APPLYING GRAPH THEORY TO PROBLEMS IN AIR TRAFFIC MANAGEMENT

Amir Farrahi (Universities Space Research Association)

Alan Goldberg (Kestrel Institute)

Leonard Bagasol (Universities Space Research Association)

Jaewoo Jung (NASA Ames Research Center)

OUTLINE

- Introduction and motivation
- Background
- Three ATM Problems
    - Airspace sectorization problem
    - Minimum delay scheduling in traffic flow management
    - Maximum dependent set of an aircraft in arrival scheduling
- Summary
- Conclusion

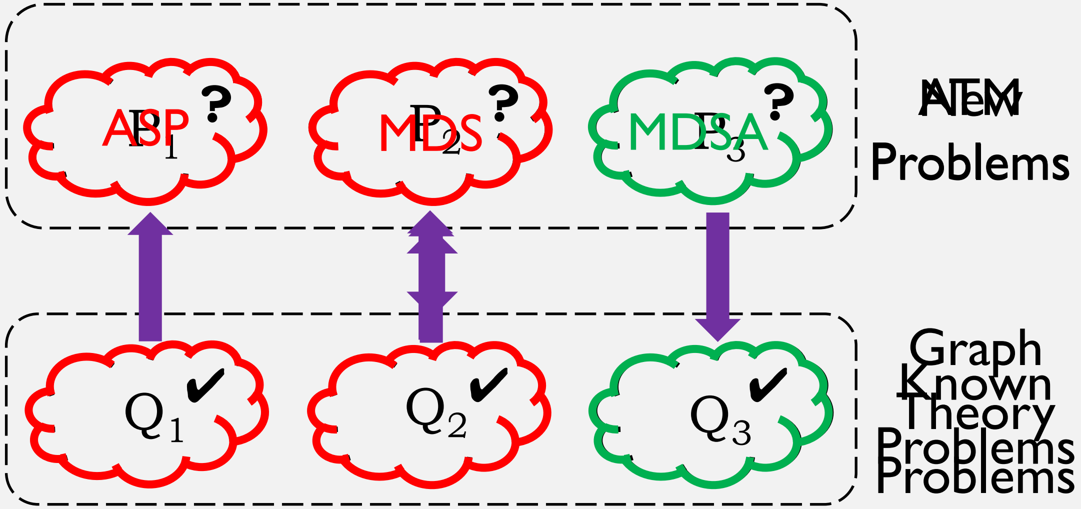
INTRODUCTION & MOTIVATION

- Use known problems to learn about new problem
- Use graph theoretic problems as a suitable substrate
- Bridging isolated islands of knowledge
- Gaining insights about inherent difficulty of new problems
- Solving new problems efficiently using what is known about related problems
- Reap the benefits of development in other technical domains

INTRODUCTION & MOTIVATION

- Learn about new problems by
  - Linking them to known problems
  - Polynomial “transformation” or “reduction”

- Three examples from graph theory to ATM
  - Airspace sectorization problem (ASP)
  - Minimum delay scheduling (MDS)
  - Maximum set of dependent aircraft (MSDA)



If we can transform Q_1 to P_1
And we know Q_1 is hard to solve
Then P_1 must be hard to solve

If we can transform P_3 to Q_3
And we know how to solve Q_3
To solve P_3 : Transform it to Q_3, then solve

BACKGROUND: COMPUTATIONAL COMPLEXITY

- Problems have different inherent difficulty
- **Example:** Sorting an array of n distinct integers
- One correct solution among $n!$ permutations
- Know an O(n log n) time algorithm
- Naive algorithm:
    - Search among all permutations
    - O (n!)

n	n!	Runtime
100	100!	10^{-6} sec.
102	101!	> 0.01 sec.
104	102!	> 1.8 min.
106	106!	> 14 days
108	108!	> 45 decades
110	**110!**	**> 5000 millennia**

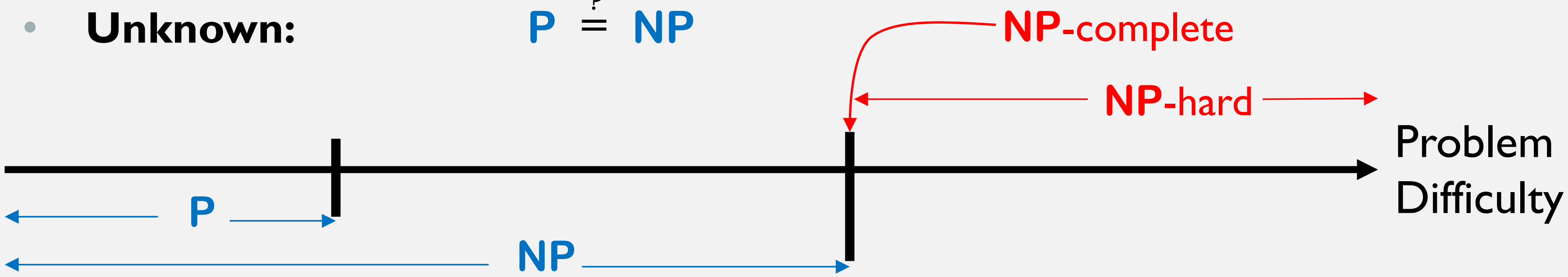
Ridiculously longer runtime

10% Increase in problem size

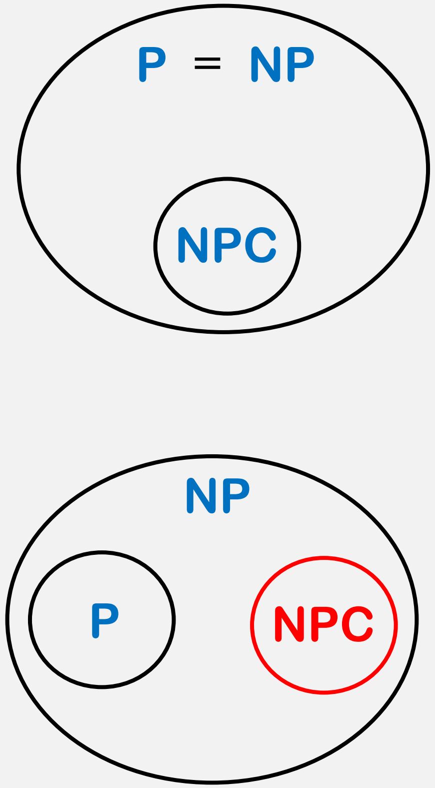
BACKGROUND: NP-COMPLETENESS

Computational complexity: Classify problems based on their inherent difficulty

- **P**: Set of problems that can be solved in polynomial time
- **NP**: Set of problems whose solution can be verified in polynomial time
- **NP-complete (NPC)**: The hardest problems in **NP**
- **NP-hard (NPH)**: Problems at least as hard as the hardest problems in **NP**
- Known: $P \subseteq NP$
- **Unknown:** $P \stackrel{?}{=} NP$



Two possibilities

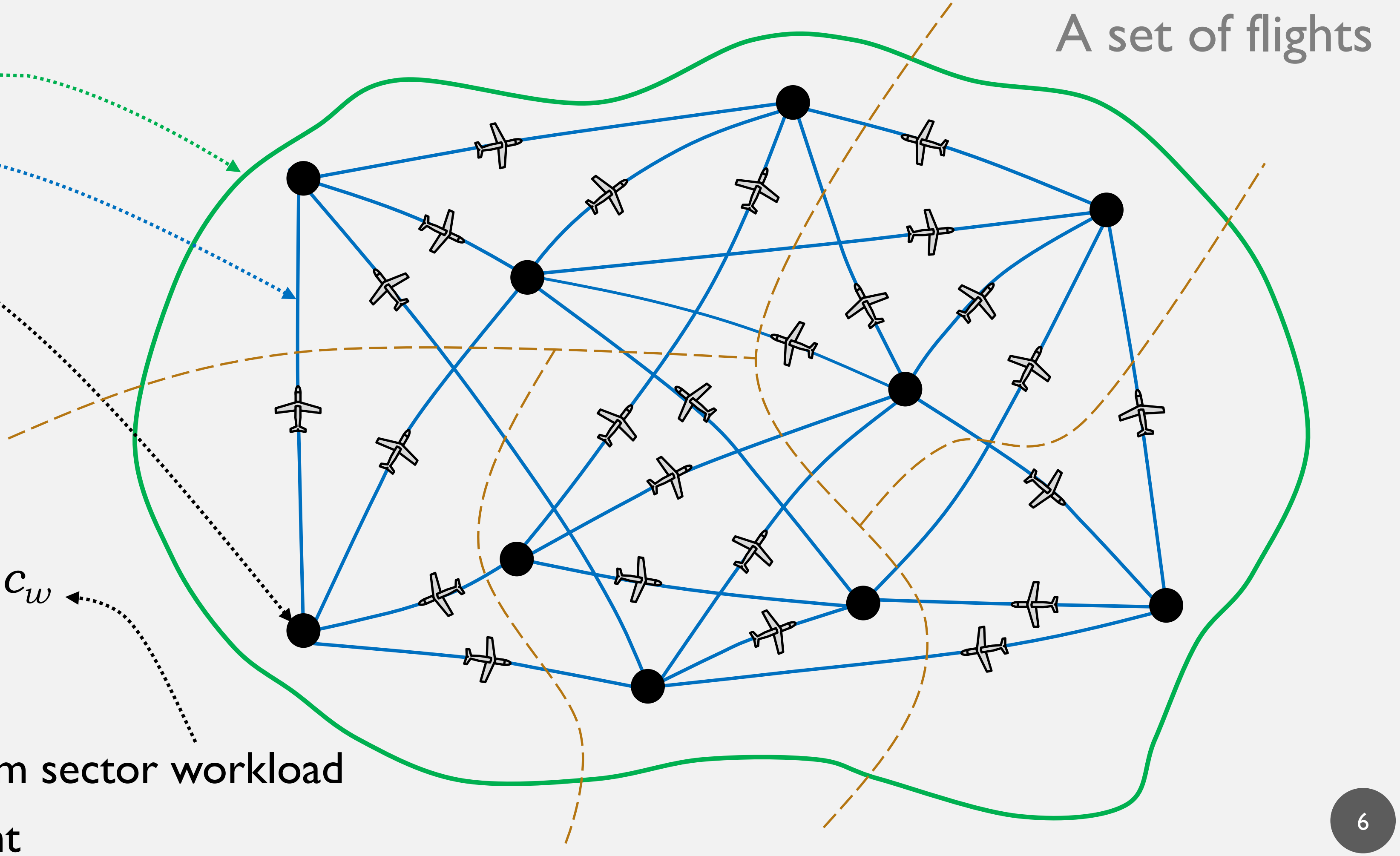


ATM PROBLEM 1: AIRSPACE SECTORIZATION PROBLEM (ASP)

Airspace

Flight paths

Airport



Partition the airspace into <u>sectors</u> such that:

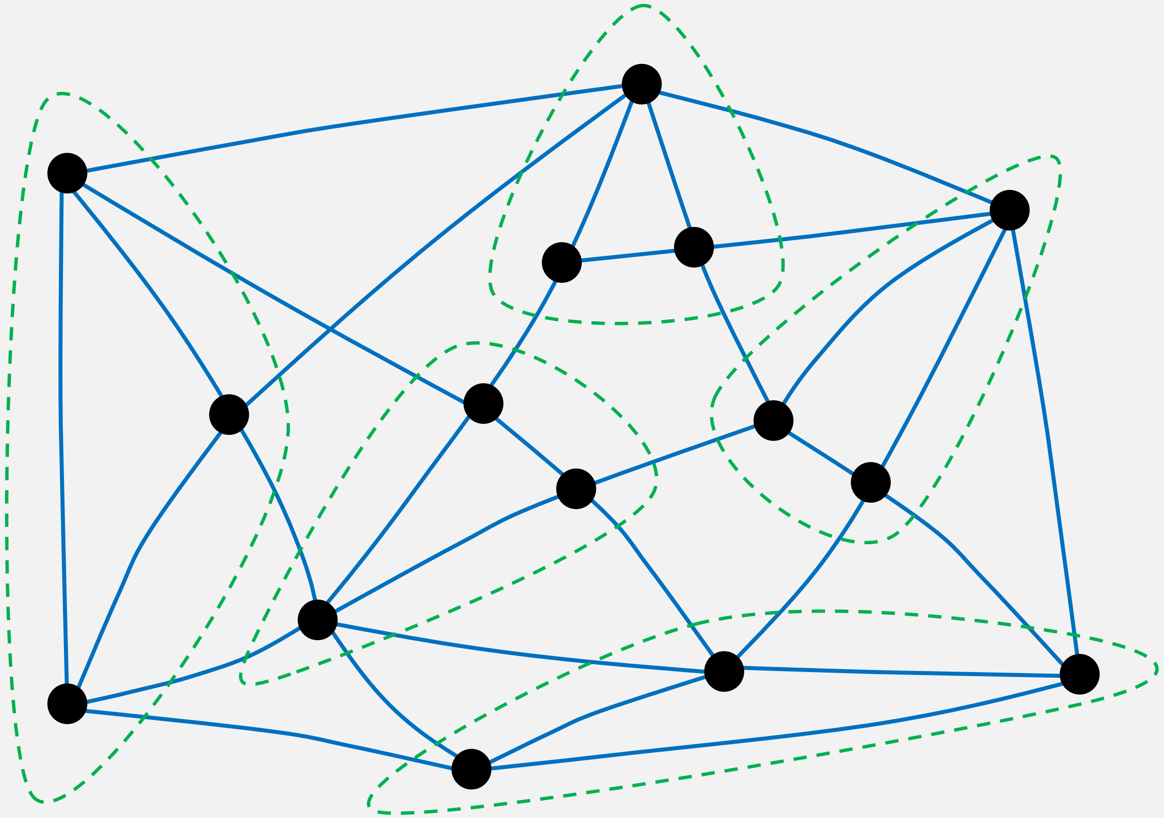
- <u>Workload</u> in each partition is ≤ c_w
- The number of sectors is ≤ c_s

Maximum sector workload

Maximum sector count

ATM PROBLEM 1: AIRSPACE SECTORIZATION PROBLEM (ASP)

- Known problem: PLANAR-P3(6):
  - Given a planar graph
  - Each node connected to no more than 6 other nodes
- Question: Can we partition the nodes into sets of 3, such that nodes in each set form a triangle)
- In this example the answer is YES
- Known to be **NP-complete**



ATM PROBLEM 1: AIRSPACE SECTORIZATION PROBLEM (ASP)

- It is known that ASP is **NP-complete** if sectors are required to be axis-aligned rectangles [Sabhnani, et al 2008]
- We transform PLANAR-P3(6) to ASP

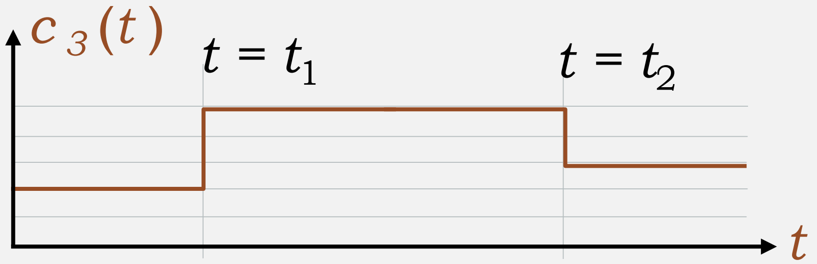
- **Theorem 1:** ASP is **NP-complete** under several workload models, in general, even if the flight paths form a planar graph, and no more than 6 flights originate or terminate at each airport.

ATM PROBLEM 2: MINIMUM DELAY SCHEDULING (MDS) IN TRAFFIC FLOW MANAGEMENT

Airspace

Partitioned into sectors $s_1, s_2, s_3, \ldots$

Each sector s_i has a capacity function $c_i(t)$



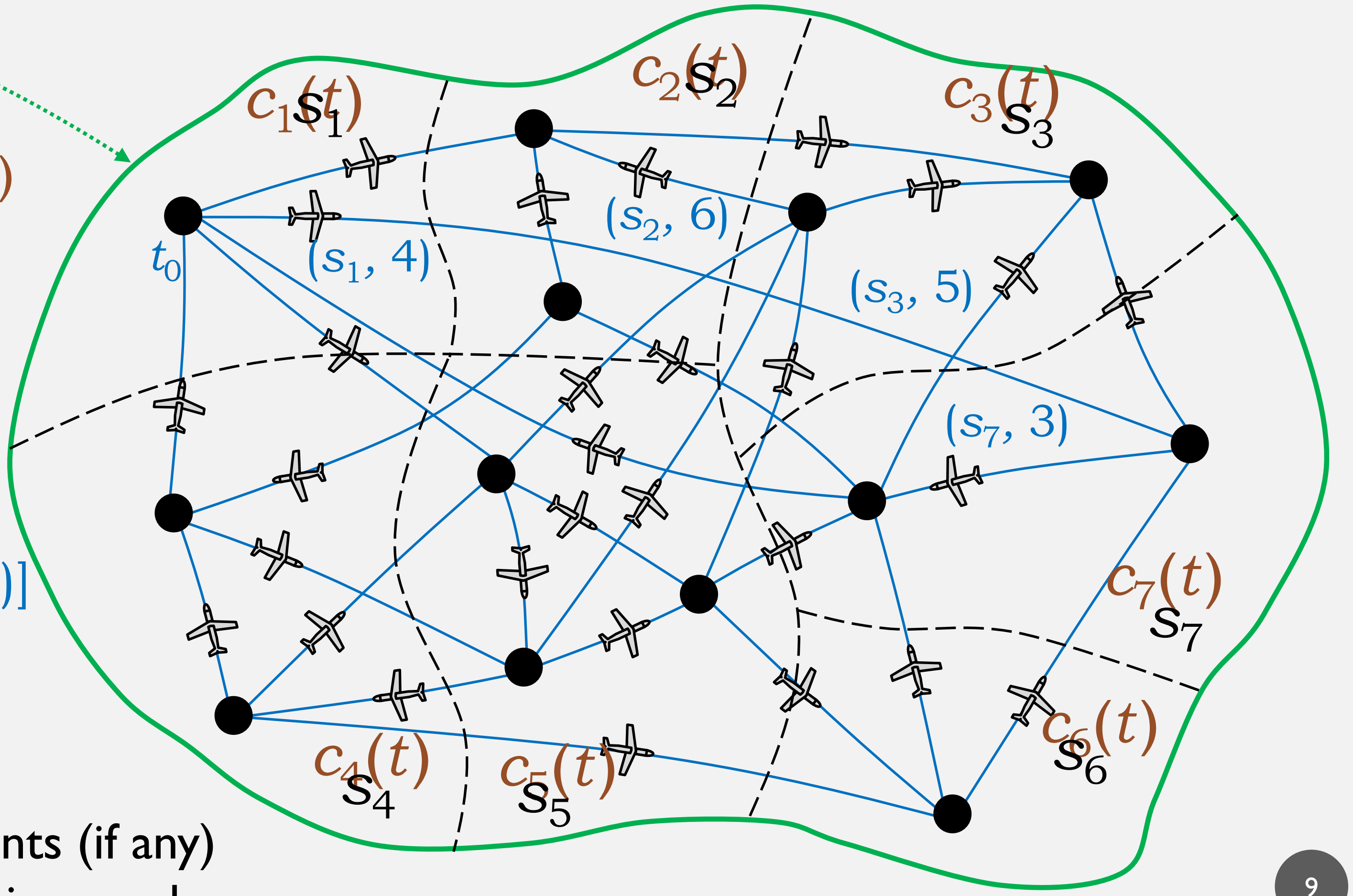
Each flight f has a schedule

$[\ t_0,\ (s_1, 4),\ (s_2, 6),\ (s_3, 5),\ (s_7, 3)]$

Assign delays to each flight on the ground or along its path to meet:

- Sector capacity constraints
- Airport arrival departure rate constraints (if any)

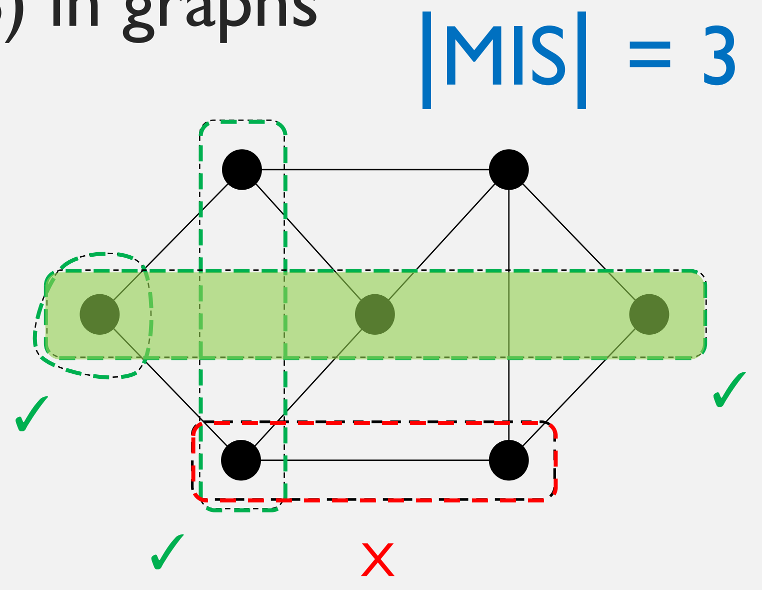
Objective: Minimize the sum of all delays imposed



ATM PROBLEM 2:
MINIMUM DELAY SCHEDULING (MDS) IN TRAFFIC FLOW MANAGEMENT

- Known problem: Maximum Independent Set (MIS) in graphs
  - Given a graph
  - Find its largest independent set

Subset of nodes, such that none of them is connected by an edge to any other node in the set



- Known to be **NP-hard**
- Known that unless **P = NP**, the problem cannot be approximated within polynomial time to within a factor $n^{1-\varepsilon}$ for any $\varepsilon > 0$

ATM PROBLEM 2: MINIMUM DELAY SCHEDULING (MDS) IN TRAFFIC FLOW MANAGEMENT

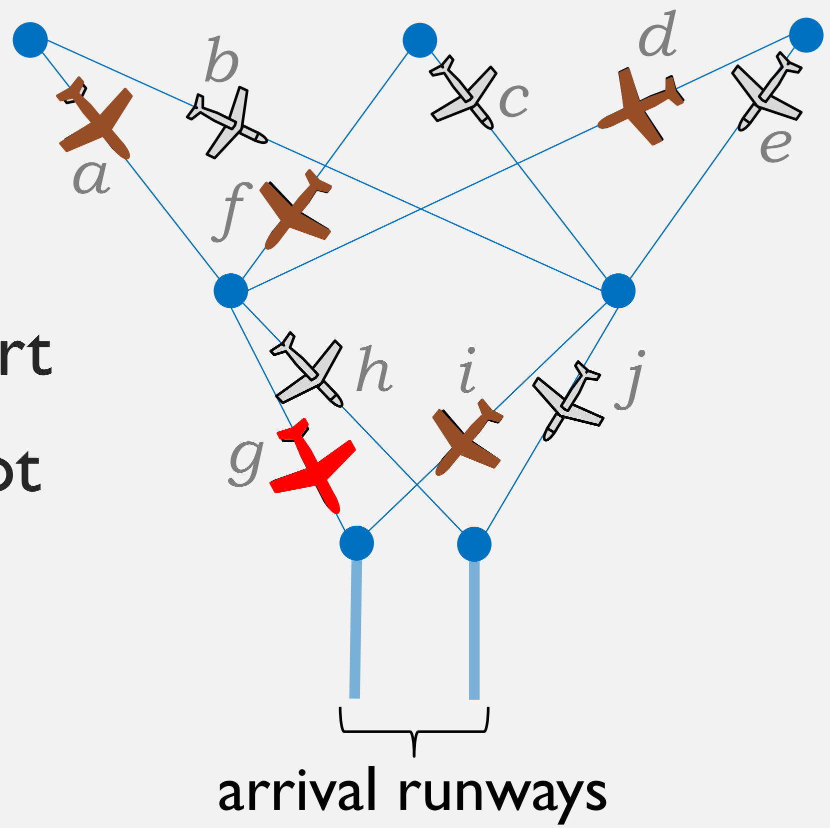
- It is known that the MDS problem is **NP-hard** [Bertsimas et al 1998]

- We transform graph MIS problem to a simplified version of MDS problem

- **Theorem 2:** Unless **P = NP**, the MDS problem cannot be approximated in polynomial time to within a factor $n^{1-\varepsilon}$ for any $\varepsilon > 0$, where n is the number of aircraft in the problem instance, even if all the delays are to be taken on the ground prior to takeoff.

ATM PROBLEM 3: MAXIMUM SET OF DEPENDENT AIRCRAFT (MSDA) IN PRECISION ARRIVAL SCHEDULING

- Consider a set of aircraft $a, b, c, \ldots$
- Flying along their arrival routes following their prescribed schedules
- To land on their designated runways at the airport
- Due to off-nominal conditions, an aircraft may not meet its scheduled time slot and needs to be rescheduled
- We need to identify only the set of **dependent aircraft** who need to be rescheduled along with it

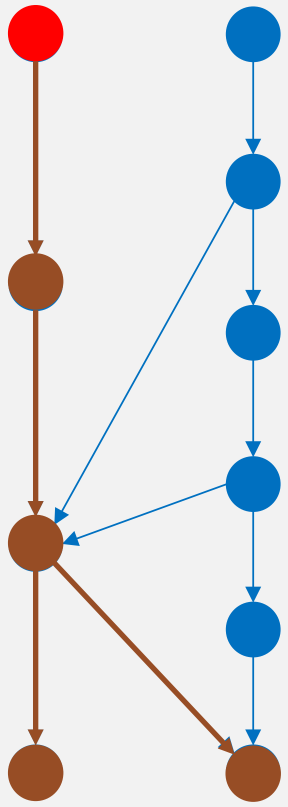


ATM PROBLEM 3: MAXIMUM SET OF DEPENDENT AIRCRAFT (MSDA) IN PRECISION ARRIVAL SCHEDULING

- Known graph algorithm: Graph reachability (e.g. Breadth-First Search, Depth-First Search)

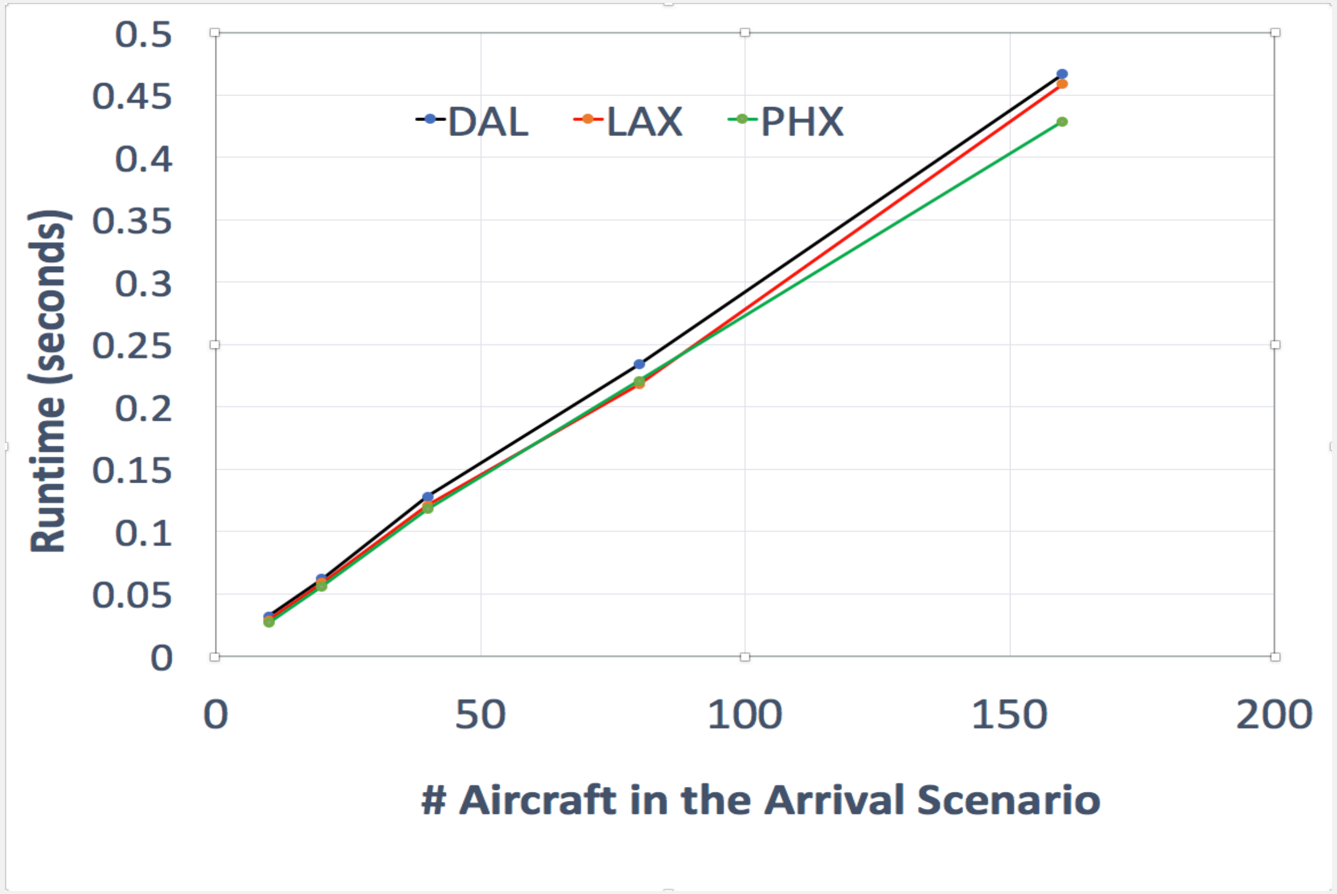
- Given a directed graph $G = (V, E)$ and a node $v \in V$ find the set of all nodes reachable from v

- Can be solved in $O(|V| + |E|)$



ATM PROBLEM 3: MAXIMUM SET OF DEPENDENT AIRCRAFT (MSDA) IN PRECISION ARRIVAL SCHEDULING

- Transformed MSDA into graph reachability and solved it
- Monte Carlo Simulation
- 3 Airports: DAL, LAX, PHX
- 100 random scenarios
- 10, 20, 40, 80, 160 aircraft
- randomly chosen target
- Run-time < 0.5 sec.

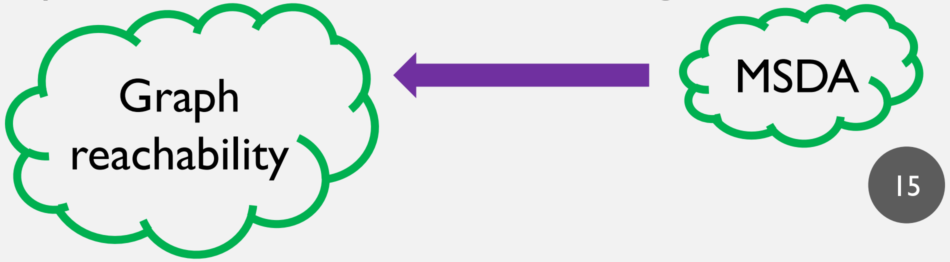


SUMMARY

- Studied three problems arising in ATM:
  - Airspace Sectorization Problem (ASP): Showed it is **NP-complete**

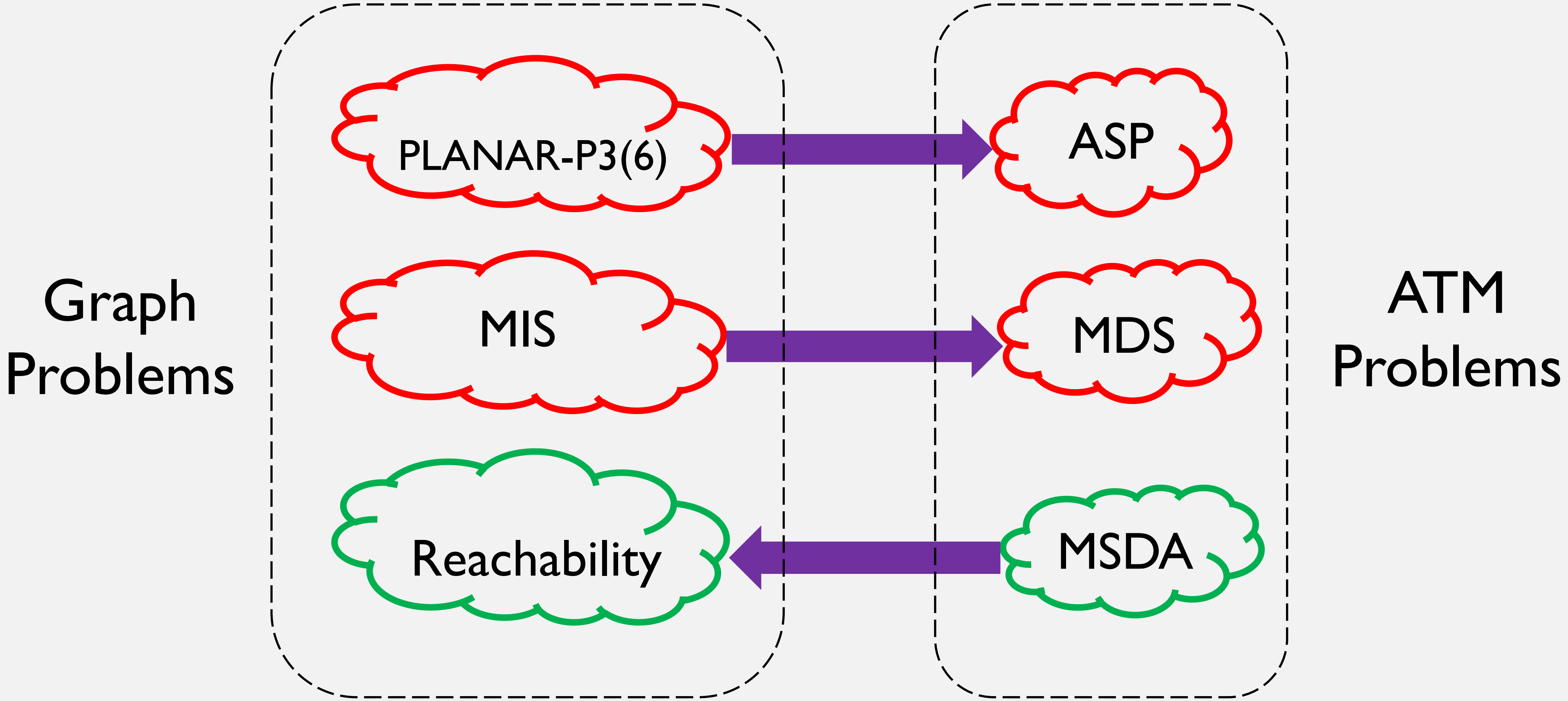
  - Min Delay Scheduling (MDS): Showed unless **P** = **NP** the problem cannot be approximated in polynomial time

  - Maximum Set of Dependent Aircraft (MSDA) in arrival scheduling: Solved using a very efficient algorithm



CONCLUSION

- Graph theory is a natural abstraction for many ATM problems
- Used known graph problems to learn about ATM problem
- Polynomial transformation can be used to
  - Gain insights about inherent difficulty of new problems
  - Solve new problems efficiently
- Linking problems allows:
  - Reap the benefits of earlier or future development
  - Fertilization across different technical disciplines



Questions?

amir.h.farrahi@nasa.gov

APPLYING GRAPH THEORY TO ATM