WALKING THE FILAMENT OF FEASIBILITY:

GLOBAL OPTIMIZATION OF HIGHLY-CONSTRAINED, MULTI-MODAL INTERPLANETARY TRAJECTORIES USING A NOVEL STOCHASTIC SEARCH TECHNIQUE

Arnold C. Englander[1] and Jacob A. Englander[2]

August 23rd, 2017

[1]Englander & Associates, LLC, Concord, NH, ace@aenglander-assocates.net
[2]NASA Goddard Space Flight Center, Greenbelt, MD, jacob.a.englander@nasa.gov

Motivation

- Monotonic Basin Hopping (MBH) has been shown to be very effective optimization methodology for interplanetary space-probe trajectories – which are often very challenging constrained global optimization problems in high-dimensional solution spaces.
- When trajectory optimization methods are autonomous, “when to stop” optimizing is an important question.
- Despite the many benefits of MBH, we do not see how it can autonomously determine when the global search is complete.
- The goal of Filament Walking (FW) optimization is to address the “when to stop,” which is related to “thoroughness” in finding and walking filaments.
- This presentation is a progress report. It does not present the solution to the “thoroughness” issue, or – thereby – the “when to stop” question.
- It presents FW as an alternative to MBH, demonstrates that the assumptions of FW are realistic and it’s logic is sound, brings new precision to the “when to stop” question, and outlines future work.

The basic concept of FW

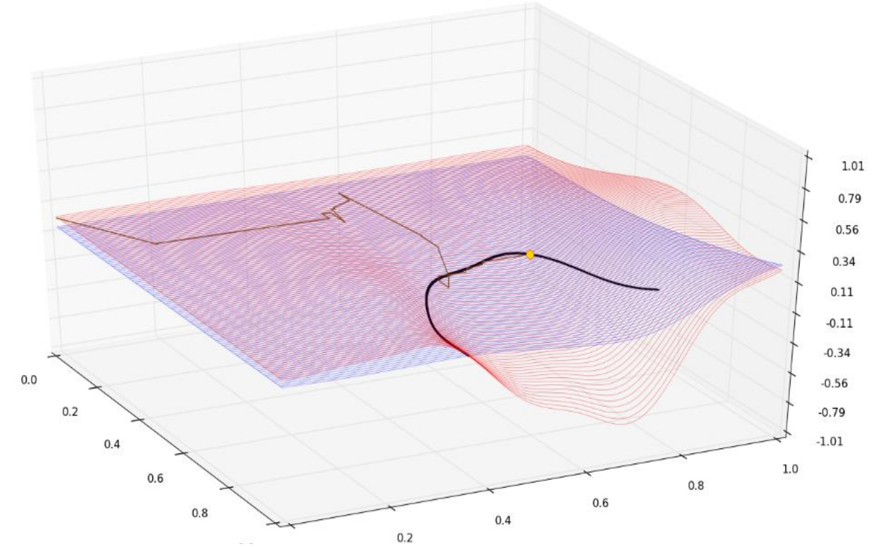
- Optimization of interplanetary space-probe trajectories is a difficult and complicated constrained global optimization problem in high-dimensional solution spaces.
- Such problems are more challenging when low-thrust propulsion is used.
- Often, the massive optimizations that are required must be totally autonomous. They do not allow time for human intervention (e.g., for initial guesses).
- We “shrink” the high-dimensional problem space and its complicated constraints, down to a small number of 1-dimensional search with simpler constraints.
- We do this by transforming the set of candidate-solution points, that are feasible with respect to equality constraints, into one (or a few) one-dimensional search filaments that can be found and walked along “thoroughly” with extremely high probability.
- At each step in the walk along a filament the objective function at that point is sampled. Then we applying the equality constraints to the points on the filament.

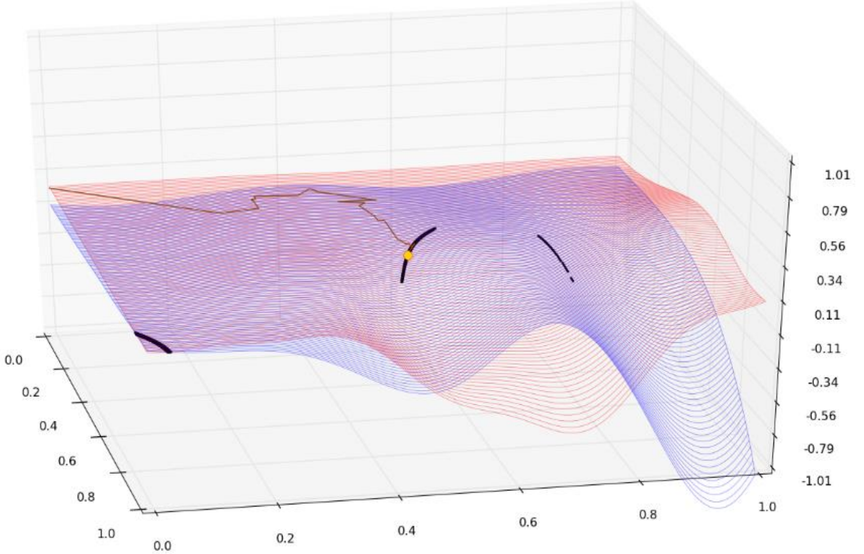
Formal problem statement

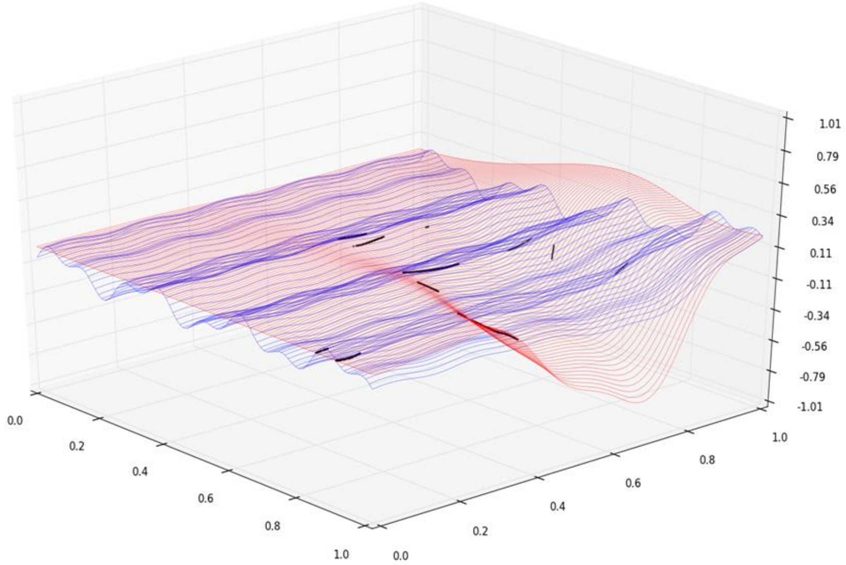
- We define **x** as a point in a dense set of points **X** in solution space $\mathcal{S}$ in $\mathbb{R}^N$
  - In typical problems, N is approximately 10 to 100
- We seek global min(f(**x**)), subject to:
  - $g_i(\mathbf{x})-c_i = 0$ for all I equality constraints
  - $h_j(\mathbf{x})-k_j \leq 0$ for all J inequality constraints
- In other words, we seek global min(f($\mathbf{x}^{++}$)), where $\mathbf{x}^{++}$ are strictly feasible points
- We say that points $\mathbf{x}^{+}$ are feasible at least with respect to the set of all equality constraints $g_i(\mathbf{x})-c_i$
- Because we require the $\mathbf{x}^{+}$ to satisfy all of the equality constraints, they can only reside at the intersections of the $g_i(\mathbf{x})-c_i = 0$. We call these intersections "filaments."
- Because the functions $g_i(\mathbf{x})$ that determine the equality constraints are known to be first-differentiable, and they are usually are reasonably "smooth", the gradients of $g_i(\mathbf{x})$ near an $\mathbf{x}^{+}$ usually "point" to that $\mathbf{x}^{+}$.
- The $\mathbf{x}^{++}$ may then be found by filtering the $\mathbf{x}^{+}$ to remove all cases where $h_j(\mathbf{x}^{+})-k_j > 0$.
- $\mathbf{x^*}$ is then found by evaluating all of the f($\mathbf{x}^{++}$)

Our initial concerns about filaments

Their existence, their lengths, and the number of them





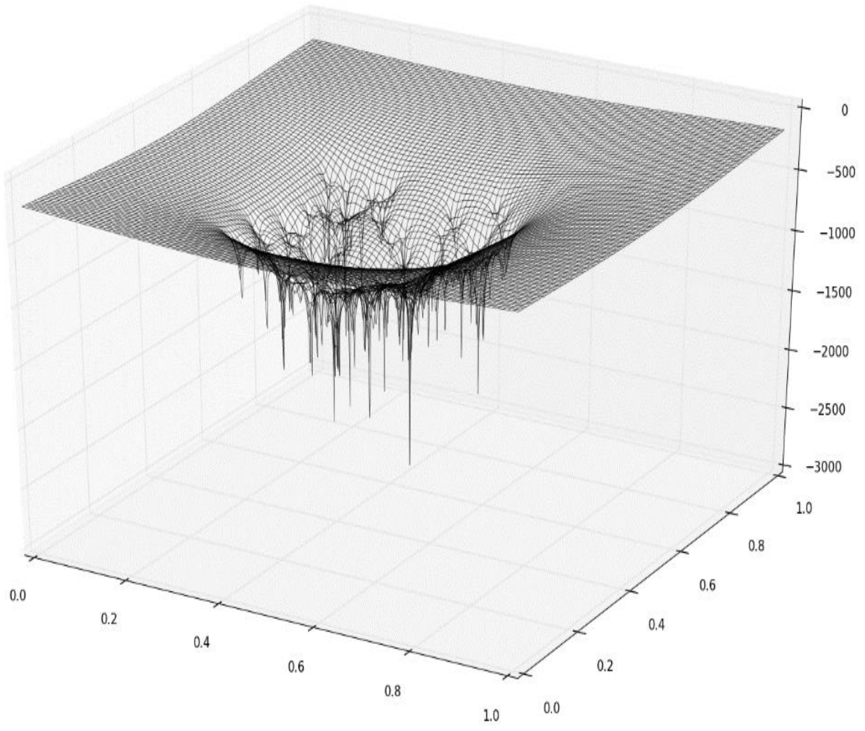


The apparent nature of real problems

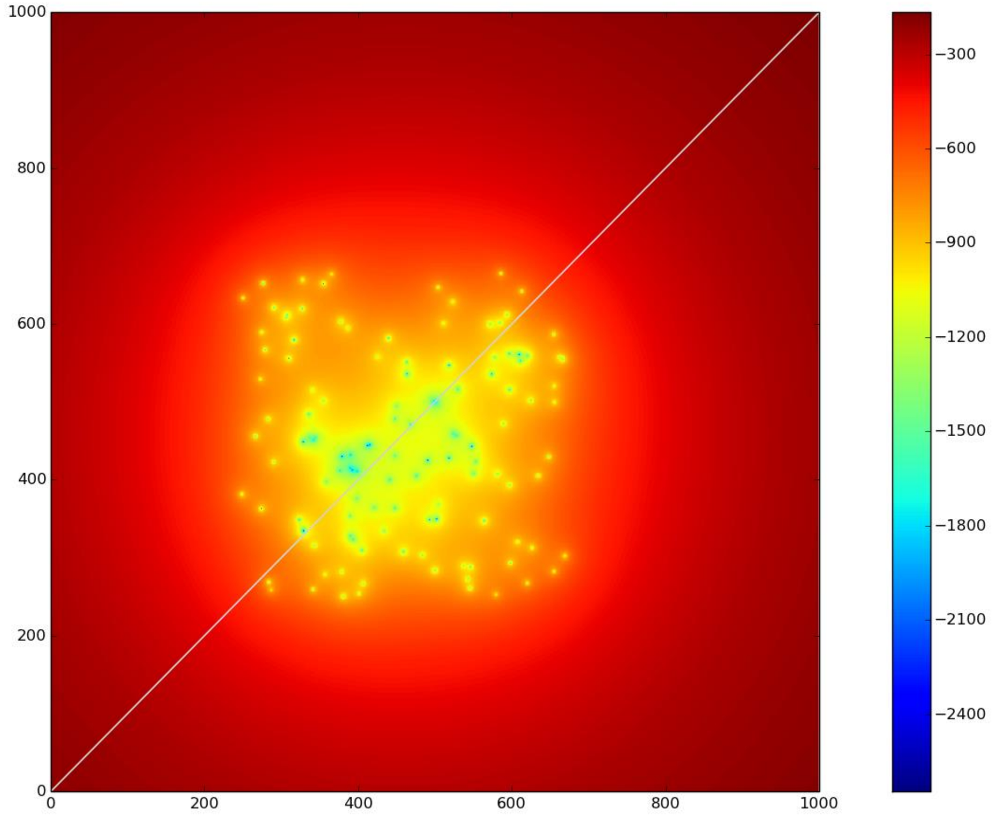
- So far, filaments do not seem to be too "short" or numerous in real problems.
- Gradients do not seem to be "rippled," "rugged," roughly textured, or locally flat.
- We are left with the challenge of "thoroughness:"
  - How can we know that we have "almost surely" found all the filaments?
  - How can we know that the walker has walked, "almost surely," each filament thoroughly enough to have found that filament's min(f($\mathbf{x}^{+}$))?
- Can we build autonomous quantitative measures of "thoroughness?"

Simplified example

Wireframe of objective function f

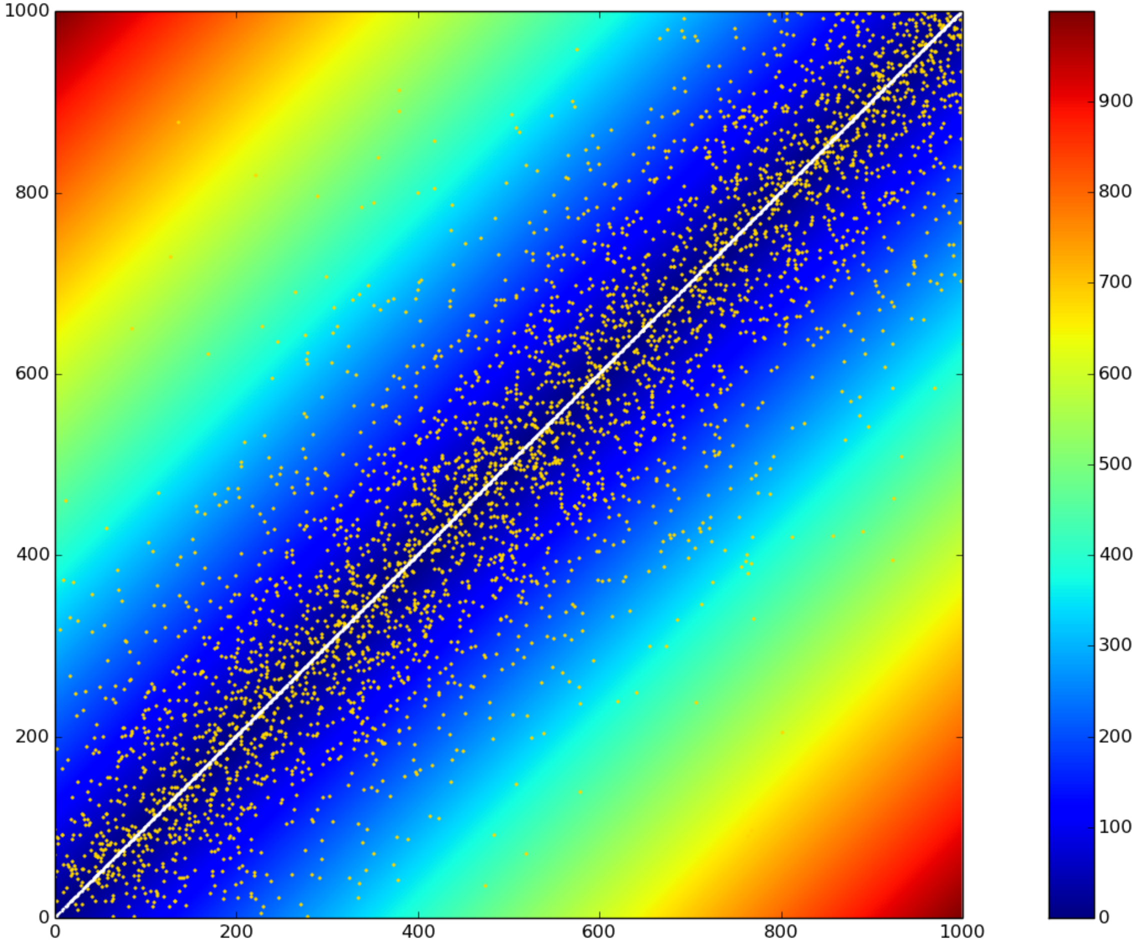


Heatmap of f with filament shown



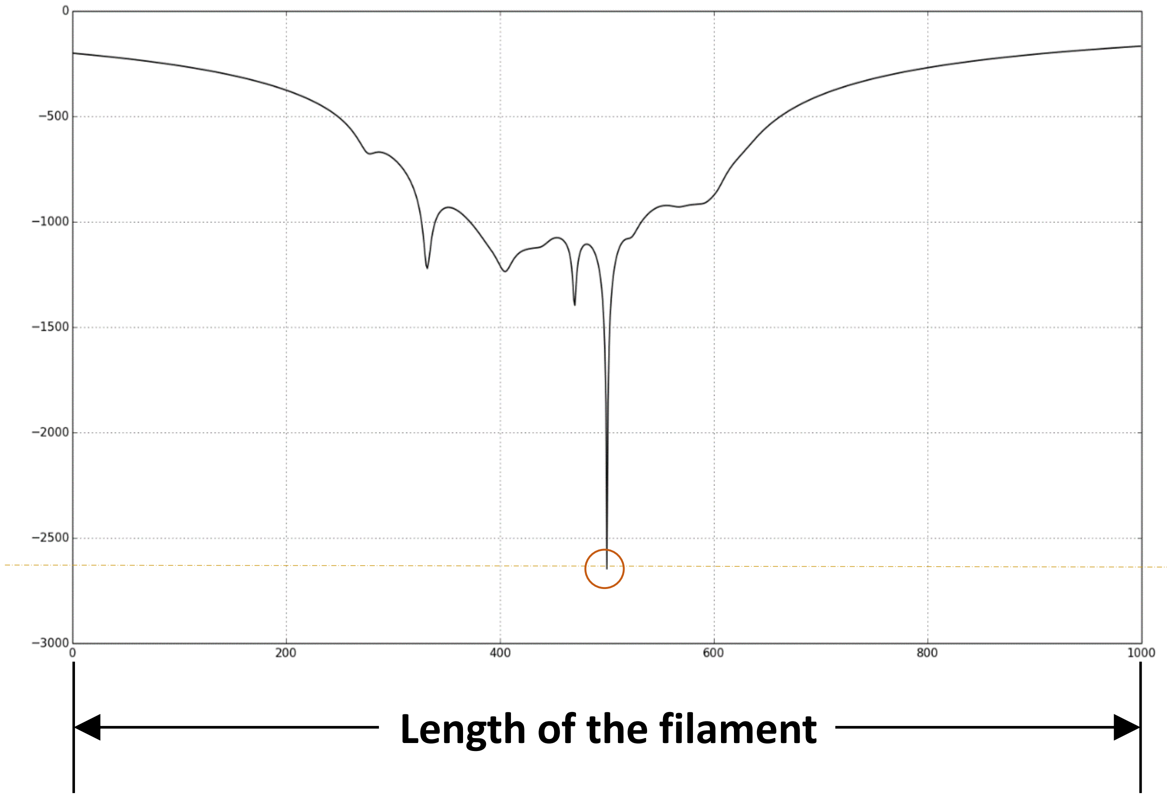
Testing our simple example

5,000 steps along the filament

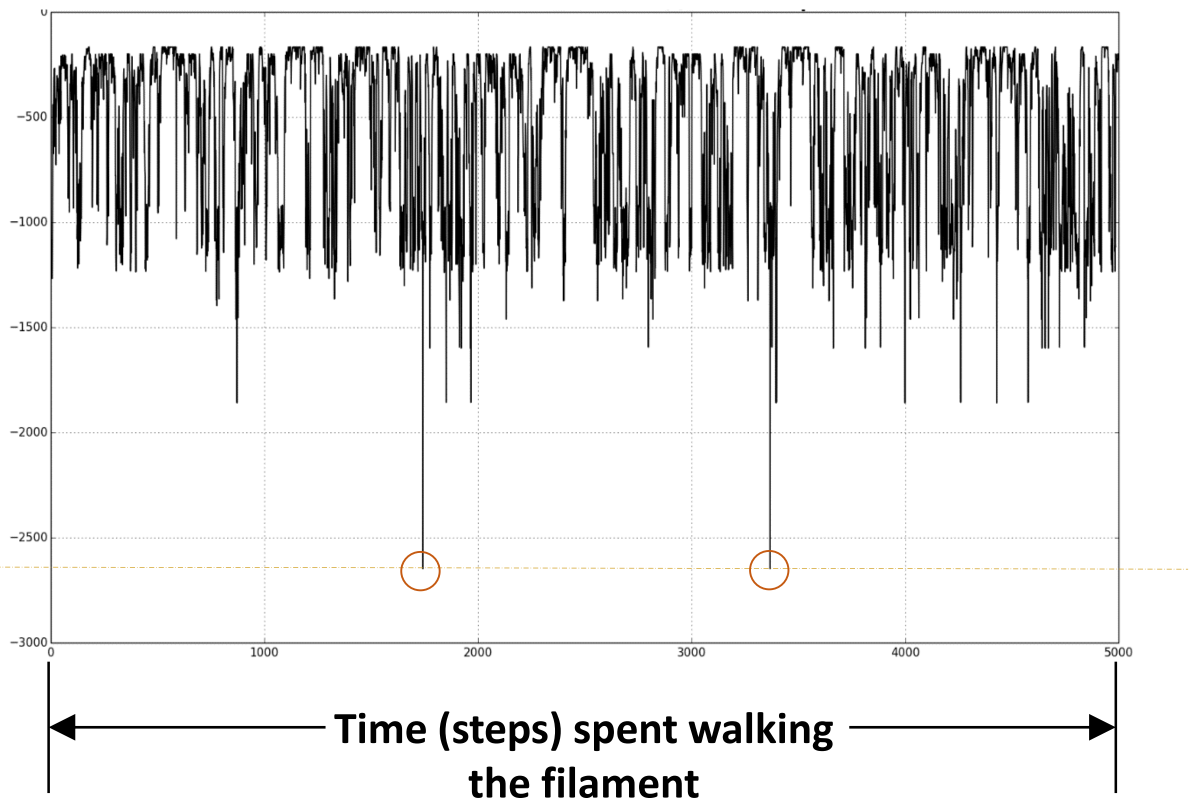


min(f($\mathbf{x}^+$)) **has been found (twice in 5,000 steps)**

f along the filament



f along the 5,000-step filament walk

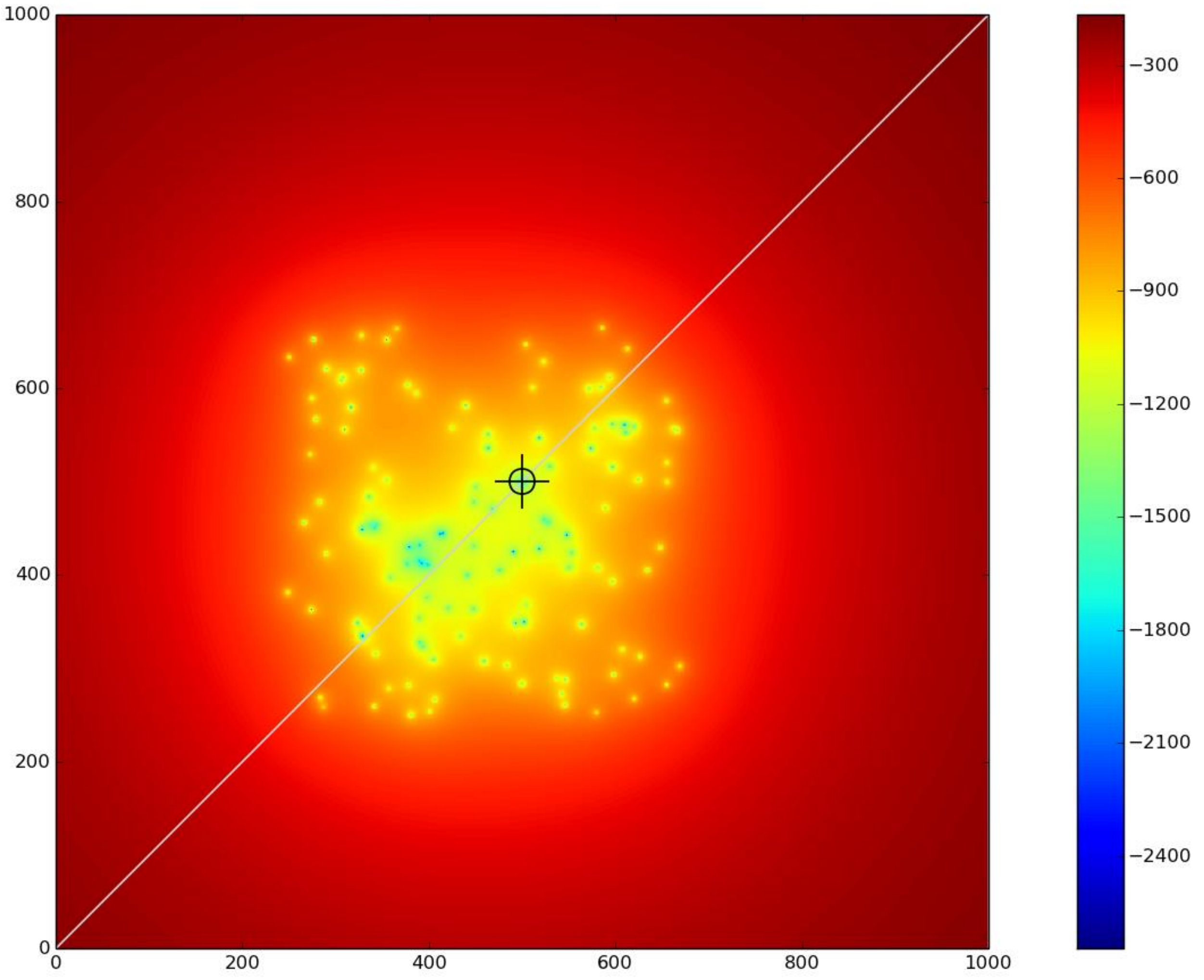


True (*a prior* known) min(f) = -2646.099011
True (*a prior* known) argmin_x(f) = 500 on x = (0, 1000]
True (*a prior* known) argmin_y(f) = 500 on y = (0, 1000]

min(f) along the filament found by the walker = -2646.099011
argmin(s)_t(f along the filament) = 1742 and 3251 on t = (0, 5000]
argmin_x(f along the filament) = 500 on x = (0, 1000]
argmin_y(f along the filament) = 500 on y = (0, 1000]

Success with the simple example

After 5,000 steps along the filament

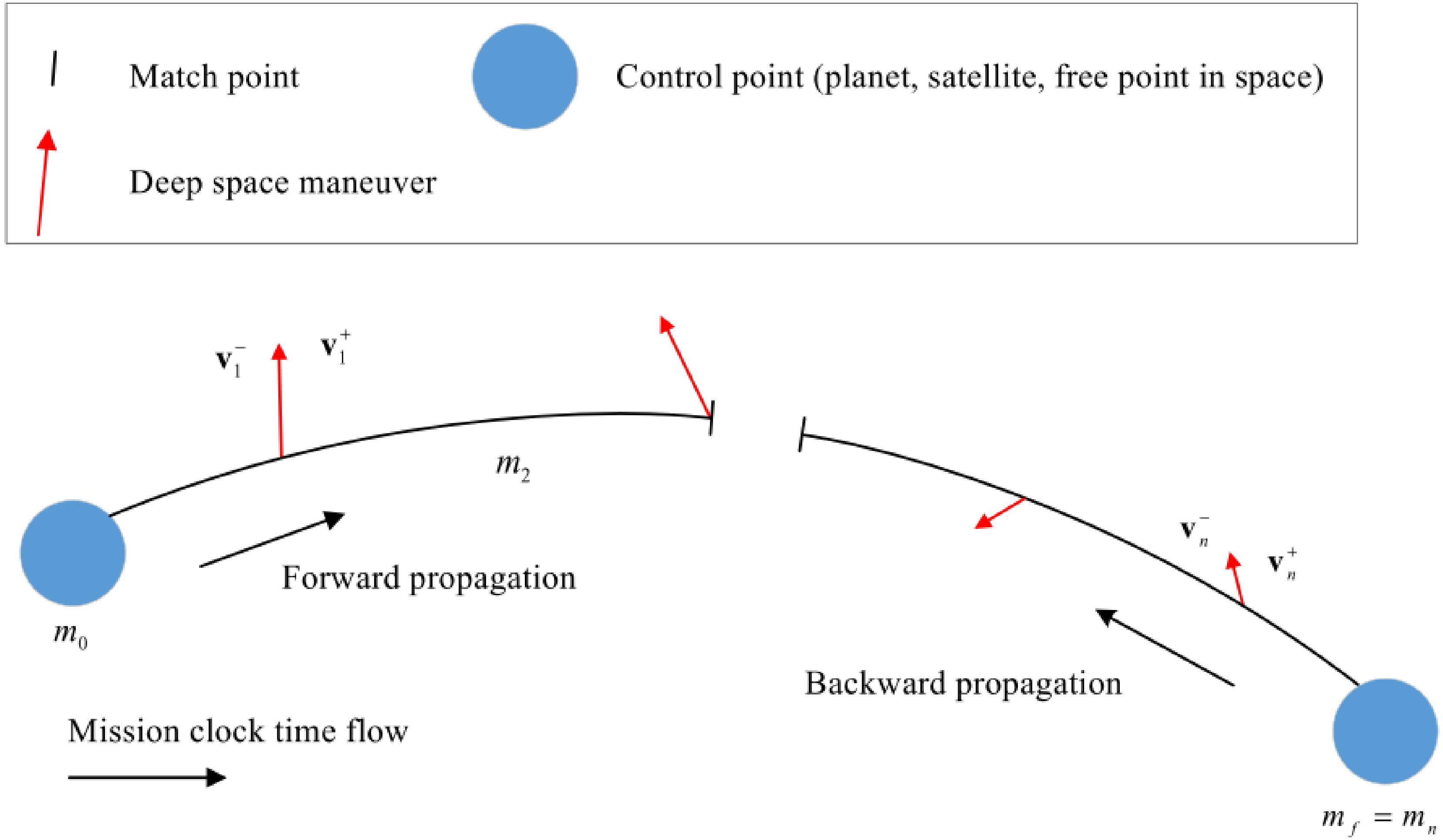


Applicability: Our initial concerns and what we found

- Do filaments really exist in important trajectory optimization problems? Yes
- Can they be found? Yes
- Are they walkable? Yes
- Are they few in number? So far, there at most a few
- Are they unique? Not always
- Are they sufficiently “long,” so as to be “walk-worthy” Yes, so far
- Are the gradients that “point” towards them sufficiently smooth that if the walker gets “close” to a filament, or falls not-too-far off of it, the gradients will “guide” it back to the filament? Apparently yes. (Much better than one of us thought)
- If filaments exist, can be found, are long, and are walkable, how will we visualize them? You will see in the next slides

Context for the real problem we are about to show:

Multiple Shooting Transcription for High-Thrust Chemical Propulsion



Courtesy of Donald Ellison and Matthew Vavrina

Earth to Mars Transfer Trajectory Example

Transcription	MGAnDSMs* [10]
Launch date bounds	2/16/2006 - 11/12/2008
Departure type	launch with $C_3 \leq 48.5\ km^2/s^2$
Arrival type	chemical rendezvous
Objective function	minimize Δv
Flight time upper bound	500 days
Known global optimum (from MBH)	2.343 km/s

**Multiple Gravity Assist with n Deep Space Maneuvers using shooting*

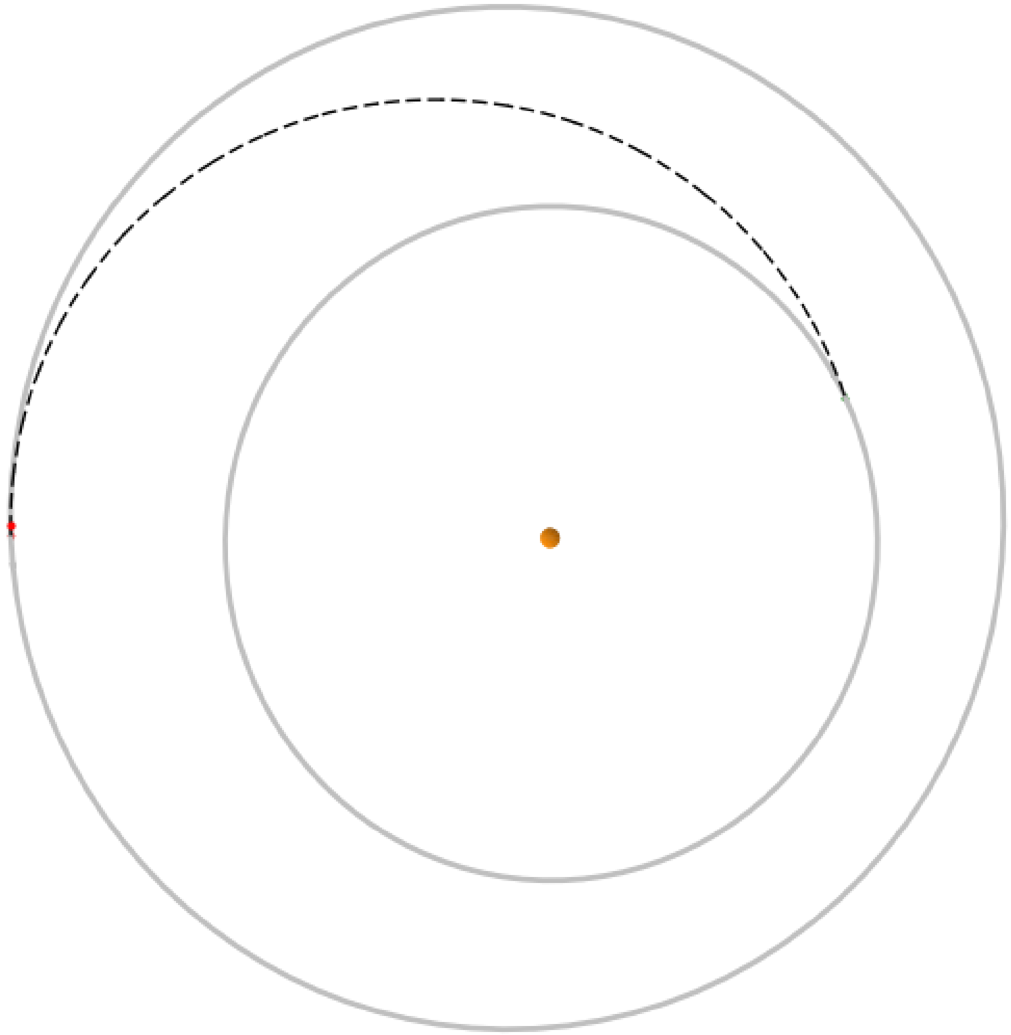
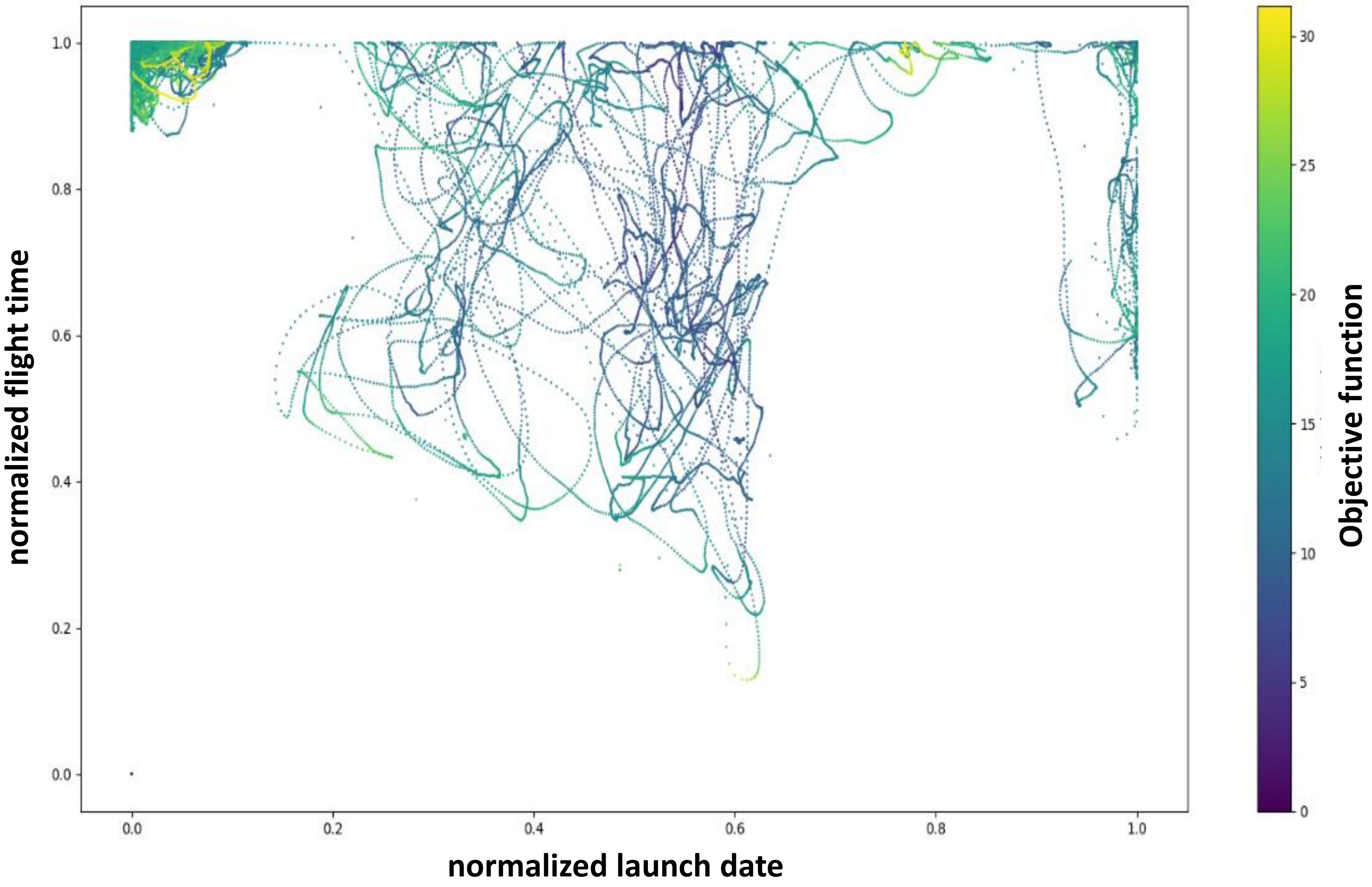


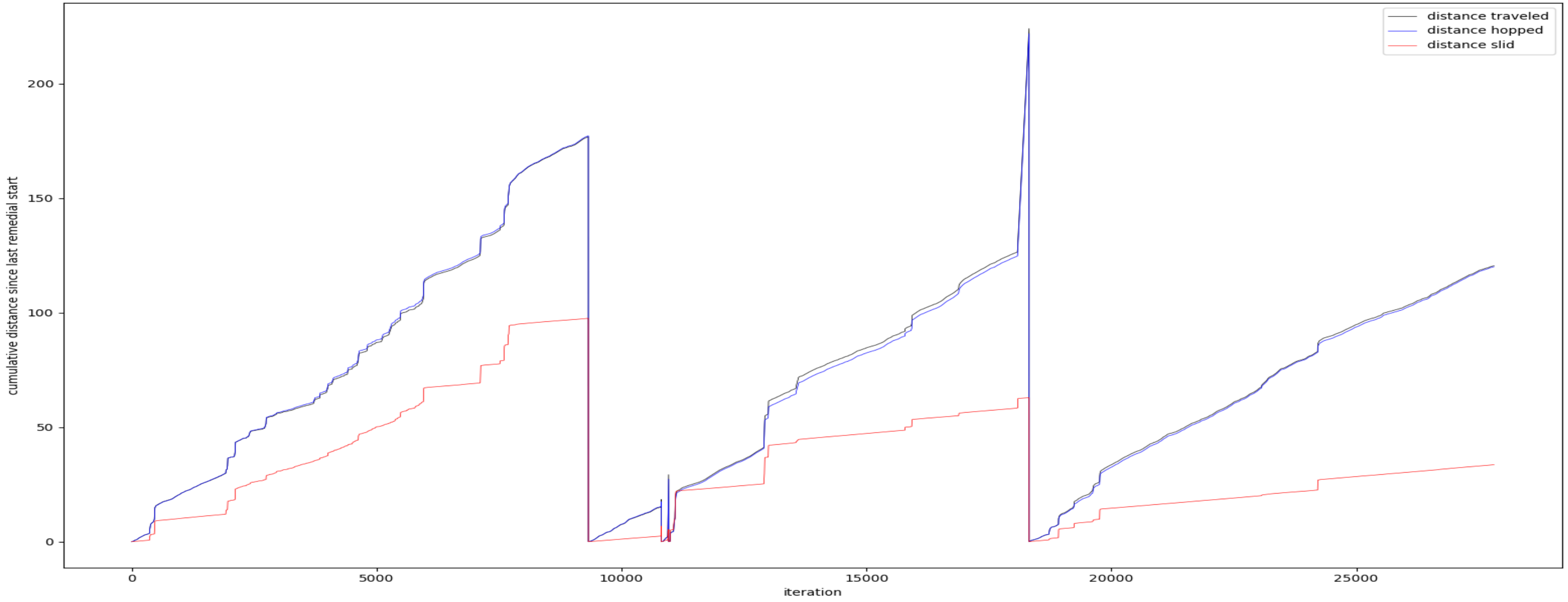
Image of a filament from the first real problem

- **A projection of a 13-dim space onto to two dimensions (normalized launch date versus normalized flight time)**

- **This is the filament walking equivalent of the so-called “pork-chop” plot**



Unraveling the filament
A distance-preserving homeomorphism of the walk



The above suggests:

- Only one filament exists
- It was walked reasonably thoroughly, mostly in a single direction
- Occasionally, the walker "turned around" (switched directions)
- When the walker fell off a filament, it was usually guided back, quickly, onto that (or another) filament by local gradients

Homeomorphism

Future work

- Can we assure that all of the filaments were be found “almost surely?”
- Can we assure that a walker has walked each found filament “thoroughly” enough to find min(f($\mathbf{x}^+$)) on that filament “almost surely?”
- Can we predict the number and “length” of filaments that should be to expected, based on the structure of the problem?
- Can we build a autonomous quantitative measures of confidence in whether the true global min(f($\mathbf{x}^+$)) has been found?
- Can one design transcriptions that are especially well-suited to filament walking?
- Benchmarking FW against MBH in terms of accuracy, speed, and reliability

Summary

- The concept and assumptions of FW appear to be valid and applicable.
- The "when to stop problem" is not yet solved but, for the first time, we have a promising framework in which to solve it.
- This is the first phase of a long journey.

Thank you

Arnold C. Englander

ace@aenglander-assocates.net

Jacob A. Englander

jacob.a.englander@nasa.gov

References

- **A. Bryson and Y. Ho**, *Applied Optimal Control*, Taylor and Francis, 1975.
- **M. Vavrina, J. Englander, and D. Ellison**, “Global Optimization of N-Manuever, High-Thrust Trajectories Using Direct Multiple Shooting,” AAS/AIAA Space Flight Mechanics Meeting, February 2016.
- **R.H. Leary,**. “Global Optimization on Funneling Landscapes”, Journal of Global Optimization, Vol. 18, No. 4, 2000, pp. 367-383, 10.1023/A:1026500301312.
- **M. Locatelli and F. Schoen**, “Efficient Algorithms for Large Scale Global Optimization: Lennard-Jones Clusters”, lj2002-rev. text;20/03/2003; 13:18.
- **P. Amengual, A. Allison, R. Toral and D. Abbott**, “Discrete-time ratchets, the Fokker-Planck equation and Parrondo’s paradox”, Proceedings of the Royal Society of London, A **460** (2004) 2269 – 2284
- **P. E. Gill, W. Murray, and M. A. Saunders**, “SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization,” *SIAM Rev.*, Vol. 47, No. 1, 2005, pp. 99 – 131.
- **J. A. Englander and A. C. Englander**, “Tuning Monotonic Basin Hopping: Improving the Efficiency of Stochastic Search as Applied to Low-Thrust Trajectory Optimization,” 24th International Symposium on Space Flight Dynamics (ISSFD), Laurel, MD, May 2014. Available as a NASA Technical Report at https://ntrs.nasa.gov/search.jsp?R=20140007521.