

***A Review of the Proposed  $K_{Isi}$  Offset-Secant Method for Size-Insensitive Linear-Elastic Fracture Toughness Evaluation***

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Recently proposed modifications to ASTM E399 would provide a new size-insensitive approach to analyzing the force-displacement test record. The proposed size-insensitive linear-elastic fracture toughness,  $K_{Isi}$ , targets a consistent 0.5mm crack extension for all specimen sizes by using an offset secant that is a function of the specimen ligament length. The  $K_{Isi}$  evaluation also removes the  $P_{max}/P_Q$  criterion and increases the allowable specimen deformation. These latter two changes allow more plasticity at the crack tip, prompting the review undertaken in this work to ensure the validity of this new interpretation of the force-displacement curve. This paper provides a brief review of the proposed  $K_{Isi}$  methodology and summarizes a finite element study into the effects of increased crack tip plasticity on the method given the allowance for additional specimen deformation. The study has two primary points of investigation: the effect of crack tip plasticity on compliance change in the force-displacement record and the continued validity of linear-elastic fracture mechanics to describe the crack front conditions. The analytical study illustrates that linear-elastic fracture mechanics assumptions remain valid at the increased deformation limit; however, the influence of plasticity on the compliance change in the test record is problematic. A proposed revision to the validity criteria for the  $K_{Isi}$  test method is briefly discussed.

## INTRODUCTION

To determine  $K_{Ic}$ , ASTM E399 [1] uses an offset secant to the force-displacement record to identify the load at which crack extension occurs during a test. This secant construction line, offset to 95% of the slope of the linear portion of the test record, corresponds to the compliance change when crack extension equals approximately 2% of the specimen's original crack length under the assumption that all compliance change is due to crack extension. When the material exhibits stable tearing with a rising R-curve—corresponding to a Type I curve per ASTM E399-12e3, Figure 7—this method provides a toughness result that is specimen size dependent.

Wallin [2] proposed modifications to E399 to provide a new approach to analyzing an ASTM E399 test record utilizing a secant construction line with an offset slope that is a function of the specimen size based on the remaining ligament,  $b_o$ , thereby minimizing the influence of specimen size on the test result. The proposed size-insensitive linear-elastic fracture toughness, recently labeled  $K_{Isi}$ , targets a consistent 0.5mm crack extension for all specimen sizes. The  $K_{Isi}$  method also increases the allowable specimen deformation, and removes the  $P_{max}/P_Q$  criterion. These latter two changes allow more plasticity at the crack tip before a test result is deemed invalid.

In the current version of E399, the deformation limit for ligament plasticity (also called the specimen size requirement) is expressed in terms of the ligament validity criterion as  $b_o \geq M_K (K/\sigma_{ys})^2$ , where  $K$  is the linear elastic stress intensity factor,  $\sigma_{ys}$  is the 0.2% offset engineering yield strength, and the constant  $M_K = 2.5$ . Wallin [2] proposed that the deformation limit be extended to allow higher deformation with  $M_K = 1.1$  based on an evaluation of a variety of data sets available at the time of that paper. To make these limiting measures of deformation more tangible, note that with  $M_K = b_o * \sigma_{ys}^2 / K^2$ ,  $M_K$  is simply the ratio of ligament length to plastic zone size ( $r_p$ ) with a proportional factor, such that  $M_K \approx 0.15 b_o / r_p$  [3]. Using this engineering estimate, at  $M_K = 2.5$ ,  $r_p$  is approximately 6% of  $b_o$ , and at  $M_K = 1.1$ ,  $r_p$  extends to 14% of  $b_o$ . When evaluating the proposed changes for standardization of  $K_{Isi}$ , concerns arose that with  $M_K = 1.1$ , the contribution of crack tip plasticity to compliance change in the test record may be sufficient to influence the test result. In the E399 test method, without unloading compliance checks during the test, it is not possible to distinguish compliance change due to crack extension from that due to plasticity.

This paper summarizes a finite element study with the following two objectives: first, evaluate the continued validity of linear-elastic fracture mechanics assumptions to describe the conditions at the crack tip using  $K$  at  $M_K = 1.1$  and, second, quantify the effects of crack tip plasticity on the compliance change in the force-displacement record and evaluate its influence on interpretation of the test record for identifying crack extension. If required, the analytical study should provide results to formulate appropriate validity criteria to avoid detrimental plasticity.

## FINITE ELEMENT ANALYSIS APPROACH

In this study, the common compact specimen (C(T)) with  $W = 50.8$  mm (2.0 inch),  $a/W = 0.5$ ,  $W/B = 4$ , and no side-grooves is used as the reference geometry. The C(T) analysis model was developed using the FEACrack [4] finite element modeling software and solved with WARP3D [5] v16.2.7. The model used a 1/4 symmetric mesh with 56863 nodes and 12305, 20-noded hex, small-strain elements. The crack tip was modeled using collapsed elements with untied duplicate nodes. The J-integral was calculated over 15 domains to gauge convergence while using the

WARP3D Type D domain evaluation for a bulk-average J-integral. Forces were applied at the center of the pin mesh, with pin rotation allowed, and elastic pin material. Figure 1 is an illustration of the finite element mesh.

The material constitutive model used incremental plasticity and the isotropic Mises flow rule. The stress-strain relationship was modeled using linear behavior up to the proportional limit followed by a power law relation for plastic strain. Expressed in a normalized form with the proportional limit,  $\sigma_o = 1$ , this allows easy evaluation of a broad material space with stiffness to proportional limit ratio varying  $100 \leq E / \sigma_o \leq 1000$  and strain hardening exponent varying  $3 \leq n \leq 20$  with Poisson's ratio,  $\nu = 0.3$ . The calculation of  $M_K$ , uses the engineering yield strength defined at 0.2% plastic strain. This material space covers practically all engineering alloys.

ASTM E399 allows a variety of specimens for determining linear-elastic fracture toughness. This study focuses on the C(T) specimen with the understanding that specimens of different geometry will need to be evaluated to quantify the effects of specimen geometry on plasticity-induced compliance change. The slender ligament aspect ratio,  $W/B = 4$ , is used to represent the E399 lower limit on specimen thickness and is the worst-case for allowing plasticity to influence the non-linearity in the force-displacement record.

#### ***The $K_{Isi}$ offset secant***

For this work, the change in compliance ( $\Delta C$ ) is defined as a percent increase in compliance (or percent decrease in slope) of the force-displacement record with respect to the initial linear portion, as defined by the first, fully-elastic analysis load step. See Figure 2. Consistent with Wallin [2], the offset compliance where fracture toughness is evaluated,  $\Delta C$ , is proposed to follow the convention yielding  $\Delta C$  in percent:

$$\Delta C = A/(W-a), \quad (1)$$

with  $A = 135$  for the C(T) specimen for dimensions in mm, which corresponds to crack extension of 0.5mm. For common E399 C(T) specimen sizes with  $a/W = 0.5$ :

$W = 25.4 \text{ mm (1 inch)}$	$\Delta C = 10.6\%$	$W = 50.8 \text{ mm (2 inch)}$	$\Delta C = 5.3\%$
$W = 63.5 \text{ mm (2.5 inch)}$	$\Delta C = 4.25\%$	$W = 101.6 \text{ mm (4 inch)}$	$\Delta C = 2.7\%$

For a  $K_{Ic}$  assessment per the E399 standard, the offset secant is fixed at  $\Delta C = 5\%$  for all specimens. Thus, to target 0.5mm crack extension, this corresponds to a C(T) with  $W = 54.0$  mm. At this specimen size, the  $K_{Ic}$  and  $K_{Isi}$  methods are equivalent. Note that the coefficient  $A$  listed here is different from that in Wallin [2] because in that case, the coefficient was developed for the ASTM E1820 [6] specimen that measures displacement at the load line, whereas in the current paper the coefficient is developed for specimens that measure displacement at the specimen front face.

In experiments for  $K_{Isi}$ , the change in compliance throughout a test is estimated as a summation of crack extension, plasticity, and experimental error:

$$\Delta C = \Delta C_{crack \text{ ext}} + \Delta C_{plasticity} + \Delta C_{experimental \text{ error}} \quad (2)$$

Currently, experimental methods for  $K_{Isi}$  assume  $\Delta C_{plasticity}$  and  $\Delta C_{experimental\ error}$  are sufficiently small relative to  $\Delta C_{crack\ ext}$  such that the offset secant will properly identify crack extension of 0.5mm. The magnitude of  $\Delta C_{plasticity}$  is easily estimated in the finite element study where the crack length is fixed at  $a/W = 0.5$ , such that  $\Delta C_{crack\ ext} \equiv 0$ . This study does not provide insight into the magnitude of  $\Delta C_{experimental\ error}$  though this could be of importance as specimen size increases and the offset secant for  $K_{Isi}$  gets smaller.

Though not shown herein for brevity, the authors demonstrate that all calculations related to compliance change and deformation are fully scalable with geometry, thus one model provides results for all size specimens of the same proportion.

## RESULTS

The main result of the analytical work is represented in a plot of change in compliance,  $\Delta C$ , versus specimen deformation,  $M_K = b_o * \sigma_{ys}^2 / K^2 \approx 0.15 b_o / r_p$ . Figure 3a illustrates this plot and its interpretation. The abscissa axis,  $M_K$ , is plotted log scale and reversed, left to right—decreasing values of  $M_K$  correspond to increasing force and deformation. The green box indicates the compliance and deformation requirements for  $K_{Ic}$  fixed at  $\Delta C = 5\%$  and  $M_K = 2.5$ , respectively. The solid line marked with open circles represents a typical result from the analysis, in this case representative of aluminum alloy 2219-T8. Recall the analysis reflects only the  $\Delta C$  contribution from plasticity—compliance increases non-linearly with plastic zone size. In an experiment, the measured  $\Delta C$  includes all sources as considered in Eq 2, though for the current discussion, we will consider experimental error contributions negligible. The dashed line in Figure 3a represents a typical experimental result, in this case, one that is valid according to the deformation limit for  $K_{Ic}$  per E399, i.e.,  $M_K \geq 2.5$ . Prior to the onset of crack extension,  $\Delta C_{plasticity}$  is the only source of compliance change. Once crack extension begins,  $\Delta C_{crack\ ext}$  contributes strongly to the total  $\Delta C$ . In this example, the  $K_{Ic}$  test is valid for deformation because  $\Delta C = 5\%$  is reached while  $M_K \geq 2.5$ . The pertinent detail is that, in this example, nearly 2/5<sup>th</sup> of  $\Delta C$  is due to plasticity, not crack extension.

Figure 3b illustrates the key findings of the analysis effort needed to answer two questions regarding the consequence of allowing the deformation measure to extend to  $M_K = 1.1$  as first proposed for  $K_{Isi}$ : 1) Does the increased deformation invalidate LEFM assumptions for the use of  $K_I$ ; and 2) Can plasticity alone create sufficient  $\Delta C$  to reach the  $K_{Isi}$  limit before crack extension begins? Either case is undesirable, leading to a non-relevant underestimate of linear-elastic toughness. In Figure 3b, the analysis result of  $\Delta C_{plasticity}$  remains shown by the solid line with open circles. The contribution to  $K_I$  from plasticity ( $K_{J-plastic}$ , given in percent) is shown by the curve marked with solid circles and the proposed deformation limit of  $M_K = 1.1$  is shown by the vertical dotted line. Recall that analysis results plotted here are examples representative of Al 2219-T8 material; however, the findings are fully supported by results from the larger material space.

The  $K_{J-plastic}$  result answers question 1. In this example,  $K_{J-plastic} = 4.4\%$  at  $M_K = 1.1$ . In the full material space of the analysis set, at  $M_K = 1.1$ ,  $K_{J-plastic}$  ranges from 3% and 6.5%. In the LEFM, E399 test method, the plastic contribution to fracture energy is not captured ( $K_I$  computed from only force, not absorbed energy in the specimen). The LEFM toughness measurements that ignore the small plastic contribution are low, or conservative, by this percentage. This range of

conservative error would generally not be grounds to consider the result invalid for LEFM assumptions; therefore, the use of  $M_K = 1.1$  appropriately maintains LEFM assumptions.

The  $\Delta C_{\text{plasticity}}$  result answers question 2. The  $\Delta C$  limits for  $K_{Isi}$  are shown for various size specimens by horizontal dashed lines in Figure 3b. It is clear that the  $\Delta C_{\text{plasticity}}$  result crosses many of these lines before reaching  $M_K = 1.1$ . In the example given by Figure 3b, only the smallest specimen of  $W = 25$  mm avoids having  $\Delta C_{\text{plasticity}}$  cause the test record cross the  $K_{Isi}$  offset secant prior to  $M_K = 1.1$ . If non-linearity in the test record due to  $\Delta C_{\text{plasticity}}$  causes the test record to cross the  $K_{Isi}$  offset secant prior to crack extension, the result is not a measurement of toughness, but merely a measure of yielding in the specimen. Additional limits on specimen deformation are needed to control plasticity to ensure crack extension has occurred at the secant crossing. The proposed remedy is to make  $M_K$  a function of specimen size, for example,  $M_K \geq b_o/12.5\text{mm}$ , which is equivalent to a fixed maximum size for  $r_p$ . Using  $M_K \approx 0.15b_o/r_p$ , this criterion equates approximately to  $r_p \leq 1.9$  mm for all specimen sizes.

## CONCLUSIONS

This body of work has provided a concise and convenient way to visualize and quantify the relationship between compliance change in the C(T) specimen due to plasticity and the deformation level in the specimen. By using non-dimensional metrics, the conclusions from the single analysis are valid for all specimen sizes of identical proportion. Though not detailed herein, the material space covered by the analysis allows the evaluation to extend to all practical engineering alloys. Two key conclusions were developed:

1. An increase in the allowable limit for deformation to  $M_K = 1.1$  would not invalidate LEFM assumptions, rendering only small conservative errors in the evaluation of  $K$ .
2. An increase in the allowable limit for deformation to  $M_K = 1.1$  would create sufficient compliance change in the force-displacement record due to plasticity to result in potential misidentification of toughness prior to crack extension for specimens larger than approximately  $W = 25\text{mm}$ . The proposed remedy is to make  $M_K$  a function of specimen size, e.g.  $M_K \geq b_o / 12.5\text{mm}$ , which equates to a fixed maximum plastic zone size for all specimens.

## REFERENCES

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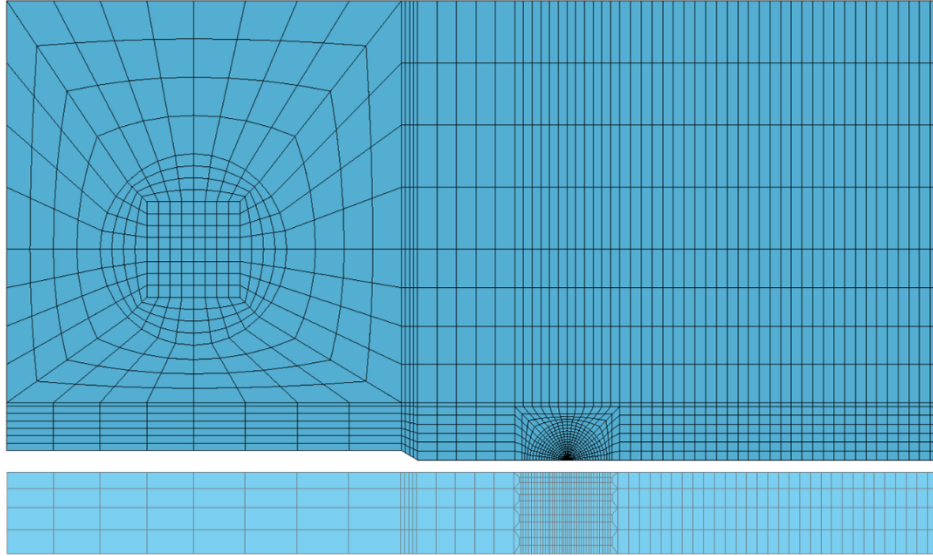


Figure 1. Finite element C(T) mesh exploiting 1/4 symmetry (side and fracture-plane views).

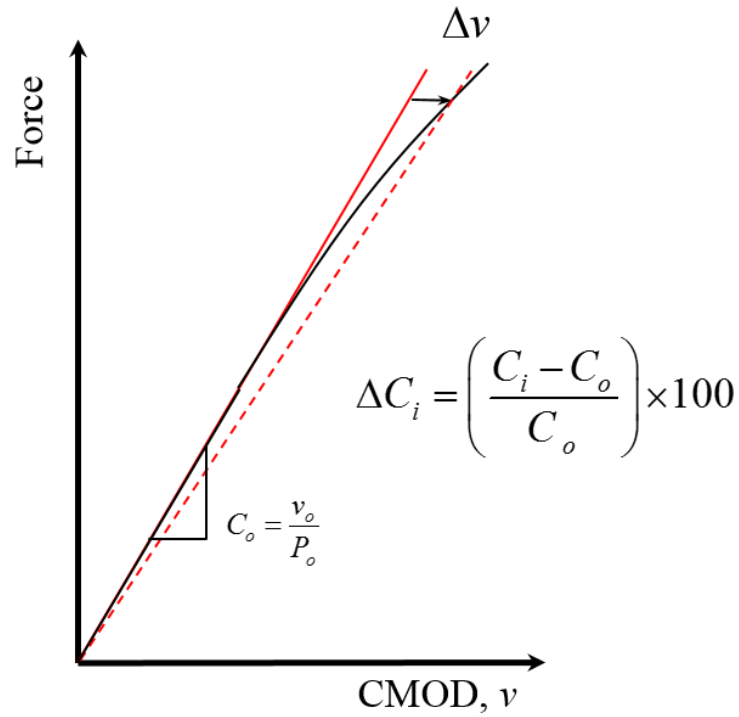


Figure 2. The change in compliance ( $\Delta C$ ) is defined as a percent increase in compliance (or percent decrease in slope) of the force versus displacement trace with respect to the initial linear portion as defined by the first analysis load step.

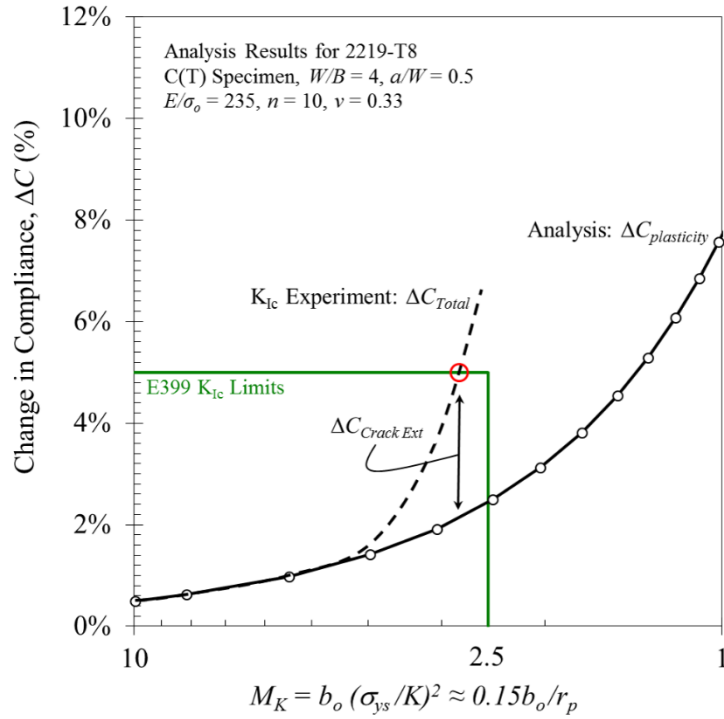


Figure 3a. Percent change in compliance versus normalized loading level, illustrating sources of compliance change.

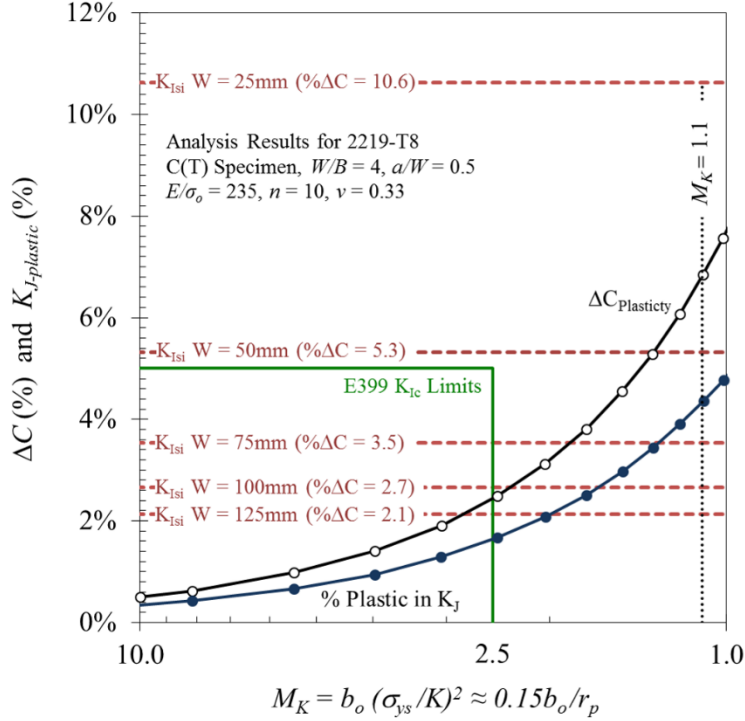


Figure 3b. Percent change in compliance and percent plastic contribution to  $K_I$  versus normalized loading level, illustrating issues with  $\Delta C_{\text{plasticity}}$  at high deformation.