A Smoothed Eclipse Model for Solar Electric Propulsion Trajectory Optimization

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Motivation

Solar electric propulsion (SEP) is the primary means of employing low-thrust on a space mission.

Eclipse constraints are discontinuous and problematic for gradient-based trajectory optimization.

Contribution

A smoothed eclipse model suitable for gradient-based trajectory optimization.

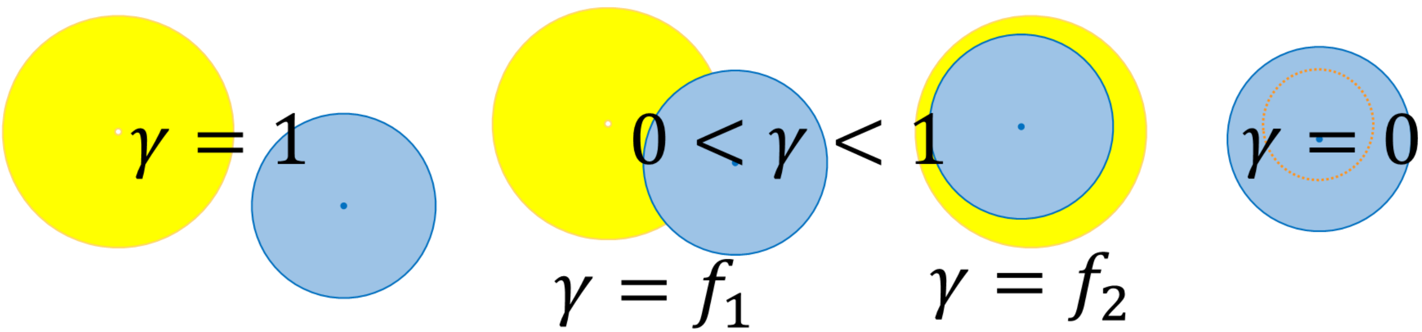
Demonstrated improvement in mass delivered from LEO to GEO using a second-order optimizer.

Approach

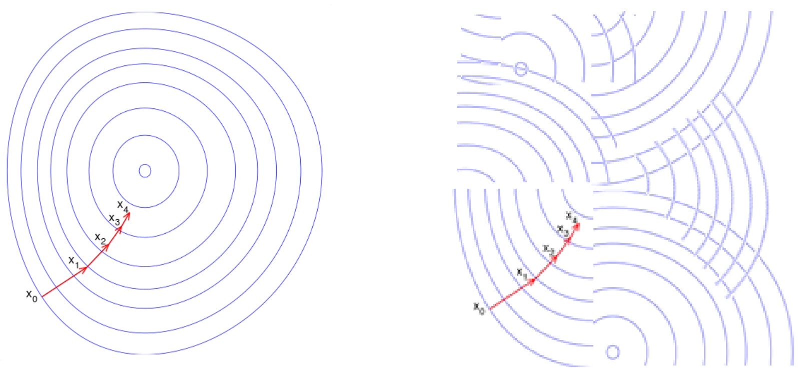
- Introduce the *sunlight fraction*, γ, to include eclipse effects

$$power\ available = \gamma\ (computed\ power)$$

- Sunlight fraction is a piecewise function.

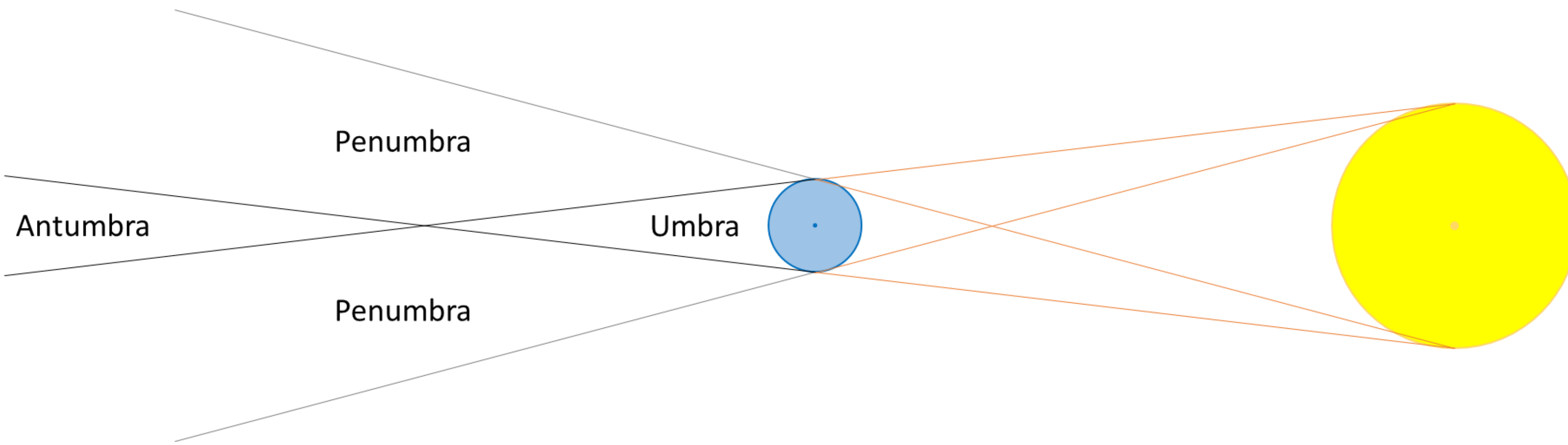


- Discontinuous derivatives are a problem for gradient-based optimization.

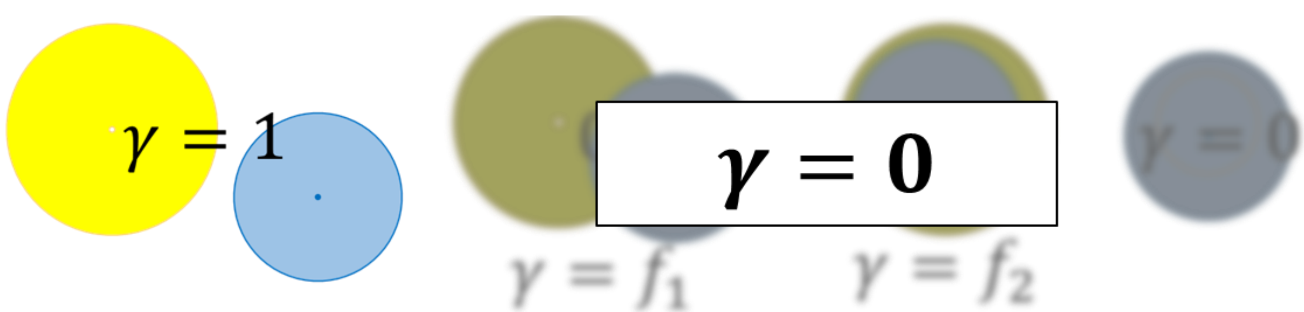


The Penumbra Constraint

- Disallow thrusting in shadow, even partial eclipse

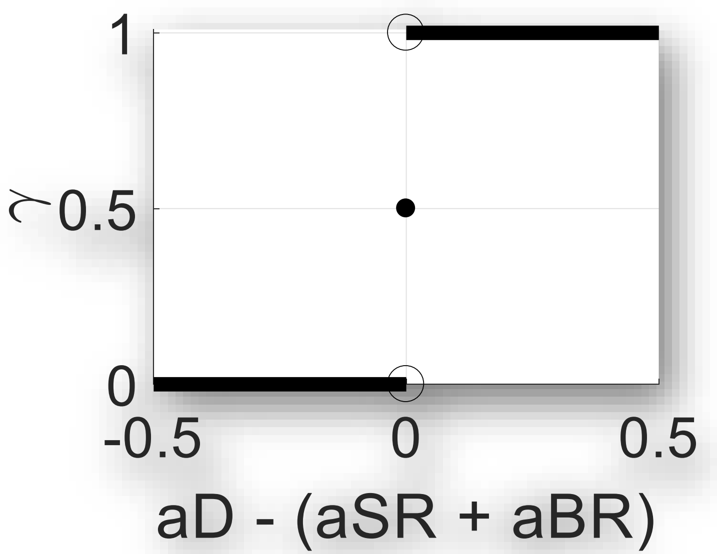


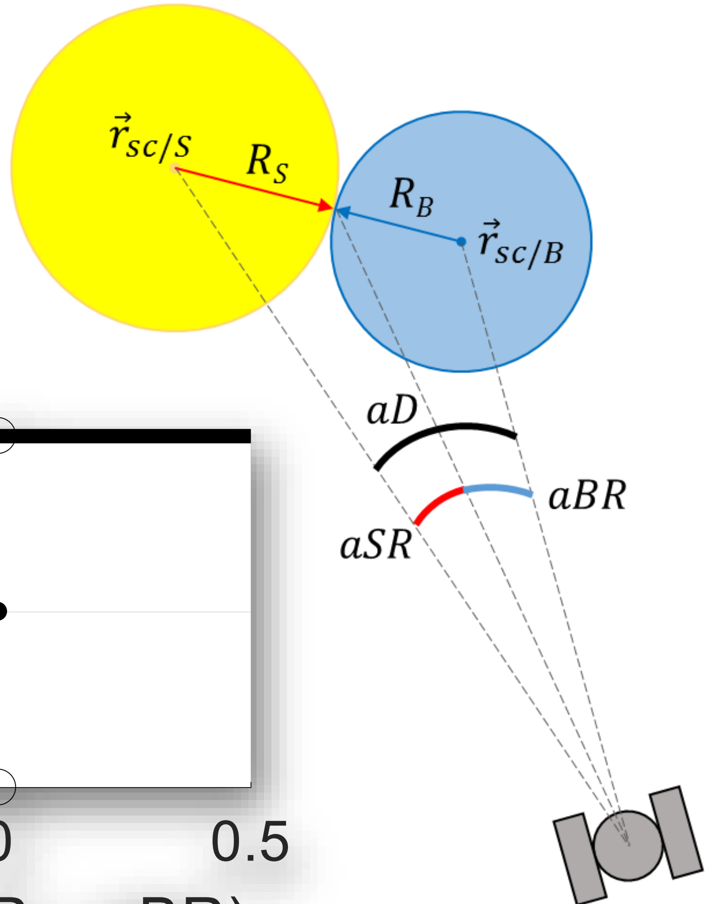
- Percent sunlight is now a step function



Heaviside Sunlight Fraction

- The Heaviside step function is half-valued at the transition.





Apparent Sun Radius

$$aSR = \sin^{-1} \frac{R_S}{r_{sc/S}}$$

Apparent Body Radius

$$aBR = \sin^{-1} \frac{R_B}{r_{sc/B}}$$

Apparent Distance

$$aD = \cos^{-1} \left(\frac{\vec{r}_{sc/B} \cdot \vec{r}_{sc/S}}{r_{sc/B} r_{sc/S}} \right)$$

Eclipse occurs when

$$aD < aSR + aBR$$

Logistic Sunlight Fraction

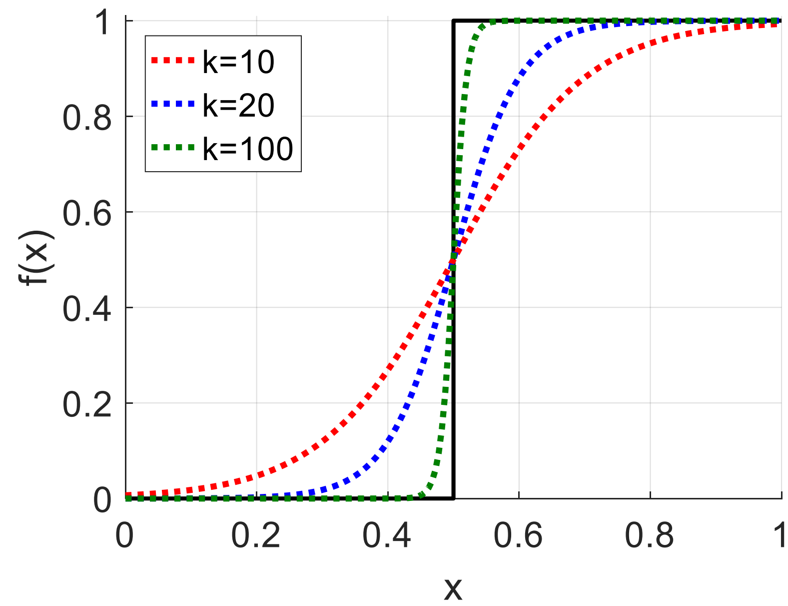
- The logistic function is a smooth approximation to the Heaviside step function,

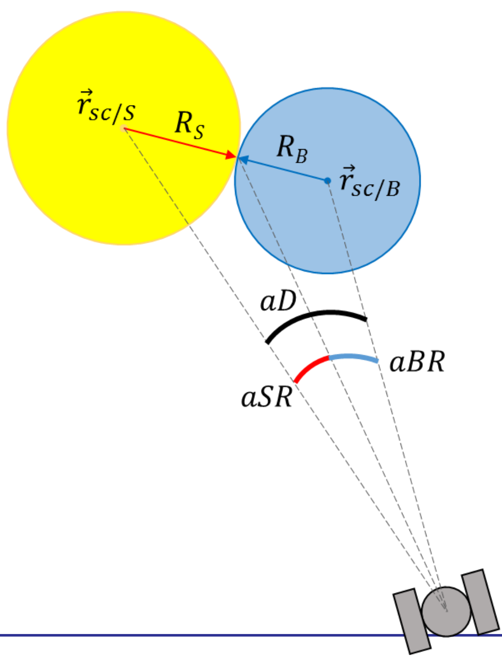
$$f(x) = \frac{1}{1+e^{-k(x-x^*)}}$$

and is continuously differentiable.

Transition occurs at $x = x^*$.

Eclipse occurs at $aD = aBR + aSR$:

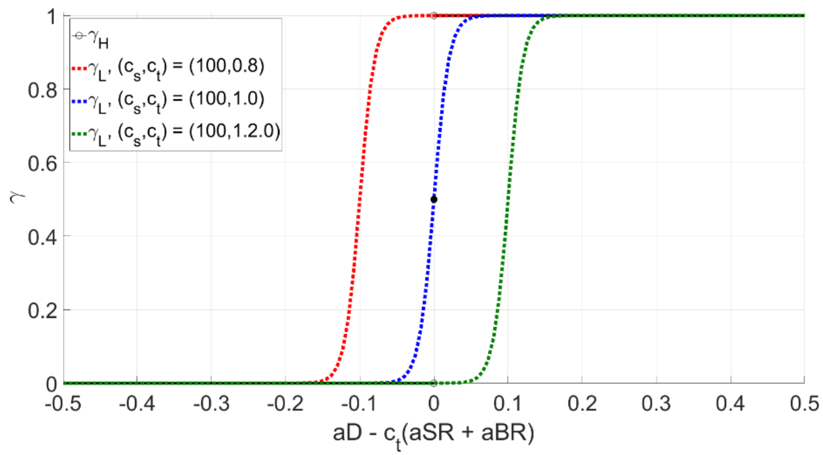


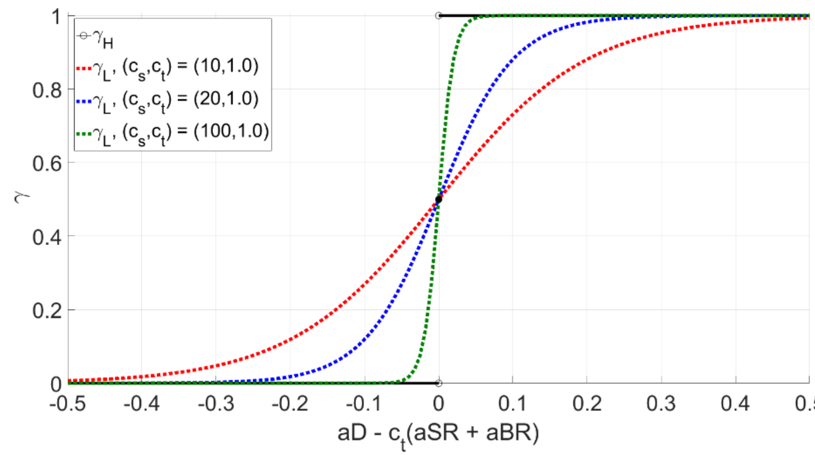


$$\gamma_L = \frac{1}{1 + e^{-c_s[aD - c_t(aSR + aBR)]}}$$

Sharpness and Transition Coefficients

- Sharpness coefficient, c_s, and transition coefficient, c_t, permit tuning of the smoothed eclipse model

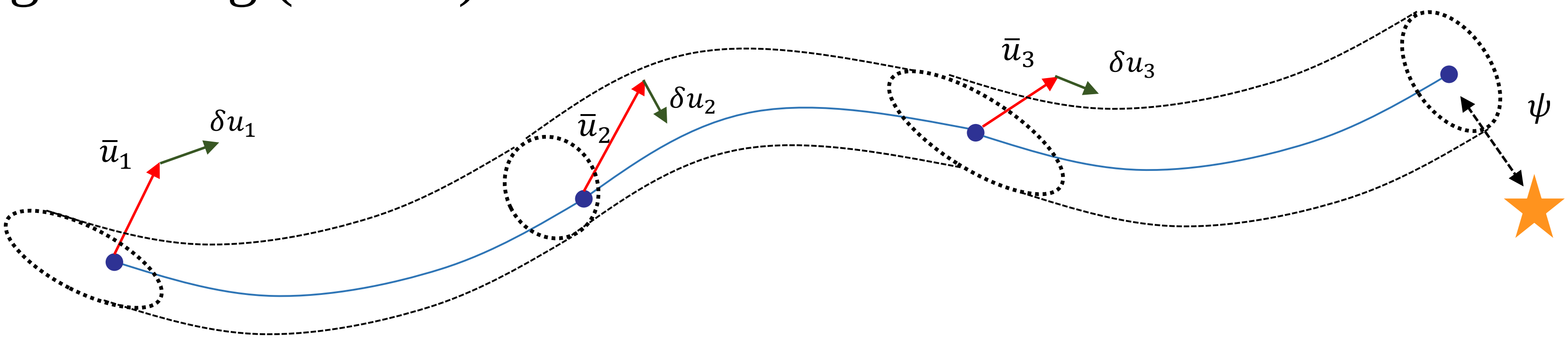




- Can select (c_s, c_t) to minimize error, numerical reasons, or operational considerations (early/late power down/up).
- For $c_t = 1.0$, find optimal c_s~289.78 at Earth, c_s~432.35 at Mars
  - holds across a range of spacecraft semi-major axes and eccentricities
  - central body and its heliocentric orbit are the drivers

LEO to GEO Transfer

- Reproduce transfer from Betts (2014)
  - min-fuel LEO to GEO in in 248.5 revolutions
  - construct initial guess with Lyapunov control thrust arcs and forced coast arcs through eclipse
  - refine solution with Sparse Optimization Suite (SOS)
    - direct transcription and sequential nonlinear programming
- Now use smoothed eclipse model and Hybrid Differential Dynamic Programming (HDDP)



  - HDDP control updates minimize local quadratic models
  - 1st and 2nd derivatives of cost function and dynamics w.r.t states, controls, multipliers, parameters

Problem Setup

- Minimize propellant consumed for LEO to GEO transfer in 248.5 revolutions

Table 1.: Spacecraft Parameters.

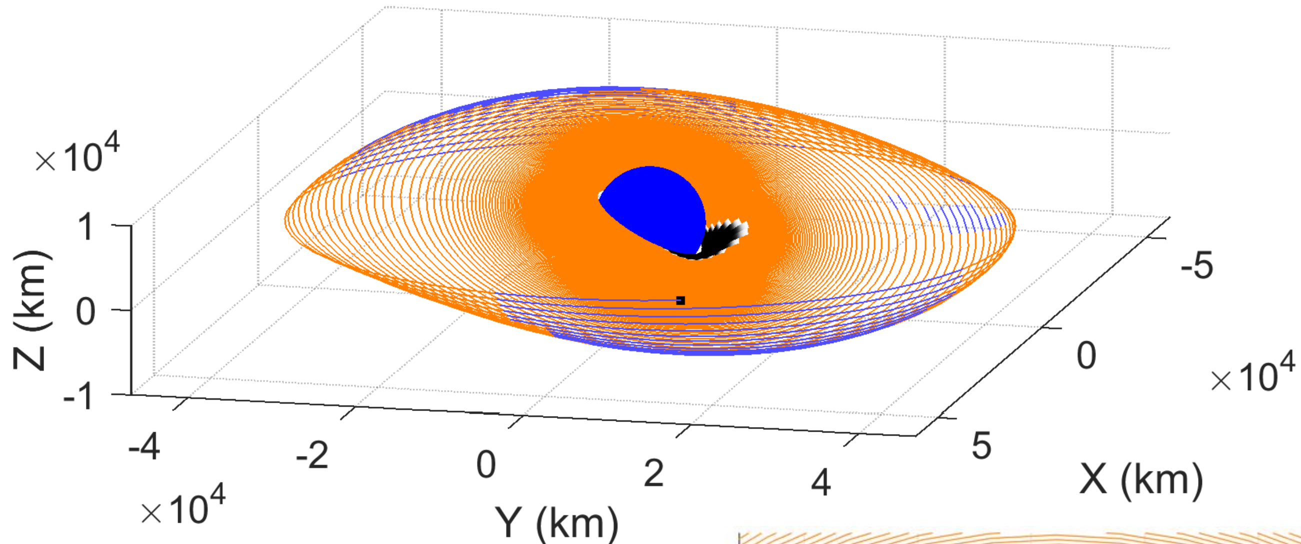
m_0	1000 kg	T_{max}	1.445 N
I_{sp}	1849.3477486671852 s	P_0	13.1031921568649 kW

Table 2.: Initial and Target Orbit States.

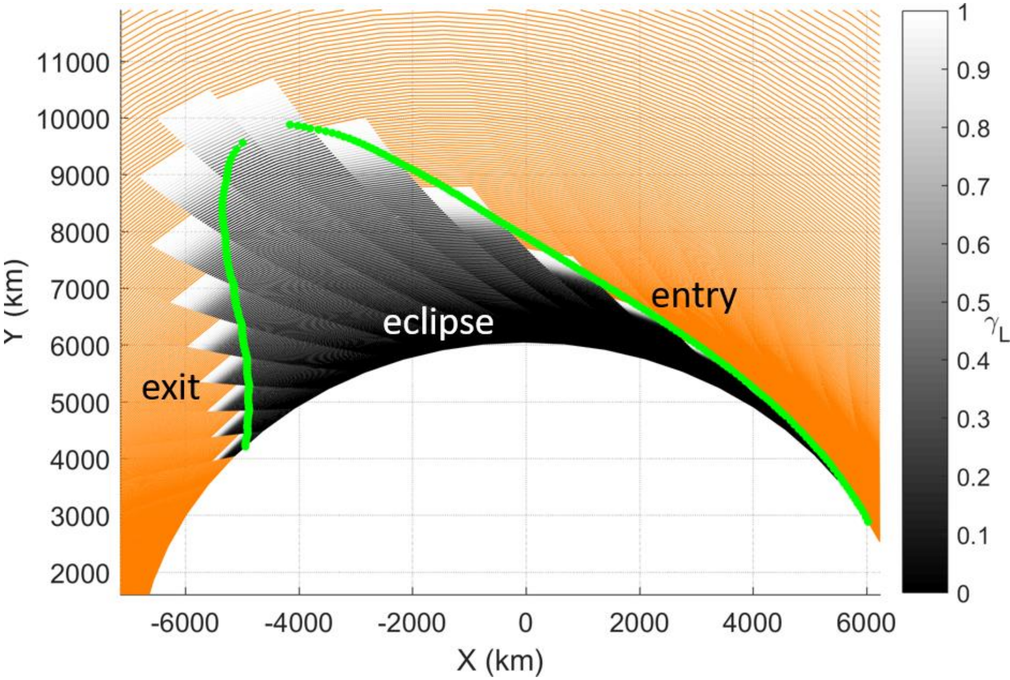
p_0	6878.14 km	p_f	42241.095482827557 km
f_0	0.0	f_f	0.0
g_0	0.0	g_f	0.0
h_0	-0.2539764647494369	h_f	0.0
k_0	0.0	k_f	0.0
L_0	π		
t_0	0.0		

- Modified Equinoctial Elements above (500 km circular 28.5° LEO)
- Perturbations: Lunar and Solar gravity, Earth $J_2 - J_4$

Smoothed Eclipse Model Result

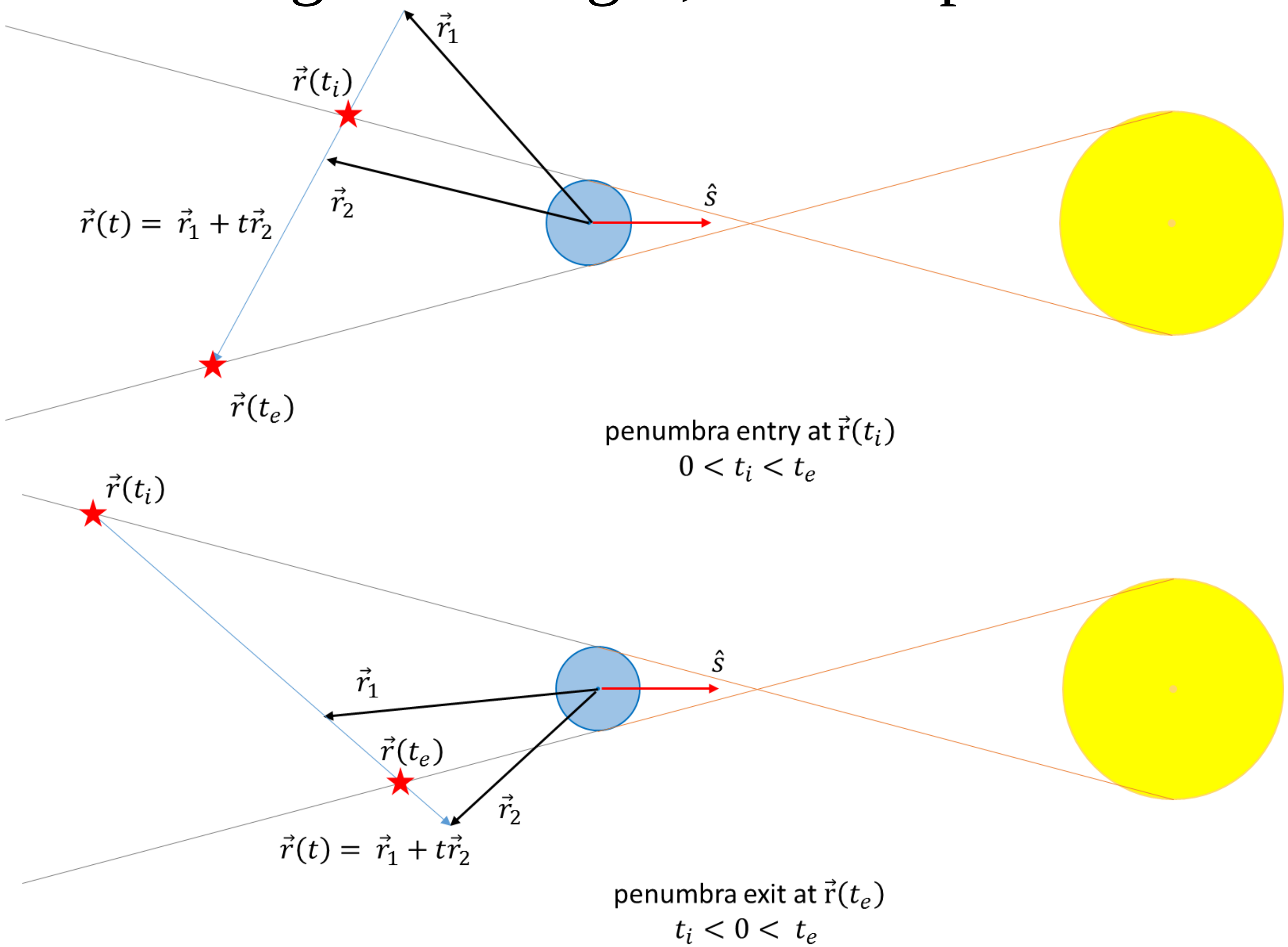


- Improved final mass to 733.29 kg from Betts' 718.79 kg
- Smoothed model eclipse detection is 'automatic', but suffers discretization error
- Violate penumbra constraint stepping through entry, and powers up late after exit



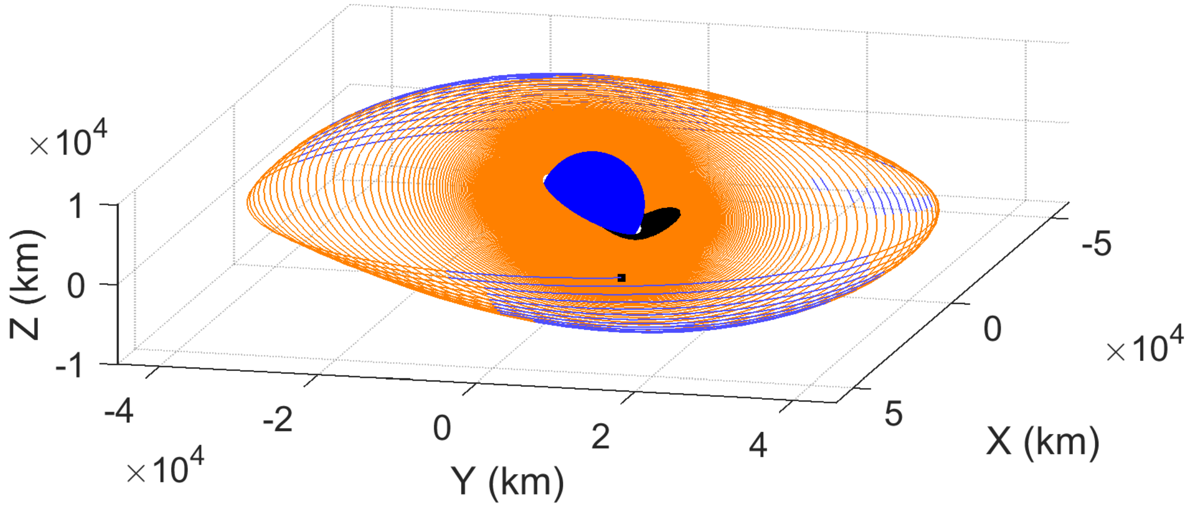
Penumbra Entry/Exit Detection

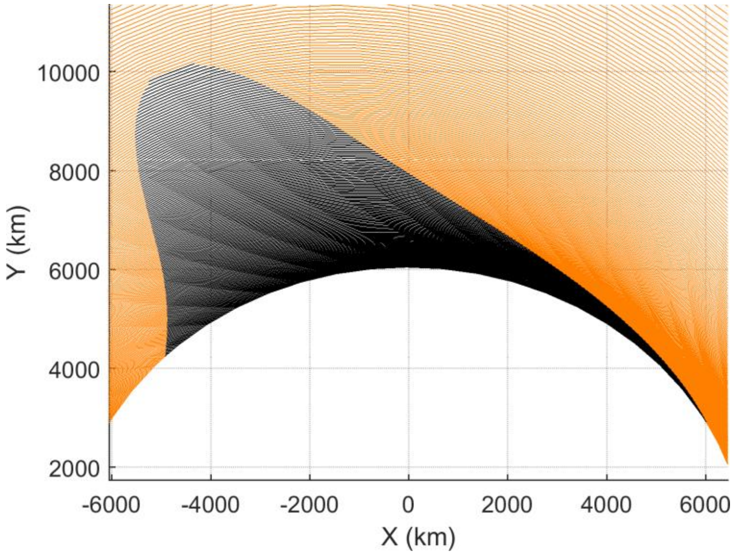
- Compute penumbra entry/exit locations as intersection of line between integration stages, and the penumbral cone



Refined Solution

- Insert integration stages at the penumbra entry/exit locations, use first solution as initial guess, assign $T_{max} = 0$ for shadow





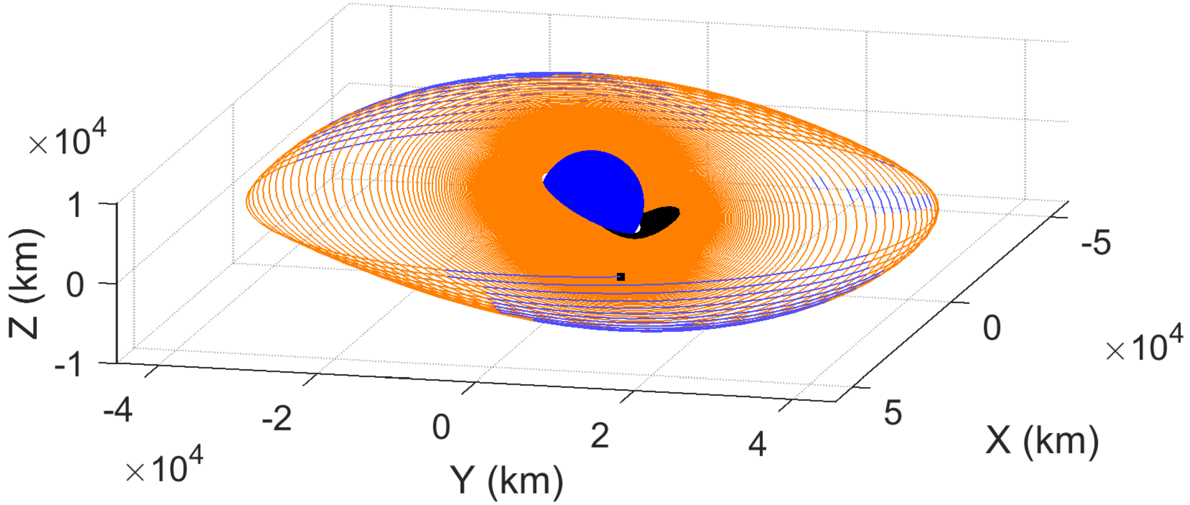
Iteration	m_f (kg)	t_f (days)
1	733.29	45.78
2	732.61	45.58
3	732.41	45.51
4	732.29	45.49
5	732.24	45.46
Betts[3)]	718.79	43.13

Conclusion

- Smoothed Eclipse Model presented as the logistic sunlight fraction:

$$\gamma_L = \frac{1}{1 + e^{-c_s[aD - c_t(aSR + aBR)]}}$$

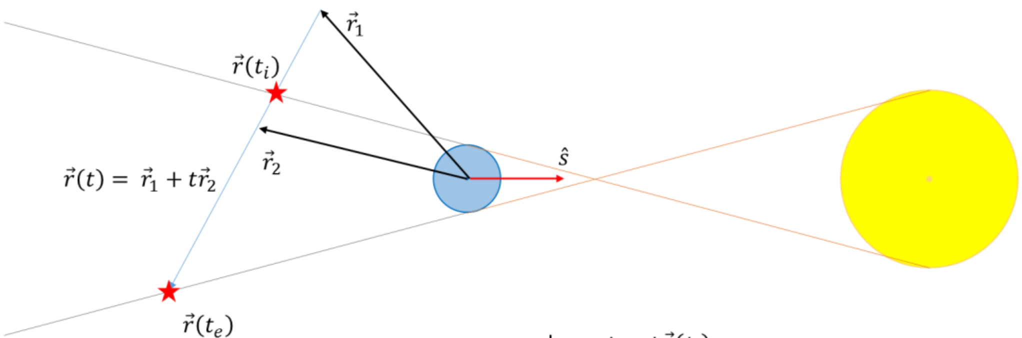
- Demonstrated in a second-order gradient-based trajectory optimization algorithm, HDDP
- Improved delivered mass for LEO to GEO transfer



Backup Slides

Penumbra Entry/Exit Detection

- Compute penumbra entry/exit locations as intersection of line between integration stages, and the penumbral cone



penumbra entry at $\vec{r}(t_i)$

$0 < t_i < t_e$

$$\boldsymbol{r}(t) = \boldsymbol{r}_1 + t\boldsymbol{r}_2$$

$$(\boldsymbol{r} - \boldsymbol{X}_p)^T(\hat{\boldsymbol{s}}\hat{\boldsymbol{s}}^T - I\cos^2\alpha_p)(\boldsymbol{r} - \boldsymbol{X}_p) \geqslant 0$$

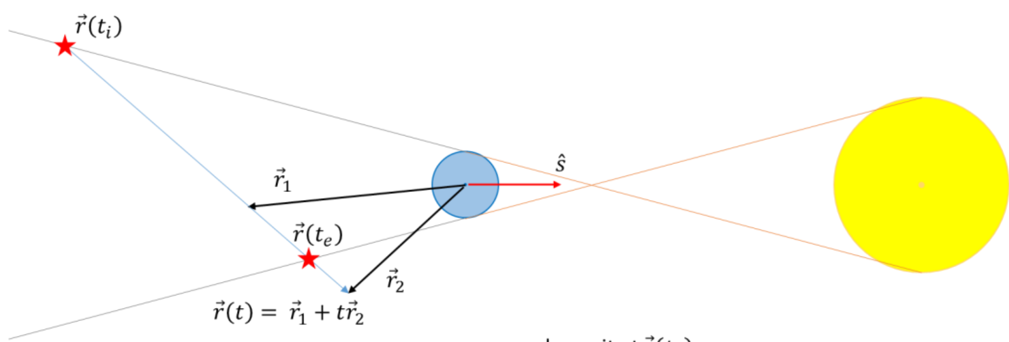
$$M = (\hat{\boldsymbol{s}}\hat{\boldsymbol{s}}^T - I\cos^2\alpha_p)$$

$$a = \boldsymbol{r}_2{}^T M \boldsymbol{r}_2$$

$$b = \boldsymbol{r}_2{}^T M(\boldsymbol{r}_1 - \boldsymbol{X}_p)$$

$$c = (\boldsymbol{r}_1 - \boldsymbol{X}_p)^T M(\boldsymbol{r}_1 - \boldsymbol{X}_p).$$

$$t = (-b \pm \sqrt{b^2 - ac})/a$$



penumbra exit at $\vec{r}(t_e)$

$t_i < 0 < t_e$

Refined Solution

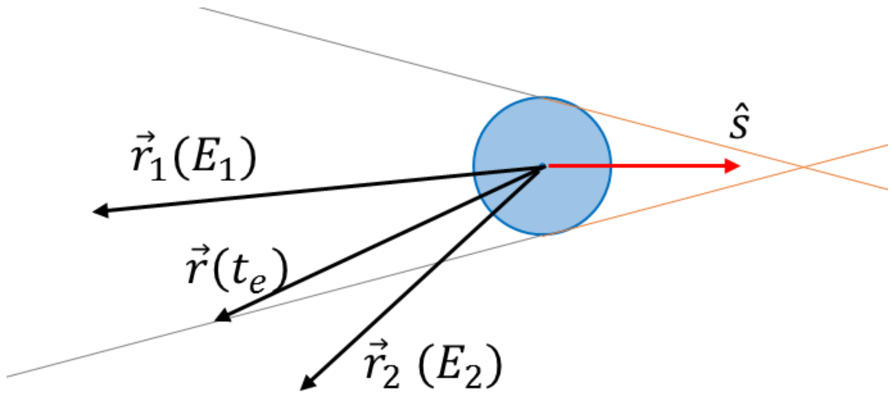
- Insert integration stages at the penumbra entry/exit locations, use first solution as initial guess, assign $T_{max} = 0$ for shadow

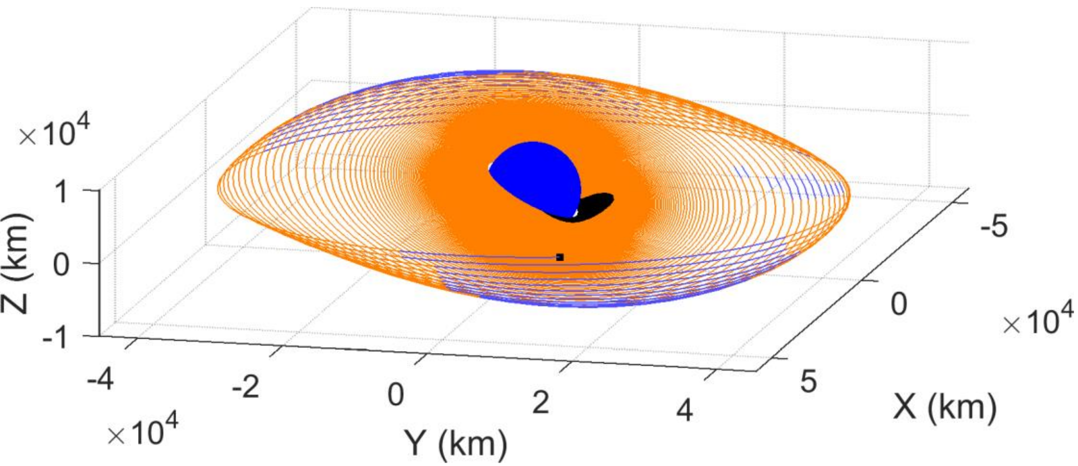
$$\Delta\nu = \cos^{-1}\left(\frac{\boldsymbol{r}_e{}^T\boldsymbol{r}_1}{\|\boldsymbol{r}_e\|\|\boldsymbol{r}_1\|}\right)$$

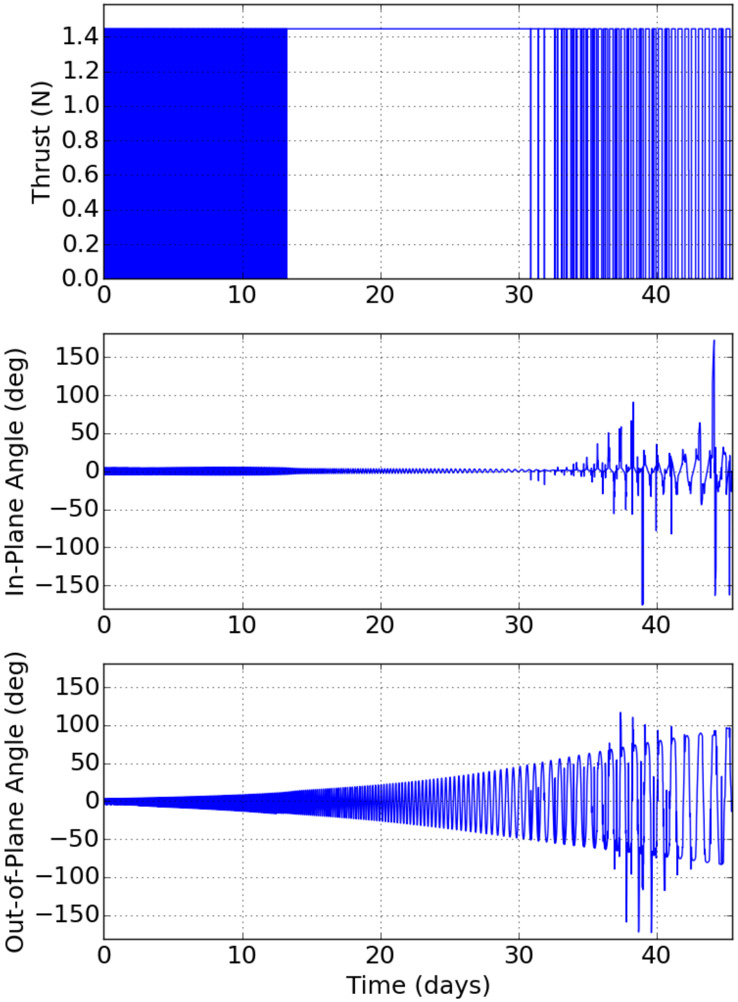
$$\tau_e = \tau(\boldsymbol{X}_1, \tau_1, \Delta\nu)$$

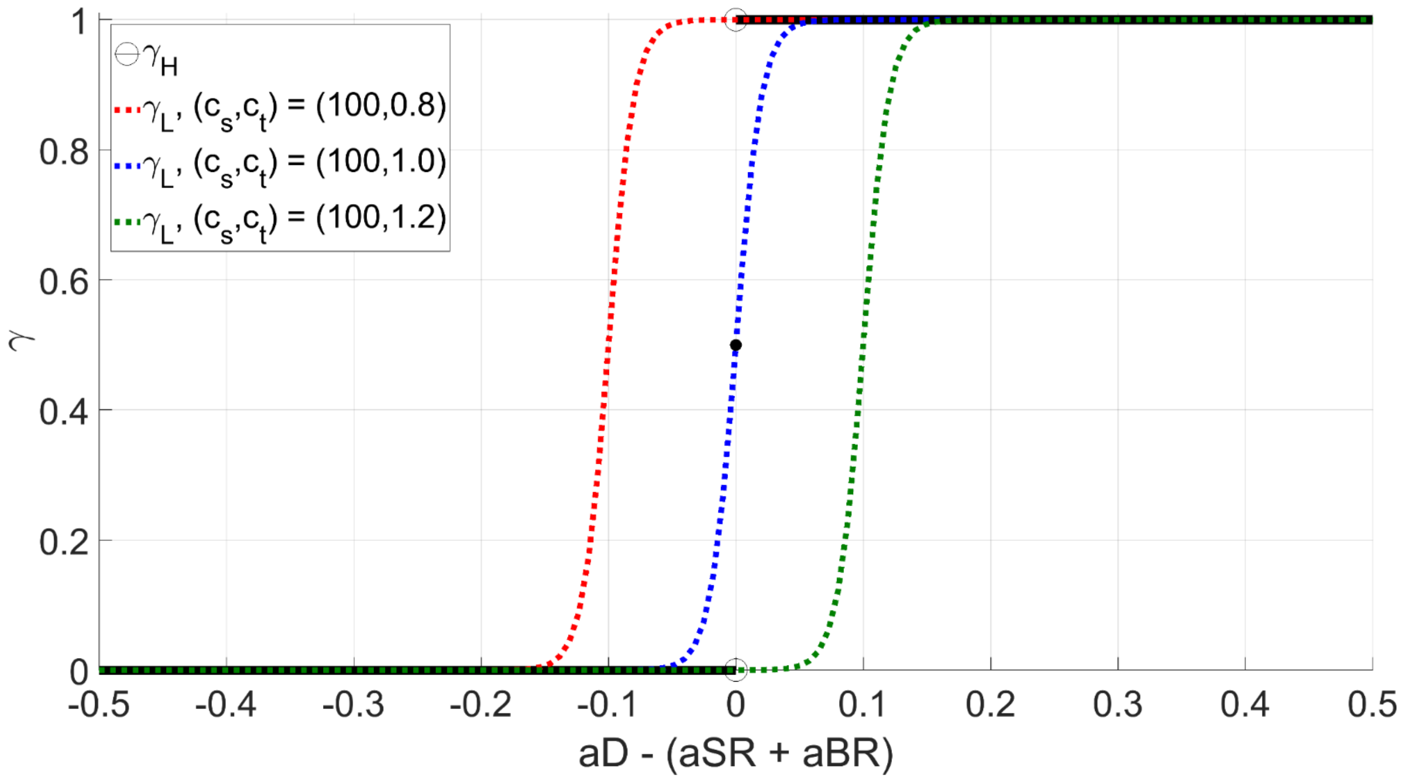
$$\boldsymbol{X}_e = \boldsymbol{f}(\boldsymbol{X}_1, \tau_e - \tau_1)$$

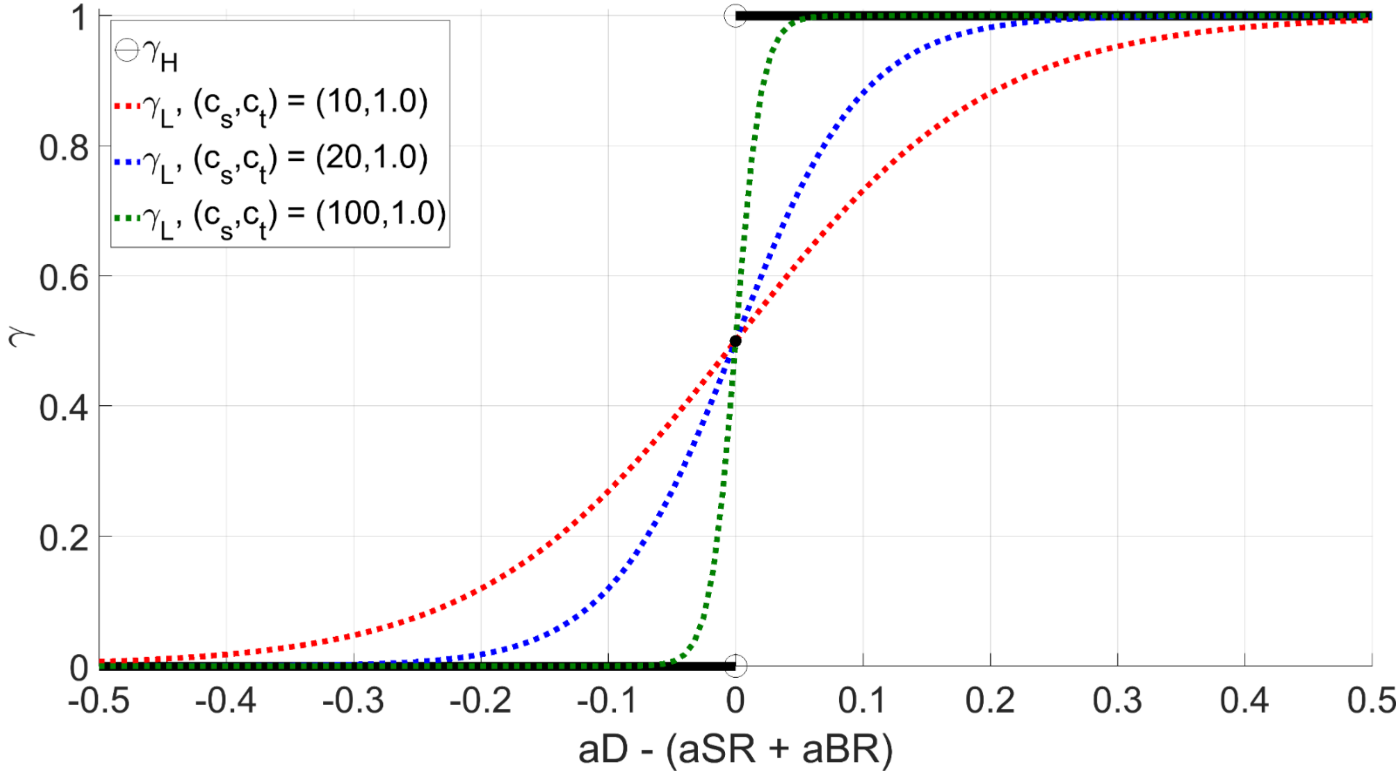
$$\boldsymbol{X}_2 = \boldsymbol{f}(\boldsymbol{X}_e, \tau_2 - \tau_e)$$




Z (km)
Y (km)
X (km)
×10^4
1
0
-1
-4
-2
0
2
4
-5
0
5


Thrust (N)
In-Plane Angle (deg)
Out-of-Plane Angle (deg)
Time (days)
0
10
20
30
40
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
150
100
50
0
−50
−100
−150


γ_H
γ_L, (c_s,c_t) = (100,0.8)
γ_L, (c_s,c_t) = (100,1.0)
γ_L, (c_s,c_t) = (100,1.2)
γ
1
0.8
0.6
0.4
0.2
0
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
aD - (aSR + aBR)


γ_H
γ_L, (c_s,c_t) = (10,1.0)
γ_L, (c_s,c_t) = (20,1.0)
γ_L, (c_s,c_t) = (100,1.0)
1
0.8
0.6
0.4
0.2
0
γ
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
aD - (aSR + aBR)