



## Two-Pendulum Model of Propellant Slosh in Europa Clipper PMD Tank

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NASA GSFC 597



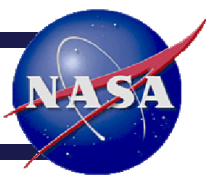
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NASA Marshall Space Flight Center  
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# Outline



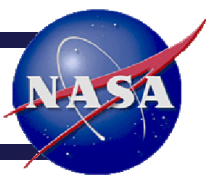
- Objective
- Background
- Results and literature verification
  - Mass
  - Frequency
  - Damping ratio
  - Hinge location
- Conclusions



# Objective



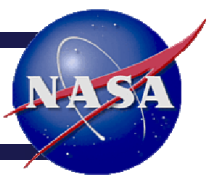
Model propellant slosh for Europa Clipper using two pendulums such that controls engineers can predict slosh behavior during the mission.



# BACKGROUND



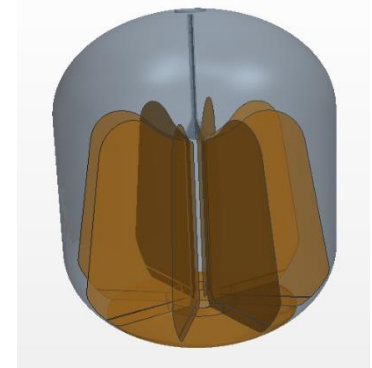
# Motivation



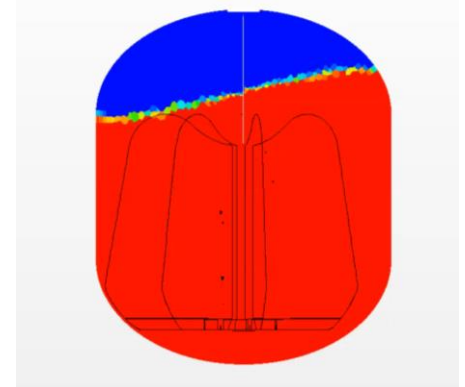
- Importance of predicting propellant slosh
  - Sloshing changes CM (center of mass) of spacecraft and exerts forces and torques on spacecraft
  - Avoid natural frequencies of structures
  - Size ACS (Attitude Control Systems) thrusters to counteract forces and torques
- Can model sloshing fluid as two pendulums with specific parameters (mass, length, damping)

- Europa Clipper tanks
  - Bipropellant system
  - Cylindrical with domed top and bottom
  - 8-vane PMD (propellant management device)
- CFD (computational fluid dynamics) data used as “real” slosh behavior
  - Have data for two propellants at three fill fractions each
  - Initial condition of 15 degree free surface offset, released and allowed to settle
  - CFD requires long computing time -> Need a computationally simple model

**Notional tank and PMD**

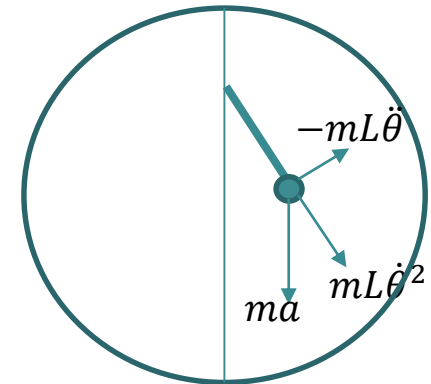


**CFD Simulation**  
*Solution Time 2 (s)*



- Pendulum model
  - Model fluid movement as two pendulums attached to central axis of the tank
  - For each CFD data set, find parameters: mass, frequency, damping ratio, attachment height

Forces exerted on tank by fluid

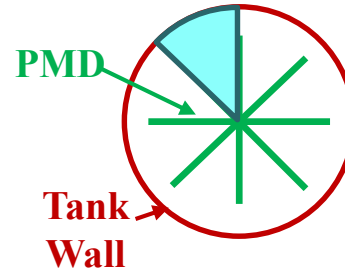


$$CM(t) = mL \sin \theta(t)$$

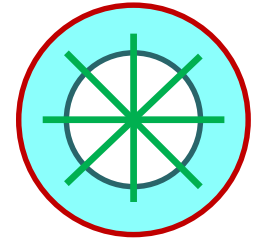
$$= mL \sin \theta_0 e^{-\xi \omega t} \left( \frac{\xi \omega}{\omega \sqrt{1 - \xi^2}} \sin \left( \omega \sqrt{1 - \xi^2} t \right) + \cos \left( \omega \sqrt{1 - \xi^2} t \right) \right)$$

- SP-106 (1966), SwRI (2000): **Analytical equations and empirical correlations** for damping and frequency
  - Includes bare cylindrical (no PMD), sector, and annular tanks
- Cassini slosh paper (1994): **Two pendulum model**
  - Slosh around PMD was modeled as combination of sector and annular slosh modes
  - Two separate pendulums to model two slosh modes
  - Static mass component at bottom that experiences little movement

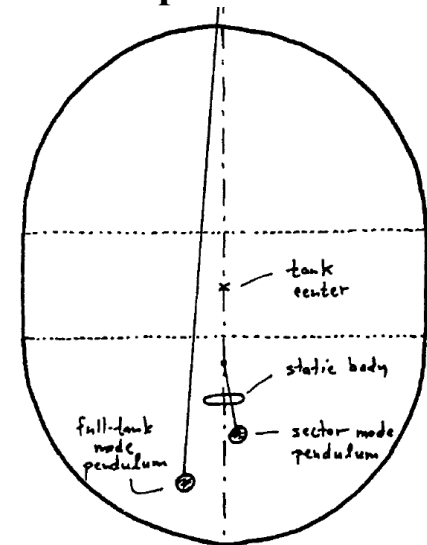
Sector tank mode  
(top view)



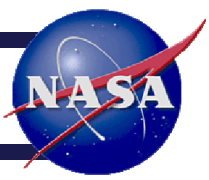
Annular tank mode  
(top view)



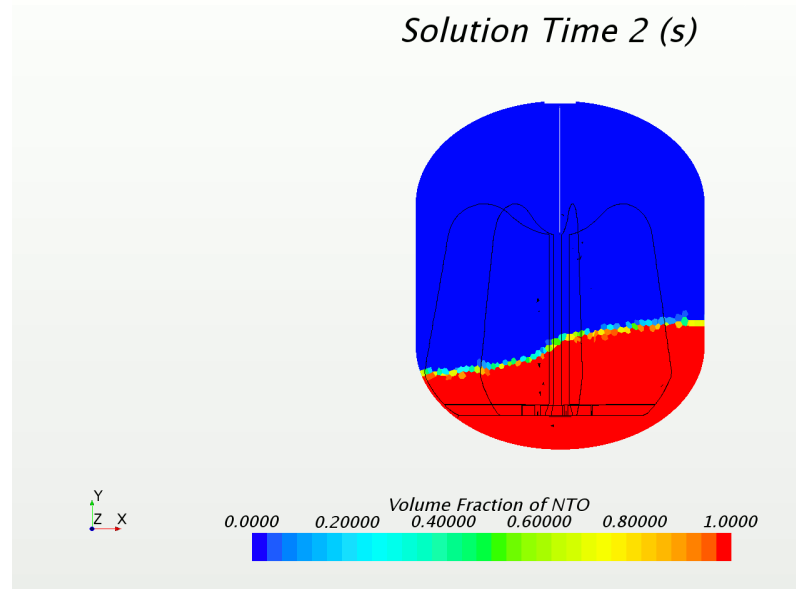
Cassini paper illustration of  
double pendulum model



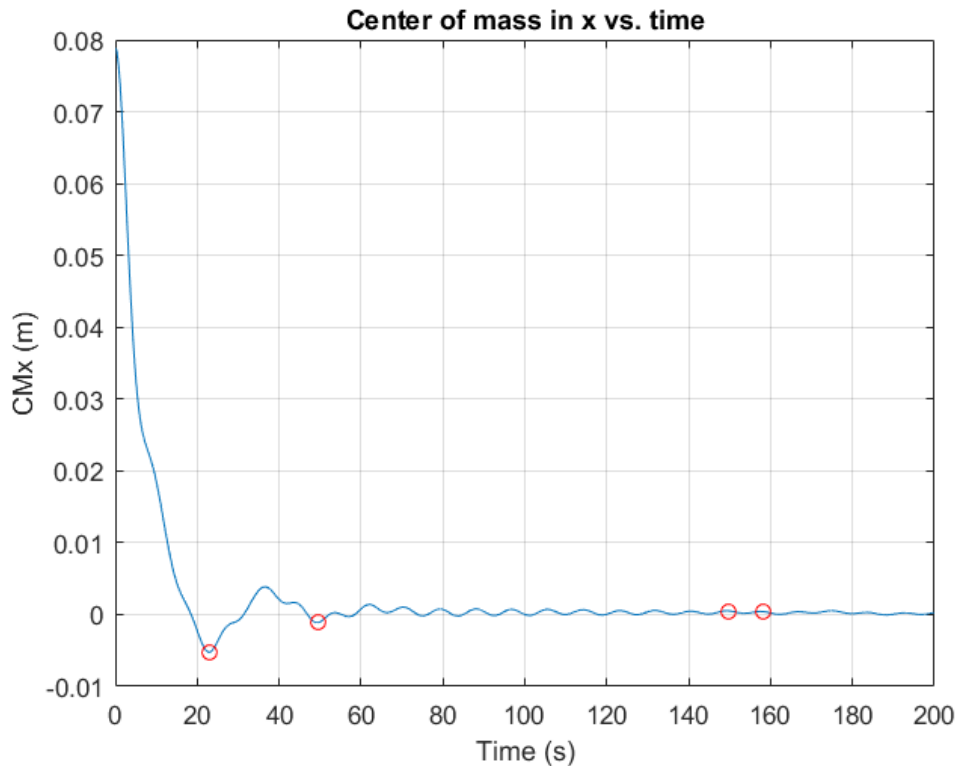




# METHODS OVERVIEW

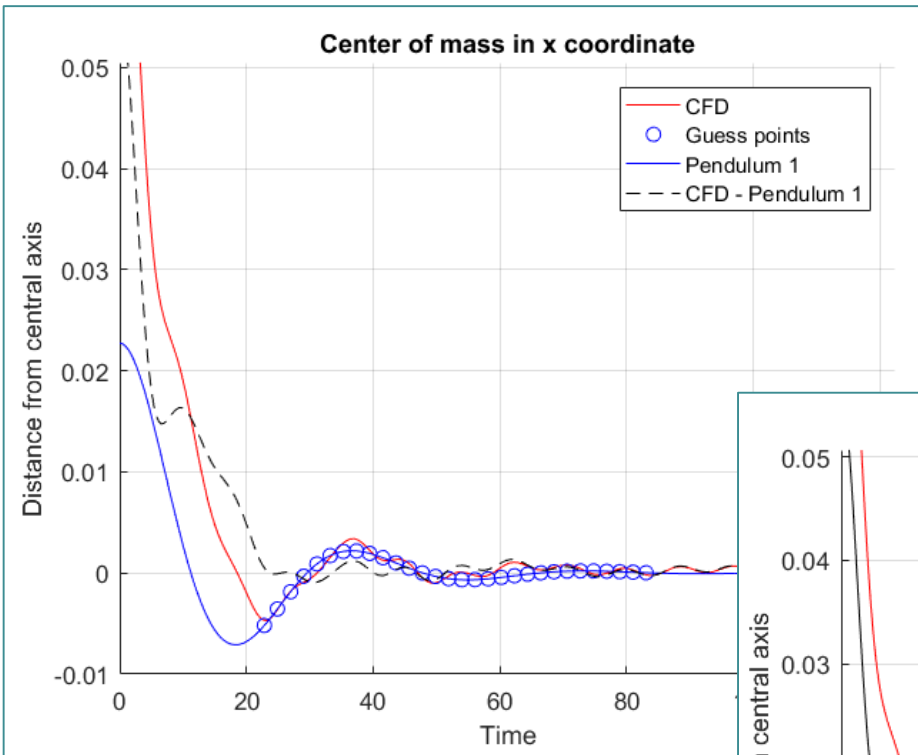


- Propellants: NTO and MMH
- Fill fractions: 25%, 50%, 85%
- Data: CM, Force, Moment (all 3 axes)

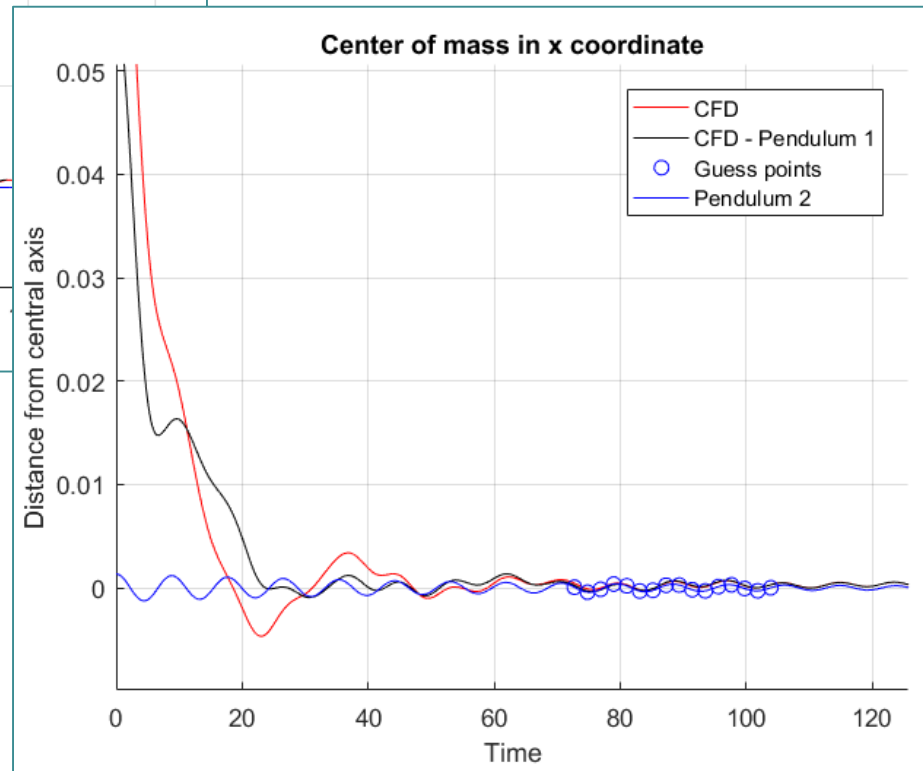


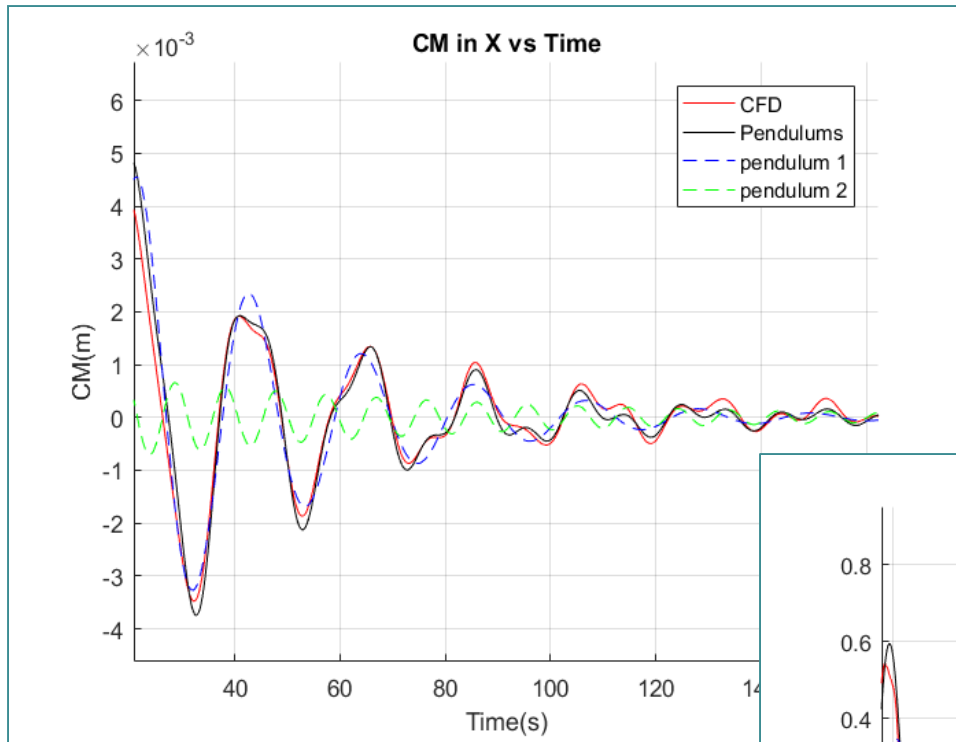
- Curve fitting by finding parameters in pendulum equation that most closely match CFD
- Trying to resolve CFD into two pendulums
- Peak-to-peak values ->
- Initial guesses for damping and frequency of each pendulum
- Note much higher damping before first peak

# Find Parameters to Fit CM Data

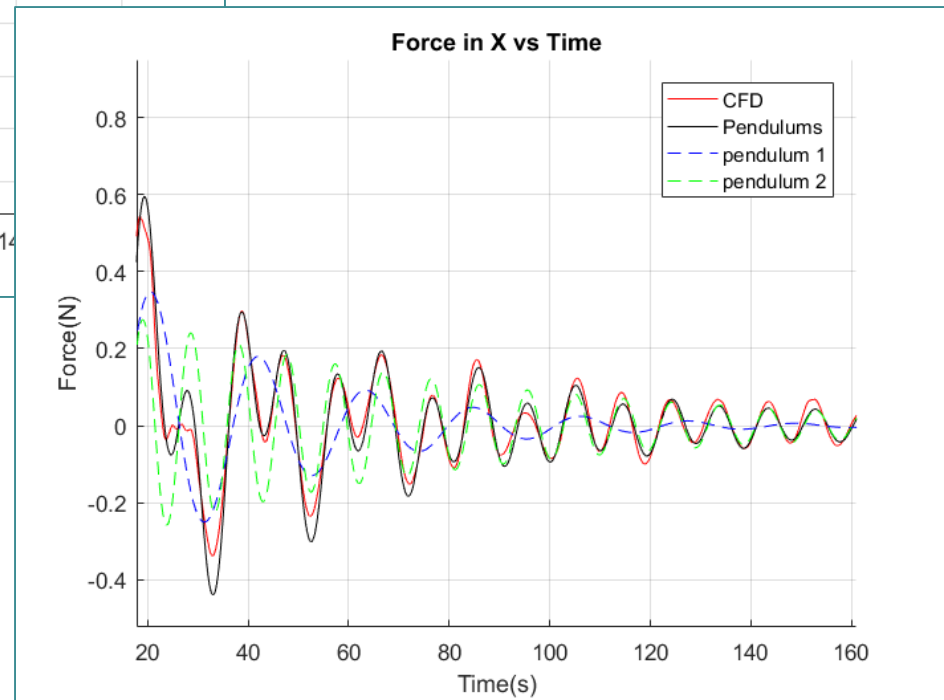


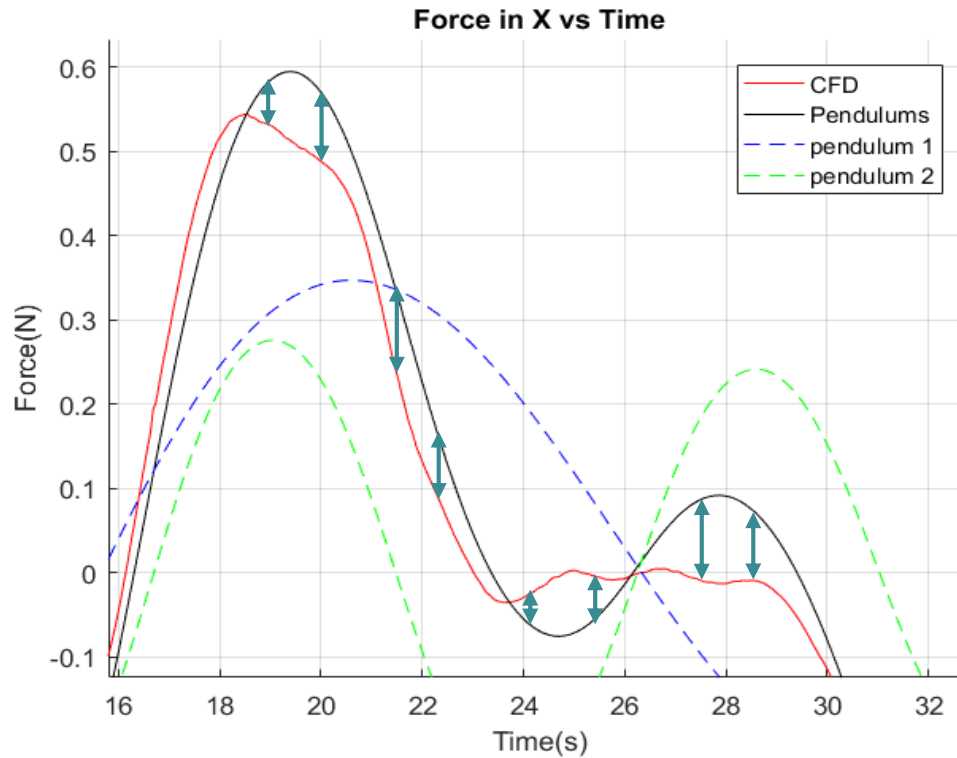
- Matlab's `fsolve(x)` ->
- Mass, damping, and frequency parameters to fit **CMx CFD data**
- Refine and iterate





- Sum of two pendulums generates model for propellant slosh
- Should match both CM and Force data

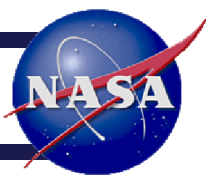




- Metric to quantify accuracy of fit: mean absolute difference between CFD force and pendulum model force

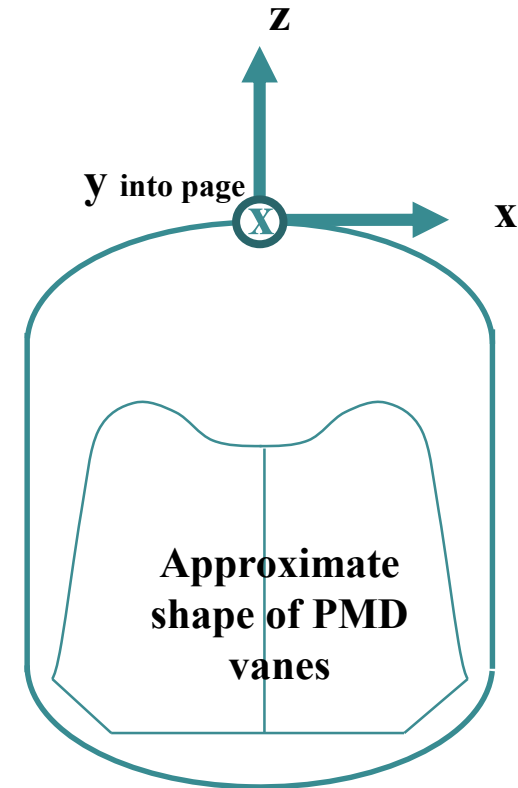
$$\frac{1}{n} \sum_{1}^{n} \text{abs}(CFD - \text{pendulum})$$

- Select methods that minimize this



# RESULTS AND LITERATURE COMPARISON

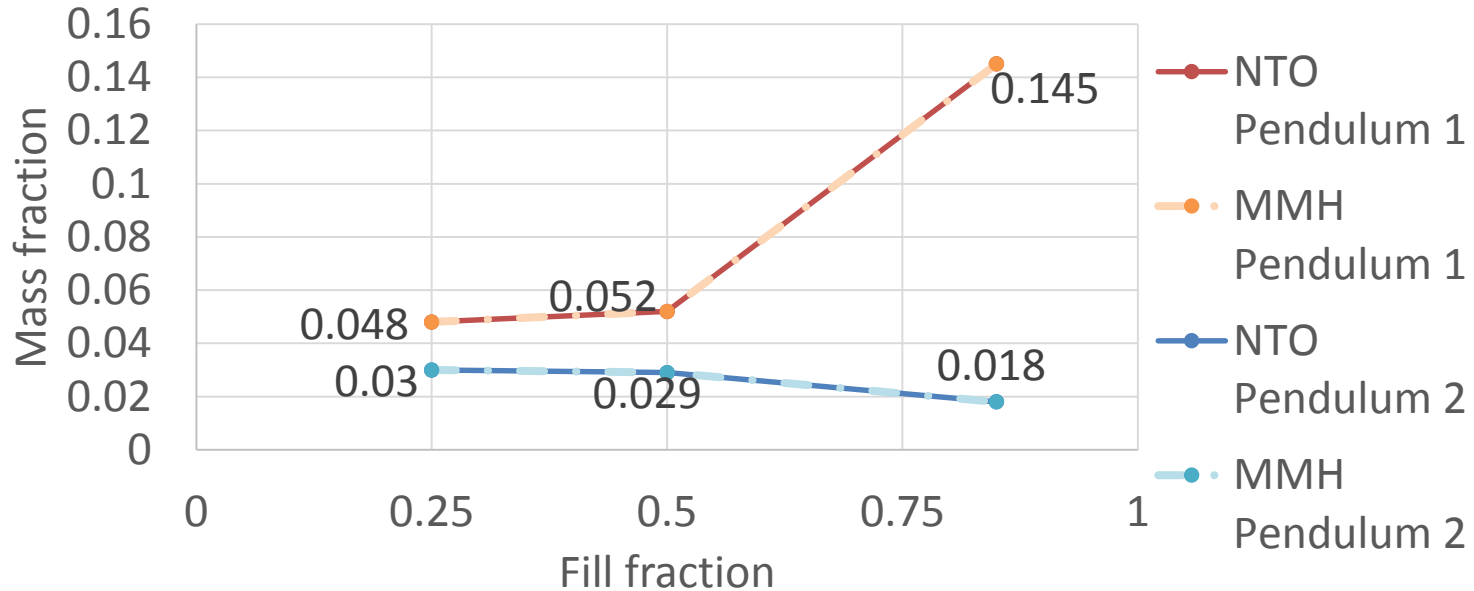
- Coordinate system – origin at top of tank
- Parameters prioritized fitting the behavior after the first peak
- Two pendulum model is an approximation only
  - PMD does not create a perfectly sector nor annular tank and is only a fraction of tank height
  - Parameters not constant over time
  - Model does not scale well with high fluid displacements





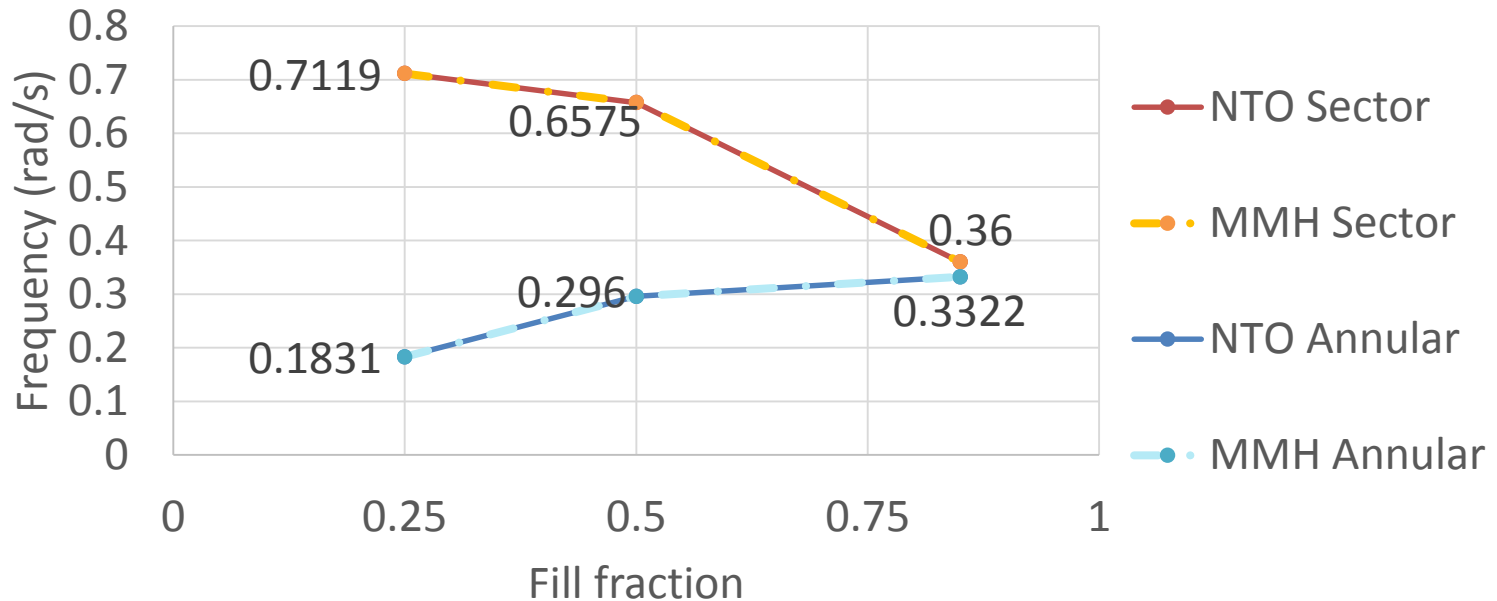
# Mass Participation Fraction

Mass participation fraction vs. fill fraction



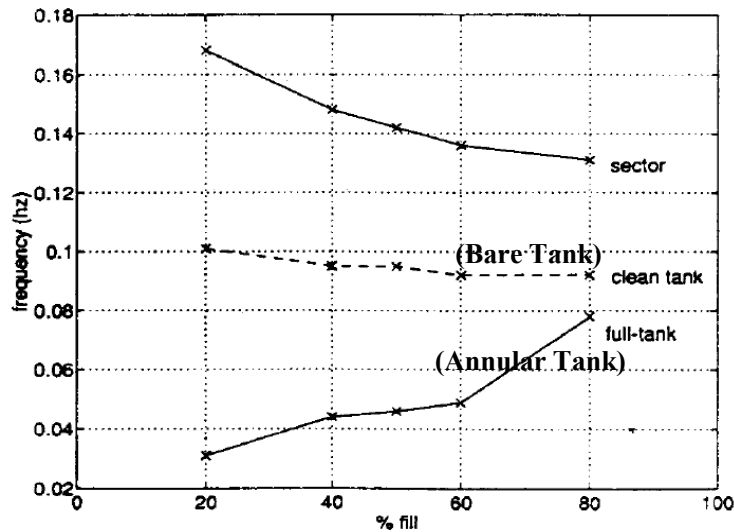
- Pendulum mass as a fraction of total fluid mass
- **Monotonic** trends
- Mass fractions are **identical between NTO and MMH**
- **Piecewise linear fit**
  - First two fill fractions – fluid partially submerges PMD, sloshing occurs between vanes
  - Last fill fraction – fluid completely submerges PMD, different slosh behavior

Frequencies vs. Fill Fraction

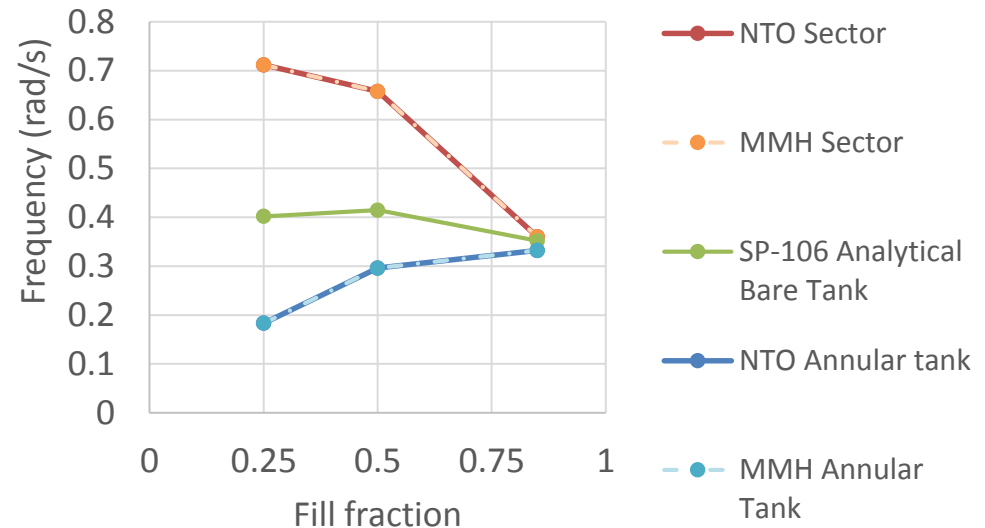


- Function of pendulum's length and acceleration
- Monotonic trends
- Frequencies are identical between NTO and MMH
- Frequencies for the two pendulums converge as fill fraction increases
  - Sector and annular slosh modes become less distinct as PMD becomes fully submerged

## Cassini Paper Frequencies vs. Fill Fraction

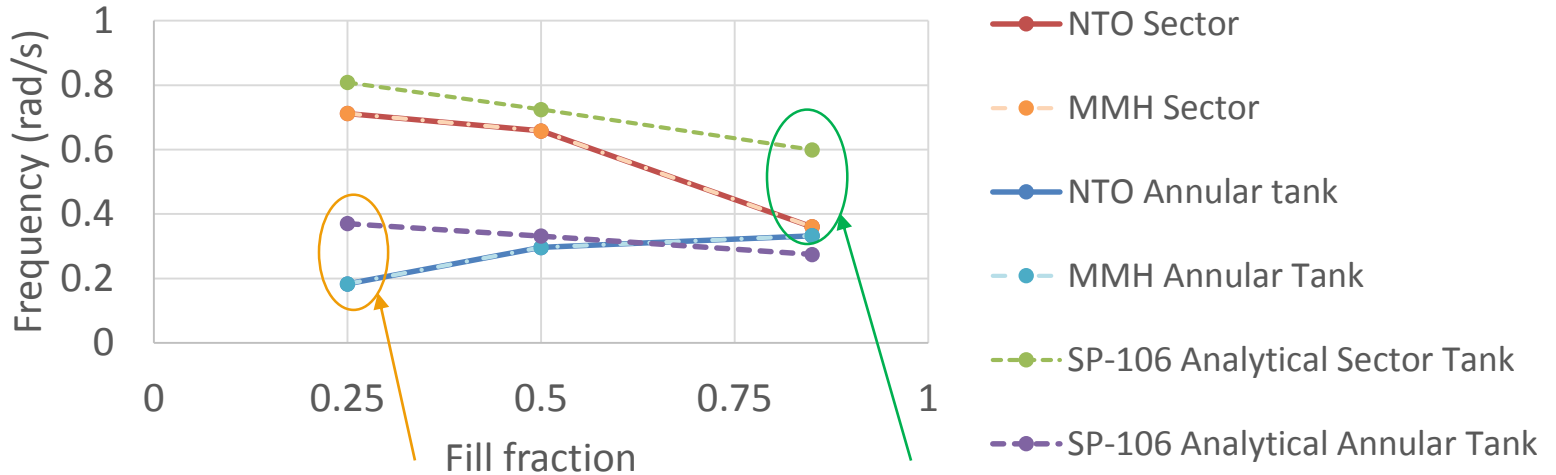


## Frequencies vs. Fill Fraction, Comparing to SP-106 Bare Tank



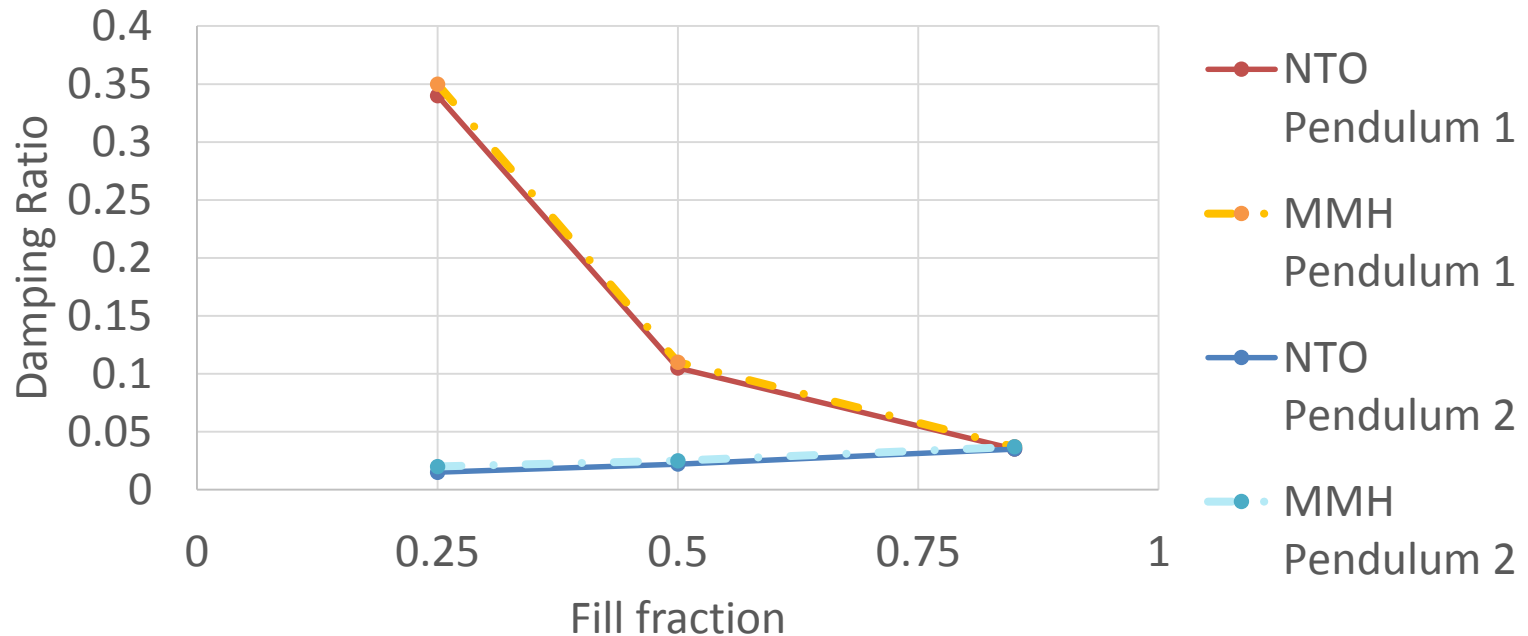
- Left: Cassini paper referenced SP-106 for an analytical equation for slosh frequency in a bare tank (cylindrical tank with no PMD) and compared it to the frequencies of their two pendulums
- Right: Similar trends to Cassini found in Europa pendulum model frequencies
- Sector and annular slosh modes converge towards bare tank frequency as PMD becomes more submerged (fully submerged at 85% fill fraction for Europa tank)

Frequencies vs. Fill Fraction,  
Comparing to Analytical Sector and Annular Tanks



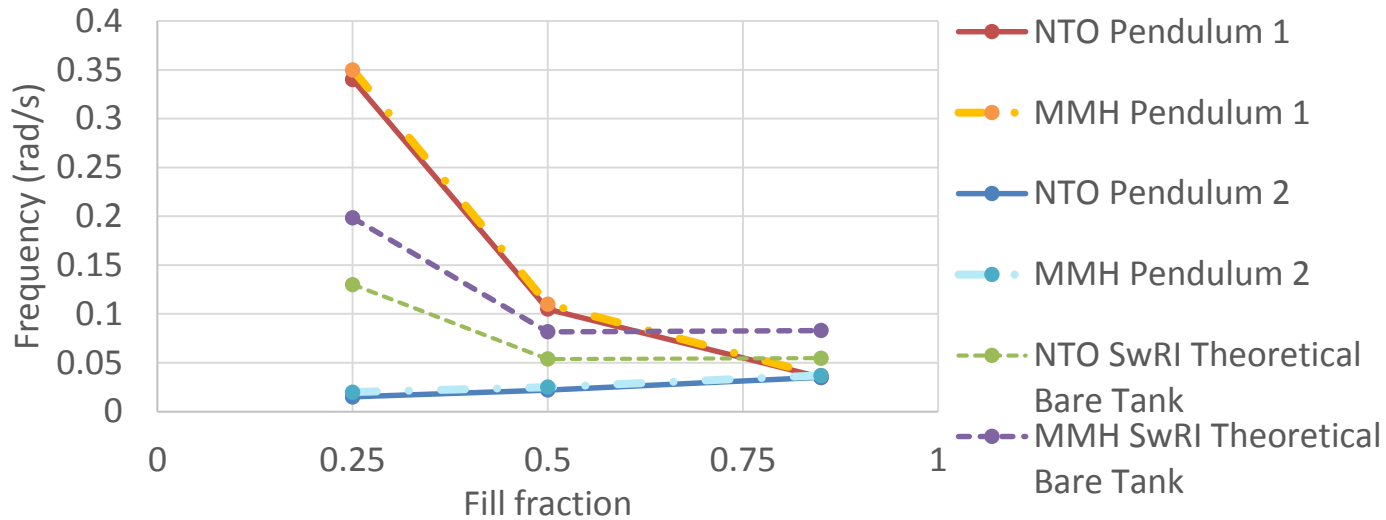
- SP-106 references tables (Bauer, 1963) for an analytical equations for sector and annular slosh frequency
- Function of acceleration, geometry, and fluid height
- Pendulum frequencies are close to analytical equation frequencies
- Differences between analytical and pendulum fits due to:
  - PMD is not exactly a sector/annular tank
  - Half-dome bottom approximated as flat bottom – at 25% fill fraction, sloshing fluid is almost entirely in the dome
  - PMD doesn't include entire height of tank – at 85% fill fraction, PMD is completely submerged

Damping Ratio vs. Fill Fraction

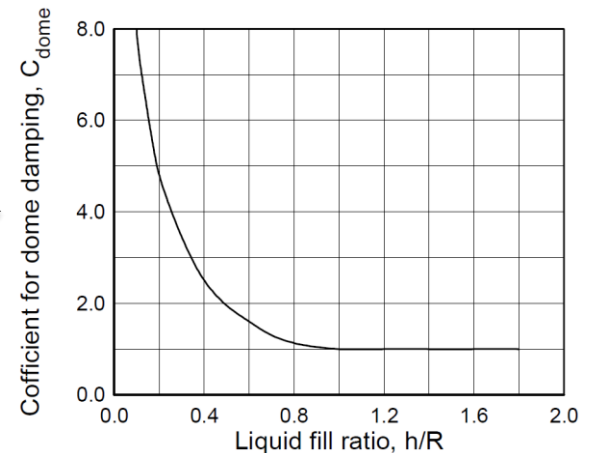


- Monotonic trends
- Slightly higher damping ratio for higher dynamic viscosity (MMH)

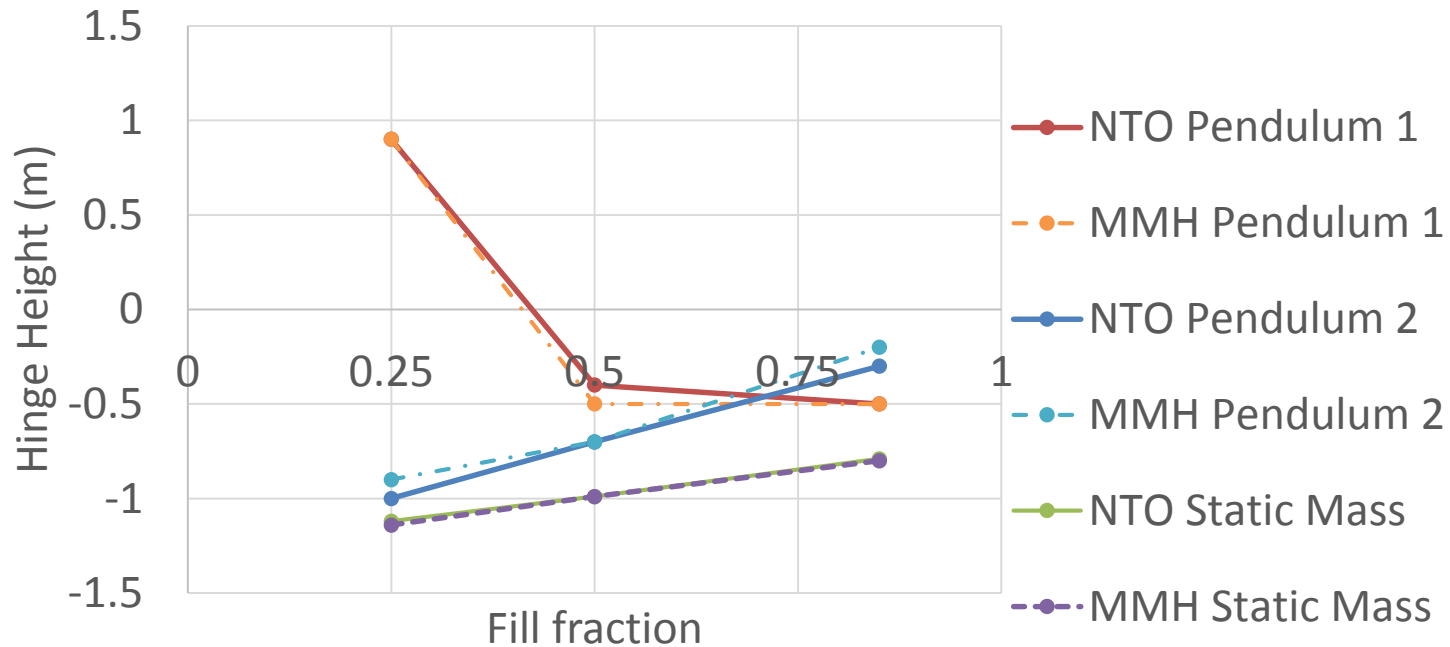
Damping Ratio vs. Fill Fraction  
CFD Fits and Analytical



- Mikishev and Dorozhkin found correlation for damping in a bare tank
- Function of geometry, acceleration, viscosity, and fluid height
- Scales by correction coefficient for domed bottom →
- Pendulum damping within order of magnitude of analytical prediction
- Pendulum damping less sensitive to viscosity than analytical prediction – viscous vs. drag forces

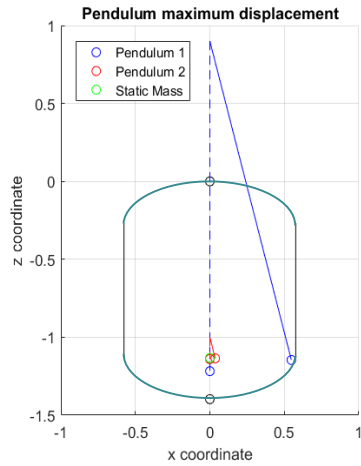


Hinge height vs. fill fraction

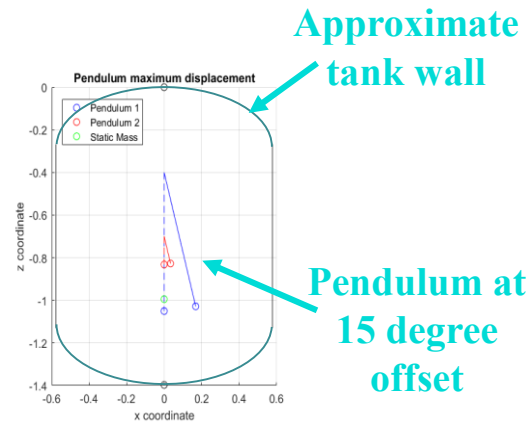


- Origin is top of tank
- Pendulum bobs stay within fluid
- Monotonic values for pendulum heights
- NTO and MMH heights are close but not identical

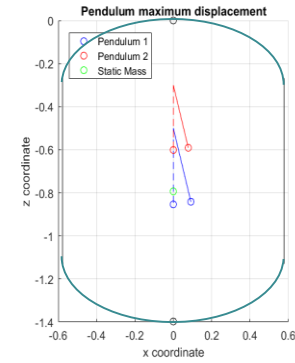
## NTO 25% fill



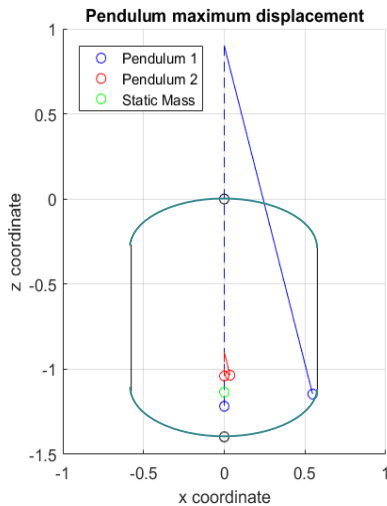
## NTO 50% fill



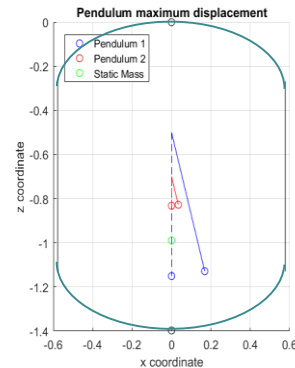
## NTO 85% fill



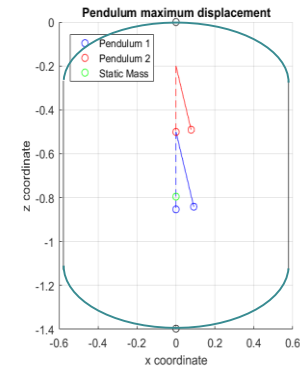
## MMH 25% fill



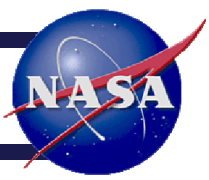
## MMH 50% fill



## MMH 85% fill

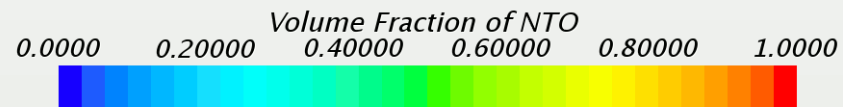
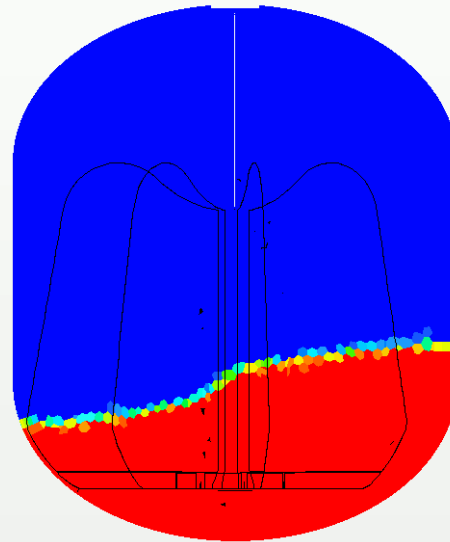


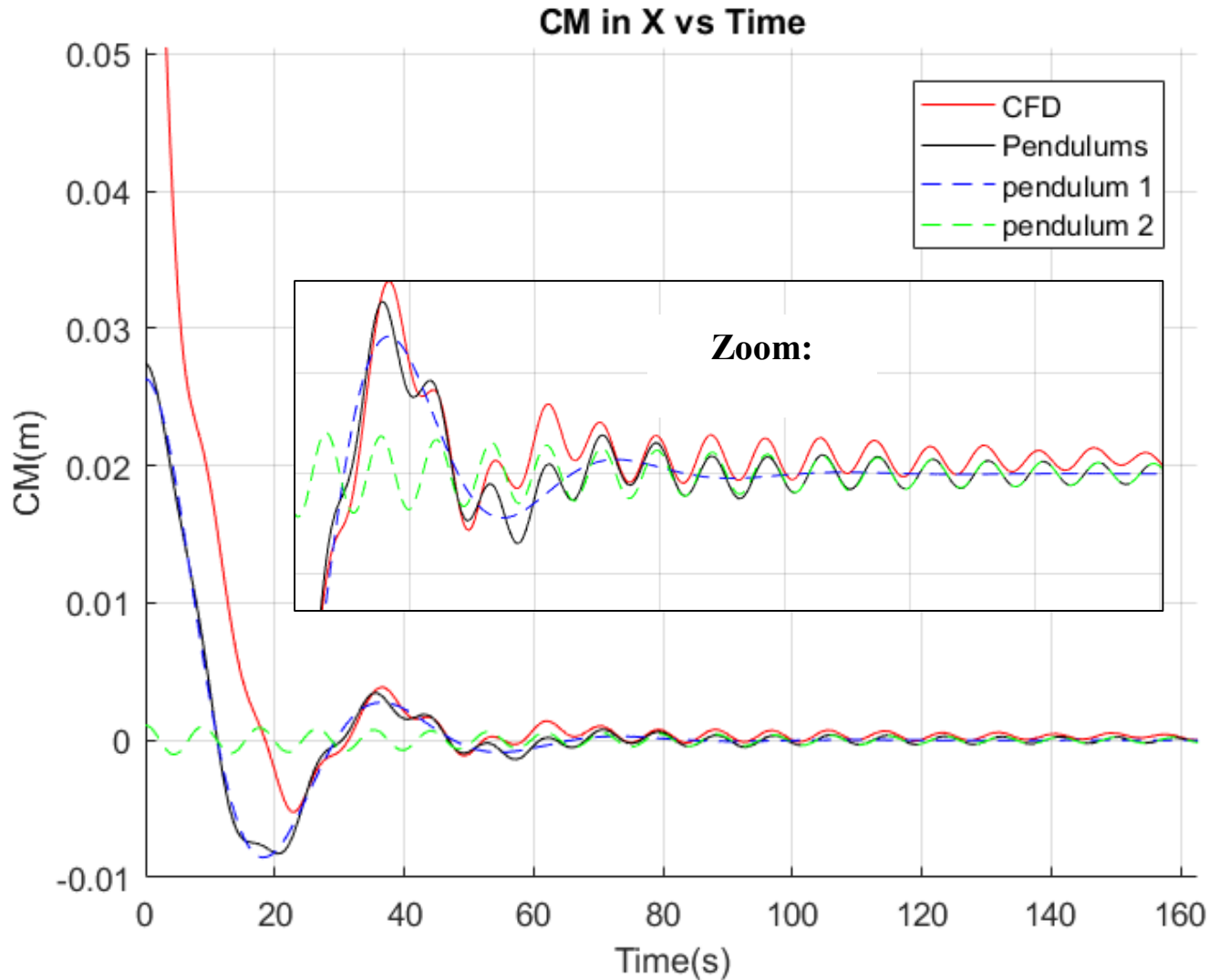


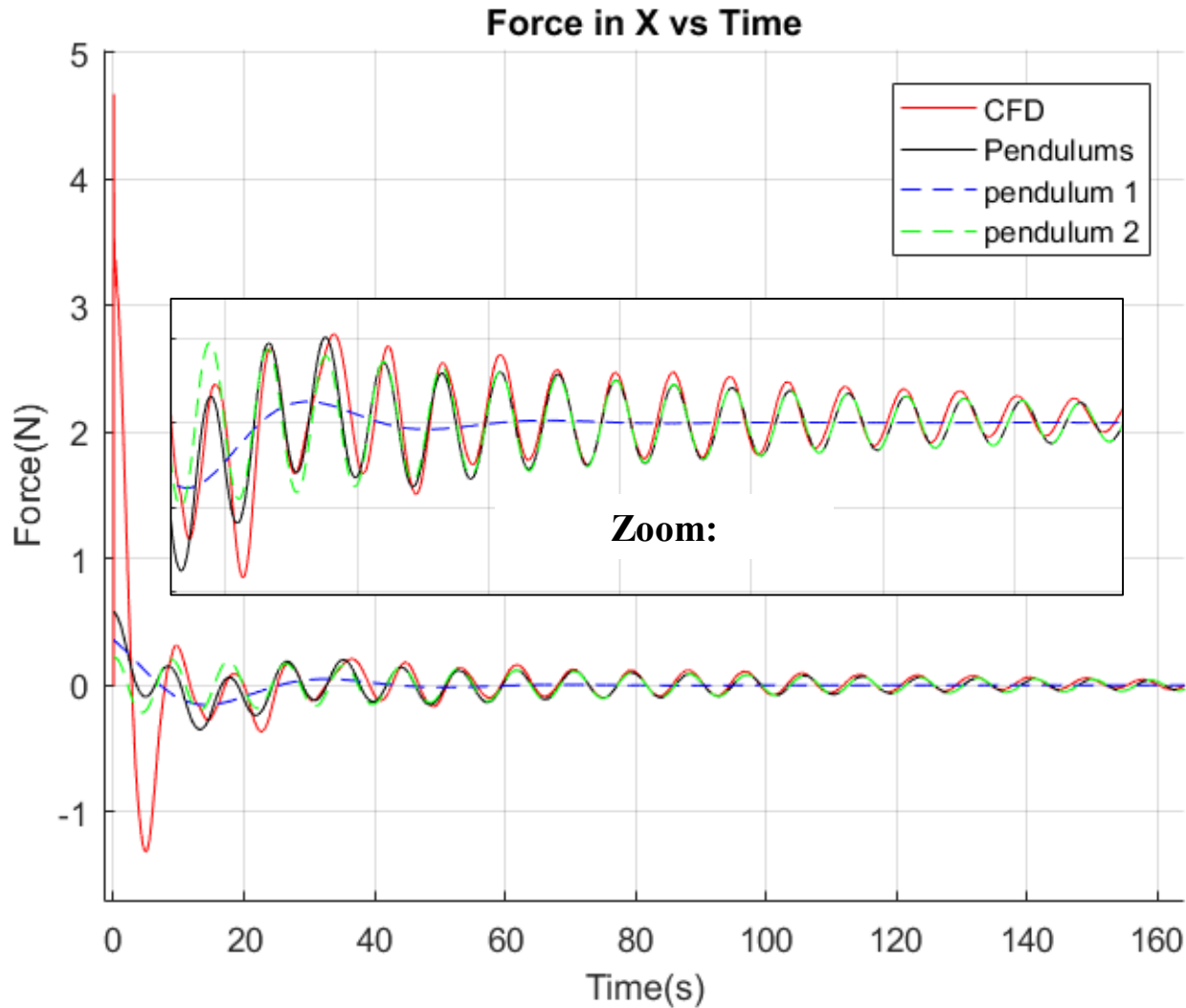


# PLOTS COMPARING PENDULUM MODELS AND CFD DATA

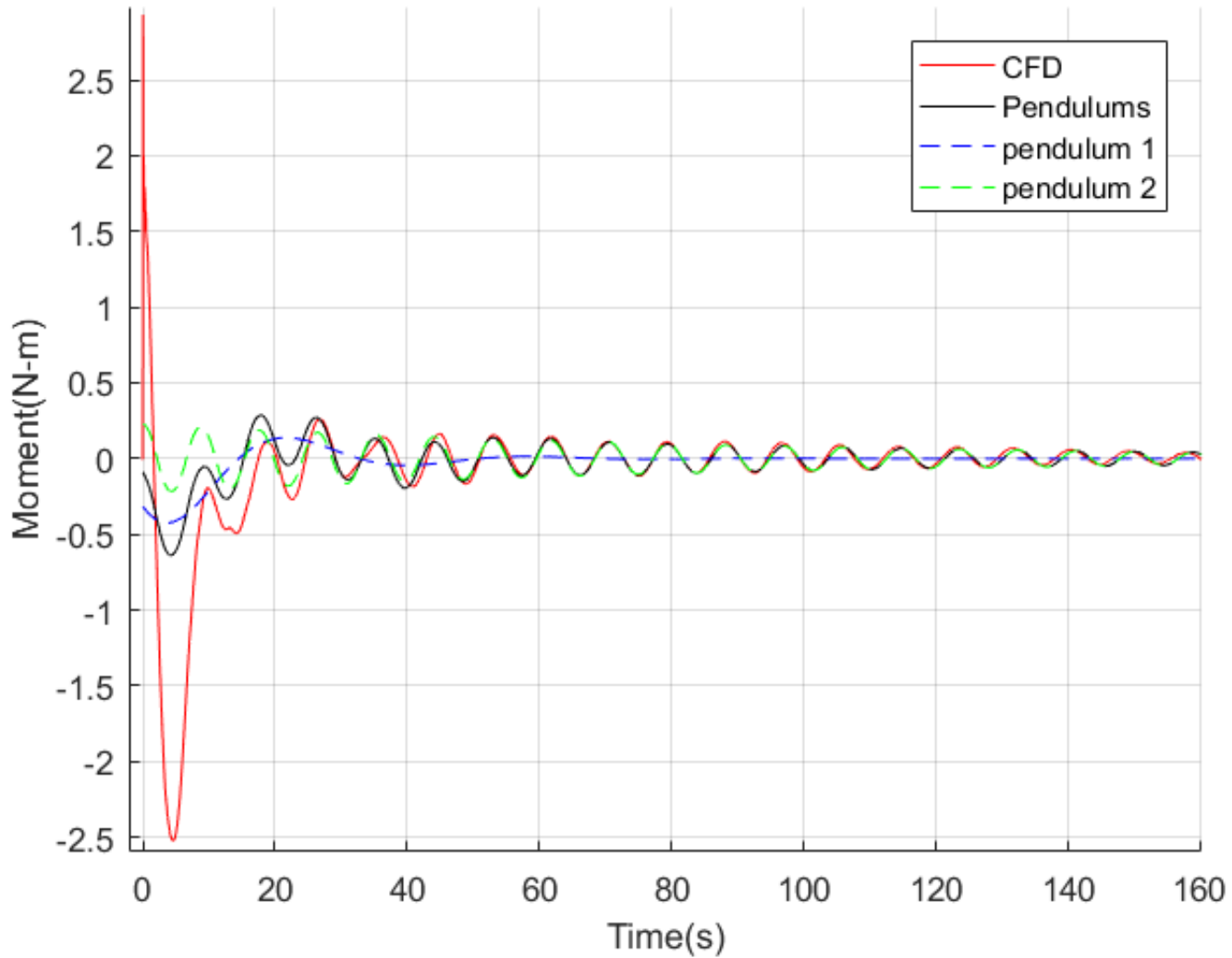
*Solution Time 2 (s)*



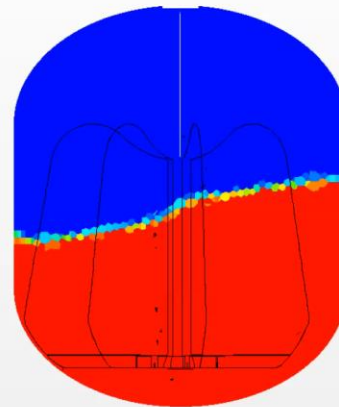




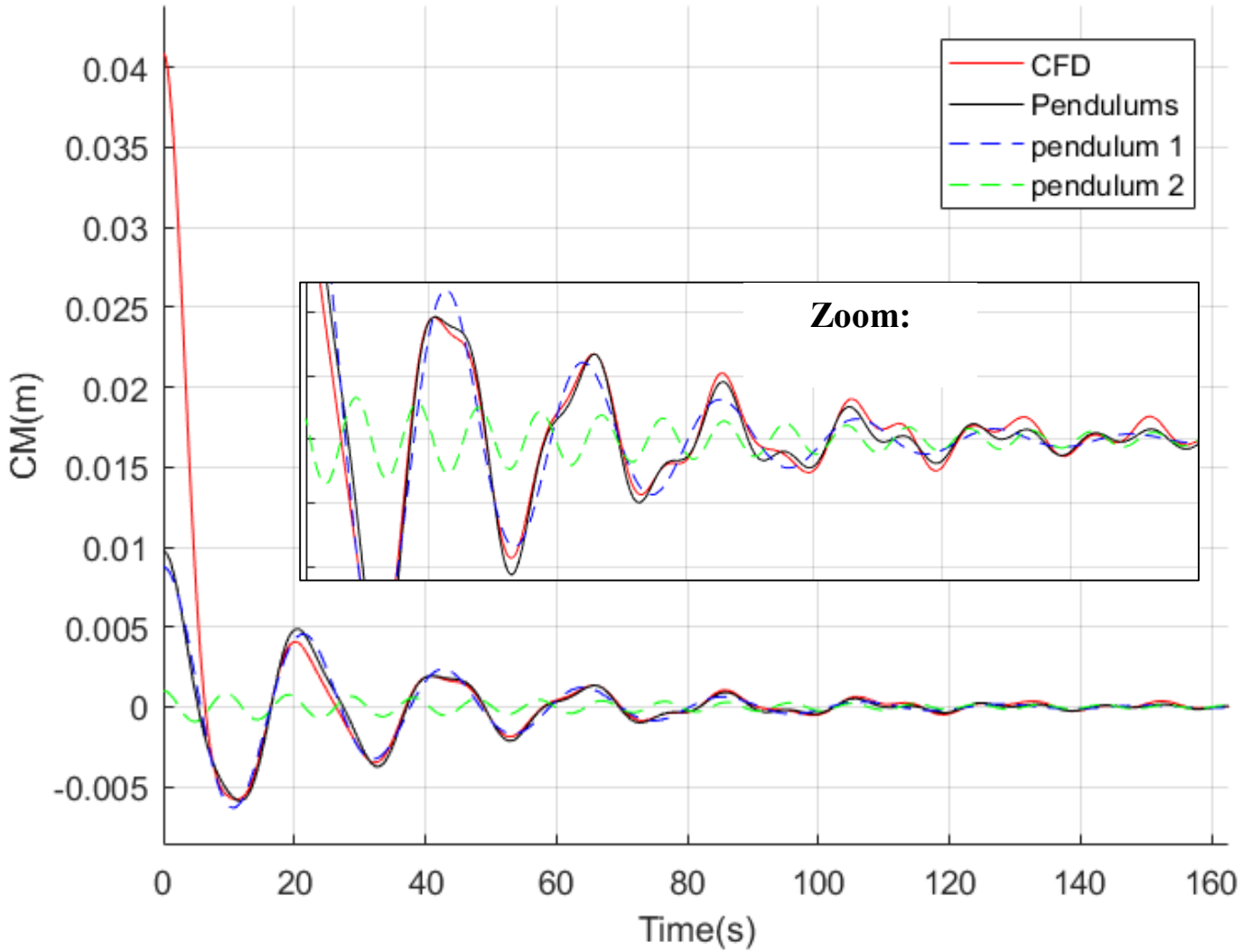
Moment about Y vs Time

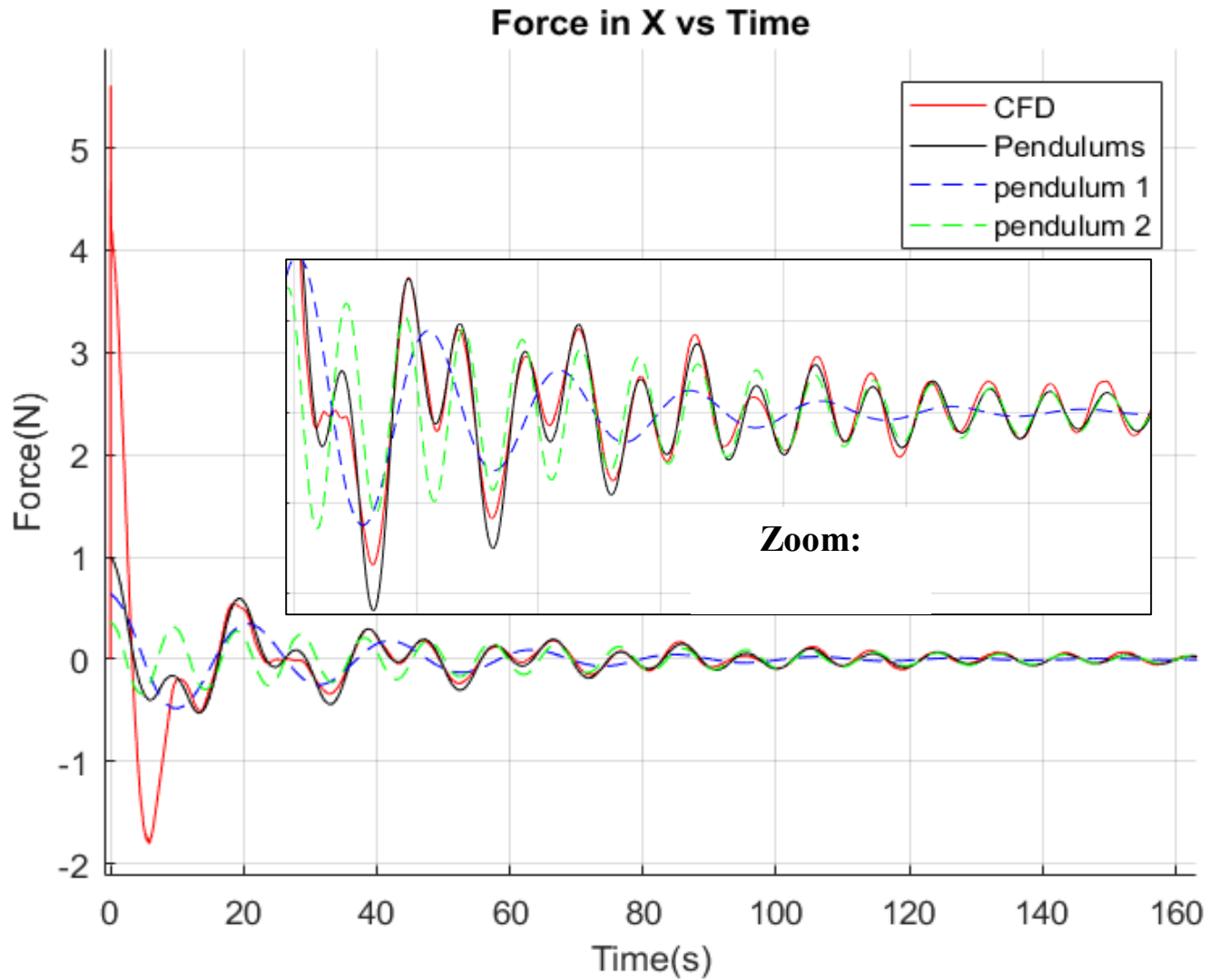


*Solution Time 2 (s)*



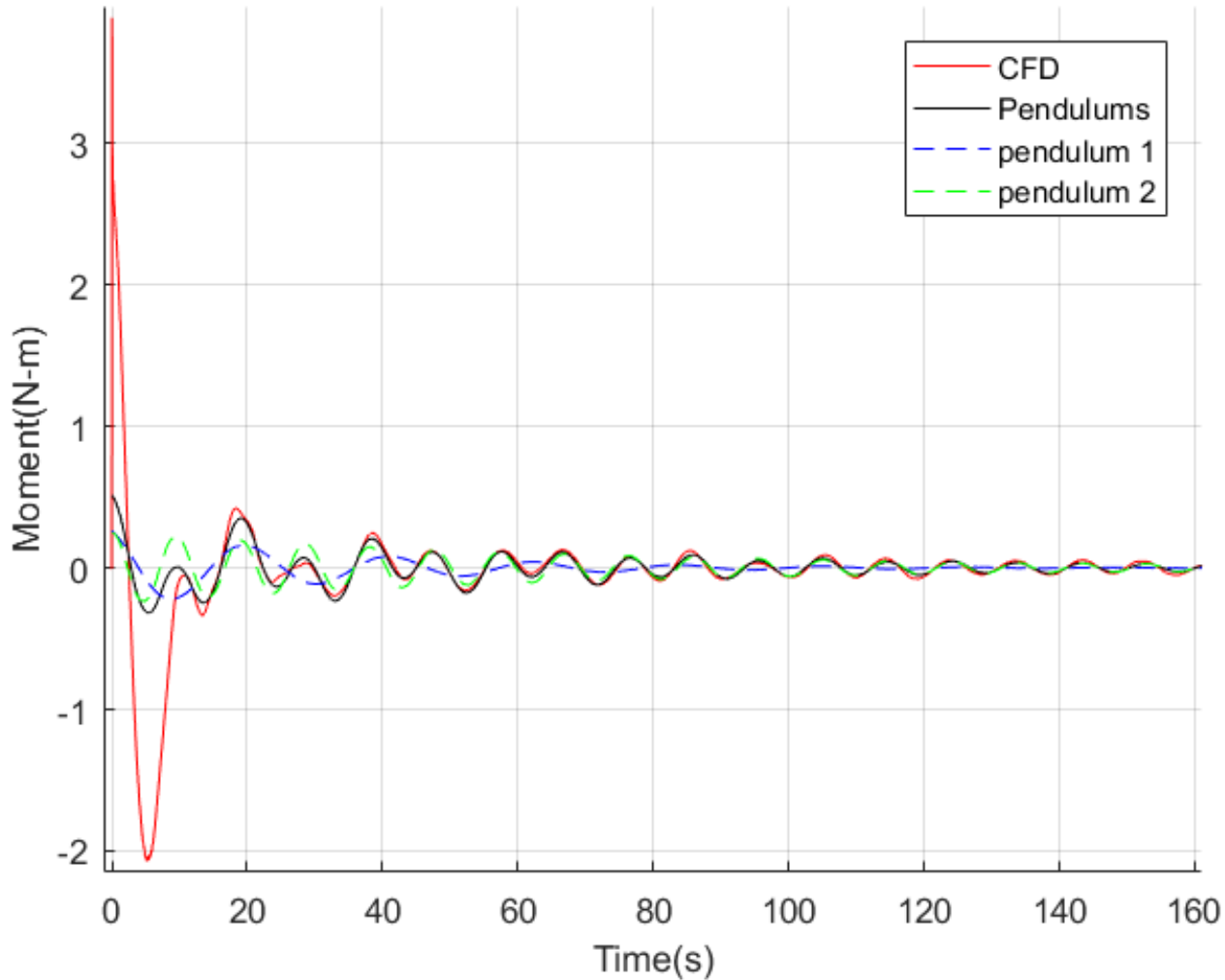
CM in X vs Time



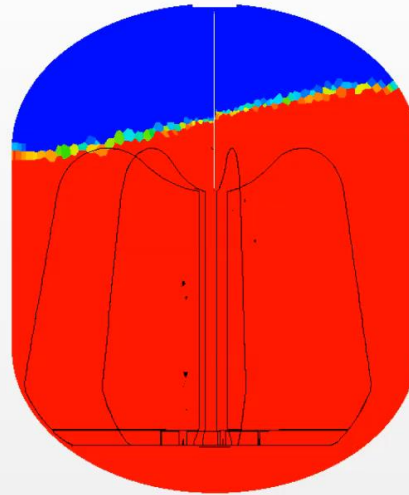


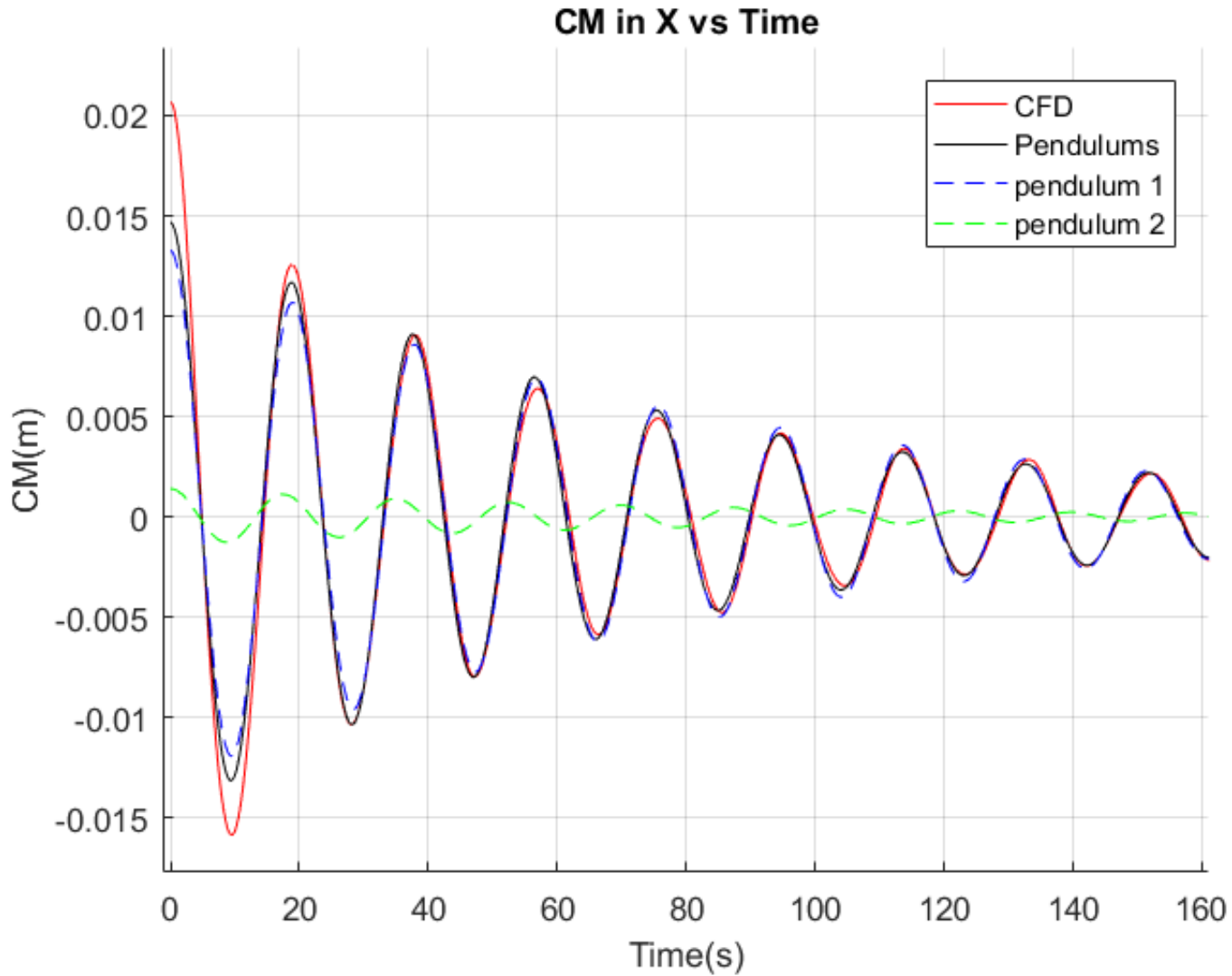


### Moment about Y vs Time

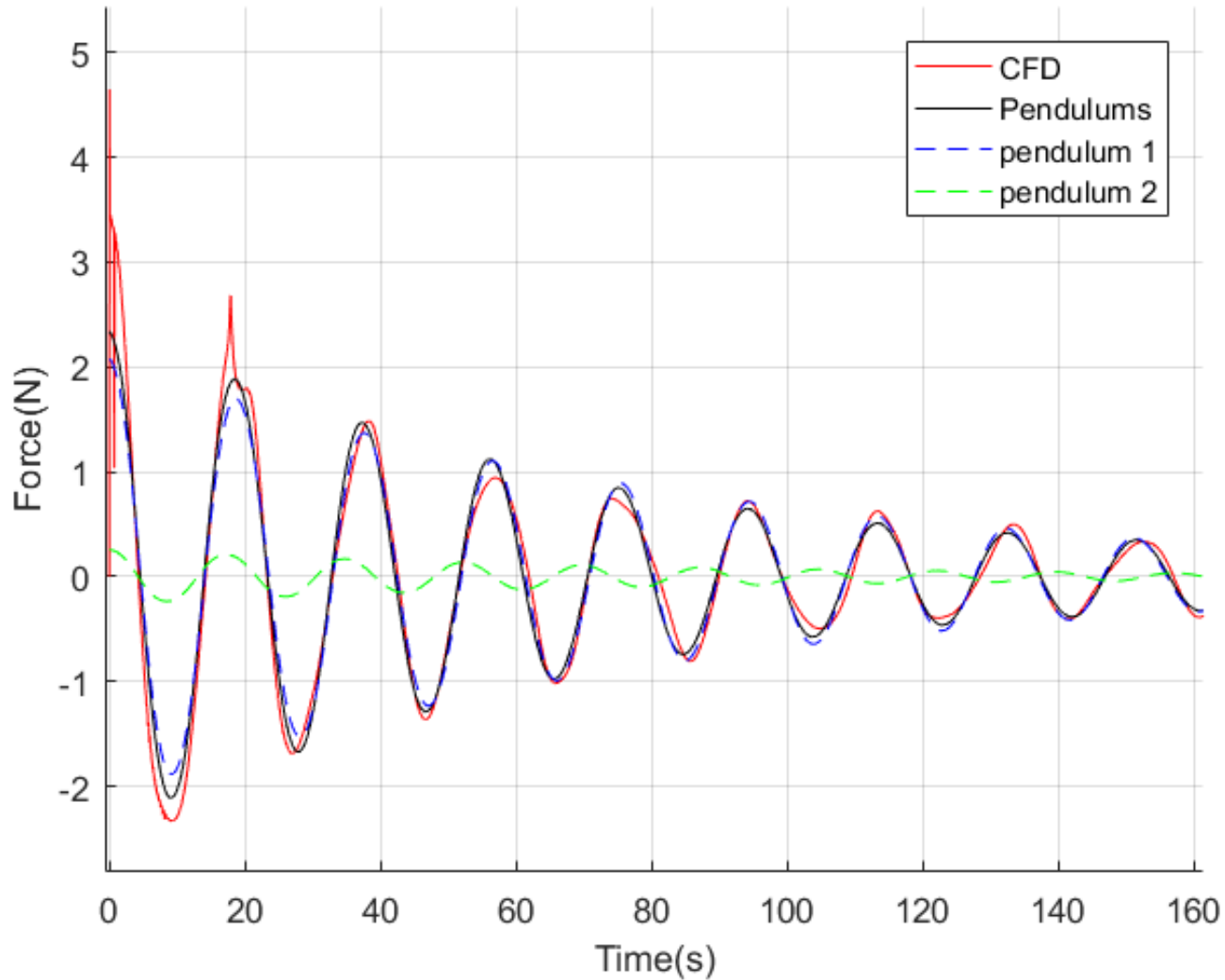


*Solution Time 2 (s)*

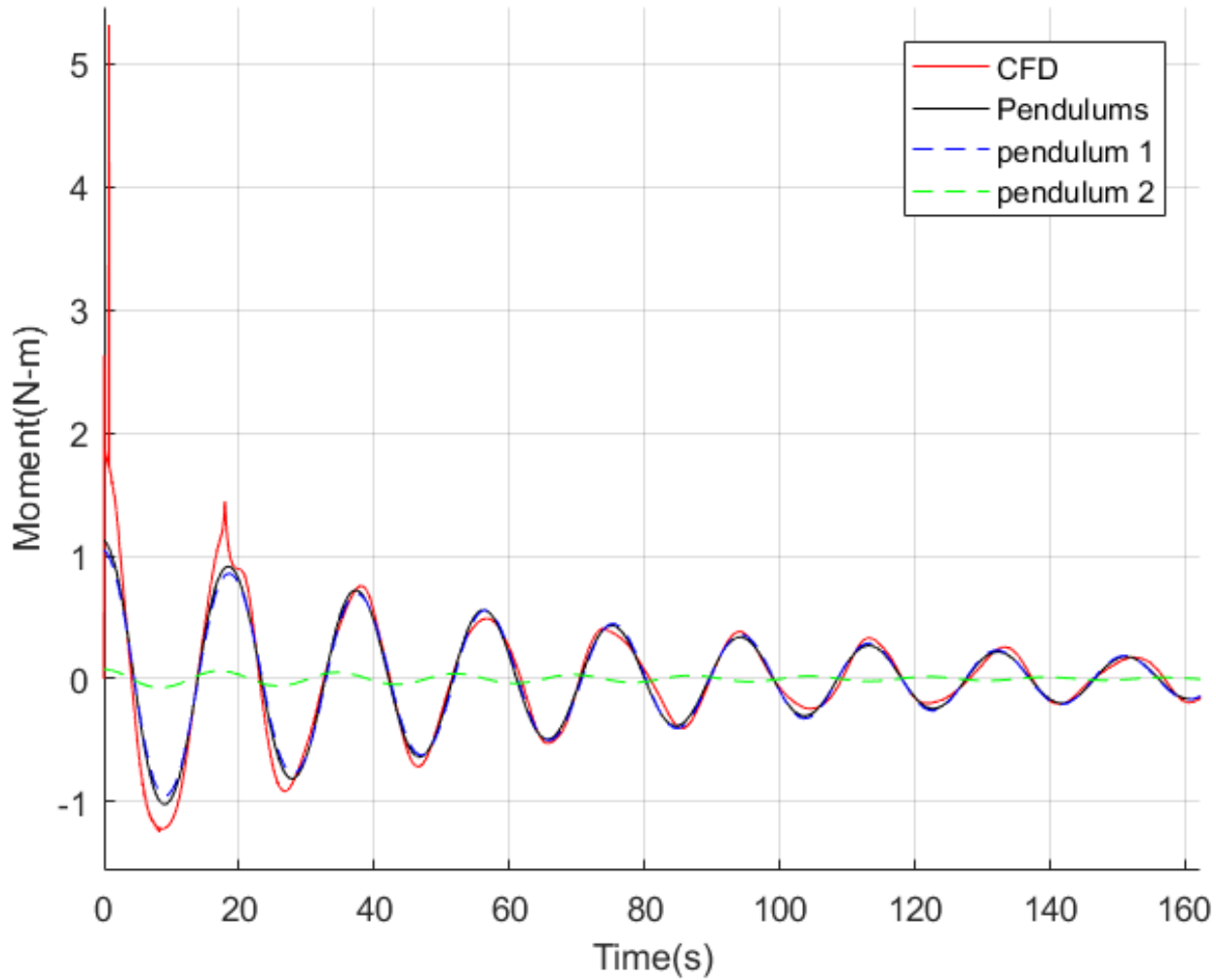




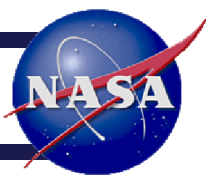
Force in X vs Time



Moment about Y vs Time



	NTO (nitrogen tetroxide)			MMH (monomethyl hydrazine)		
	25% fill	50% fill	85% fill	25% fill	50% fill	85% fill
<b>Mass fraction 1</b>	0.048	0.052	0.145	0.048	0.052	0.145
<b>Mass fraction 2</b>	0.03	0.029	0.018	0.03	0.029	0.018
<b>Mass 1 (kg)</b>	20.09	44.49	210.87	12.12	26.69	126.53
<b>Mass 2 (kg)</b>	12.56	24.81	26.18	7.58	14.89	15.71
<b>Frequency 1 (rad/s)</b>	0.1831	0.296	0.3322	0.1831	0.296	0.3322
<b>Frequency 2 (rad/s)</b>	0.7119	0.6575	0.36	0.7119	0.6575	0.36
<b>Damping Ratio 1</b>	0.34	0.105	0.035	0.35	0.11	0.037
<b>Damping Ratio 2</b>	0.015	0.022	0.035	0.02	0.025	0.037
<b>Hinge Height 1 (m)</b>	0.9	-0.4	-0.5	0.9	-0.5	-0.5
<b>Hinge Height 2 (m)</b>	-1.0	-0.7	-0.3	-0.9	-0.7	-0.2
<b>Static Mass Height (m)</b>	-1.12	-0.99	-0.79	-1.14	-0.99	-0.8
<b>Mean Force Error from t=0</b>	0.0716	0.075	0.1055	0.0398	0.0447	0.0679
<b>Mean Force Error from First Peak</b>	0.0241	0.018	0.0775	0.0118	0.0119	0.0518

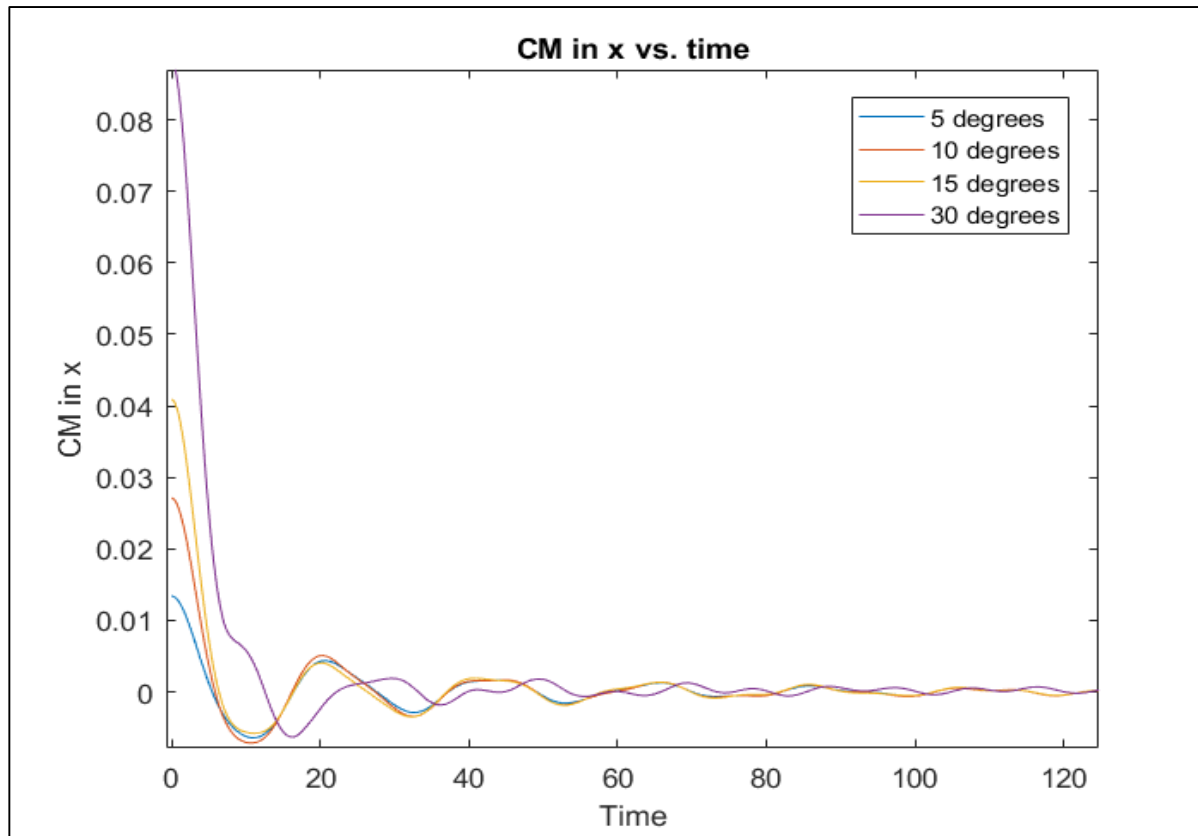


# CONCLUSIONS

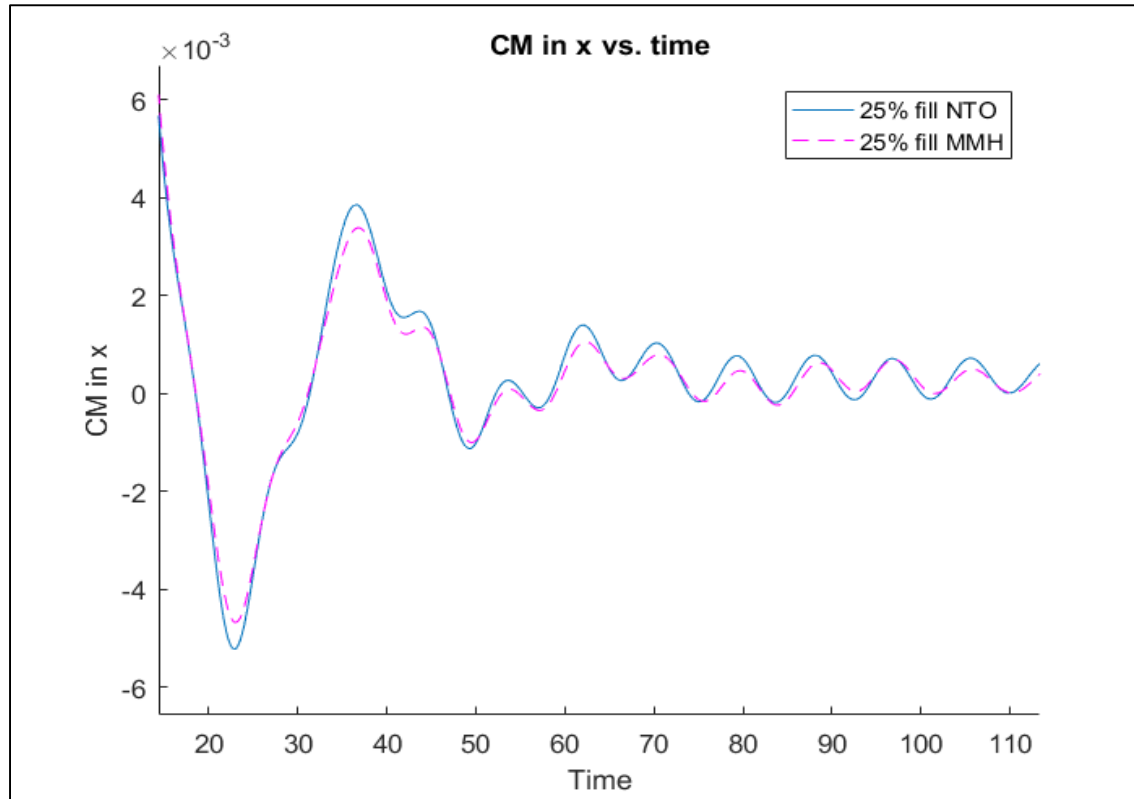
# Accuracy of Fit

- Two-pendulum model can accurately capture either before or after first peak
- High confidence on frequencies except 85% fill pendulum 2
- Moderate confidence on mass, damping, and hinge location
  - Sometimes several sets of parameters could have provided good matching to CFD
  - Selected parameters that made physical sense
- Model parameters may reflect inaccuracies in CFD
- Pendulum model does not scale well for high fluid disturbance angles
- Damping is actually a function of time and distance traversed by moving fluid
  - Pendulum model assumes damping is constant over time





- Small initial fluid displacements: Changes have **little impact on long-term CFD results**
- Large initial displacements: **behavior differs drastically**



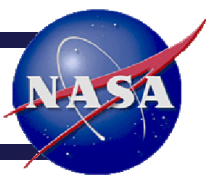
- Changing density (NTO vs MMH) **only slightly changes damping**, has little impact on CFD results



# Areas for Further Investigation



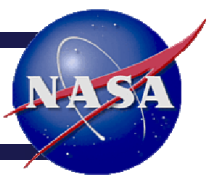
- Find **literature** to support mass fraction parameters
- Potentially to capture first peak – add **third pendulum** with damping ratio of one
- Validate with **more CFD** data:
  - At intermediate fill fractions
  - At different initial fluid offset angles - 5 degree offset is more conservative than 15, will be used for deliverable in May
- Validate with **experiments**



**Thank You**



# Sources



- Abramson, N.H.: The Dynamic Behavior of Liquids in Moving Containers. NASA SP-106, 1966
- Bauer, H.F.: Tables and Graphs of Zeros of Cross Product Bessel Functions. MTP-AERO-63-50 NASA-MSFC, June 1963
- Dodge, F.T.: The New “Dynamic Behavior of Liquids in Moving Containers”. Southwest Research Institute, 2000
- Enright, P.J. and Wong, E.C.: Propellant Slosh Models for the Cassini Spacecraft. AIAA-94-3730-CP, 1994