BENCHMARKING THE COLLOCATION STAND-ALONE LIBRARY AND TOOLKIT (CSALT)

STEVEN HUGHES*

JEREMY KNITTEL*

WENDY SHOAN*

YOUNGKWANG KIM[+]

CLAIRE CONWAY^

DARREL CONWAY^

INTERNATIONAL SYMPOSIUM ON SPACE FLIGHT DYNAMICS

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AGENDA

- **Motivation**
- Optimal Control Problems
- CSALT
- General Optimal Control Benchmarking Results
- Low-Thrust Trajectory Design Benchmarking Results
- Summary/Future Work

MOTIVATION

- Physical systems which involve time varying control decisions can rarely be solved analytically
- Low-thrust spacecraft trajectories cannot be designed using intuition
- NASA Goddard's General Mission Analysis Toolkit (GMAT) has limited capability to solve low-thrust problems
- **Goals:**
  - **Demonstrate that CSALT can solve industry-standard optimal control problems**
  - **Demonstrate that CSALT can solve optimal control low-thrust trajectory problems**
  - **Compare CSALT execution efficiency to other optimal control software packages**

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OPTIMAL CONTROL PROBLEMS

- Minimize a cost function of the form:

$$J = \Phi\left(\mathbf{x}(t_o), t_o, \mathbf{x}(t_f), t_f\right) + \int_{t_o}^{t_f} \lambda(x(t), u(t), t)dt$$

- Subject to the following set of ordinary differential equations:

$$\dot{\mathbf{x}}(t) = \mathbf{a}(\mathbf{x}(t), \mathbf{u}(t), t)$$

- Subject to algebraic path constraints:

$$\mathbf{p}(\mathbf{x}(t), \mathbf{u}(t), t) \leq \mathbf{0}$$

- Subject to boundary conditions:

$$\mathbf{b}(\mathbf{x}(t_o), \mathbf{u}(t_o), \mathbf{x}(t_f), \mathbf{u}(t_f), t_o, t_f) \leq \mathbf{0}$$

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COLLOCATION STAND-ALONE LIBRARY AND TOOLKIT

- 17000 original source lines of code (SLOC) written in C++
- Uses roughly 17000 SLOC from GMAT's utility code base
- Uses Boost C++ library for sparse matrix arithmetic
- Uses SNOPT for nonlinear programming optimization
- Will be open source released with GMAT eventually (Fall 2017 or Spring 2018)

CURRENT CSALT CAPABILITY

- Multiple collocation transcriptions
  - Trapezoid
  - Hermite-Simpson
  - Lobatto IIIa of order 4,6 and 8
  - Radau Orthagonal
- Multiple cost-function formulations
  - Mayer
  - Lagrange
  - Bolza
- Algebraic path and point constraints
- Decision vector, cost and constraint scaling
- Analytical collocation derivatives with finite differenced user point and path functions
- Automatic sparsity pattern determination
- Mesh refinement (Radau transcription only)

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HIGH LEVEL RESULTS

- Three comparison tools:
  - SOS - Sparse Optimization Suite written by Dr. John Betts
  - GPOPS-II – Gauss Pseudospectral OPtimization Software written by Dr. Anil Rao
  - PSOPT – PseudoSpectral OPTimization written by Dr. Victor Becerra
- 17 problems selected for objective function comparison
- 3 problems selected for detailed state and control comparison

Test Case	"Truth" Source	Difference
Rayleigh (Control Contraint)	SOS	-4.03E-08
Rayleigh (Control + State Constraint)	SOS	6.59E-08
Goddard Rocket	SOS /GPOPS	-8.57E-09
Hypersensitive	SOS /GPOPS	-1.39E-06
Conway Orbit	SOS	-2.11E-08
Linear Tangent Steering	SOS /GPOPS	-1.19E-09
Brachistichrone	SOS /GPOPS	2.37E-09
Bryson Denham	GPOPS /PSOPT	-9.07E-07
Schwartz	PSOPT	4.38E+00
Interior Point	PSOPT	4.53E-08
Bryson Max Range	PSOPT	-6.57E-07
Obstacle Avoidance	PSOPT	0.00E+00
Moon Lander	PSOPT	-4.09E-05
Rau Automatica	Analytic	4.70E-12
Hull Problem 9.5	Analytic	4.00E-12

Problem Name	Tool	Max. Rel. State Error	Max. Rel. Control Error
Goddard	SOS	6.863e-07	2.191e-08
Hypers.	GPOPSII	4.305e-02	2.911e-01
Conway	SOS	9.814e-06	1.610e-02

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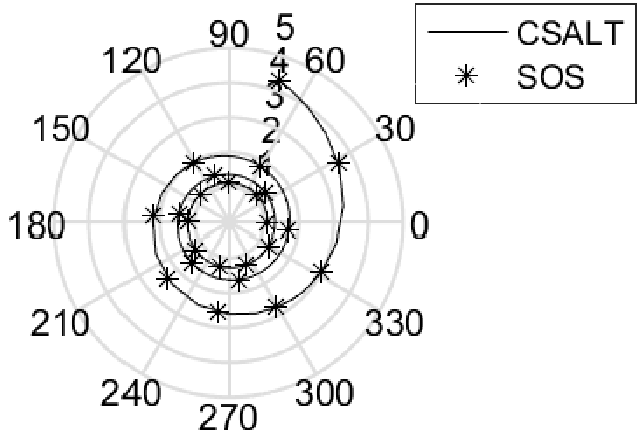
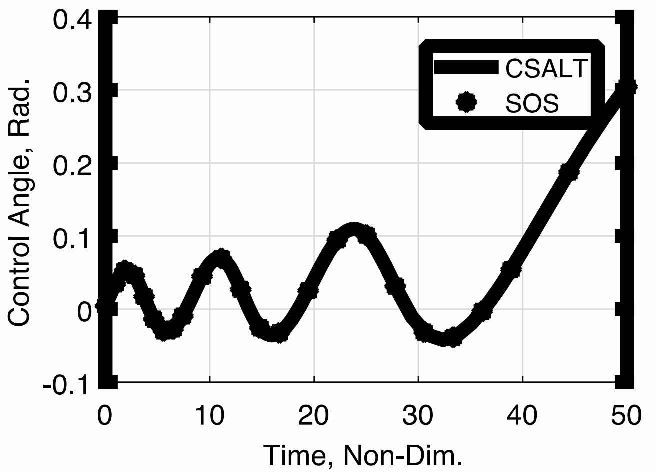
CONWAY ORBIT

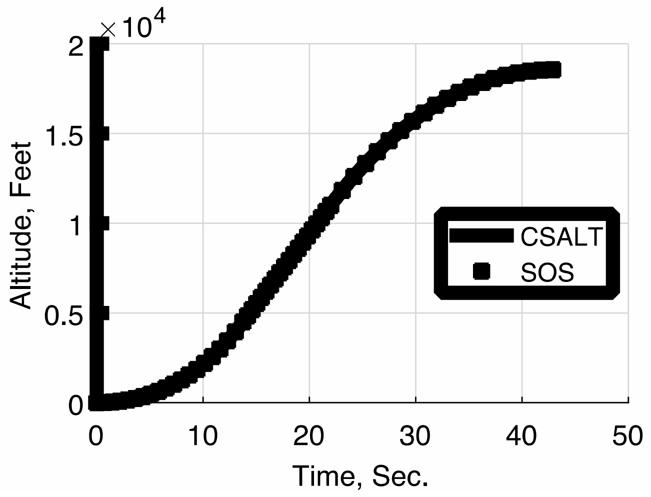
- Classic problem in the literature
- Finite-thrust orbit raising
- Optimal solutions match perfectly to the scale of the graphics
- Maximum relative error in state is better than control
  - 9.841e-6 in state
  - 1.610e-2 in control

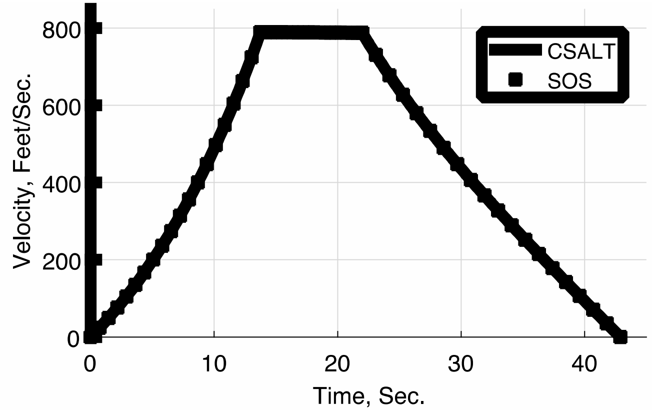


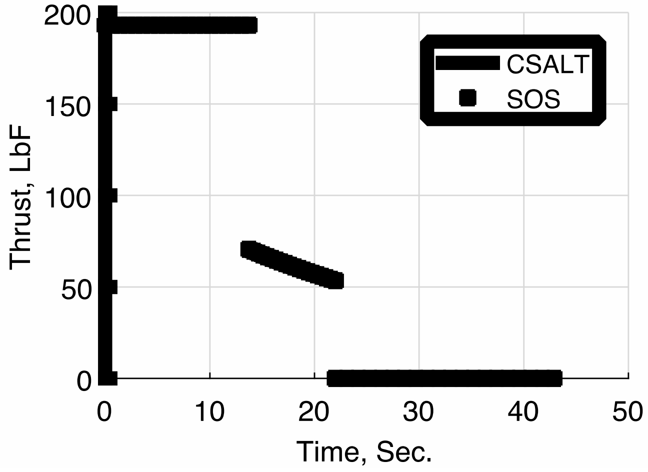


GODDARD ROCKET

- Multi-phase problem
- Control in optimal solution is discontinuous and has bang-off structure
- Control varies in the middle phase to maintain constant terminal velocity
- CSALT solution matches very well with reference software in terms of relative error
  - 6.863e-7 in state
  - 2.191e-8 in control

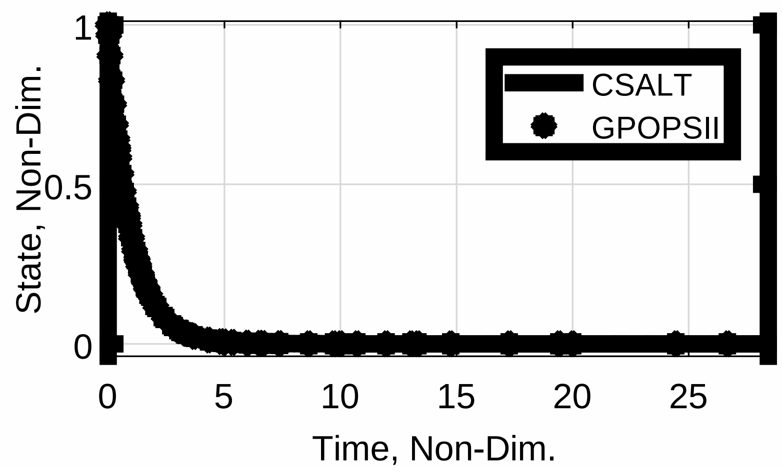


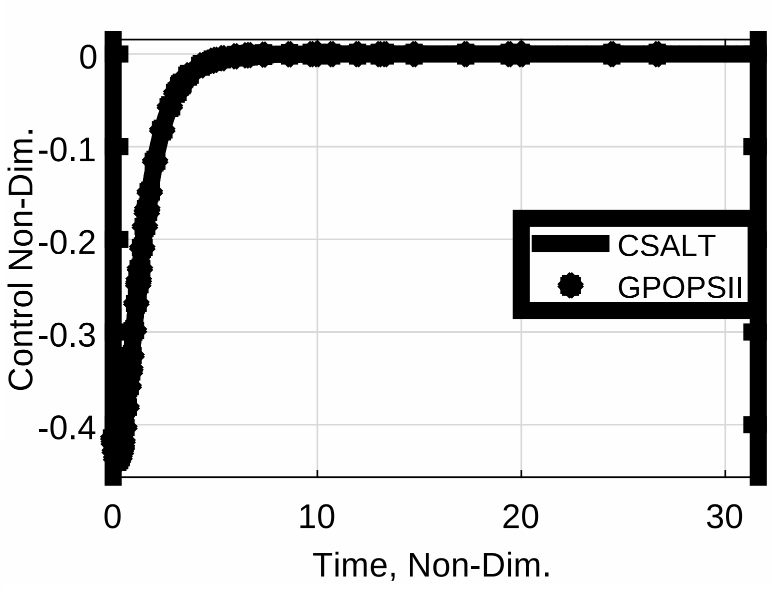




HYPERSENSITIVE PROBLEM

- Stressing case to the mesh refinement algorithm
- Dynamics rapidly change near the boundary conditions, but are nearly constant in between
- Relative errors are larger than desirable
  - 4.305e-2 in state
  - 2.911e-1 in control
  - Believed to be due to interpolation to common discretization times (and away from the collocation points)
  - This is supported by the relative error between SOS and GPOPS-II (3.54e-1 and 1.037 in state and control respectively)





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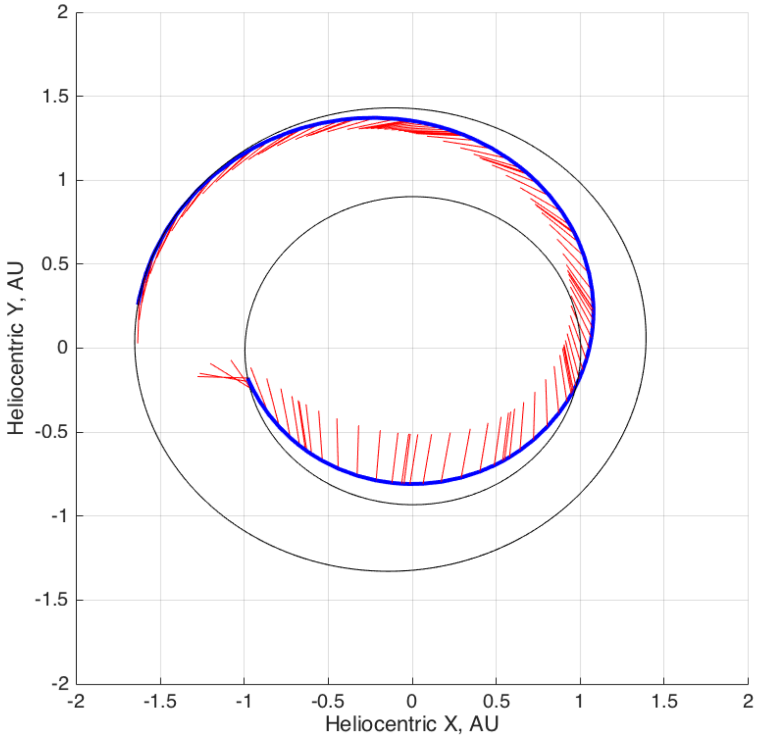
MARS TRANSFER

- CSALT was compared to solutions presented by Horsewood and Dankanich*
- Direct transfer from Earth to Mars using 2 ion engines
- Qualitatively indentical solutions were found using all 3 software tools
- The final solutions all match to the expected margin of error of the modeling techniques

Parameter	Malto	HILTOP	CSALT
Launch date	3/29/2022	4/7/2022	4/4/2022
Launch C_3 (km^2/s^2)	32.26	32.57	35.06
Launch declination (deg)	-5.1	-3.776	-5.067
Launch mass (kg)	2,105.4	2,187.3	2,143.0
Cruise flight time (days)	458.2	455.7	452.0
Cruise propellant (kg)	324.3	338.7	336.9
Mars Arrival date	7/1/2023	7/6/2023	7/1/2023



* Horsewood, J.L., Dankanich, J.W.: Heliocentric Interplanetary Low-thrust Optimization Program Capabilities and Comparison to NASAs Low-thrust Trajectory Tools, 31st International Electric Propulsion Conference, University of Michigan, Sept. 20–24, 2009.

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DAWN

- CSALT was compared to solutions presented by Horsewood and Dankanich*
- Repeat of the Dawn mission
  - Earth to Mars low-thrust transfer
  - Mars flyby
  - Rendezvous with Vesta
  - Low-thrust transfer to Ceres
- CSALT found a qualitatively improved solution compared to the other two tools
  - At a cost of ~220 days of additional flight time, 160 extra kg of dry mass could be delivered to Ceres using less propellant

Horsewood, J.L., Dankanich, J.W.: Heliocentric Interplanetary Low-thrust Optimization Program Capabilities and Comparison to NASAs Low-thrust Trajectory Tools, 31st International Electric Propulsion Conference, University of Michigan, Sept. 20–24, 2009.

Parameter	Malto	HILTOP	CSALT
Leg 1 Earth-Mars			
Launch date	9/27/2007	9/27/2007	9/27/2007
Launch C_3 (km^2/s^2)	5.1529	5.2285	1.4819
Launch declination (deg)	28.5	28.5	11.7803
Launch mass (kg)	1,114.4	1,105.2	1243.0
Flight time (days)	510	510	527
Arrival mass (kg)	1,039.8	1,032.7	1180.2
Propellant used (kg)	74.6	72.4	62.8
Leg 2 Mars-Vesta			
Swingby date	2/18/2009	2/18/2009	3/7/2009
Swingby v_∞ (km/s)	4.10	4.11	4.49
Passage altitude (km)	300	300	300
Flight time (days)	894	827.7	903.1
Arrival date	8/1/2011	5/26/2011	8/27/2011
Arrival mass (kg)	907.3	901.4	1,079.0
Propellant used (kg)	132.5	131.3	101.2
Stay time (days)	270	336.3	270
Leg 3 Vesta-Ceres			
Departure date	4/27/2012	4/27/2012	5/23/2012
Flight time (days)	1,038	1,038	1,230
Arrival date	2/28/2015	2/28/2015	10/4/2015
Arrival mass (kg)	807.2	802.3	960.1
Propellant used (kg)	100.1	99.1	118.9
Total Propellant (kg)	**307.2**	**302.8**	**282.9**
Mission duration (days)	**2,711**	**2,711**	**2,930**

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SUMMARY AND FUTURE WORK

- CSALT is a mature software package capable of solving a variety of optimization problems with high accuracy
- CSALT has been successfully benchmarked against industry standard tools in both general engineering problems and for the solution of low-thrust trajectories
- Future Work
  - Static and integral decision parameters and integral constraints
  - Second derivatives of the collocation
  - Improve performance
  - Full integration into GMAT