-Twister algorithm. The second generator was a website called Random.org, which processes atmospheric noise measured using radios to create the random numbers. The third generator was based on synthesis of the Fourier series, where the random number sequences are constructed from prescribed amplitude and phase spectra. A total of 200 sequences, each having 601 random numbers, for each generator were collected and analyzed in terms of the mean, variance, normality, autocorrelation, and power spectral density. These sequences were then applied to two problems in aircraft dynamic modeling, namely estimating stability and control derivatives from simulated onboard sensor data, and simulating flight in atmospheric turbulence. In general, each random number generator had good performance and is well-suited for aircraft dynamic modeling applications. Specific strengths and weaknesses of each generator are discussed. For Monte Carlo simulation, the Fourier synthesis method is recommended because it most accurately and consistently approximated Gaussian white noise and can be implemented with reasonable computational effort.

# Contents

Nomenclature . . . . .	3
1 Introduction . . . . .	5
2 Random Number Generators . . . . .	7
3 Single Sample Records . . . . .	9
4 Ensembles of Sample Records . . . . .	12
5 Application to Parameter Estimation . . . . .	13
6 Application to Simulation in Turbulence . . . . .	15
7 Concluding Remarks . . . . .	16
8 Acknowledgements . . . . .	17
References . . . . .	19
Appendix A – Variance of the Finite Fourier Series . . . . .	20
Appendix B – Autocorrelation of the Finite Fourier Series . . . . .	21
Tables . . . . .	22
Figures . . . . .	24

# Nomenclature

## Roman

$a_z$	vertical accelerometer output, g
$c_k$	Fourier series amplitude
$\text{cov}(\cdot), \text{var}(\cdot)$	covariance and variance
$E[\cdot]$	expectation operator
$f$	frequency, Hz
$f_N$	Nyquist frequency, = $1/2\Delta t$ Hz
$G_{vv}$	one-sided power spectral density
$g$	gravitational acceleration, = 32.174 ft/s <sup>2</sup>
$i$	sample index variable
$J$	cost function
$j$	imaginary number, = $\sqrt{-1}$
$k$	frequency index variable
$l$	lag index variable
$M$	number of frequencies
$M_\alpha, M_q, M_{\delta_e}$	pitching moment stability and control derivatives
$N$	number of samples
$\mathcal{N}(0, 1)$	normal distribution having zero mean and unit variance
$P(\cdot)$	Probability
$q$	pitch rate, rad/s
$R^2$	coefficient of determination
$r_{..}$	autocorrelation function
$s$	Laplace operator
$s(\cdot)$	standard error
$T$	record length, s
$t$	time, s
$V$	airspeed, ft/s
$\mathbf{v}$	noise sequence
$w$	vertical body-fixed velocity, ft/s
$\mathbf{X}$	regressor matrix
$Z_\alpha, Z_q, Z_{\delta_e}$	vertical body-axis force stability and control derivatives
$\mathbf{z}$	dependent modeling variable

## Greek

$\alpha$	angle of attack, rad
$\Delta$	perturbation quantity
$\Delta t$	sampling period, s
$\delta_e$	elevator deflection, deg
$\theta$	model parameters
$\rho$	pairwise correlation coefficient
$\sigma$	standard deviation
$\Phi$	cumulative distribution function
$\phi_k$	Fourier series phase angle, rad
$\omega$	angular frequency, rad/s

### Superscripts

*	complex conjugate
·	time derivative
^	estimated value
-	mean value
-1	inverse

### Subscripts

0	trim value
<i>g</i>	gust

# 1 Introduction

Random number sequences are often used in aircraft dynamic modeling applications to approximate Gaussian, band-limited, white noise sequences having the following ideal characteristics:

- Zero mean
- Unit variance
- Gaussian/normal distribution
- Uniform power spectrum
- No serial correlations
- Finite energy

For example, this type of noise is often added to simulated output signals to mimic sensor measurement noise. White noise can also be used to drive a dynamic system to simulate sources of process noise. Shaping or coloring filters use white noise as input to produce random signals with a specified power spectral density, for example to simulate neglected deterministic dynamics [1,2] or stochastic turbulence [3]. Drift in computer clocks, such as in GPS receivers, or drift in sensor calibration parameters, for example due to temperature sensitivities, can be modeled as a random walk using white noise [4]. In these modeling applications, many different white noise sequences are used in repeated Monte Carlo experiments to gain confidence in the predictive capability of a model, to evaluate handling qualities or control law performance, or to develop and test new methods of analysis before application to flight test data. In identification applications, the performance of parameter estimation algorithms using the extended Kalman filter improve when artificial white noise is used in the analysis to allow the parameter estimates to vary [5].

Given the ubiquity of random numbers in aircraft dynamic modeling problems, it is important to have fast access to many random number sequences of high quality and potentially long duration. Algorithms for generating random number sequences have been improving since the advent of the modern computer [6]. During the 1980's and 1990's, as Monte Carlo simulations of aerospace vehicles became more widespread, many methods for generating random numbers were developed and evaluated. More recently, the focus has been on generating random numbers from measurements of seemingly random physical processes, especially for cryptography applications.

Despite this steady growth of research, no recent studies on evaluating random number generators for aircraft dynamic modeling applications were found in the literature. A casual polling of colleagues indicated that most users had not investigated the quality of their random number generators, but generally felt comfortable with the results for their applications. On this topic of using random number generators to simulate noise, Hamming writes [7]

*Experience seems to indicate that the careless use of a random-number generator will trap the unwary many more times than at first seems reasonable, but also that with care random-number generators give very satisfactory results. It is a field for the beginner to be wary and suspicious of and he should make a few careful checks that all is going as he thinks it should before he plunges into the details of using the simulation as a working tool of design optimization.*

This report documents such an investigation. The purpose is to examine a few interesting random number generators, using relevant statistical tools, and report on the suitability of the generators for use in dynamic modeling applications.

Three sources of random number sequences were considered. The first was the Mersenne-Twister algorithm implemented as the default random number generator in the MATLAB<sup>®</sup> software suite. The second was the website Random.org, which streams random numbers processed from atmospheric noise measured using radios. The third was a method involving the construction of random sequences through Fourier synthesis.

The following section describes the random number generators. Results are then shown for one sample record, to illustrate the characteristics of these sequences using statistical tools commonly used for dynamic modeling. To examine the asymptotic nature of the generators, results using ensembles of sequences are then presented. These same random sequences are then applied to two example aircraft modeling applications in a Monte Carlo simulation framework. The first example is estimating stability and control derivatives from simulated onboard sensor data using linear regression. The second example is adding atmospheric turbulence to an aircraft flight dynamics simulation. Concluding remarks then complete the report. In general, each random number generator performed well and was considered suitable for Monte Carlo simulation. The method of Fourier synthesis performed better than the other two methods in that it more consistently and more precisely approximated Gaussian white noise.

Software for the multisine input design and much of the analysis are available in a MATLAB toolbox called System IDentification Programs for AirCRAFT, or SIDPAC [8], which is associated with Ref. 9.

## 2 Random Number Generators

### MATLAB / Mersenne-Twister

The MATLAB [10] software suite has a built-in random number generator. The current software version (R2016b) invokes the Mersenne-Twister method [11], which was developed specifically for Monte Carlo simulation. The software creates a sequence of 32-bit random integers, which are then transformed to the open interval  $(0, 1)$ . The function `randn.m` takes this uniformly-distributed sequence and maps it to a zero-mean, unit-variance, normally-distributed sequence using the Ziggurat algorithm [12]. Other non-default settings for the MATLAB random number generator were not investigated because they implement legacy algorithms or other distributions.

This generator was considered in this report for several reasons. Because MATLAB is widely used in the analysis of aerospace vehicles, its native random number generator is used often. Mersenne-Twister is a deterministic algorithm known as a pseudo-random number generator, and previously-used sequences can be regenerated or new sequences can be generated. This is advantageous for debugging software, but it also means that the sequences are not truly random, and that there is a finite period before the sequence repeats. However, this method can create many long sequences of random numbers quickly, which facilitates Monte Carlo simulation.

### Random.org

The website Random.org [13] streams random numbers created from measurements of atmospheric noise using radio receivers. The radios are set to unused frequencies to register static noise, which is digitized as a sequence of 8-bit numbers. All bits in these numbers are discarded, except the least significant bit, which has a high level of entropy. Retained bits from many different recorded numbers are then combined to form a uniformly-distributed number. For a Gaussian sequence, the Box-Muller method [14] is used to transform the distribution.

This random number generator was considered because it is regarded as a true random number generator, which should be free of the problems associated with pseudo-random number generators. In addition, the website contains documentation by the Trinity College, Dublin evaluating the quality of the results from the website. Therefore this generator was expected to be of high quality and useful for comparison. Furthermore, access to the website was free of charge, and the user interface provided enough control to facilitate a fair comparison with other methods, e.g., distribution type, number of significant digits, etc.

A relatively large amount of time was needed to query the website for random numbers, and then to store and recall the results for analysis. As Monte Carlo simulations involve repeated calculations and require relatively large amounts of time, any additional delay due to obtaining or handling random numbers is not desirable. Another drawback is that some true random number generators are susceptible to environmental factors that degrade performance, such as the aging of electrical components that may bias the numbers. Random.org guards against these factors by using multiple radios in different locations and monitoring statistical metrics in real time.

## Fourier Synthesis

Lánczos and Bellai [15] expound on an earlier observation by Lánczos [16] that an ideal Gaussian white noise sequence is characterized by a constant amplitude spectrum and a random phase spectrum, and that such a sequence can be synthesized using the Fourier series. Specifically, given  $N$  equidistant sample times

$$t_i = (i - 1) \Delta t, \quad \text{for } i = 1, 2, \dots, \frac{T}{\Delta t} + 1 \quad (1)$$

and  $M$  harmonic frequencies up to but not including the Nyquist frequency

$$f_k = \frac{k}{T}, \quad \text{for } k = 1, 2, \dots, \frac{T}{2\Delta t} - 1 \quad (2)$$

a Gaussian random noise sequence can be constructed using the finite Fourier series as

$$v(t_i) = \sum_{k=1}^M c_k \sin \left( \frac{2\pi k}{T} t_i + \phi_k \right) \quad (3)$$

The amplitudes  $c_k$  are set equal to a constant to give a flat amplitude spectrum, characteristic of white noise. Choosing this constant as  $\sqrt{2/M}$  yields a sequence with unit variance, as shown in Appendix A. The phase angles  $\phi_k$  are drawn from a uniform distribution over the interval  $(0, 2\pi)$ . Any method of obtaining uniformly-distributed random numbers can be used for this step. In this report, the MATLAB function `rand.m` was used to implement the Mersenne-Twister algorithm described in Section 2. The resulting sequences have, to numerical accuracy, zero mean because the bias term in the Fourier series was omitted and because the harmonic sinusoids had complete cycles over the time duration  $[0, T]$ . When the sequence is long enough that many harmonic sinusoids are present in the sequence, the central limit theorem suggests the sequence attains a Gaussian distribution [15].

This generator was considered because of its excellent potential qualities. It also retains the favorable properties of the pseudo-random number generator in that sequences can be constructed quickly and can be regenerated for software debugging or randomized for Monte Carlo simulation. Another benefit is that the Fourier series is used to both construct the sequence and to analyze its spectral character. As discussed later, this results in an excellent approximation of Gaussian white noise in the frequency domain. If this method is to be used in real time, for example with flight simulators or parameter estimation using extended Kalman filters, a large value of  $T$  can be selected to synthesize a noise sequence. If that value of time is reached, a new sequence can be constructed using another random drawing of phase angles.

### 3 Single Sample Records

In this section, a single sample record from each random number generator is studied to illustrate typical statistical characteristics of interest in dynamic modeling. The next section examines the asymptotic qualities of the generators.

Standardized tests for random number generators are typically intended for uniform distributions or binary number sequences, and evaluate a variety of characteristics such as spectral content, pattern reoccurrence, total draws of 0's and 1's, and furthest drift incurred from random walk processes [17, 18]. These tests are not all directly applicable or relevant to the random number sequences studied here. Instead, this report uses analysis tools commonplace in aircraft dynamic modeling applications to judge the precision in which these sequences approximate Gaussian white noise. In this context, a successful generator will closely match all the prescribed characteristics of Gaussian white noise mentioned previously.

Each sequence contains 601 random numbers in double-precision format. The length of the sequence was selected to reflect typical aircraft modeling data records. Specifically, these sequences correspond to 12 s of data recorded at 50 Hz, which could be a maneuver involving the short period or dutch roll motions of an aircraft and standard onboard instrumentation. Each set of random numbers was generated as a Gaussian, zero-mean, unit-variance, white noise sequence. Figure 1 shows each of the three sample records. The plot markers are used consistently throughout this paper to distinguish the random number generators. Summary statistics for these sequences are listed in Table 1.

#### Mean and Variance

The random number sequences were intended to have a mean of zero and a variance of unity. For the sample records shown, the MATLAB sequence had the largest error in the mean, and the Random.org sequence had the largest error in the variance. The Fourier synthesis sequence had almost no error in these regards, by design. The small amount of error in the mean was due to numerical rounding errors, which could be removed by enforcing boundary conditions.

Although the variances of the sample records are approximately the same, Fig. 1 shows that there is a greater dispersion of the data in the sequences from Random.org and Fourier synthesis than from MATLAB. These points are not statistical outliers, as discussed in the following section. The comments given here applied in general, not just for these particular sample records.

#### Normality

Normal probability plots show data and assumed theoretical distributions, drawn as straight lines [19]. These plots are useful for detecting systematic and large deviations from a theoretical distribution, and do not require any adjustments by the analyst, such as selecting bin sizes and locations for histograms.

For a normal distribution, the transformed data are [9]

$$\Phi^{-1} [P(i)] \tag{4}$$

where

$$P(i) = \left(i - \frac{1}{2}\right) / N \quad (5a)$$

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-u^2/2} dt \quad (5b)$$

Here  $P(i)$  is the empirical quantile quantifying the fraction of data less than its argument, and  $\Phi(z)$  is the cumulative distribution function. When the transformed data are plotted against the ordered data, the slope quantifies the variance and the intercept quantifies the mean. The degree to which the transformed data diverge from the straight line describes the inaccuracy of the assumed distribution.

Normal probability plots are shown in Fig. 2 for the sample records. Also shown with the data is a straight line, corresponding to the normal distribution with zero mean and unit variance. Agreement of these plots indicates that each sequence was approximately Gaussian. Coefficients of determination

$$R^2 = \frac{\hat{\mathbf{v}}^T \mathbf{v} - N\bar{v}^2}{\mathbf{v}^T \mathbf{v} - N\bar{v}^2} \quad (6)$$

listed in Table 1 indicate that fits to the theoretical distribution all had less than 0.4% error. There were slight deviations from the theoretical distribution in the larger quantiles of the data, but this is expected for distributions with small tails, such as the normal distribution [19].

As remarked earlier from viewing the sample records, it is interesting that the sequences from Random.org and Fourier Synthesis have a larger dispersion of data, although a similar variance. These data correspond to the tails of the normal probability distribution function, as illustrated in Fig. 2 by data in the fourth quartiles. Because these data still lie close to the straight line, they are consistent with a normal distribution and are not statistical outliers.

## Autocorrelation

The autocorrelation of a sequence can be used to show serial correlations in the data and evaluate whiteness. An estimate of the autocorrelation is [9]

$$\hat{r}_{vv}(l) = \frac{1}{N} \sum_{i=1}^{N-l} v(i)v(i+l), \quad \text{for } l = 0, 1, 2, \dots, N-1 \quad (7)$$

where  $l$  is a lag index. This estimate is biased, but is accurate for data with large  $N$ , which is common in aircraft modeling problems [9]. The standard error of this estimate is approximated as [9, 20]

$$s[\hat{r}_{vv}(l)] = \frac{\hat{r}_{vv}(0)}{\sqrt{N}}, \quad \text{for } l \neq 0 \quad (8)$$

For white noise sequences, plotting the autocorrelation for various lag index values results in a peak at  $l = 0$ , equal to the variance of the sequence, and zero at all other lag indices. Systematic and large deviations from this description indicate the presence of serial correlations in the data. In that case, the data is representative of colored noise, not white noise.

Figure 3 shows the autocorrelations of the sample records, as well as two standard error intervals, where  $s[\hat{r}_{vv}(l)] = 1/\sqrt{601}$ . Each sequence appears approximately white. Table 1 indicates that the variance estimate attained from  $\hat{r}_{vv}(0)$  was less accurate for the Random.org sequence than for the other two.

There are appreciable differences between the autocorrelations of the sequences for non-zero lag indices. The MATLAB sequence had 0.5% of the lagged autocorrelation values outside the error bound, and the Random.org sequence had 2.16% of its values outside the error bound. While these values are in statistical agreement with the 95% confidence interval, these points could be confused with colored noise sequences or unmodeled dynamics in dynamic modeling problems. The Fourier synthesis sequence had all of its lagged autocorrelation values remain well within the error bounds. The reason for this is discussed in Appendix B

## Power Spectral Density

White noise has a uniform power spectral density (PSD), which can be computed as

$$G_{vv}(j\omega_k) = \frac{2}{T} v^*(j\omega_k)v(j\omega_k) \quad (9)$$

where  $\omega_k = 2\pi f_k$ ,  $v(j\omega_k)$  is computed from the finite Fourier transform of the sequence [9, 21], and  $v^*(j\omega_k)$  denotes the complex conjugate of  $v(j\omega_k)$ . The transform was evaluated using the chirp z-transform at the frequencies given by Eq. (2), which were the same frequencies used in the method of Fourier synthesis to create the noise sequence.

Figure 4 shows the PSD for each of the three sample records. These plots are multiplied by the Nyquist frequency so that the values shown approximate the variance of the signal. In addition, the means of these data (averaged over frequency) are shown, as well as two standard deviation intervals about those mean values. On average, each of the three sample records approximated unit variance white noise well in this regard; however, the performance of the Fourier synthesis method was clearly superior. This was a result of designing the random sequences in the frequency domain to have a uniform amplitude spectrum.

The variance of the amplitude spectrum was significantly lower for the sample record generated using Fourier synthesis. This was in part due to its frequency-based design, and also because the same harmonic frequencies used to construct the sequence were used in the Fourier transform to analyze it. If, for example, the frequency resolution of the chirp z-transform were increased by a factor of 2, more scatter would be seen in the PSD of the Fourier synthesis data due to spectral leakage caused by using a finite duration of data. This was investigated, and the mean over frequency decreased to 0.9968 and the standard deviation increased to 0.5412. Comparison with the other methods in Table 1 shows this is still a better approximation of white noise than the sample records obtained from MATLAB and Random.org, from this perspective. One may attempt to improve the quality of the noise sequence by using a finer spacing of frequencies in its construction, i.e., a frequency resolution finer than  $1/T$  in Eq. (2), but this forfeits the orthogonality property of the sinusoid series and introduces serial correlations into the sequence. Instead, a longer noise sequence could be designed and truncated to give the same result using a finer frequency resolution.

## 4 Ensembles of Sample Records

The results in the previous section examined a single sample record from each random number generator. Analysis of ensembles of sequences are discussed in this section to examine the asymptotic nature of the random number generators. In total, 200 different sequences of 601 random numbers were collected from each random number generator. The MATLAB Mersenne-Twister sequences and the Fourier synthesis sequences were generated at the time of analysis. The sequences from Random.org were queried thirty sequences at a time over seven days, and at different times of day, to remain within the allowed website quota and to attempt to remove any correlations due to time of day, weather, etc.

The results of the analysis generally followed the same trends as those exhibited from the single sample records discussed in the previous section. Summary statistics are listed in Table 2. The words “average” and “scatter” are used here to describe the mean and standard deviation of the Monte Carlo ensembles, to distinguish them from the words “mean” and “variance,” which were reserved for the evaluation of the individual sequences.

In addition, the pairwise correlation coefficient between two sequences  $v_1(t_i)$  and  $v_2(t_i)$

$$\rho = \frac{\sum_{i=1}^N [v_1(t_i) - \bar{v}_1] [v_2(t_i) - \bar{v}_2]}{\sqrt{\sum_{i=1}^N [v_1(t_i) - \bar{v}_1]^2} \sqrt{\sum_{i=1}^N [v_2(t_i) - \bar{v}_2]^2}} \quad (10)$$

was investigated. This metric ranges between  $\pm 1$  and is a measure of similarity between two sequences, which could arise from periodicities in the data or similar starting seed values for pseudo-random number generators, or from deterministic fluctuations manifesting in the true random number generators. Metrics listed in Table 2 indicated this was not the case and the sequences were not appreciably correlated with each other. Figure 5 shows two sequences plotted against each other for each generator. Because these data do not form a straight line, they visually indicate a low correlation. Each block of thirty sequences from Random.org showed similar values of correlation as the full ensemble.

The MATLAB and Random.org generators achieve a normal distribution through an explicit mapping of the random numbers. In contrast, the Fourier synthesis method only suggests a Gaussian distribution via the central limit theorem. To investigate the asymptotic convergence of the sample distribution, a separate Monte Carlo analysis was performed using only the Fourier synthesis method. Sequences with  $N = 1, 2, \dots, 601$  samples were generated 200 different times each, and the  $R^2$  values for the normal probability plots were evaluated. Figure 6 shows the mean and two standard deviations of scatter in the results. A normal sample distribution was achieved relatively quickly using Fourier synthesis. On average, approximately 1% error was attained after using  $N = 100$  (50 sinusoids), which represents 2 s of data sampled at 50 Hz.

Overall, all three of the random number generators performed well in approximating Gaussian white noise. The method of Fourier synthesis most consistently simulated the desired characteristics per the statistics shown. The Fourier synthesis method also efficiently attains a Gaussian distribution.

## 5 Application to Parameter Estimation

One use of measured flight test data is to estimate aerodynamic stability and control derivatives. These estimates can be used in a variety of applications, such as providing physical insight into the aircraft, constructing flight dynamics simulations, developing feedback control laws, evaluating handling qualities, or computing performance metrics.

In the development and testing of a parameter estimation method, a simulation model is often used first before applying the estimator to measured flight test data. As confidence in the analysis is gained, various sources of error can be included to assess performance and robustness in realistic conditions. It is therefore important to accurately replicate the effects expected in measured flight test data.

To investigate the effect of different random number generators on parameter estimation, a simple example was investigated. The simulation model was the short-period dynamics of an aircraft about a reference flight condition, given in state-space form as [9, 22]

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1 + Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \Delta\delta_e \quad (11a)$$

$$\begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta a_z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{V_0}{g} Z_\alpha & \frac{V_0}{g} Z_q \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{V_0}{g} Z_{\delta_e} \end{bmatrix} \Delta\delta_e \quad (11b)$$

where  $\alpha$  is the angle of attack,  $q$  is the pitch rate,  $\delta_e$  is the elevator deflection, and  $a_z$  is the vertical acceleration at the center of mass. The goal here is to estimate values of the stability and control derivatives, such as  $Z_\alpha$  and  $M_{\delta_e}$ , from measurements of the input and output data.

The model was simulated for 12 s and sampled at 50 Hz, which corresponded to  $N = 601$  samples, as before. The elevator input included a 10 s multisine excitation [9] with frequency content between 0.3 Hz and 2.1 Hz<sup>1</sup>. The model parameters for this simulation are listed in Table 3, and pertain to the T-2 subscale aircraft, shown in Fig. 7, in a typical flight condition. The frequency of the short period mode was 1.04 Hz. The time histories resulting from simulating Eqs. (11) are shown in Fig. 8.

This estimation problem can be posed as the least squares problem

$$\mathbf{z} = \mathbf{X}\boldsymbol{\theta} + \mathbf{v} \quad (12)$$

where  $\mathbf{z}$  are the dependent variables,  $\mathbf{X}$  are the explanatory variables,  $\boldsymbol{\theta}$  are the unknown model parameters, and  $\mathbf{v}$  is a Gaussian white noise sequence. Solving the least-squares cost function

$$J(\boldsymbol{\theta}) = \frac{1}{2} (\mathbf{z} - \mathbf{X}\boldsymbol{\theta})^T (\mathbf{z} - \mathbf{X}\boldsymbol{\theta}) \quad (13)$$

yields the optimal estimate

$$\hat{\boldsymbol{\theta}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{z} \quad (14)$$

The uncertainty in this estimate is quantified by the covariance matrix

$$\text{cov}(\hat{\boldsymbol{\theta}}) = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T E[\mathbf{v}\mathbf{v}^T] \mathbf{X} (\mathbf{X}^T \mathbf{X})^{-1} \quad (15)$$

---

<sup>1</sup>Note that these multisines are also described by Eq. (3), and therefore can also be thought of as a truncated Fourier series expansion.

where  $E[\mathbf{v}\mathbf{v}^T]$  is the autocorrelation matrix, formed using Eq. (7). The parameter covariance matrix is therefore sensitive to serial correlations in the modeling residual sequence.

The estimation was performed twice for each Monte Carlo simulation. In the first case,  $\mathbf{z}$  were the measurements  $\Delta a_z$  and  $\boldsymbol{\theta}$  included the  $Z$  derivatives. In the second case,  $\mathbf{z}$  were the measurements  $\dot{q}$ , obtained by smooth numerical differentiation [9] of the measured  $q$ , and  $\boldsymbol{\theta}$  included the  $M$  derivatives. In each case the matrix  $\mathbf{X}$  consisted of  $\Delta\alpha$ ,  $\Delta q$ , and  $\Delta\delta_e$  in column vectors.

A Monte Carlo simulation was performed using the same 200 random noise sequences from each of the three random number generators. In each run, these noise sequences were scaled to 20% of the root mean square variation of the corresponding signal, and added to  $\mathbf{z}$ . Because estimation of the  $Z$  derivatives is performed separately from the  $M$  derivatives, the same noise sequences can be used without correlating the estimation results. Noise was not added to  $\mathbf{X}$  because these data should be smoothed before estimation to remove the noise.

Model fits to one sample record of simulated  $\Delta a_z$  and  $\dot{q}$  data for each random number generator are shown in Figs. 9 and 10. The fits were generally good and had small residuals. The summary statistics for the Monte Carlo simulations are listed in Table 4. Parameter estimates were generally similar in value for each random number generator. Each set of parameter estimates were in statistical agreement with the true values, and had scatter of the parameters within the standard errors. The results using noise sequences generated using Fourier synthesis had consistently lower scatter in the estimated standard errors than the results using MATLAB and Random.org. The condensed scatter also indicates that the method of Fourier synthesis produces sequences with lower serial correlations, as evident from Table 2 and previous work [1, 2].

## 6 Application to Simulation in Turbulence

Simplified models of atmospheric turbulence are useful engineering approximations for evaluating handling qualities of piloted aircraft, assessing robustness of control laws and identification algorithms, and other applications. One such model is the continuous stochastic gust [3]. The gust history is modeled as a random variable with a specified PSD which is dependent upon variables including turbulence level, altitude, and airspeed. The spectral character of the random sequences is achieved by passing unit variance white noise through a coloring filter.

In particular, the Dryden turbulence model for the body-fixed vertical velocity  $w$  has the filter transfer function

$$\frac{\Delta w_g(s)}{v(s)} = \sigma_w \sqrt{\frac{2L_w}{\pi V_0}} \frac{(1 + 2\sqrt{3}L_ws/V_0)}{(1 + 2L_ws/V_0)^2} \quad (16)$$

where  $L_w$  is the length scale and  $\sigma_w$  is the turbulence intensity. Parameters used for the simulation are listed in Table 3. Each of the white noise sequences generated was passed through the coloring filter to produce a vertical gust time history. For the first sample records, the gust sequences are shown in Fig. 11 and the PSD estimates, again multiplied by the Nyquist frequency, are shown in Fig. 12. The sequences from MATLAB and Random.org had many data points with less power than prescribed. At the higher frequencies, the PSD from Random.org appears to begin to diverge from the specified PSD. The sequence generated using Fourier synthesis matched the specified PSD very closely and was therefore a more precise implementation of the desired turbulence spectrum.

To investigate the extent to which using different random number generators affected the resulting motion, the vertical gusts were used in the aircraft flight dynamics simulation. The vertical gusts can be transformed into an angle of attack gusts as

$$\Delta\alpha_g = \arctan\left(\frac{\Delta w_g}{V_0 \cos \alpha_0}\right) \quad (17)$$

for small perturbations about the reference condition. The relative angle of attack  $\Delta\alpha - \Delta\alpha_g$  manifests in the aerodynamic forces and moments [22] and the short period model is augmented as

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1 + Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} & -Z_\alpha \\ M_{\delta_e} & -M_\alpha \end{bmatrix} \begin{bmatrix} \Delta\delta_e \\ \Delta\alpha_g \end{bmatrix} \quad (18a)$$

$$\begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta a_z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{V_0}{g} Z_\alpha & \frac{V_0}{g} Z_q \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{V_0}{g} Z_{\delta_e} & -\frac{V_0}{g} Z_\alpha \end{bmatrix} \begin{bmatrix} \Delta\delta_e \\ \Delta\alpha_g \end{bmatrix} \quad (18b)$$

This model was simulated using each generated white noise sequence. To create variability in the initial conditions of the state variables, the turbulence sequences were run through the model with zero elevator deflections. Monte Carlo results of the vertical accelerometer perturbations for each different generator are shown in Fig. 13. The thick lines show the ensemble means for the simulations, whereas the thin lines show two standard deviations of scatter in the time histories. Similar trends were observed for the angle of attack and pitch rate outputs. The spread in the time histories were similar for each random number generator. Although sequences from MATLAB and Random.org had less precise implementations of the turbulence spectrum than Fourier synthesis, as shown in Fig. 12, these errors were averaged out over many Monte Carlo simulations.

## 7 Concluding Remarks

Three random number generators, used to create sequences of Gaussian white noise, were compared. The first generator was the MATLAB implementation of the Mersenne-Twister algorithm, which was included because of its widespread use. The second generator was a website called Random.org, which processes measurements of atmospheric noise to create random sequences. This generator was included because it claims to be a true random number generator, creating the number sequences from measurements of physical processes considered to be random. The third method was the method of Fourier synthesis by Lanczos and Bellai. This method constructs noise from definitions of amplitude and phase spectra, and was included because of its excellent advertised properties.

Ensembles of 200 sample records, each having 601 numbers, were obtained for each of the three random number generators. These sequences were analyzed individually and as an ensemble, and were used in dynamic modeling applications including parameter estimation of stability and control derivatives and simulation of atmospheric turbulence. The same sample records were used throughout this paper.

The main conclusions of this report can be summarized as follows:

1. Each random number generator was generally of good quality and sufficient for Monte Carlo simulation in dynamic modeling problems.
2. The Fourier synthesis method was consistently more accurate in approximating Gaussian white noise sequences, per the statistical tools applied.

The deficiencies of the generators exhibited in any single sample record were generally averaged out over many Monte Carlo simulations, and therefore each generator is suitable for Monte Carlo analysis in aircraft dynamic modeling applications. All three generators performed similarly on average, where the differences were negligible for the applications considered. However, the Fourier synthesis method was significantly more consistent in achieving Gaussian white noise sequences than the other two methods. This was true in terms of the sequence mean and variance, normality, autocorrelation, and power spectral density, all of which are tools relevant to the analysis of aircraft dynamic modeling.

It is therefore recommended to use Fourier synthesis for generating Gaussian white noise sequences in Monte Carlo simulation. These sequences required negligibly more time than the built-in MATLAB generator, and produced a better approximation to the white noise sequences assumed in the analysis. Although the examples shown here are themselves engineering approximations (e.g., measurement noise is not exactly white, stability and control derivatives are Taylor series approximations, and turbulence can take on a different character than the models predict), having better analysis tools that work as expected can simplify software development and possibly lead to fewer needed Monte Carlo simulations. More importantly, the analysis can be more precise and therefore the decisions based on that analysis will be clearer, more accurate, and more reliable, because of the greater precision in the white noise generation.

## 8 Acknowledgements

This work was funded by the NASA Aeronautics Research Mission Directorate (ARMD) Learn-To-Fly project. Discussions with Dr. Eugene Morelli are acknowledged and appreciated. Mr. Carey Buttrill, Dr. Eugene Morelli, and Dr. John Ryan performed technical reviews of this report.

## References

1. Maine, R.; and Iliff, K.: Use of Cramer-Rao Bounds on Flight Data with Colored Residuals. *Journal of Guidance and Control*, vol. 4, no. 2, March–April 1981, pp. 207–213.
2. Morelli, E.; and Klein, V.: Accuracy of Aerodynamic Model Parameters Estimated from Flight Test Data. *Journal of Guidance, Control, and Dynamics*, vol. 20, no. 1, January–February 1997, pp. 74–80.
3. Anonymous: Military Standard – Flying Qualities of Piloted Aircraft. MIL-STD-1797A, Department of Defense, August 2004.
4. Brown, R.; and Hwang, P.: *Introduction to Random Signals and Applied Kalman Filtering*, 3rd edition. Wiley, 1997.
5. Technical Staff, T. A. S. C.: *Applied Optimal Estimation*. M.I.T. Press, 1974.
6. von Neumann, J.: *Monte Carlo Method*, National Bureau of Standards, vol. 12 of *Applied Mathematics Series*, Various Techniques Used in Connection with Random Digits. 1951, pp. 36–38.
7. Hamming, R.: *Introduction to Applied Numerical Analysis*. Computer Science Series, McGraw-Hill.
8. Morelli, E.: System IDentification Programs for AirCRAFT (SIDPAC). <http://software.nasa.gov>.
9. Morelli, E.; and Klein, V.: *Aircraft System Identification: Theory and Practice*, 2nd edition. Sunflyte Enterprises, Williamsburg, VA, 2016.
10. MathWorks: MATLAB, R2016b. [www.mathworks.com](http://www.mathworks.com).
11. Matsumoto, M.; and Nishimura, T.: Mersenne Twister: A 623-Dimensionally Equidistributed Uniform Pseudo-Random Number Generator. *ACM Transactions on Modeling and Computer Simulation*, vol. 8, no. 1, January 1998, pp. 3–30.
12. Marsaglia, G.; and Tsang, W.: The Ziggurat Method for Generating Random Variables. *Journal of Statistical Software*, vol. 5, no. 8, October 2000, pp. 1–7.
13. Haahr, M.: Random.org. <http://random.org>.
14. Box, G.; and Muller, M.: A Note on the Generation of Random Normal Deviates. *The Annals of Mathematical Statistics*, vol. 29, no. 2, 1958, pp. 610–611.
15. Lánzos, C.; and Gellai, B.: Fourier Analysis of Random Sequences. *Computers and Mathematics with Applications*, vol. 1, no. 3–4, 1975, pp. 269–276.
16. Lánzos, C.: *Applied Analysis*. Prentice Hall, 1959.
17. Marsaglia, G.: The Marsaglia Random Number CDROM, with the Diehard Battery of Tests of Randomness. [www.stat.fsu.edu/pub/diehard](http://www.stat.fsu.edu/pub/diehard). Produced in 1985.
18. Rukhin, A.; et al.: A Statistical Test Suite for Random and Pseudorandom Number Generators for Cryptographic Applications. 800-22, National Institute of Standards and Technology, Gaithersburg, MD, April 2010.

19. Chambers, J.; Cleveland, W.; Kleiner, B.; and Tukey, P.: *Graphical Methods for Data Analysis*. Statistics/Probability Series, Wadsworth and Brooks/Cole, Pacific Grove, CA, 1983.
20. Bartlett, M.: *An Introduction to Stochastic Processes*. Cambridge University Press, 2nd ed., 1966.
21. Morelli, E.: High Accuracy Evaluation of the Finite Fourier Transform Using Sampled Data. TM-110340, NASA, Hampton, VA, June 1997.
22. McRuer, D.; Ashkenas, I.; and Graham, D.: *Aircraft Dynamics and Automatic Control*. Princeton University Press, Princeton, NJ, 1973.

## A – Variance of the Finite Fourier Series

For ideal white noise, selecting  $c_k = \sqrt{2/M}$  results in a unit-variance sequence. This can be seen by considering the variance of the continuous, zero-mean variable  $v(t)$  over the domain  $t \in [0, T]$

$$\begin{aligned}\text{var}[v(t)] &= \frac{1}{T} \int_0^T v^2(t) dt \\ &= \frac{1}{T} \int_0^T \left[ \sum_{k=1}^M c_k \sin(\omega_k t + \phi_k) \right]^2 dt\end{aligned}$$

where  $\omega_k = 2\pi k/T$  is the radian frequency.

Expanding the squared expression results in terms proportional to  $\sin^2(\omega_k t + \phi_k)$  and  $\sin(\omega_k t + \phi_k) \sin(\omega_m t + \phi_m)$ . These latter terms sum to zero because the sinusoids are harmonics, which are mutually orthogonal, regardless of the phase angles. This simplifies the variance to

$$\text{var}[v(t)] = \frac{1}{T} \int_0^T \sum_{k=1}^M c_k^2 \sin^2(\omega_k t + \phi_k) dt$$

which, due to linearity, can be rearranged as

$$\text{var}[v(t)] = \frac{1}{T} \sum_{k=1}^M c_k^2 \int_0^T \sin^2(\omega_k t + \phi_k) dt$$

Employing double-angle identities, the above integral evaluates to

$$\begin{aligned}\int_0^T \sin^2(\omega_k t + \phi_k) dt &= \frac{1}{2} \int_0^T 1 - \cos(2\omega_k t + 2\phi_k) dt \\ &= \frac{T}{2} - \frac{\sin(2\omega_k T + 2\phi_k)}{4\omega_k} + \frac{\sin(2\phi_k)}{4\omega_k} \\ &= T/2\end{aligned}$$

because  $k$  is an integer and the sinusoids are periodic in  $T$ .

Substituting this result back into the variance,

$$\begin{aligned}\text{var}[v(t)] &= \frac{1}{T} \sum_{k=1}^M c_k^2 \frac{T}{2} \\ &= \frac{1}{2} \sum_{k=1}^M c_k^2\end{aligned}$$

For white noise,  $c_k$  is the same for each harmonic frequency. For unit variance white noise, we require  $\text{var}[v(t)] = 1$  so that

$$c_k = \sqrt{\frac{2}{M}}$$

It is interesting that the orthogonality of the Fourier series means that the variance of the signal is only dependent on the number of frequencies included, and not upon the random phase angles that create constructive and destructive interference of the sinusoids.

## B – Autocorrelation of the Finite Fourier Series

The autocorrelation of the Fourier synthesis sequences were good approximations of white noise. The reason for this can be seen by extrapolation from the sum of the first two harmonic sinusoids,

$$\begin{aligned} v(t_i) &= v_1(t_i) + v_2(t_i) \\ &= c_1 \sin\left(\frac{2\pi}{T}t_i + \phi_1\right) + c_2 \sin\left(\frac{4\pi}{T}t_i + \phi_2\right) \end{aligned}$$

The autocorrelation function of this sequence can be expanded using the definition of the autocorrelation, and then rearranged as

$$\begin{aligned} r_{vv}(l) &= E[v(t_i)][v(t_i + l)] \\ &= E[v_1(t_i) + v_2(t_i)][v_1(t_i + l) + v_2(t_i + l)] \\ &= E[v_1(t_i)v_1(t_i + l)] + E[v_2(t_i)v_2(t_i + l)] + E[v_1(t_i)v_2(t_i + l)] + E[v_2(t_i)v_1(t_i + l)] \\ &= r_{v_1v_1}(l) + r_{v_2v_2}(l) + r_{v_1v_2}(l) + r_{v_2v_1}(l) \end{aligned}$$

which is the sum of the autocorrelations and crosscorrelations of the first two harmonic sinusoids. By extrapolation of this expression, the autocorrelation of the finite Fourier series is equivalent to the sum of the autocorrelations and crosscorrelations of all the sinusoids.

The autocorrelation  $r_{v_1v_1}$  and the crosscorrelation  $r_{v_1v_2}$  for a single sample record obtained using Fourier synthesis is shown in Fig. 14. The autocorrelation has a coherent peak at lag index  $l = 0$ , and decaying oscillations elsewhere. The crosscorrelation is zero at  $l = 0$ , and has oscillations at other lag indices. When many sinusoids are used in the finite Fourier series, the peak at  $l = 0$  is reinforced whereas the autocorrelation at the other lag indices averages to nearly zero. This effect creates a good approximation to the autocorrelation of a white noise sequence.

# Tables

Table 1: Summary of single sample records

Characteristic	Statistic	Matlab	Random.org	Fourier synthesis
Time history	Mean	+0.0189	+0.0065	+0.0009
	Variance	0.9899	1.1085	1.0000
Normality	$R^2$ with $\mathcal{N}(0, 1)$	0.9960	0.9962	0.9983
Autocorrelation	Variance $\hat{r}_{vv}(0)$	0.9886	1.1067	0.9983
	Percentage outside $2\sigma$	0.4992	2.1631	0.0000
Power spectral density	Mean	0.9926	1.1122	0.9991
	Variance	0.7887	1.1811	0.0010

Table 2: Summary of ensembles of sample records

Characteristic	Statistic	Matlab	Random.org	Fourier Synthesis
Time history	Average of mean	-0.0014	-0.0002	+0.0002
	Scatter of mean	0.0421	0.0420	0.0015
	Average of variance	0.9969	0.9957	1.0000
	Scatter of variance	0.0544	0.0549	0.0000
Normality	Average of $R^2$ with $\mathcal{N}(0, 1)$	0.9948	0.9948	0.9972
	Scatter of $R^2$ with $\mathcal{N}(0, 1)$	0.0029	0.0031	0.0013
Autocorrelation	Average of variance $\hat{r}_{vv}(0)$	0.9970	0.9958	0.9983
	Scatter of variance $\hat{r}_{vv}(0)$	0.0544	0.0547	0.0000
	Percentage outside $2\sigma$	0.0048	0.0032	0.0000
Power spectral density	Average of mean	0.9990	0.9972	1.0030
	Scatter of mean	0.0712	0.0679	0.0035
	Average of variance	0.9991	1.0018	0.0028
	Scatter of variance	0.1940	0.1873	0.0002
Pairwise correlations	Average of correlation	-0.0006	+0.0001	+0.0001
	Scatter of correlation	0.0411	0.0408	0.0411

Table 3: Simulation model parameters for the T-2 aircraft

Parameter	Value	Unit
$V_0$	128.91	ft/s
$Z_\alpha$	-2.5394	1/s
$Z_q$	-0.0551	-
$Z_{\delta_e}$	-0.2543	1/s
$M_\alpha$	-36.366	1/s <sup>2</sup>
$M_q$	-3.1959	1/s
$M_{\delta_e}$	-39.112	1/s <sup>2</sup>
$L_w$	500	ft
$\sigma_w$	16.878	ft/s

Table 4: Summary for Parameter estimation

Parameter	Statistic	Matlab	Random.org	Fourier synthesis
$Z_\alpha$	Average of estimate	-2.5390	-2.5403	-2.5405
	Scatter of estimate	0.0230	0.0237	0.0238
	Average of standard error	0.0212	0.0213	0.0217
	Scatter of standard error	0.0033	0.0035	0.0021
$Z_q$	Average of estimate	-0.0557	-0.0552	-0.0549
	Scatter of estimate	0.0054	0.0055	0.0046
	Average of standard error	0.0048	0.0048	0.0049
	Scatter of standard error	0.0007	0.0007	0.0004
$Z_{\delta_e}$	Average of estimate	-0.2569	-0.2537	-0.2578
	Scatter of estimate	0.0254	0.0257	0.0229
	Average of standard error	0.0231	0.0231	0.0235
	Scatter of standard error	0.0036	0.0034	0.0020
$M_\alpha$	Average of estimate	-35.8216	-35.8363	-35.8913
	Scatter of estimate	0.3226	0.3284	0.2942
	Average of standard error	0.4047	0.4064	0.4000
	Scatter of standard error	0.0963	0.0829	0.0877
$M_q$	Average of estimate	-3.0624	-3.0560	-3.0721
	Scatter of estimate	0.1150	0.1061	0.1110
	Average of standard error	0.1195	0.1192	0.1186
	Scatter of standard error	0.0235	0.0195	0.0175
$M_{\delta_e}$	Average of estimate	-37.8299	-37.8128	-37.8970
	Scatter of estimate	0.5992	0.5944	0.6122
	Average of standard error	0.6225	0.6179	0.6160
	Scatter of standard error	0.1283	0.1087	0.0898

# Figures

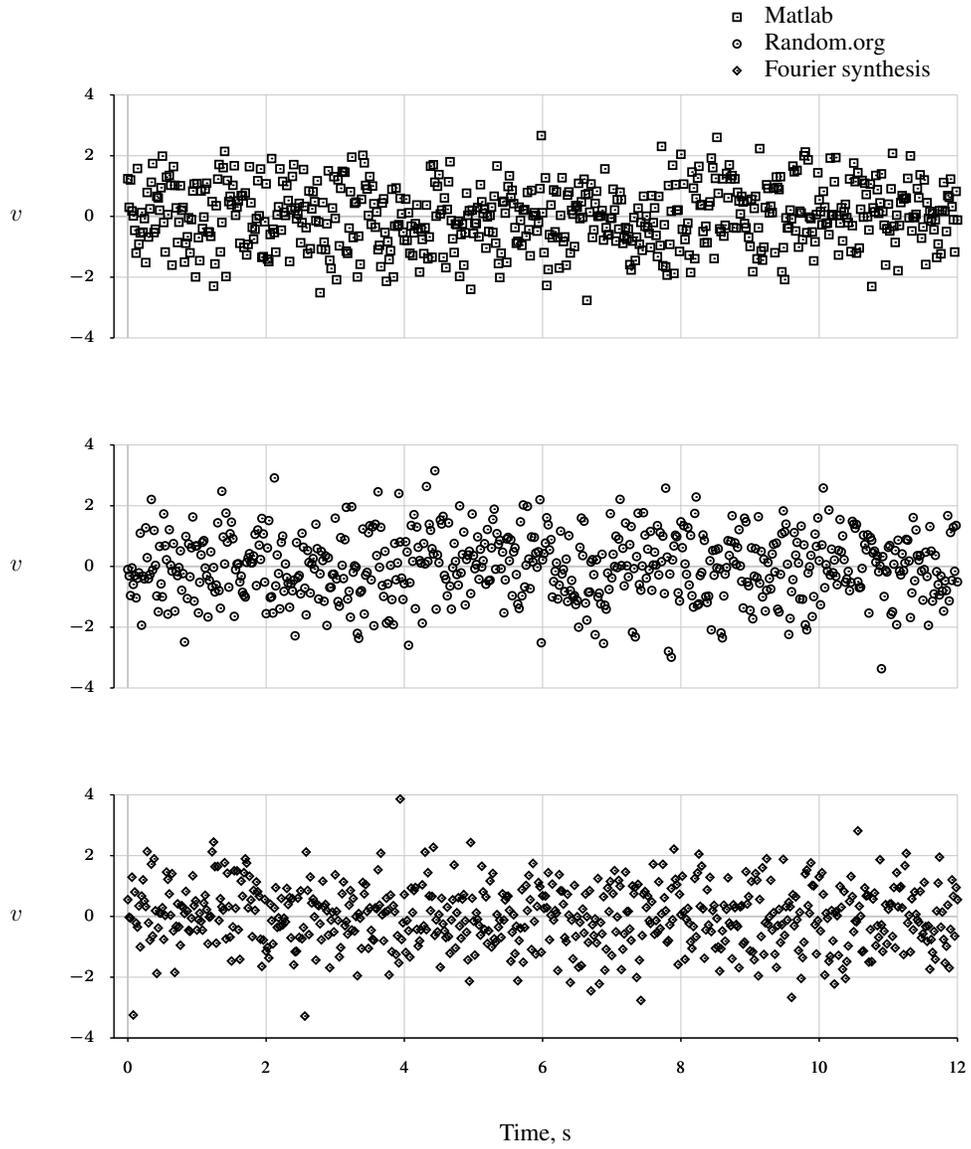


Figure 1: Single sample records from each random number generator

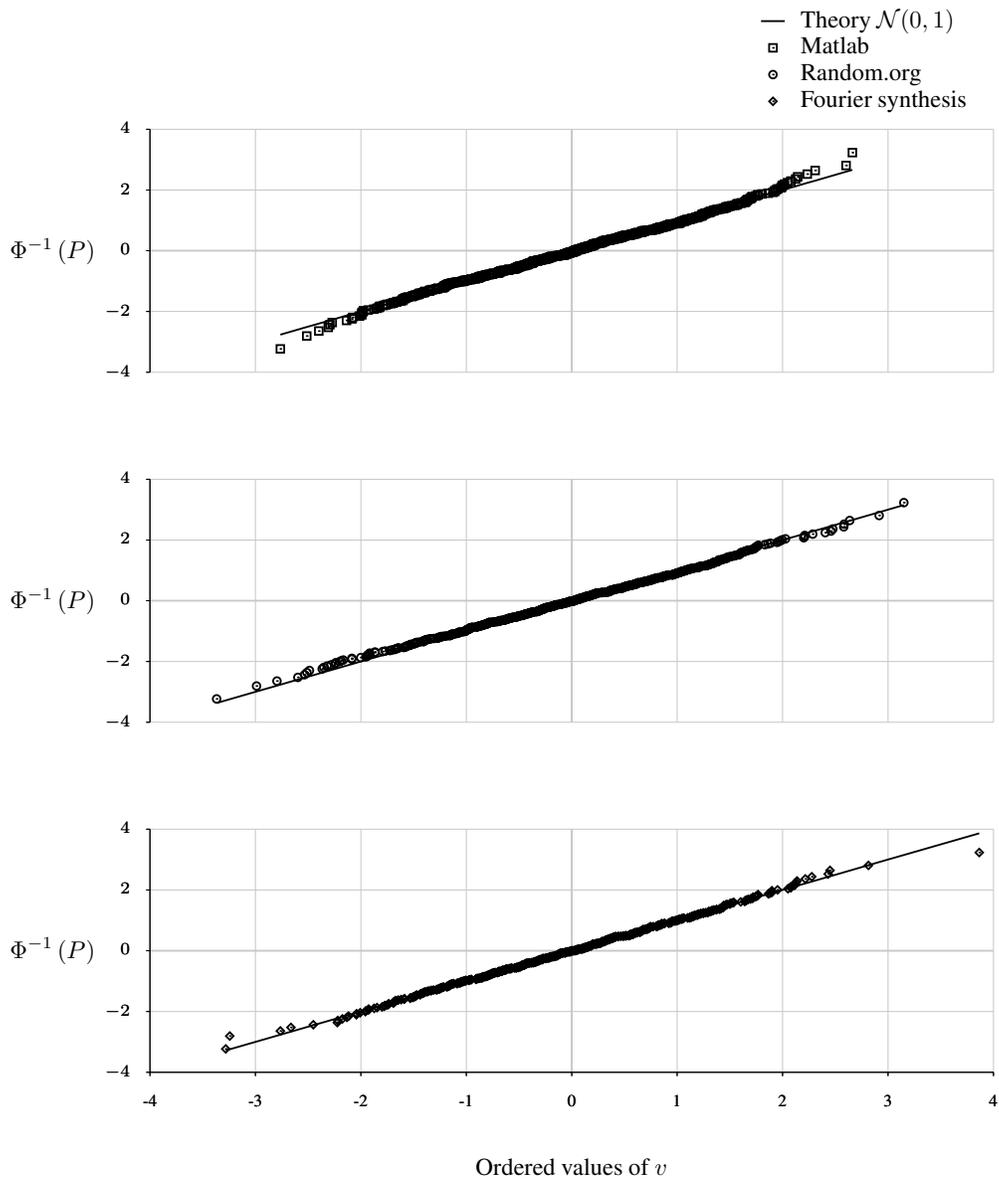


Figure 2: Normal probability plots, single sample records

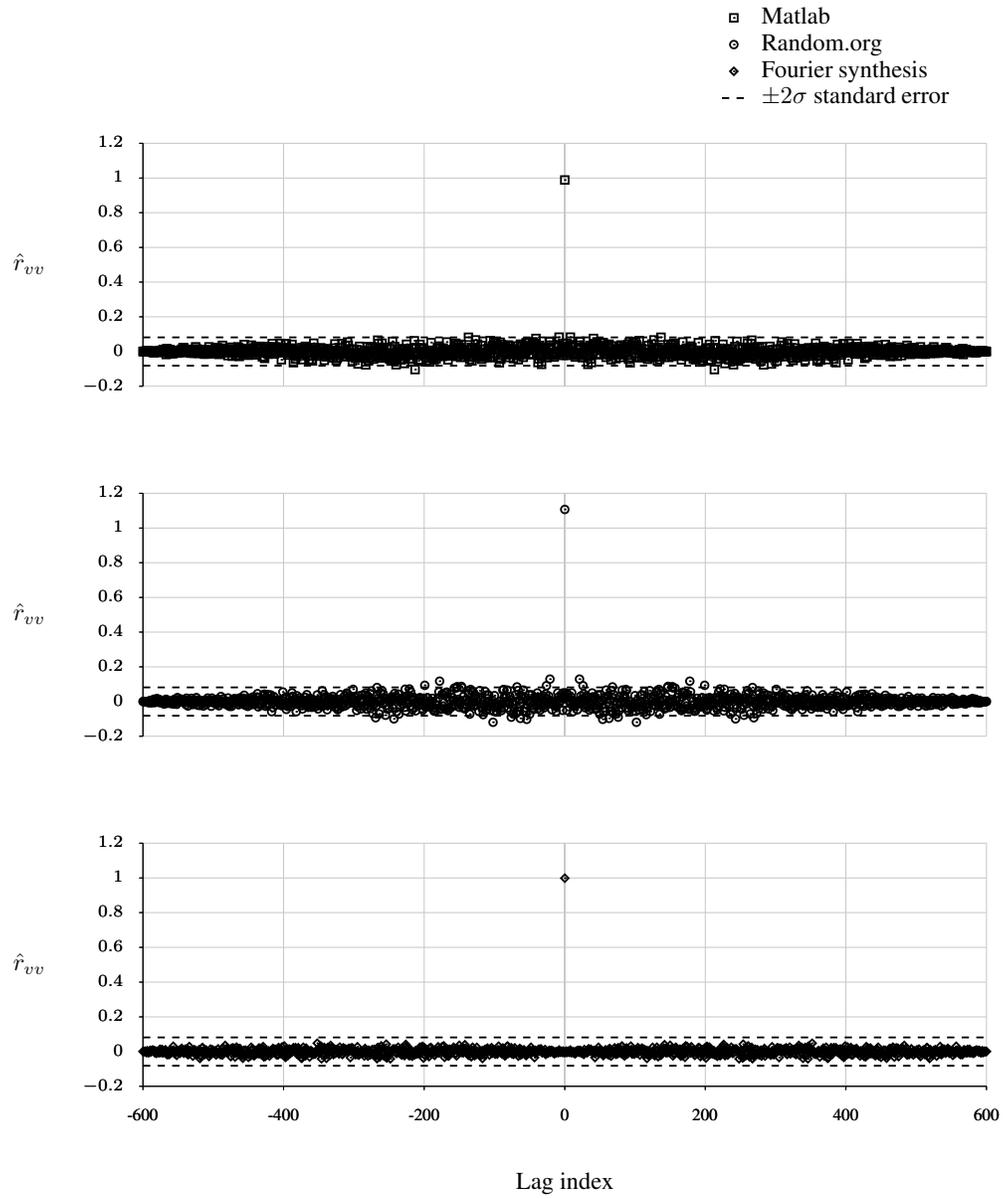


Figure 3: Autocorrelations, single sample records

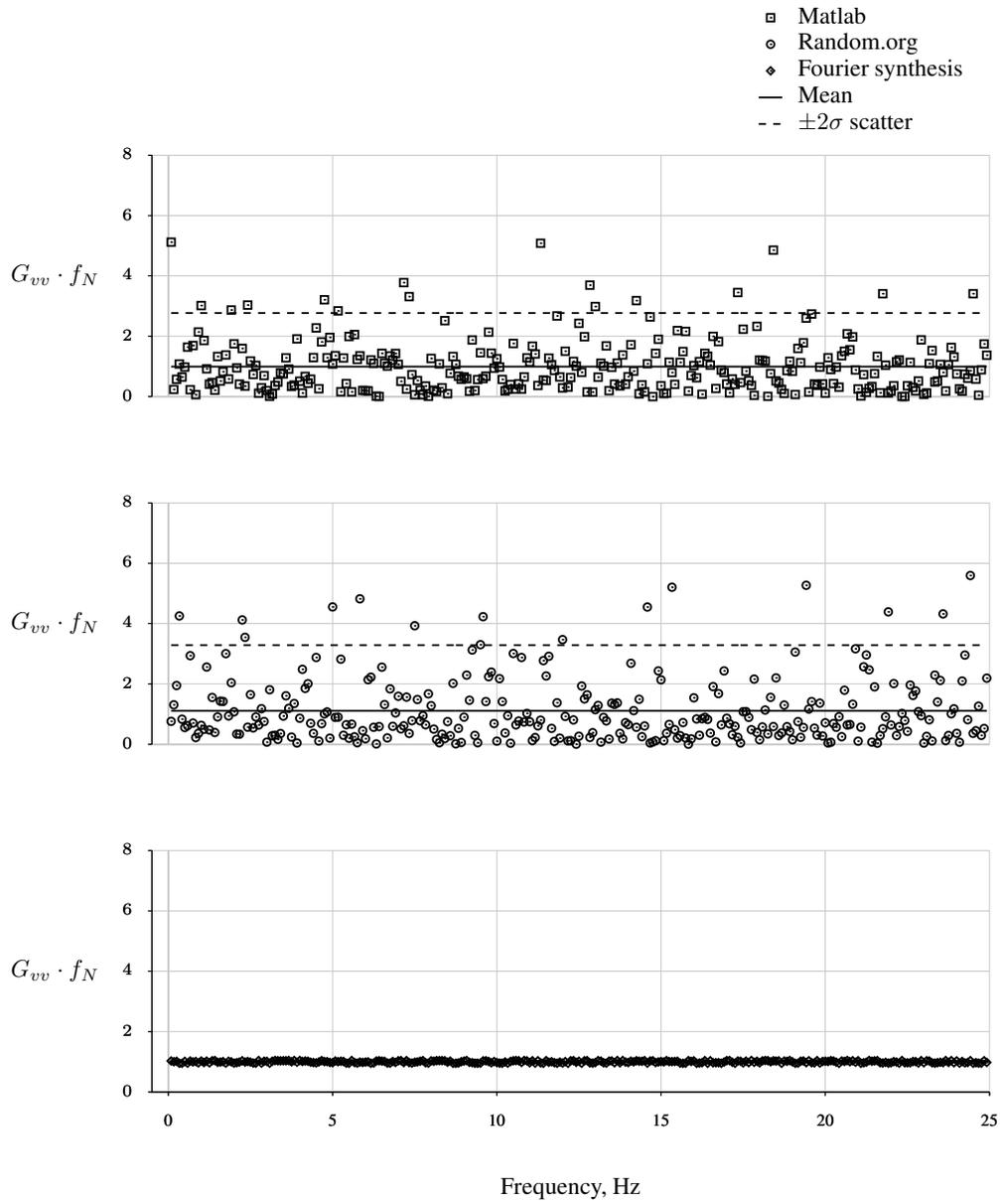


Figure 4: Power spectral densities, single sample records

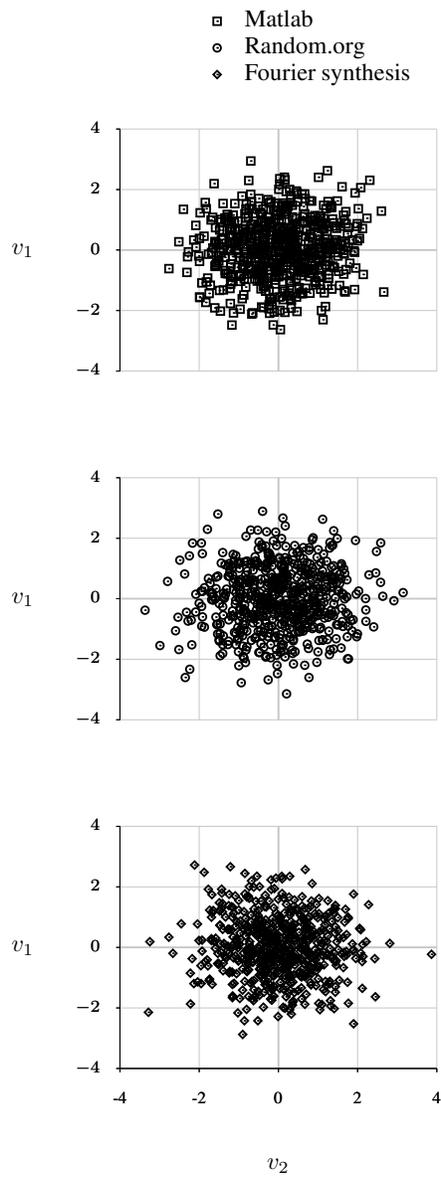


Figure 5: Cross plots of two sample records

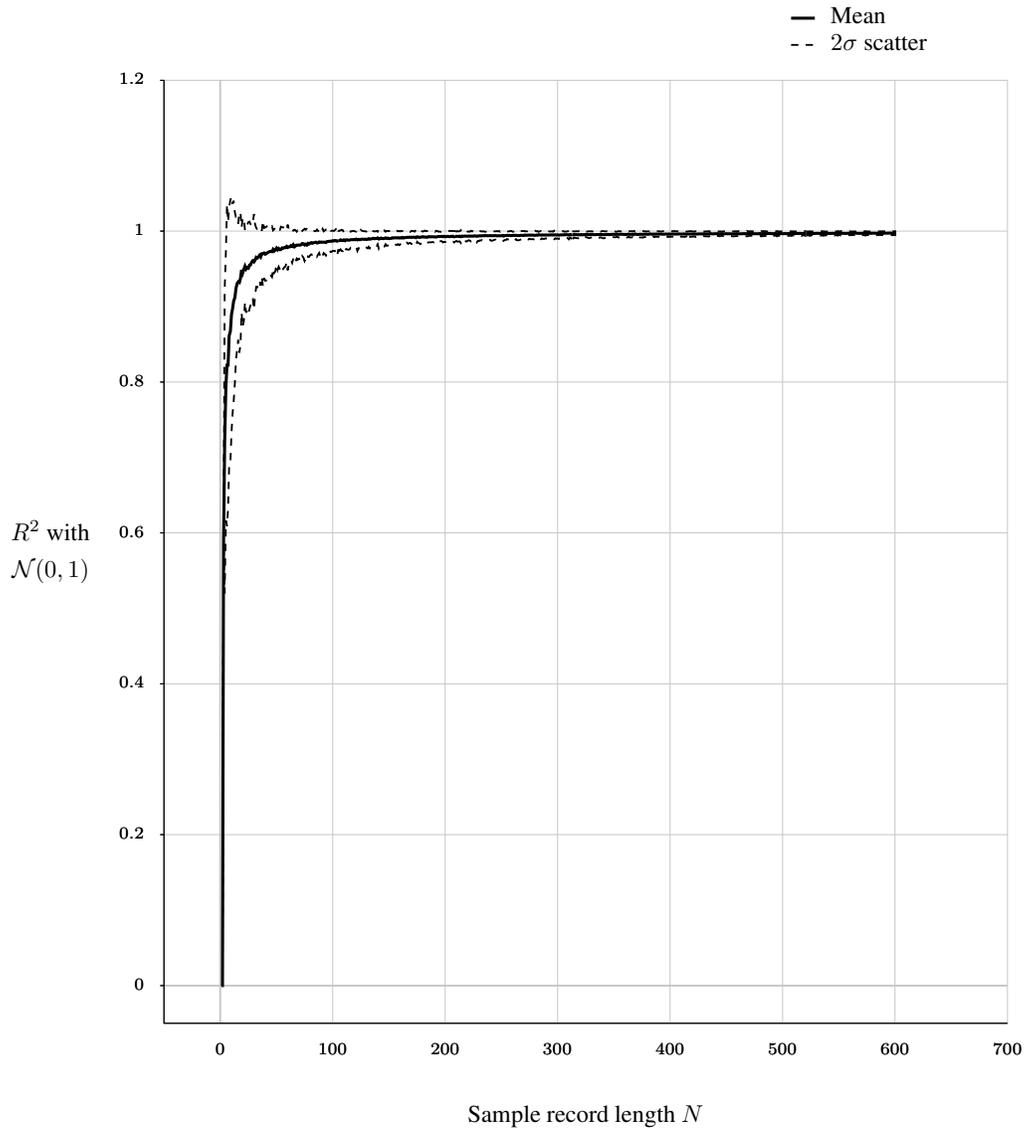


Figure 6: Efficiency of the Fourier synthesis generator in achieving a Gaussian distribution



Figure 7: T-2 airplane (credit: NASA Langley Research Center)

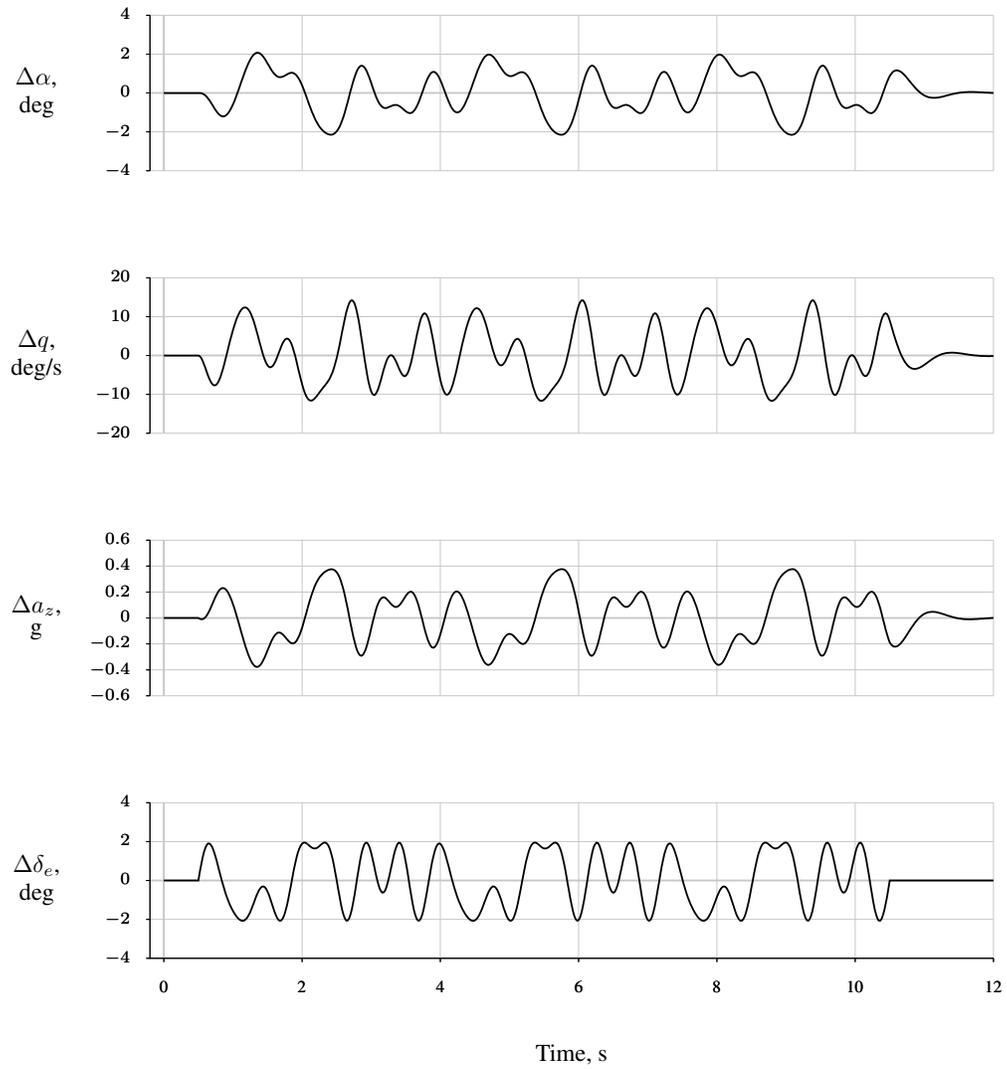


Figure 8: Simulated data for parameter estimation

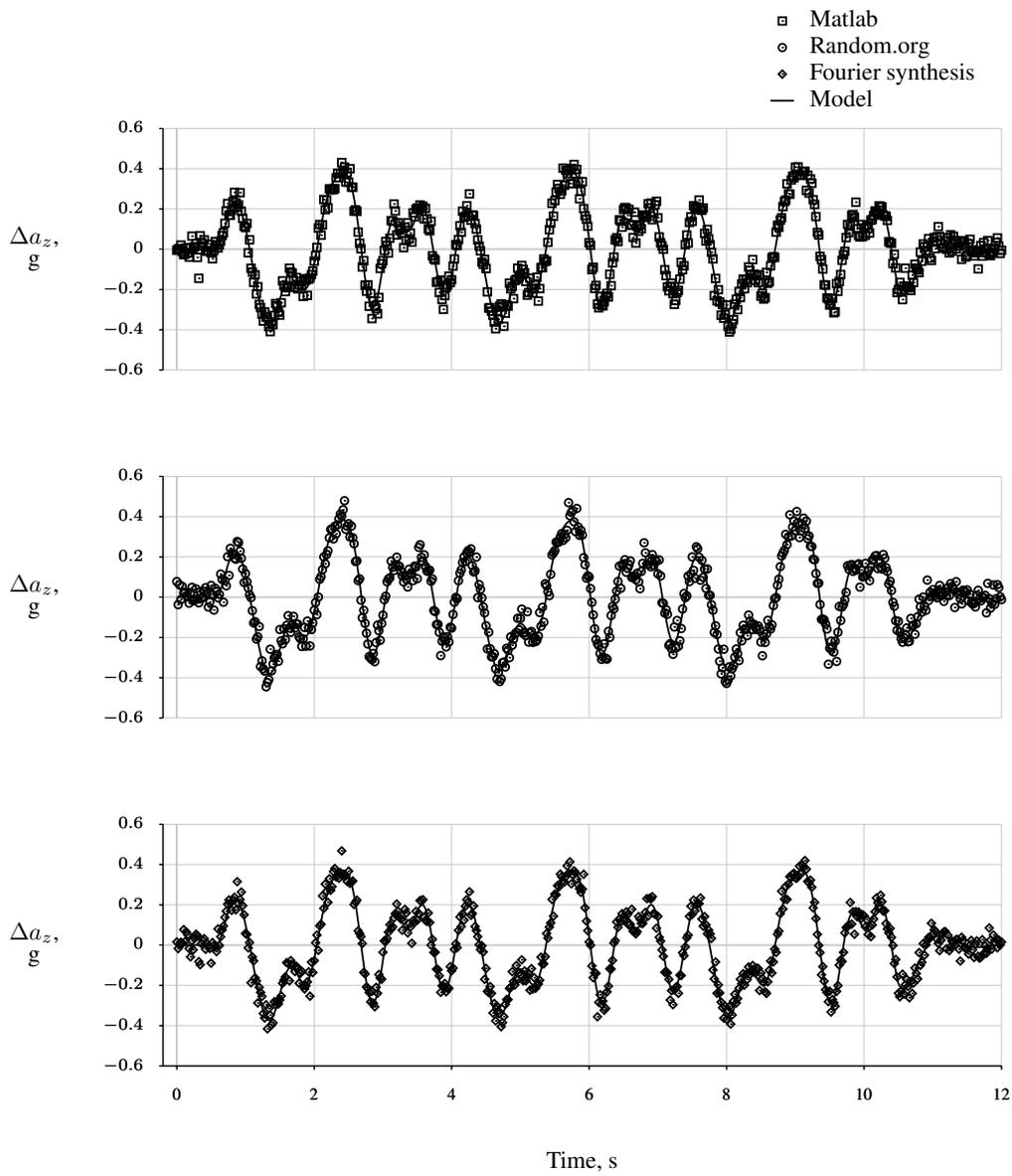


Figure 9: Model fits to noisy vertical accelerometer data, single sample records

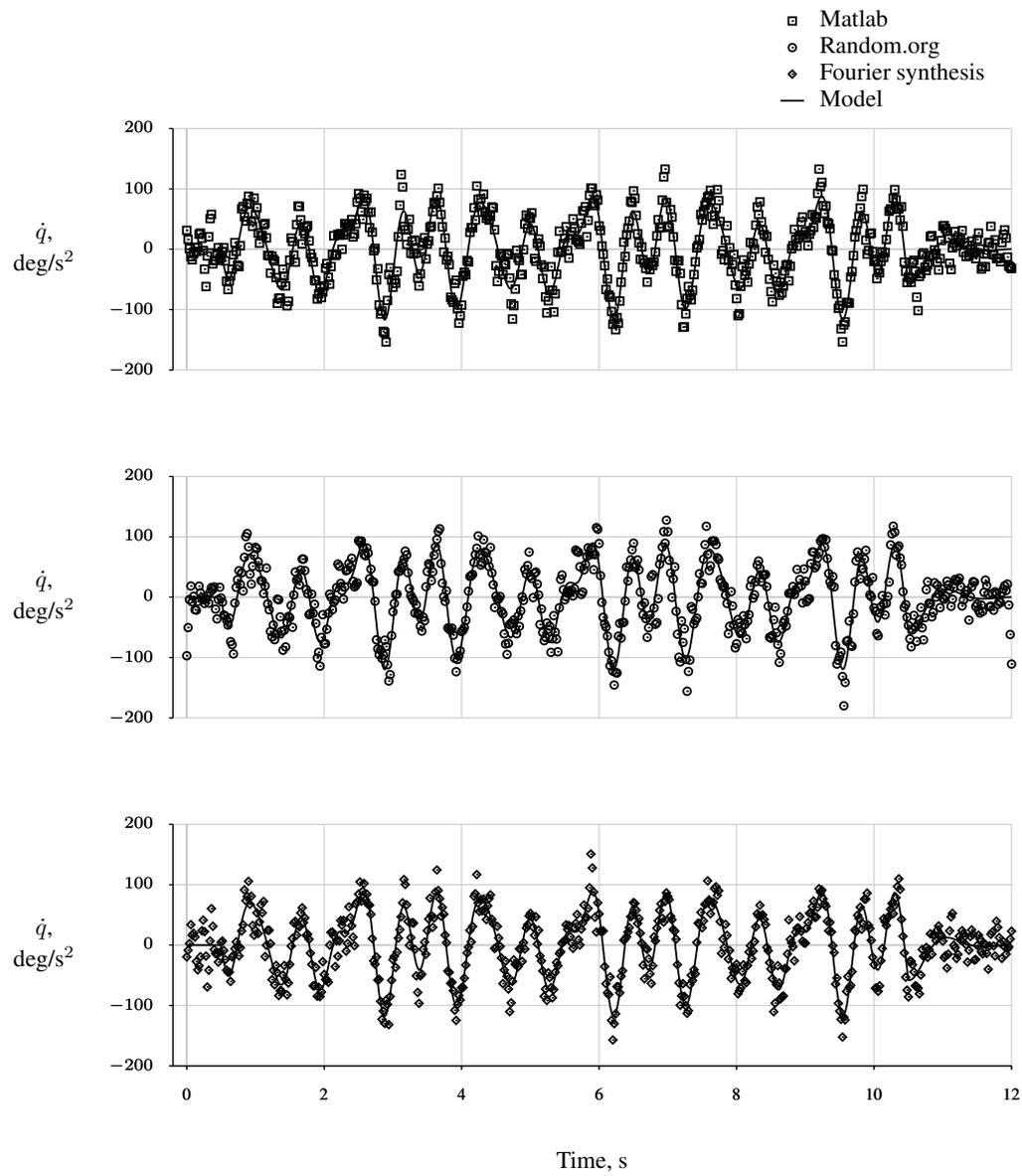


Figure 10: Model fits to noisy pitch acceleration data, single sample records

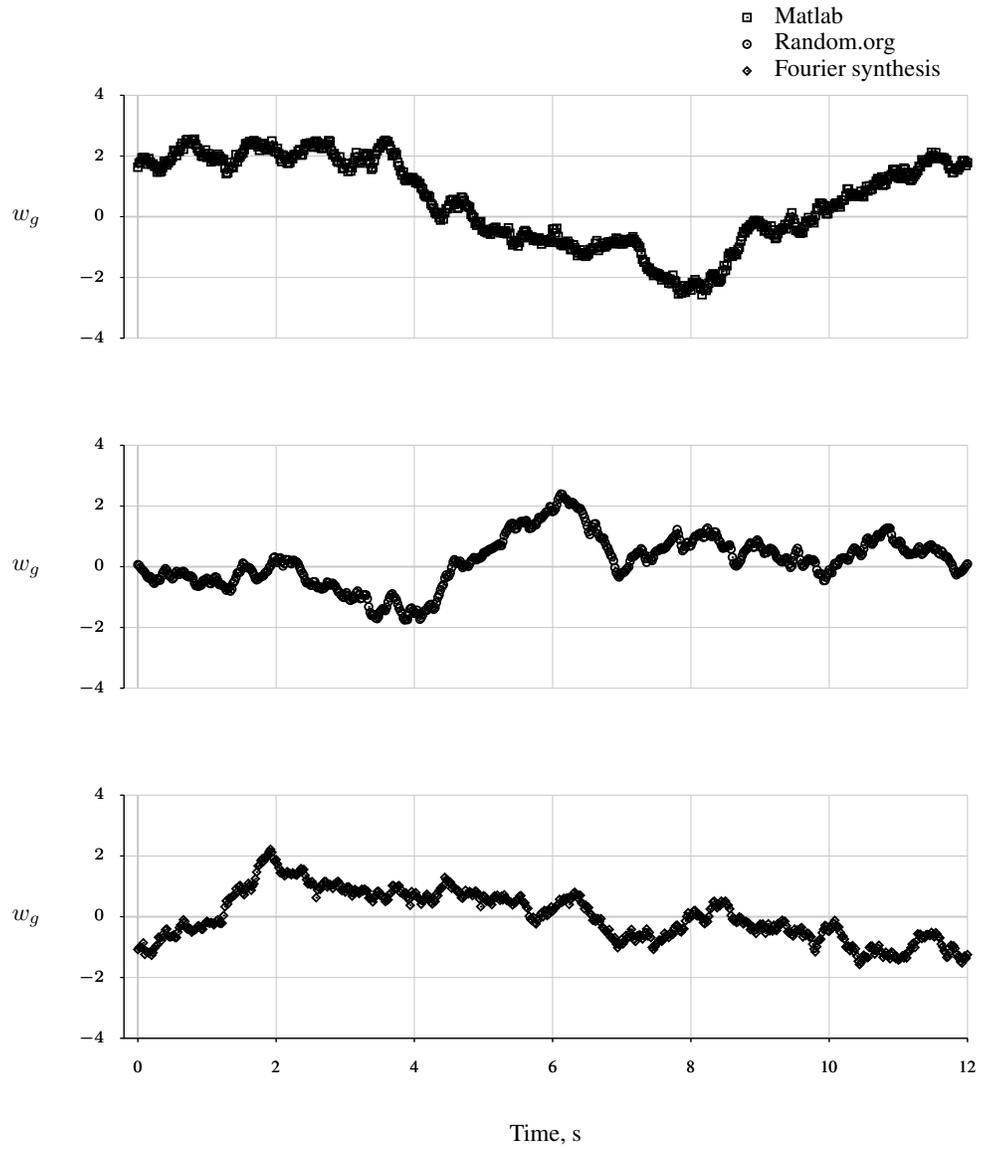


Figure 11: Vertical gust time histories, single sample records

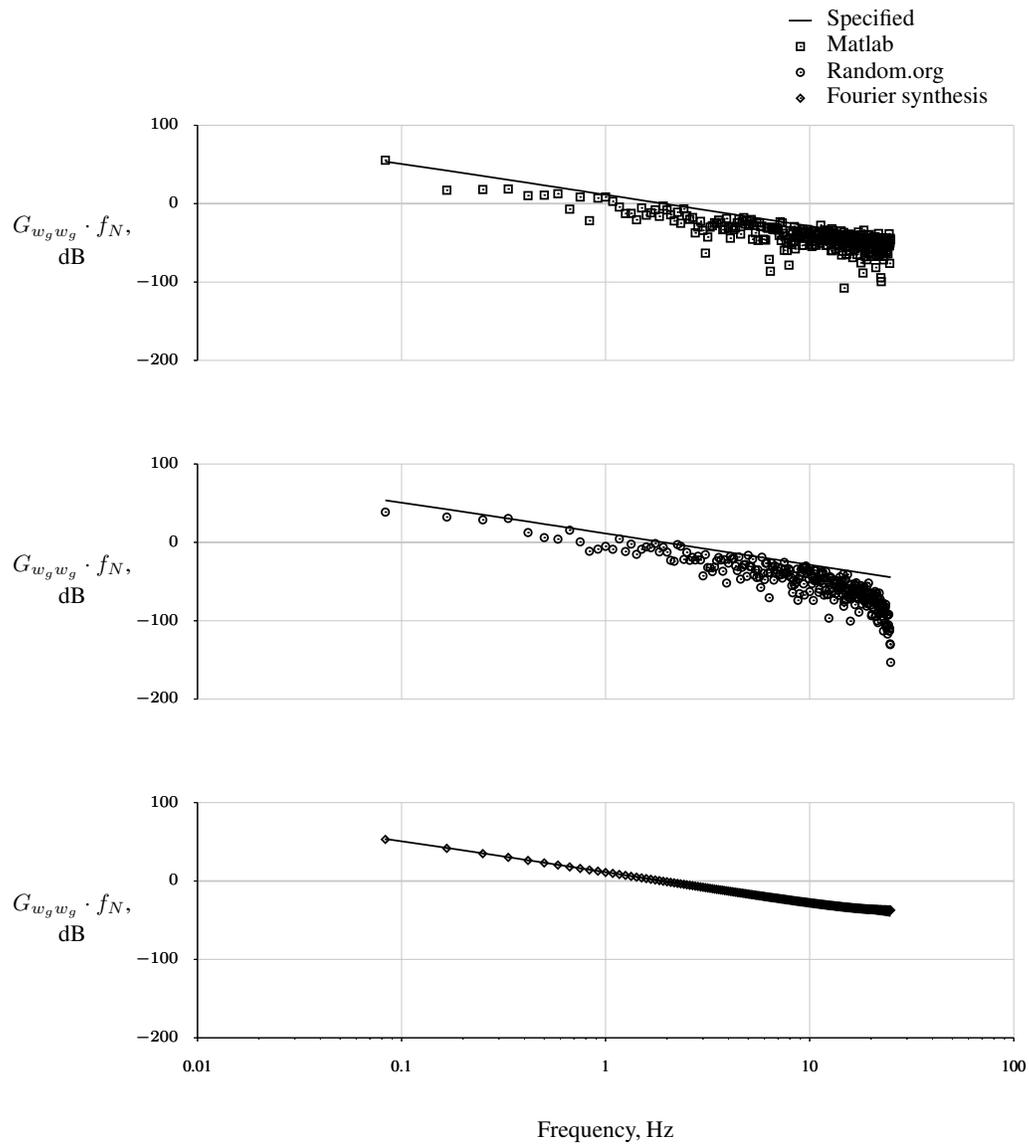


Figure 12: Vertical gust power spectral densities, single sample records

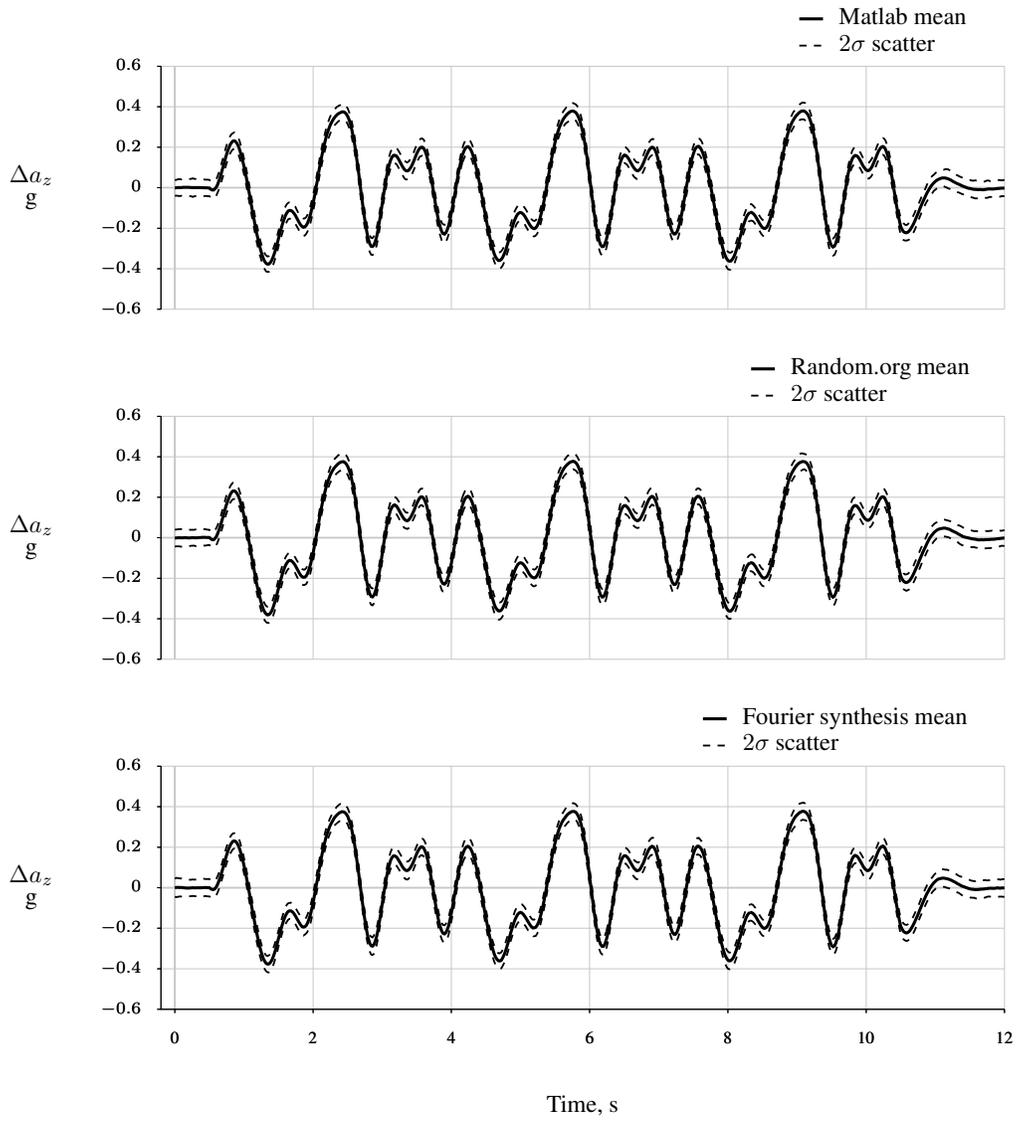


Figure 13: Simulated vertical accelerations with turbulence, Monte Carlo results

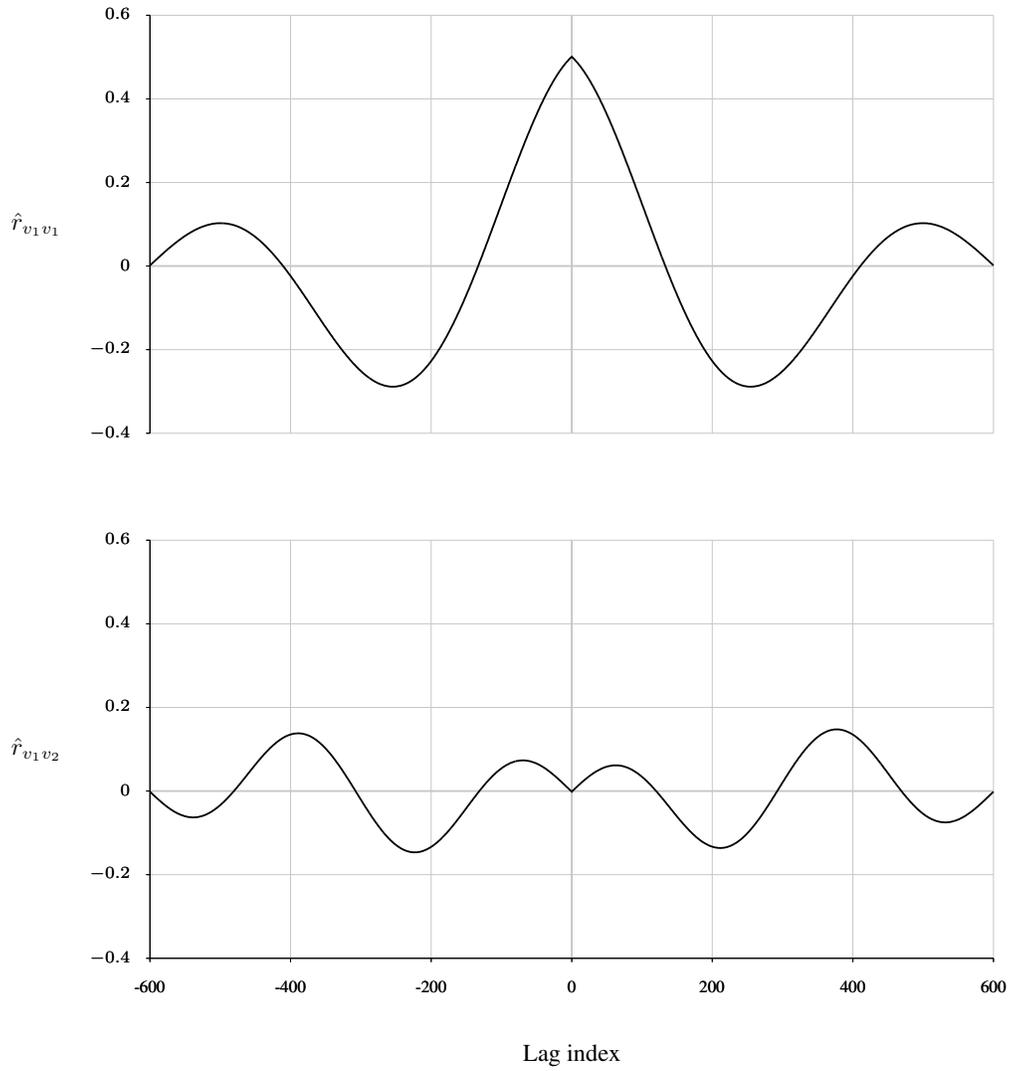


Figure 14: Autocorrelation and crosscorrelation of harmonic sinusoids

REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
<p>The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.</p> <p><b>PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.</b></p>					
1. REPORT DATE (DD-MM-YYYY) 01-05-2017		2. REPORT TYPE Technical Memorandum		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE  A Comparison of Three Random Number Generators for Aircraft Dynamic Modeling Applications				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Jared A. Grauer Langley Research Center, Hampton, Virginia				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER 109492.02.07.02.10	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) NASA Langley Research Center Hampton, Virginia 23681-2199				8. PERFORMING ORGANIZATION REPORT NUMBER L-20807	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Washington, DC 20546-0001				10. SPONSOR/MONITOR'S ACRONYM(S) NASA	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S) NASA/TM-2017-219612	
12. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified-Unlimited Subject Category 08 Availability: NASA CASI (301) 621-0390					
13. SUPPLEMENTARY NOTES An electronic version can be found at <a href="http://ntrs.nasa.gov">http://ntrs.nasa.gov</a> .					
14. ABSTRACT Three random number generators, which produce Gaussian white noise sequences, were compared to assess their suitability in aircraft dynamic modeling applications. The first generator considered was the MATLAB <sup>®</sup> implementation of the Mersenne-Twister algorithm. The second generator was a website called Random.org, which processes atmospheric noise measured using radios to create the random numbers. The third generator was based on synthesis of the Fourier series, where the random number sequences are constructed from prescribed amplitude and phase spectra. A total of 200 sequences, each having 601 random numbers, for each generator were collected and analyzed in terms of the mean, variance, normality, autocorrelation, and power spectral density. These sequences were then applied to two problems in aircraft dynamic modeling, namely estimating stability and control derivatives from simulated onboard sensor data, and simulating flight in atmospheric turbulence. In general, each random number generator had good performance and is well-suited for aircraft dynamic modeling applications. Specific strengths and weaknesses of each generator are discussed. For Monte Carlo simulation, the Fourier synthesis method is recommended because it most accurately and consistently approximated Gaussian white noise and can be implemented with reasonable computational effort.					
15. SUBJECT TERMS Random number, Gaussian white noise, system identification, Monte Carlo simulation					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			STI Help Desk (email: <a href="mailto:help@sti.nasa.gov">help@sti.nasa.gov</a> )
U	U	U	UU	42	19b. TELEPHONE NUMBER (Include area code) (301) 621-0390