3D Pc Operational Issues and Ways Forward

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2D and 3D Pc: Abbreviated Technical Background

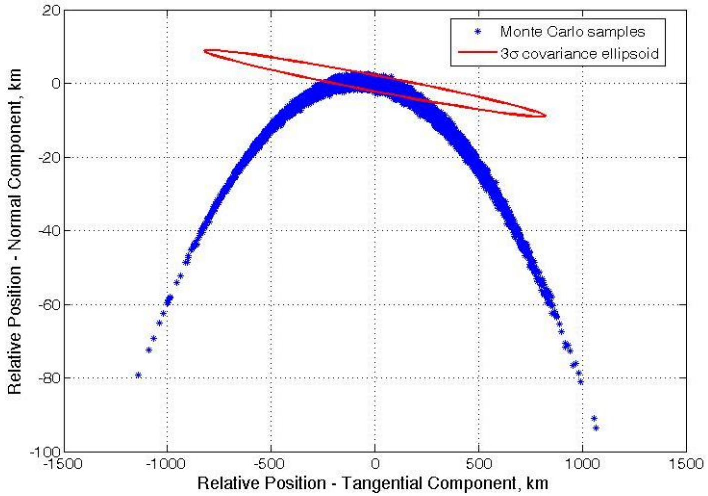
- **Two-dimensional (2D) probability of collision (Pc)**
  - Developed for Shuttle program in early 1990's
  - Presumes hyperkinetic encounter—rectilinear motion, position covariance only, and static position throughout encounter
  - Applicable to great majority of conjunctions
- **3D Pc formulated to operate when these restrictions relaxed**
  - Theory developed by V.T. Coppola of AGI; integrates time-series of instantaneous penetrations of HBR sphere by uncertainty volume
  - Allows curved rather than straight trajectories, uses full 6 x 6 covariance, and allows covariance to evolve over conjunction duration
  - Attractive methodology to expand domain of Pc analytical calculation
    - Persistent conjunctions and others that respond poorly to 2D Pc
  - Also introduces/frames concept of first derivative of Pc; useful for understanding conjunction dynamics and determining background risks
  - Operates only in reference frames in which position and velocity components can be separated (*e.g.,* Cartesian orthogonal frame)

3D Development/Validation at CARA

- **Developed full operational implementation/prototype**
- **Developed non-rectilinear TCA Monte Carlo for 3D Pc validation**
- **Executed and profiled 2D/3D comparison against ~80,000 conjunctions from latter part of 2016; results grouped as follows:**
  - Group 0: 2D and 3D Pc match to within operational tolerances (most events)
  - Group 1: Persistent Conjunctions—3D Pc substantially larger
  - Group 2: Modest improvements—3D Pc somewhat different
  - Group 3: "Distended Covariances"—3D Pc substantially larger or smaller
    - Group 3 most prevalent (~6% of significant events) and most surprising
- **All four groups validated by Monte Carlo (~40 events run)**
  - Matches for Groups 2 and 3 nearly exact; Group 1 confirmatory
- **Implemented in operations December 2016**
  - Large 2D/3D Pc differences (Group 3) believed to pose safety-of-flight risk

"Non-Gaussian" Covariance Behavior: Brief Introduction

- **Two issues with covariance for CA analytic calculation (2D or 3D)**
  - Actual error volume must be Gaussian
    - Covariance as formulated can describe only multivariate Gaussian error distributions
  - Covariance must be represented in coordinates in which position and velocity can be separated
- **Concern that, with long distended covariances, error volume not properly representable in Cartesian coordinates**
  - Actual in-track error follows curved orbit path
  - In-track component of Cartesian covariance remains tangent to orbit path
  - Disjunction possible between Cartesian error representation (used in Pc calculations) and actual error distribution



Non-Gaussian Covariance Behavior

- **In 2012, CARA performed study on the effect of potential non-Gaussian behavior on CA calculations***
  - Examined 248 high-Pc conjunctions with covariances of various levels of "in-track distention"
  - Only one of the 248 showed appreciable difference in Pc between 2D value and that with methods to correct for non-Gaussian behavior
    - And for that one case, covariance was not all that seriously distended
- **Conclusion was that non-Gaussian behavior, manifested by covariance distention, is not problematic for Pc calculation**
- **Based on 2012 study, did not suspect any non-Gaussian problem with 3D Pc**
- **However, at March 2017 Users Forum committed to investigating conjunctions with large 2D/3D Pc differences for any evidence of non-Gaussian behavior**
  - Enhanced Monte Carlo capability under development to check for this

*Ghrist, R. and Plakalovic, D. "Impact of Non-Gaussian Error Volumes on Conjunction Assessment Risk Analysis." AIAA/AAS 2012 Astrodynamics Specialist Conference, Minneapolis MN, August 2012.

3D Pc Problem Discovery

- **Enhanced Monte Carlo (MC) capabilities (two separate lineages) became available last week of April 2017**
  - Performs MC in element space (equinoctial elements); reference frame insulated from non-Gaussian behavior due to orbit curvilinearity
- **First application of new Monte Carlo was to check cases with large 2D/3D Pc differences**
  - Initial two cases disturbing—Monte Carlo Pc much closer to 2D value
- **Immediately launched high-priority study effort**
  - 1) Is the 3D Pc calculation miscarrying in cases of large 2D/3D Pc differences?
  - 2) What can we do operationally to respond while problem is studied enough to understand it (and if necessary propose remediations)?
  - 3) If 3D Pc calculation is in fact erring in some circumstances, why is this so?
  - 4) What can be done to repair/enhance the 3D Pc calculation?
- **Purpose of today's Users' Forum is to report on the four questions above**

Question 1:
Is the 3D Pc Calculation Miscarrying?

- **Profiled set of ~500,000 conjunctions (CDMs) over past year**
  – 0.4% have 2D Pc > 1E-07 and > 1 order of magnitude (OoM) difference between 2D and 3D Pc
- **33 high-Pc cases examined with 2D, 3D, and MC calculation**
  – Used both “old” MC and both lineages of enhanced (“new”) MC
  – In all 33 cases, new MC and 2D calculations very close
    - Old MC and 3D Pc also very close, but different from above, and presumably wrong
- **Conclusion is that non-Gaussian effect of some type seems to be corrupting 3D Pc (and Cartesian Monte Carlo) in certain cases**

Question 2: What Immediate Operational Response is Possible?

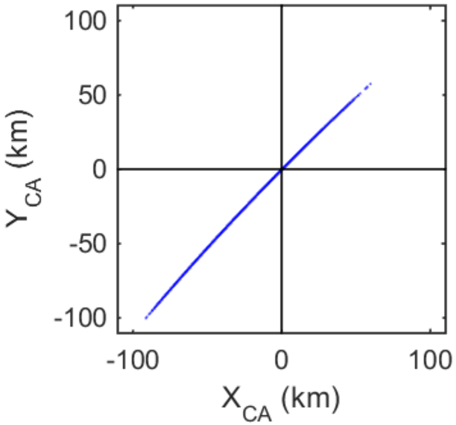
- **Once first few examples of 3D Pc miscarriage were verified by Monte Carlo, the following operational procedure was instantiated:**
  - Script written to identify all cases in which 2D and 3D Pc differed by at least an order of magnitude. This script is run daily by on-call analyst.
  - If either 2D or 3D value exceeded 1E-05, enhanced Monte Carlo is run to validate Pc and increased tasking is requested on the secondary object.
  - If “true” Pc as established by Monte Carlo exceeded worrisome level (~5E-05) and less than five days to TCA, mission is to be notified
  - No cases since procedure development have met these criteria
    - Usually natural event evolution and increased tasking shrinks covariance, and 2D and 3D Pc calculations come into alignment
- **Procedure is to be followed until fix can be put into CARA software**

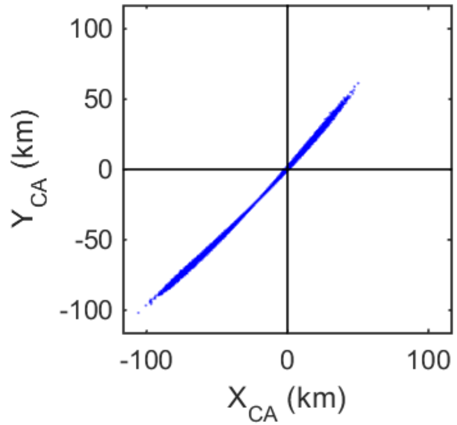
Question 3: Why does the 3D Pc Occasionally Err?

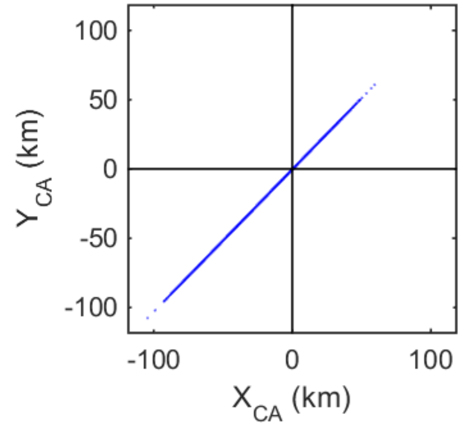
- **Initial supposition: long, thin covariances introduce non-Gaussian covariance behavior that somehow trips up 3D Pc**
  - Further analysis revealed that 2D/3D Pc mismatches are not all that strongly correlated with covariance distension
- **Current thinking: issues with the velocity portion of the covariance introduce problems in 3D Pc calculation**
  - Discovery: zeroing out the velocity portion of the covariance makes 3D Pc and old MC match 2D Pc and new MC
  - Cartesian rendering of covariance appears to overstate velocity uncertainties—causes additional dispersion that erroneously raises or lowers Pc, depending on circumstances
    - Significant finding that came as surprise to major researchers in CA discipline
    - Velocity portion of JSpOC covariances has never been studied in depth
  - Non-Gaussian behavior manifests itself through velocity uncertainties, not positional issues
    - Reason why behavior not correlated with covariance distention

Velocity Uncertainties Push 3D Pc too High

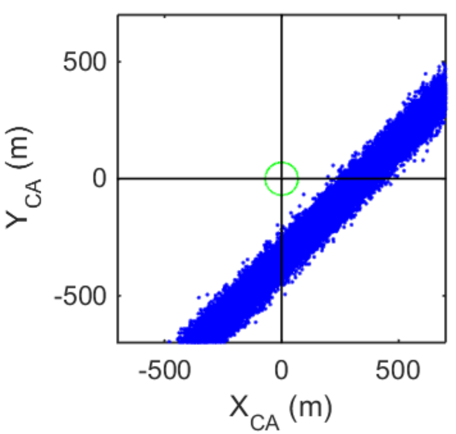
"New" MC Equinoctial Sampling	"Old" MC With Vel. Covariance	"Old" MC Zero Vel. Covariance

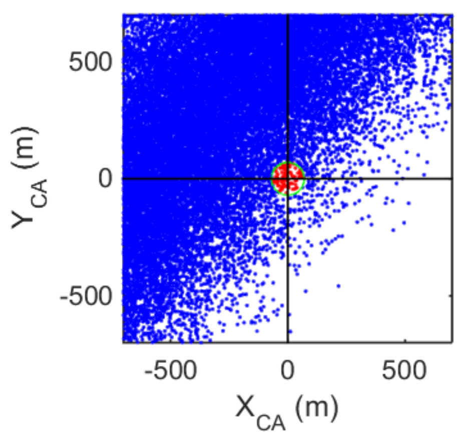


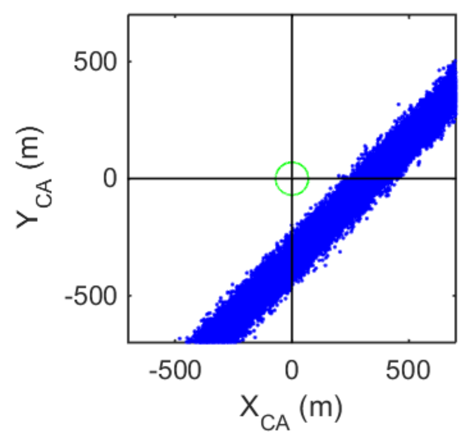




Neglecting TCA vel. covariances eliminates the erroneous curvature (mostly) and scatter



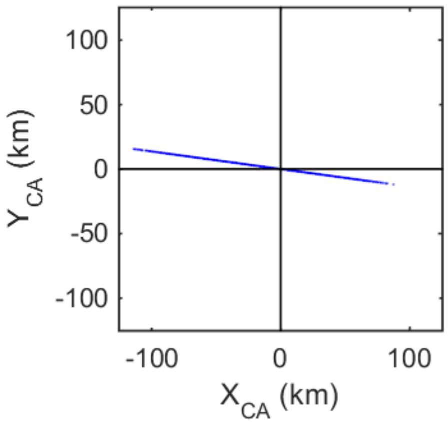


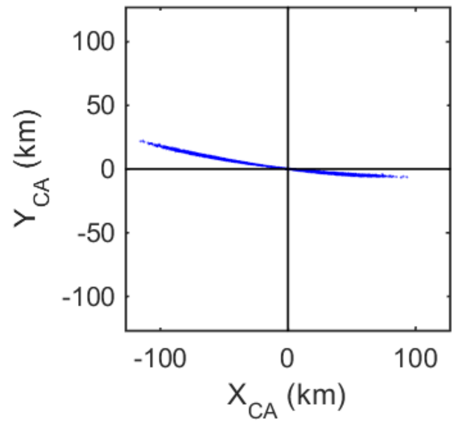


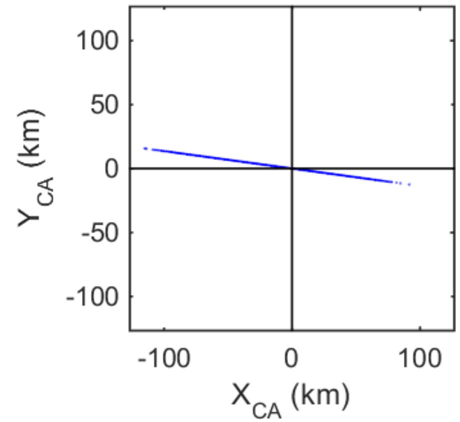
Zoom shows no close approaches occur inside the combined HBR when velocity covariances are set to zero

Velocity Uncertainties Push 3D Pc too Low

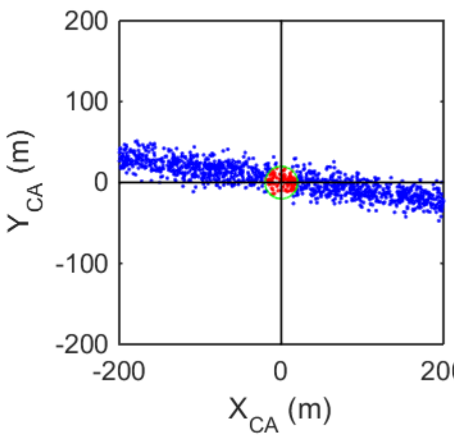
“New” MC Equinoctial Sampling	“Old” MC With Vel. Covariance	“Old” MC Zero Vel. Covariance

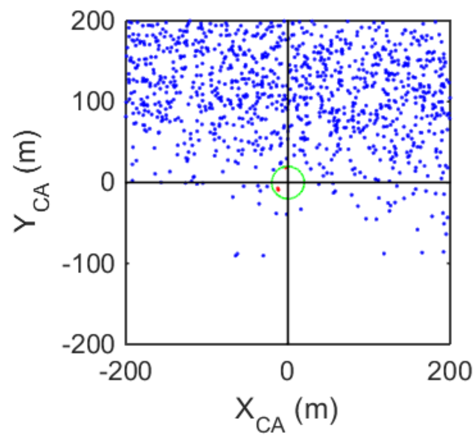


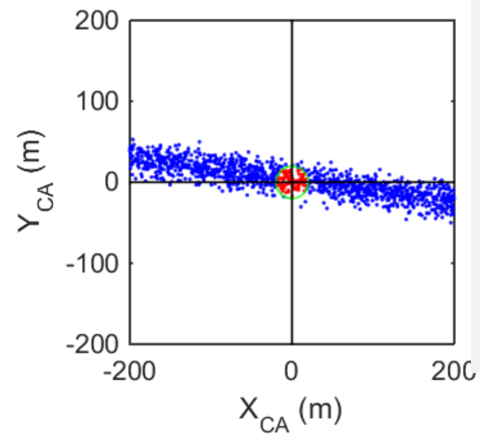




Neglecting TCA vel. covariances eliminates the erroneous curvature and scatter







Zoom shows that setting the vel. covariances to zero makes the Level II and Level III distributions match closely

Question 4: Repair to 3D Pc Algorithm (Near-Term Fix)

- **Fix to CAS Pc calculation/rendering involves two items:**
- **#1: Modify 3D Pc calculation to zero out velocity uncertainties**
  - Eliminates large differences between 2D and 3D Pc calculations
  - Smaller differences that remain have so far been shown to be correct
    - Most of these factor of 2 or smaller
- **#2: Calculate both 2D and 3D Pc for each CDM, and make the reportable Pc the larger of the two values**
  - #1 above probably sufficient; but given "discoveries" to date with velocity covariances, best to be conservative
  - Allows future enhancements to 3D Pc in a conservative framework
  - Because seeds risk assessment process with larger value, will ensure sufficient attention to perform supporting functions, such as running MC

Question 4: Fix Details

- **Summary section of report will contain "high watermark" Pc**
  - Basis for color assignment, tasking increases, and other risk assessment tasks
- **Details section of report will contain both 2D and 3D Pc values**
  - Placed together, in easy-to-access area
- **"Why not put both 2D and 3D Pc in summary section?"**
  - Wanted to get fix into CAS as quickly as possible; this is easiest change
  - As situation is studied further, will probably refine what should be shown in summary section
    - *e.g.,* may be best to show 2D, 3D, and a MC result, the latter of which would be automatically populated when 2D/3D difference exceeds given threshold
    - Don't want to jump the gun on changing summary section until understand precisely what decision information is necessary

Question 4:
Fix Status and Schedule

- **Development**
  - 3D Pc Code Correction: Complete
  - Data Integrity Correction: Complete
  - Report Corrections: Complete
  - MSA Updates: In progress
- **Testing**
  - Current completed code soaking on integration string
  - Testing Prep: Complete
  - Currently in Testing
- **Expected Delivery: week of 5 JUN 2017**

Question 4: Longer-Term Repair

- **Technical consultation held yesterday with external reviewers**
  - R. Carpenter, SSMO deputy and distinguished CA researcher
  - J. Frisbee, JSC CA senior SME
  - S. Casali, JSpOC OD and CA algorithm architect
- **Summary of findings/direction**
  - 3D Pc algorithm is technically sound and should be retained as CARA's principal analytic Pc calculation method
  - Repair used in near-term fix is acceptable for the present, but it is heavy-handed and should be replaced with a more nuanced approach
  - Transformation of 3D Pc reference frame to satellite-centered spherical coordinates may be an effective long-term solution to the non-Gaussian problem, at least for near-circular orbits
  - HEO satellites may require MC approaches as only reliable Pc calculation method

3D Pc Way Forward

- **Current operational procedure to be used until fix deployed**
- **After fix deployed, for events of significance with 2D/3D Pc discrepancy, MC will be run as a matter of course**
- **Analysis effort will continue on Pc calculation**
  - Development of a re-framed or otherwise enhanced 3D Pc approach to replace current fixed version
  - More definitive determination of when MC should be run as principal Pc calculation
  - Enhanced CAS software and reports to incorporate this expanded functionality and communicate results to users

BACKUP SLIDES

Background and History: Monte Carlo Pc Estimation

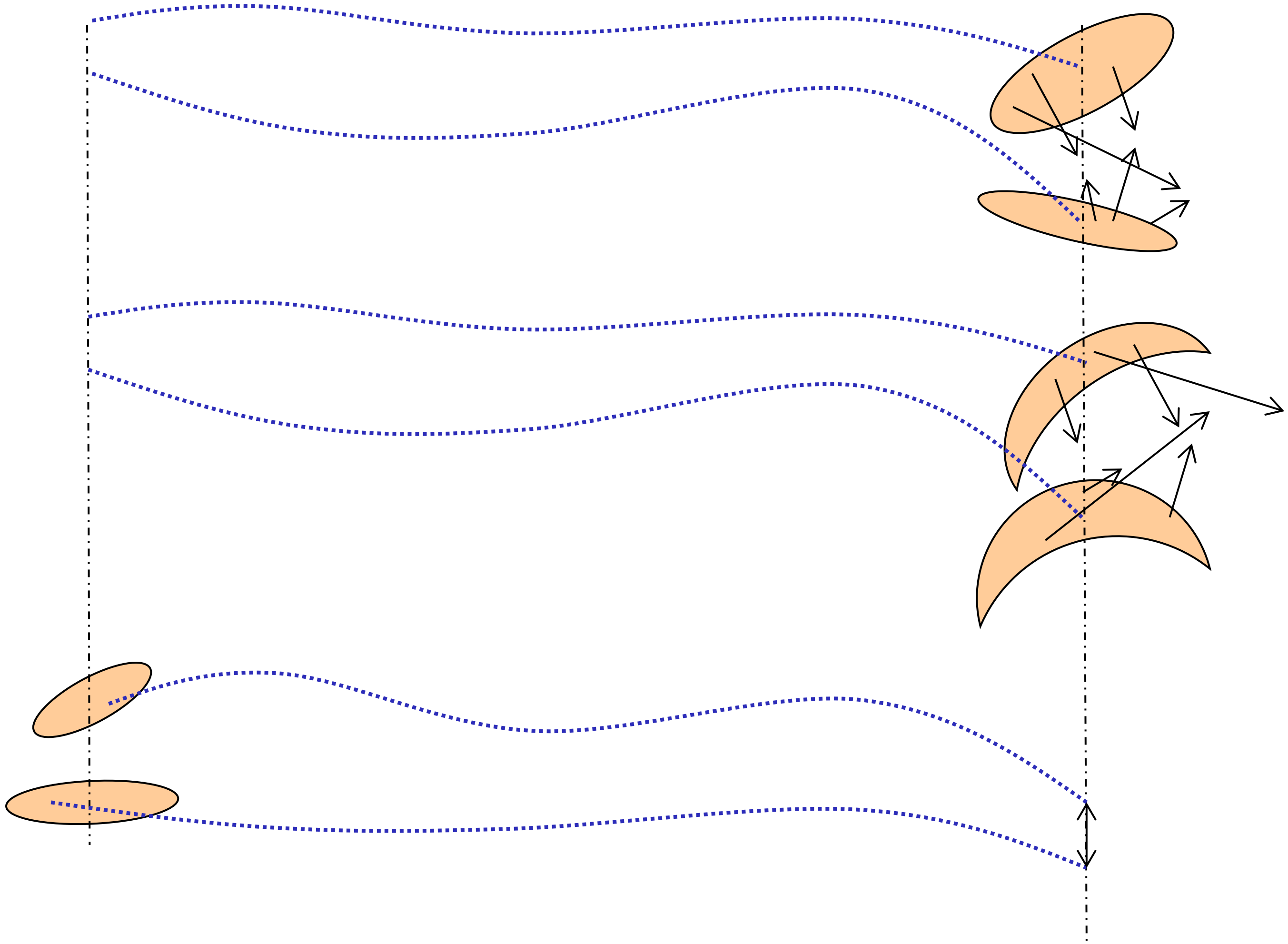
- **Level I: Legacy CAS Monte Carlo not useful to validate 3D Pc**
  - Uses rectilinear motion and position covariance; just reproduces 2D Pc
- **Level II: Upgraded MC removes these limitations ("Old" MC)**
  - 2-body propagation from TCA, using position/velocity states and covariances
  - Validated against Sal Alfano's published MC test cases
  - This version used for 3D Pc software testing and validation
- **Level III: operate in curvilinear rather than Cartesian space; better representation of the actual error volume at TCA ("New" MC)**
  - Level IIIa: Propagate covariance natively in equinoctial elements
  - Level IIIb: Use resampling technique to convert TCA covariances (Sabol 2010)
  - Both approaches pursued by CARA team in parallel
- **Level IV: Full MC from epoch ("brute force Monte Carlo")**
  - Propagates all MC trials non-linearly from epoch
  - "Gold standard" in that no simplifying assumptions used
- **Levels III and IV became available at the end of April 2017**

Different Monte Carlo Types: Cartoon Schematic

Level II: propagate covariances to TCA; generate MC samples in Cartesian space and find TCA between pairs

Level III: propagate covariances to TCA; generate MC samples in element space and find TCA between pairs

Level IV: Generate samples at epoch; propagate every pair of samples forward to its proper TCA



Monte Carlo Close-Approach Distributions

For each MC sample calculate :

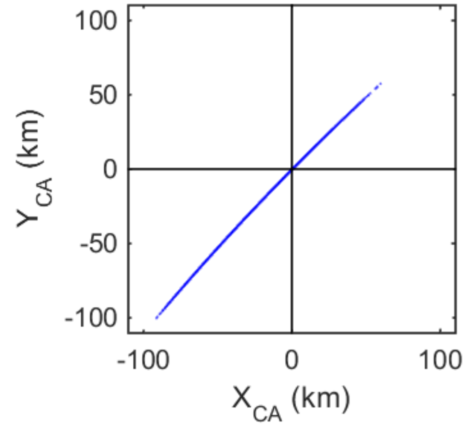
$$\hat{\mathbf{Z}}_{\mathrm{CA}} = \frac{\mathbf{v}_s(t_{ca}) - \mathbf{v}_p(t_{ca})}{\left|\mathbf{v}_s(t_{ca}) - \mathbf{v}_p(t_{ca})\right|}$$

$$\hat{\mathbf{X}}_{\mathrm{CA}} = \frac{\hat{\mathbf{Z}}_{\mathrm{CA}} \times \hat{\mathbf{z}}}{\left|\hat{\mathbf{Z}}_{\mathrm{CA}} \times \hat{\mathbf{z}}\right|} \qquad \hat{\mathbf{Y}}_{\mathrm{CA}} = \hat{\mathbf{Z}}_{\mathrm{CA}} \times \hat{\mathbf{X}}_{\mathrm{CA}}$$

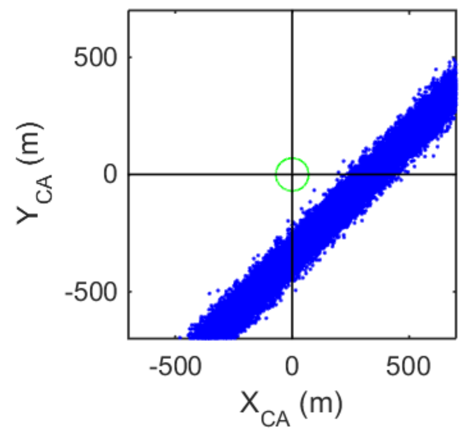
$$\mathrm{X}_{\mathrm{CA}} = \hat{\mathbf{X}}_{\mathrm{CA}} \cdot \left[\mathbf{r}_s(t_{ca}) - \mathbf{r}_p(t_{ca})\right]$$

$$\mathrm{Y}_{\mathrm{CA}} = \hat{\mathbf{Y}}_{\mathrm{CA}} \cdot \left[\mathbf{r}_s(t_{ca}) - \mathbf{r}_p(t_{ca})\right]$$

$$\mathrm{Z}_{\mathrm{CA}} = 0$$



Distribution of all close approaches for a conjunction



Zoom in to show close approaches as well as the combined HBR

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