

Spacecraft Alignment Determination and Control for Dual Spacecraft Precision Formation Flying

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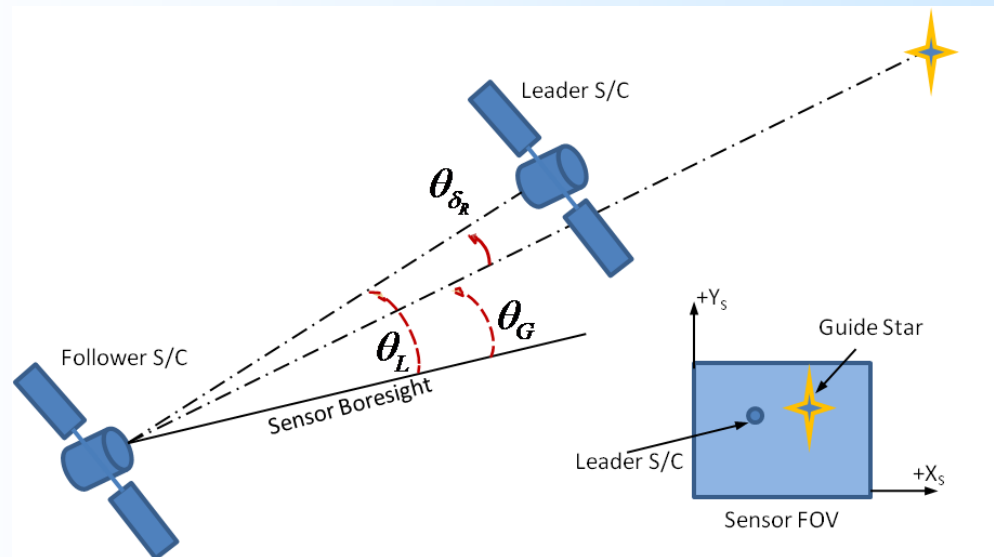


Overview

- **VT Concept**
 - Astrometric Alignment Concept for Virtual Telescopes (VT)
 - Proposed Missions (MASSIM, New Worlds Observer)
- **Stability Requirements and Measurement Models**
 - Attitude and Translation Stability Requirements
 - Optical Alignment and Ranging System Measurement Models
- **Dynamics and Controls Framework for GN&C Design**
 - Dynamics Model Formulation
 - Inertial Measurement Models (IRU, Accelerometers)
- **Case Study: GN&C Design for a Heliophysics Mission**
 - Navigation Modes for Fine Alignment Acquisition
 - GN&C Architecture Comparison
- **Conclusions**

VT Concept

- **Formation flying missions seek to advance science imaging by utilizing precision dual spacecraft formation flying. (“Virtual” Telescope (VT))**
 - Milli-Arc-Second Structure Imager (MASSIM) (Astrophysics X-ray imaging) (Sep ~ 1000 km)
 - New Worlds Observer (NWO) (exoplanet mission) (Sep $\sim 25,000$ km)
 - Heliophysics concept missions for Solar Coronagraphs and Solar imaging (Sep. 50m – 500m)



Dual Spacecraft Precision Astrometric Alignment Sensing Architecture

Objective: Develop models for a complete GN&C design framework of VT architectures

VT Attitude and Translation Stability Requirements

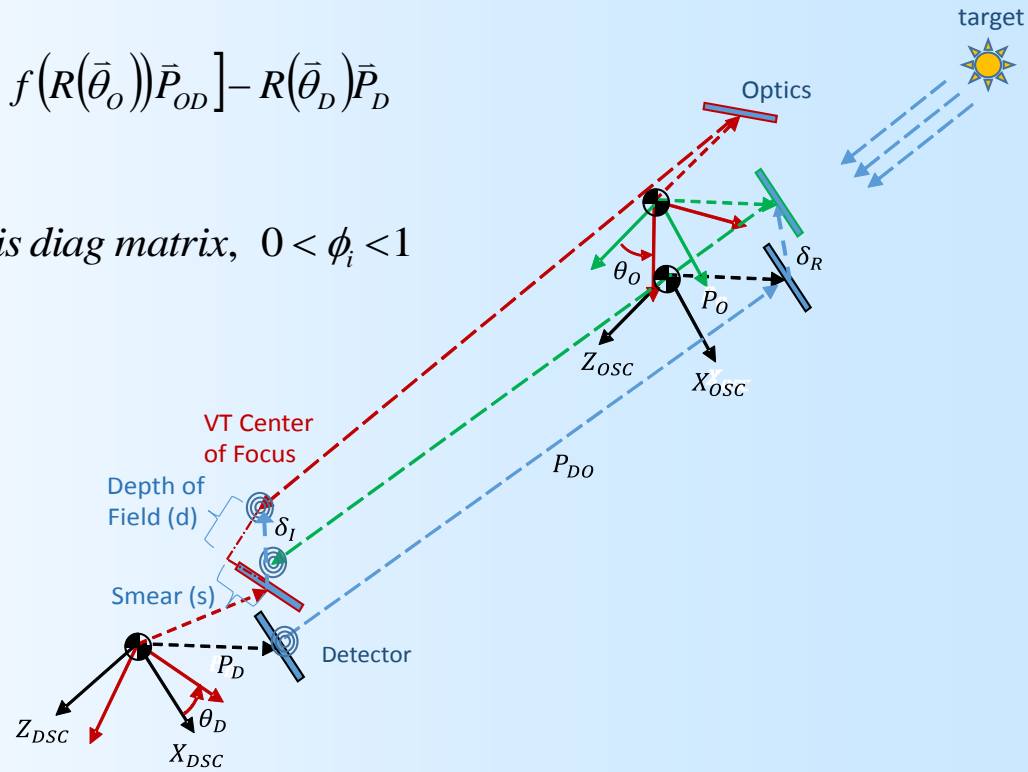
- Science detector image smear and depth of field stability model
 - Considers detector and optics not co-located with S/C mass center

$$\bar{\delta}_I = \begin{bmatrix} s_x \\ s_y \\ d \end{bmatrix} = [\bar{P}_D + \bar{P}_{DO} + \bar{\delta}_R - \bar{P}_O + R(\bar{\theta}_O)\bar{P}_O + f(R(\bar{\theta}_O))\bar{P}_{OD}] - R(\bar{\theta}_D)\bar{P}_D$$

$$f(R(\bar{\theta}_O)) = [I + {}^n\tilde{\theta}_O], \quad {}^n\bar{\theta}_O = \Phi\bar{\theta}_O, \quad \Phi \text{ is diag matrix, } 0 < \phi_i < 1$$

$$\bar{\delta}_I = \begin{bmatrix} s_x \\ s_y \\ d \end{bmatrix} = \tilde{P}_D^x \bar{\theta}_D + [\Phi \tilde{P}_{DO}^x - \tilde{P}_O^x] \bar{\theta}_O + \bar{\delta}_R$$

$$\bar{\delta}_L = \begin{bmatrix} l_x \\ l_y \\ r_l \end{bmatrix} = \tilde{P}_L^x \bar{\theta}_D + [\Phi \tilde{P}_{LB}^x - \tilde{P}_B^x] \bar{\theta}_O + \bar{\delta}_R$$



- Same model is used for laser centration and ranging measurements

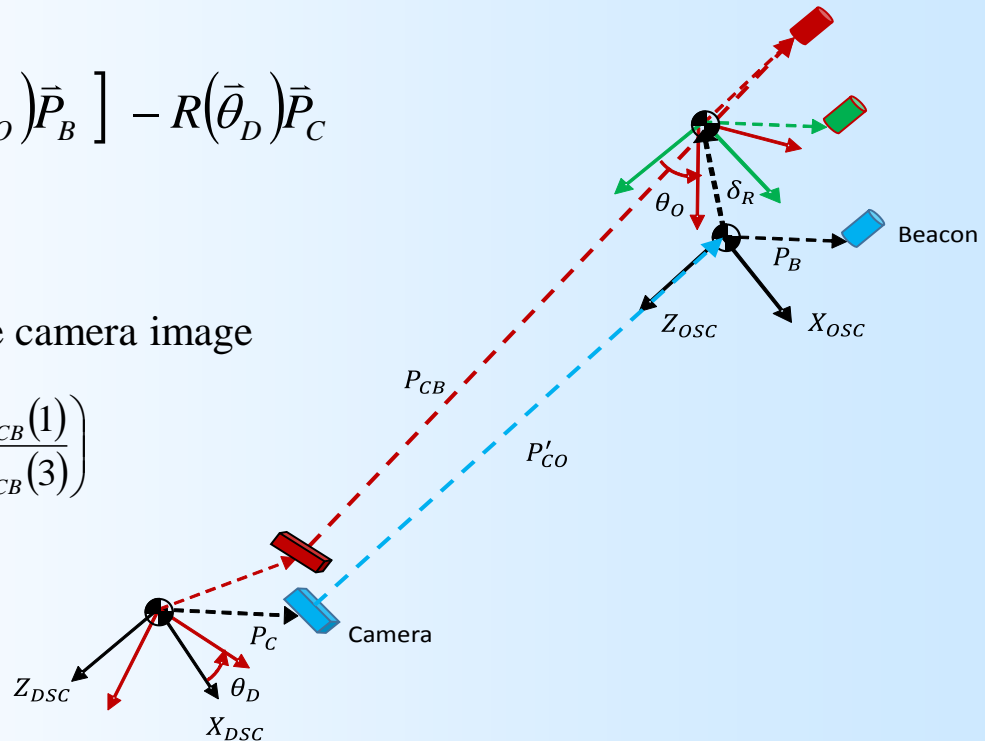
Alignment Camera Measurement Model

- **Measurement model for Alignment Camera (AC) to track laser beacon bearing angles.**
 - AC used for acquisition of Laser Centration and Ranging elements.

$$\bar{P}_{CB} = \left[\bar{P}_C + \bar{P}'_{CO} + \bar{\delta}_R + R(\bar{\theta}_O) \bar{P}_B \right] - R(\bar{\theta}_D) \bar{P}_C$$

Laser beacon centroids on the camera image

$$[\theta_x, \theta_y] = \left[\text{atan}\left(\frac{\bar{P}_{CB}(2)}{\bar{P}_{CB}(3)}\right), \text{atan}\left(\frac{\bar{P}_{CB}(1)}{\bar{P}_{CB}(3)}\right) \right]$$



VT Dynamics Framework for GN&C Design

- **Dual Spacecraft Relative Dynamics (Based on Luquette's work)**
 - Restricted Three-body Framework
 - Mods: Additional gravitational bodies, and express equations in terms of Follower differential acceleration, expressed in an inertial frame,

$$\ddot{\bar{\mathbf{x}}} = \ddot{\bar{\mathbf{r}}_F} - \ddot{\bar{\mathbf{r}}_L}$$

$$\ddot{\bar{\mathbf{r}}_F} = -\sum_{i=1}^n \mu_i \frac{\bar{\mathbf{r}}_{iF}}{\|\bar{\mathbf{r}}_{iF}\|^3} + \bar{\mathbf{f}}_{solar,F} + \bar{\mathbf{f}}_{pert,F} + \bar{\mathbf{u}}_{thrust,F}$$

$$\ddot{\bar{\mathbf{r}}_L} = -\sum_{i=1}^n \mu_i \frac{\bar{\mathbf{r}}_{iL}}{\|\bar{\mathbf{r}}_{iL}\|^3} + \bar{\mathbf{f}}_{solar,L} + \bar{\mathbf{f}}_{pert,L} + \bar{\mathbf{u}}_{thrust,L}$$

Can be simplified in terms of follower S/C, following derivation by Luquette.

- Assume $\|\bar{\mathbf{x}}\| \ll \|\bar{\mathbf{r}}_{iF}\|$, $\bar{\mathbf{x}} = \bar{\mathbf{R}}^{ref} + \bar{\boldsymbol{\delta}}_R$ and remove higher order terms.

$$\ddot{\bar{\boldsymbol{\delta}}_R} = \Gamma_{GG} \bar{\boldsymbol{\delta}}_R + \Gamma_{GG} \bar{\mathbf{R}}^{ref} + \bar{\mathbf{u}}_R$$

$$\Gamma_{GG} = -\sum_{i=1}^n \frac{\mu_i}{\|\bar{\mathbf{r}}_{iF}^{ref}\|^3} \left([I] - 3\hat{\mathbf{r}}_{iF}^{ref} [\hat{\mathbf{r}}_{iF}^{ref}]^T \right)$$

VT Dynamics Framework for GN&C Design

- Inertial Measurement Sensor (Accelerometers)**

The acceleration, $\ddot{\delta}_F^m$, at a specific sensor location, \bar{r}_A , can be represented as,

$$\ddot{\delta}_F^m = \ddot{\delta}_F + \bar{\omega}_F \times (\bar{\omega}_F \times \bar{r}_A) + \dot{\bar{\omega}}_F \times \bar{r}_A + \bar{b}_A + \bar{v}_A$$

Acceleration can be expressed in terms of forces / torques on S/C

$$\ddot{\delta}_F = \bar{u}_{F_{T_0}} + \bar{\delta} \bar{u}_{F_T} + \bar{u}_{F_E} \quad \dot{\bar{\omega}}_F = I_F^{-1} \left(\bar{T}_{F_{T_0}} + \bar{\delta} \bar{T}_{F_T} + \bar{T}_{F_E} \right)$$

And reduced to following linear form,

$$\begin{aligned} \ddot{\delta}_F^m = & \left([I] - r_A^x I_F^{-1} r_T^x m_F \right) \bar{u}_{F_{T_0}} + \left([I] - r_A^x I_F^{-1} r_T^x m_F \right) \bar{\delta} \bar{u}_{F_T} \\ & + \left([I] - r_A^x I_F^{-1} r_E^x m_F \right) \bar{u}_{F_E} + \bar{b}_A + \bar{v}_A \end{aligned}$$

- Inertial Reference Unit (Accelerometers)**

$$\dot{\bar{\theta}} = \bar{\omega}_F^m - \bar{b}_{\omega_F} + \bar{v}_{\omega_F}$$

Case Study: GN&C Design for Heliophysics Mission

Closed Loop GN&C Simulation Case Study – Photon Sieve

- GN&C design framework applied to an example problem to illustrate trades inherent in PFF for VT.
- Photon Sieve Optics (diffractive optics, ~ 0.5 m aperture). Solar Imaging at milli-arc-sec level

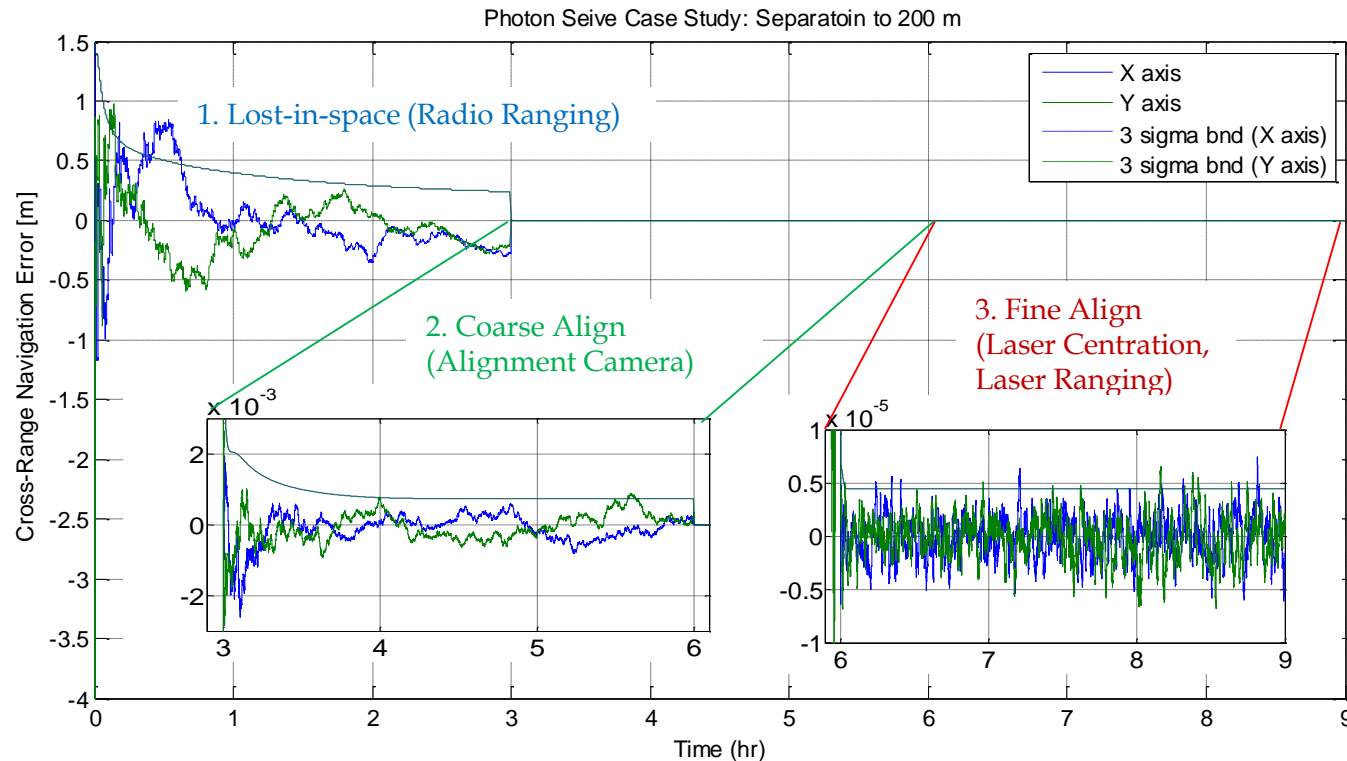
Table 1, Photon Sieve VT Alignment Requirements and Component Specifications

Parameter	Requirement (3s)		Component	Specification (3s)
Image Smear	6 microns		Laser Centration	30 microns
Depth of Field	1 mm		Laser Ranging	0.5 cm
S/C separation	200 m		Microthruster	5 mN-sec (min Impulse)
Pointing Stability (Optics S/C)	5 milli-arc sec (Sun) 10 arc-sec (roll)		Fine Sun Sensor	30 milli arc-sec
Pointing Stability (Detector S/C)	10 arc-sec		Star Tracker	6 arc-sec (transverse) 30 arc-sec (boresight)

- **State estimation:** Extended Kalman Filter, continuous state propagation, discrete measurements
 - Sequential Measurement updates to avoid numerical issues of large matrix inverses
- Separate PID Control for each of 9 DoF, (3) Relative translation, (3) Optics S/C Att, (3) Optics S/C Att
- All Measurement and Actuator models include random + systematic (1st order Markov) errors

Case Study: GN&C Design for Heliophysics Mission

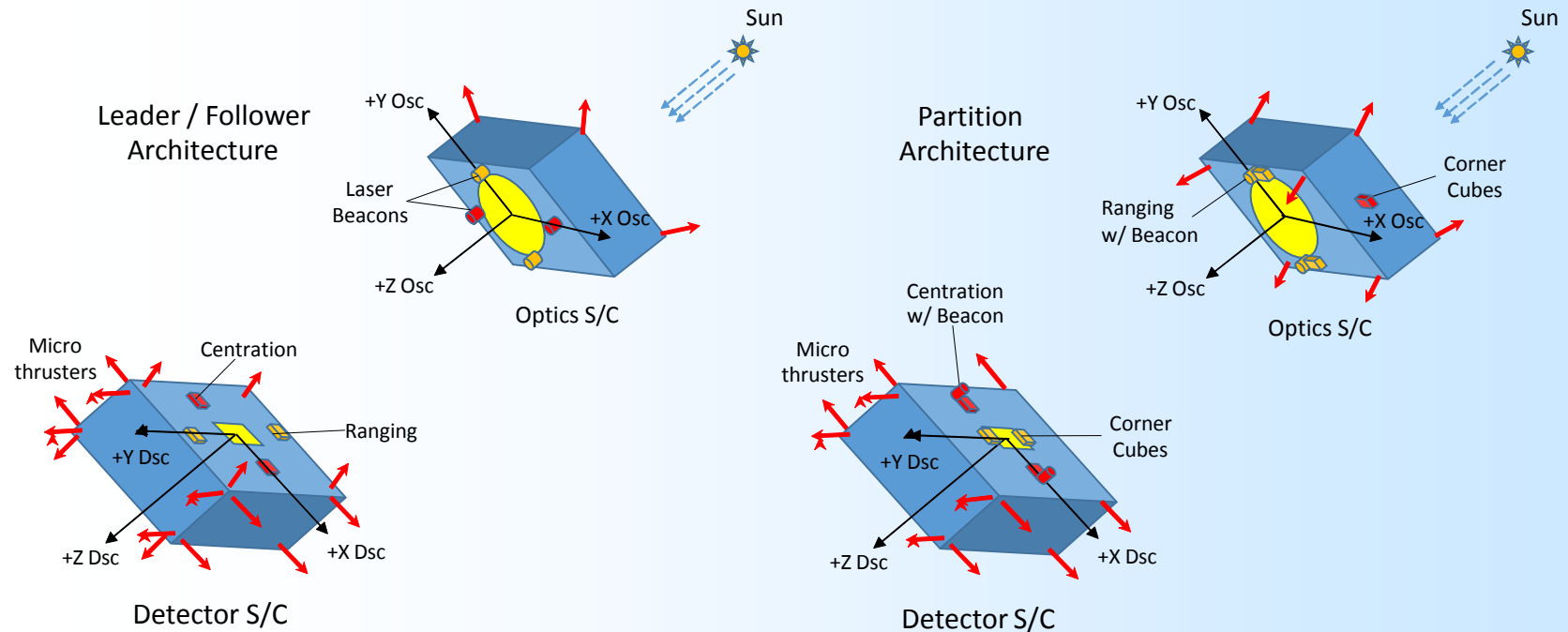
- Simulation of Navigation modes (Leader/Follower) illustrates fine align acquisition



1. Lost-in-space: Radio Range (60 cm) , Radio bearing (9 deg)
2. Coarse Align: Alignment Camera (50 arc-sec), Star Tracker (ST) (6, 6, 30 arc-sec)
3. Fine Align: Laser Centration (30 μ m), Laser Ranging: (1 cm), ST(6, 6, 30 arc-sec), Sun Sen (10e-3 arc-sec)

Case Study: GN&C Design for Heliophysics Mission

- Evaluation of Leader/Follower and Partition Architecture illustrates GN&C trades



Leader/Follower architecture has two possible deficiencies

- Comm link required to send Optics S/C Attitude to EKF on Detector S/C. Comm delay/timing sync
- Requires Full 6 DoF control on Detector S/C. Thruster coupling may result in poor performance

Partition architecture: Control / Estimation is partitioned among the two S/C

- Avoids multi-platform attitude coupling in the measurement process

Case Study: GN&C Design for Heliophysics Mission

- Evaluation of Leader/Follower and Partition Architecture illustrates GN&C trades

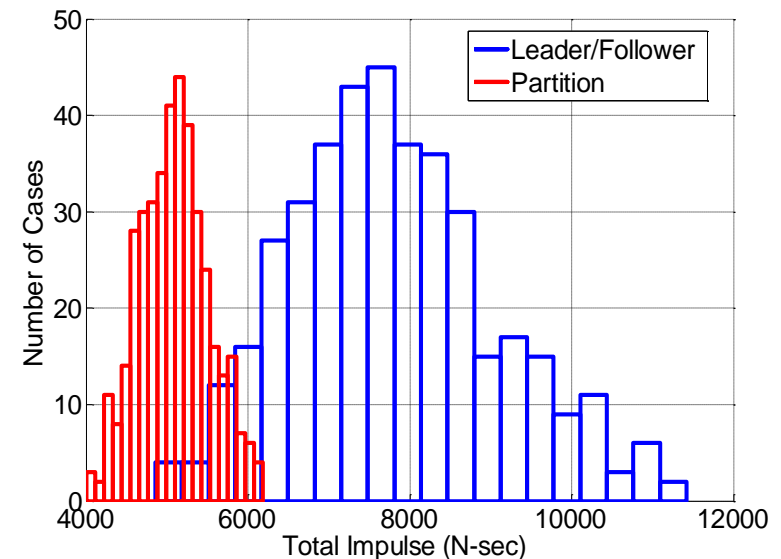
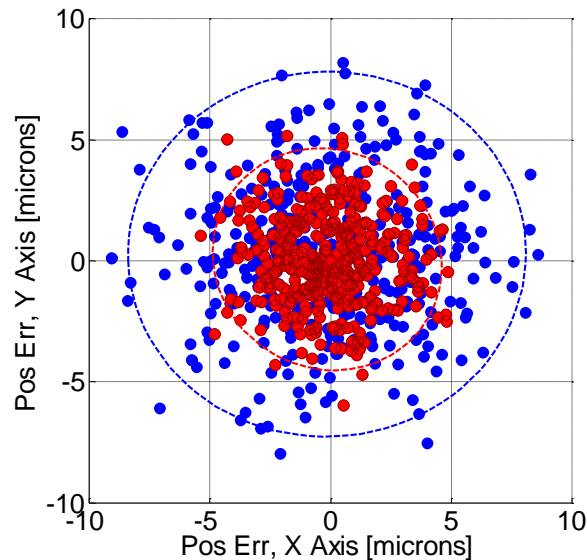


Figure 5 – Performance results of two representative GN&C architectures for the VT

- **Decoupling of laser centration/ranging measurements from the S/C attitude, (sensor positioning)**
 - Improved transverse alignment observability / performance in the Partition architecture.
- **Partition architecture performance meets Photon Sieve alignment requirements**
 - ~ 5x error reduction obtained from model-based estimation over laser centration measurements.
 - Total impulse for PFF (5 year) is reduced 35%. Solar pres along VT axis (Optics S/C is 1/2 mass of Detector S/C)

Conclusions

- **Developed General 9 DoF GN&C framework dual S/C PFF for application to VT missions**
- **GN&C performance assessment for a representative Heliophysics VT imaging mission concept illustrates the potential trade-offs inherent in the choice of system architecture for GN&C design and mission concept.**

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