Paper Number: **145**

Title: **Effects of Adiabatic Heating on the High Strain Rate Deformation of Polymer Matrix Composites**

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ABSTRACT

Polymer matrix composites (PMCs) are increasingly being used in aerospace structures that are expected to experience complex dynamic loading conditions throughout their lifetime. As such, a detailed understanding of the high strain rate behavior of the constituents, particularly the strain rate, temperature, and pressure dependent polymer matrix, is paramount. In this paper, preliminary efforts in modeling experimentally observed temperature rises due to plastic deformation in PMCs subjected to dynamic loading are presented. To this end, an existing isothermal viscoplastic polymer constitutive formulation is extended to model adiabatic conditions by incorporating temperature dependent elastic properties and modifying the components of the inelastic strain rate tensor to explicitly depend on temperature. It is demonstrated that the modified polymer constitutive model is capable of capturing strain rate and temperature dependent yield as well as thermal softening associated with the conversion of plastic work to heat at high rates of strain. The modified constitutive model is then embedded within a strength of materials based micromechanics framework to investigate the manifestation of matrix thermal softening, due to the conversion of plastic work to heat, on the high strain rate response of a T700/Epon 862 (T700/E862) unidirectional composite. Adiabatic model predictions for high strain rate composite longitudinal tensile, transverse tensile, and in-plane shear loading are presented. Results show a substantial deviation from isothermal conditions; significant thermal softening is observed for matrix dominated deformation modes (transverse tension and in-plane shear), highlighting the importance of accounting for the conversion of plastic work to heat in the polymer matrix in the high strain rate analysis of PMC structures.

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INTRODUCTION

Polymer matrix composites (PMCs) are increasingly being used in the aerospace industry for high velocity ballistic impact applications. One such application is jet engine fan blade containment systems, which are required by the FAA to completely contain loose fan blades in the event of a blade-out. To demonstrate the capability for engine cases to contain loose fan blades and certify them for commercial use, manufacturers must perform destructive testing, which is time consuming and expensive. Therefore, to reduce the amount of destructive testing necessary to certify fan blade containment systems, accurate modeling of the high strain rate deformation response of PMCs is of utmost importance.

Despite the benefits offered by PMCs, including superior specific strength and stiffness, as well as the ability to tailor the mechanical properties to specific applications, the development of predictive computational models for high strain rate loading conditions is complicated by the material heterogeneity and anisotropy, as well as the well-known strain rate, temperature, and pressure dependence of the polymer matrix constituent. Moreover, as the rate of deformation increases, the conditions transition from isothermal to adiabatic. In the case of high strain rate ballistic impact events, heat is generated locally within the material due to the conversion of plastic work to heat. However, the extremely short duration of impact events does not allow sufficient time for heat to conduct away from regions of localized plastic deformation within the material.

The work of Chou et al. [1] is largely considered to be the first attempt at measuring heat generation due to mechanical deformation of polymers; four hard plastics were tested across a range of strain rates and the corresponding temperature rises were measured using thermocouples. Rittell [2] used embedded thermocouples to investigate temperature rises due to plastic deformation of polycarbonate at high strain rates, reporting temperature rises up to 40°C. Garg et al. [3] used infrared imaging techniques to measure temperature rises in an amorphous thermoplastic and a thermoset epoxy during high strain rate compression. Infrared imaging of preliminary impact experiments conducted by Johnston et al. [4] on triaxially braided composite panels with a toughened thermoset polymer matrix have shown local temperature rises above the glass transition temperature of the matrix constituent. These temperature rises cause thermal softening of the matrix and subsequent deformation localization. Thus, in order to develop predictive models for the high strain rate response of PMCs, it is necessary to accurately model heat generation in the polymer matrix due to the conversion of plastic work to heat.

In this work, a strain rate and pressure dependent viscoplastic polymer constitutive model [5] is further extended to nonisothermal conditions to model adiabatic heating due to high strain rate plastic deformation. The ability of the modified constitutive model to capture the thermal softening associated with plastically deforming polymers at high rates of strain is demonstrated by simulating the tensile and shear response of Epon 862 (E862) thermoset epoxy resin over a range of strain rates and temperatures. The modified constitutive model is then implemented within a strength of materials based micromechanics framework to study the effects of adiabatic heating on the high strain rate response of a T700/E862 unidirectional composite. The micromechanics model is suitable for implementation in a finite element software as a user-defined material subroutine, whereby it can be invoked at each macroscale

integration point in the finite element mesh. The modified constitutive model is shown to be able to capture the thermal softening associated with plastically deforming polymers at high rates of strain as well as capture the effects of adiabatic heating on the high strain rate response of a T700/E862 unidirectional composite.

POLYMER CONSTITUTIVE MODEL

The unified state variable viscoplastic constitutive model, originally developed by Bodner and Partom [6] to model the strain rate dependent deformation response of metals above one half of their melting temperature, was modified by Goldberg et al. [5] to include hydrostatic stress effects, which can be significant in polymers. The model is unified in the sense that there is no distinction between creep strain and plastic strain [7]; instead, a single (unified) inelastic strain that is assumed to be very small during elastic deformation, is used for all levels of stress. Additionally, the model does not require a defined yield stress; strain rate dependent yield is captured by the evolution of state variables with the effective deviatoric inelastic strain rate. A brief overview of this model is as follows. An inelastic potential function is defined as

$$f = \sqrt{J_2} + \alpha\sigma_{kk} \tag{1}$$

based on the Drucker-Prager yield criterion, where J_2 is the second invariant of the deviatoric stress tensor, σ_{kk} is the trace of the Cauchy stress tensor, and α is a time-dependent state variable that controls the influence of hydrostatic stress effects. An associative flow rule is used, where the components of the inelastic strain rate tensor, $\dot{\epsilon}_{ij}^{I}$, are assumed to be equal to the product of the partial derivative of the inelastic potential function, f, with respect to the components of the Cauchy stress tensor, σ_{ij}, and the scalar rate of the plastic multiplier, $\dot{\lambda}$, as follows:

$$\dot{\varepsilon}_{ij} = \dot{\lambda}\frac{\delta f}{\delta\sigma_{ij}}. \tag{2}$$

To determine the final form of the expression for the components of the inelastic strain rate tensor, $\dot{\varepsilon}_{ij}$, the partial derivative of the inelastic potential function (Equation 1) with respect to the components of the Cauchy stress tensor is evaluated and substituted into Equation 2. By taking the tensor product of the resulting expression with itself, solving for the rate of the plastic multiplier, substituting it back into Equation 2, and defining the effective stress, σ_e, and effective deviatoric inelastic strain rate, $\dot{e}_e^I$, respectively, as

$$\sigma_e = \sqrt{3}f = \sqrt{3J_2} + \sqrt{3}\alpha\sigma_{kk} \tag{3}$$

$$\dot{\varepsilon}_e^I = \dot{e}_e^I = \sqrt{\frac{2}{3}\dot{e}_{ij}^I\dot{e}_{ij}^I} \tag{4}$$

$$\dot{e}_{ij}^I = \dot{\varepsilon}_{ij}^I - \dot{\varepsilon}_m^I\delta_{ij}, \tag{5}$$

where $\dot{\varepsilon}_m^I$ is the trace of the inelastic strain rate tensor, $\dot{\varepsilon}_e^I$ is the effective inelastic strain rate, and δ_{ij} is the Kronecker delta, an expression for the rate of the plastic multiplier is obtained. By defining

$$\frac{\sqrt{3}}{2}\dot{\varepsilon}_e^I = D_0 exp\left[-\frac{1}{2}\left(\frac{Z}{\sigma_e}\right)^{2n}\right] \tag{6}$$

based on the work of Bodner and Partom [6] & Stouffer and Dame [8], substituting the rate of the plastic multiplier back into Equation 2, and simplifying, the final form of the expression for the components of the inelastic strain rate tensor are obtained as

$$\dot{\varepsilon}_{ij}^I = 2D_0 exp\left[-\frac{1}{2}\left(\frac{Z}{\sigma_e}\right)^{2n}\right]\left(\frac{S_{ij}}{2\sqrt{J_2}} + \alpha\delta_{ij}\right). \tag{7}$$

In Equations 6 and 7, n is a state variable that controls strain rate sensitivity, S_{ij} are the components of the deviatoric stress tensor, Z is a state variable that represents the resistance to internal stress (captures strain hardening), and D_0 is a state variable that represents the maximum inelastic strain rate. The time-dependent state variables, Z and α, evolve with the effective deviatoric inelastic strain rate as follows:

$$\dot{Z} = q(Z_1 - Z)\,\dot{e}_e^I \tag{8}$$

$$\dot{\alpha} = q(\alpha_1 - \alpha)\,\dot{e}_e^I, \tag{9}$$

where Z_1 and α_1 are the final (maximum) values of Z and α, respectively, and q is a constant that represents the strain hardening rate. Note the initial values of Z and α are denoted as Z_0 and α_0, respectively. The model requires the determination of seven state variables, D_0, n, Z_0, Z_1, q, α_0, and α_1. Further details of the model can be found in Goldberg et al. [5].

It is important to note that the polymer constitutive model in its current state is isothermal. However, as aforementioned, many researchers [1-4] have demonstrated that a significant amount of heat is generated in plastically deforming polymers at high strain rates, leading to thermal softening. Therefore, explicitly modeling heat generation due to high strain rate plastic deformation is necessary to accurately predict the behavior of polymers and PMC structures subjected to dynamic loading conditions.

Temperature Effects

To account for the temperature dependence of plastic flow, the components of the inelastic strain rate tensor are now modified to explicitly capture temperature-dependence based on the Arrhenius equation for nonisothermal processes

$$\dot{\varepsilon}\ \alpha\ exp(\frac{-Q}{KT}), \tag{10}$$

where Q is the activation energy, K is Boltzmann's constant, and T is the absolute temperature. By inserting the dimensionless expression $(\frac{-Q}{KT})$ into the exponential term in the original expression for the components of the inelastic strain rate and defining a new state variable $\bar{Z}$ as

$$\bar{Z} = \frac{QZ}{K}, \tag{11}$$

the new temperature-dependent components of the inelastic strain rate tensor can be expressed as

$$\dot{\epsilon}_{ij}^{I} = 2D_0 \exp\left(\frac{-1}{2}\left(\frac{\bar{Z}}{T\sigma_e}\right)^{2n}\right)\left(\frac{S_{ij}}{2\sqrt{J_2}} + \alpha\delta_{ij}\right), \tag{12}$$

where $\bar{Z}$ is a temperature-dependent state variable that controls the resistance to internal stress *at a given temperature*, and the other parameters are the same as defined previously. Following the work of Bhattachar and Stouffer [9] it is assumed that $\bar{Z}$ evolves in the same way as Z, that is,

$$\dot{\bar{Z}} = q(\bar{Z}_1 - \bar{Z})\dot{e}_e^I, \tag{13a}$$

which can be integrated in time to yield

$$\bar{Z} = \bar{Z}_1 - (\bar{Z}_1 - \bar{Z}_0)\exp(-qe_e^I), \tag{13b}$$

where $\bar{Z}_0$ and $\bar{Z}_1$ are the temperature-dependent initial and final values of $\bar{Z}$, respectively, and q is the hardening rate. Under isothermal conditions, Equations 13a and 13b are identical. However, under nonisothermal conditions, Equation 13b should be used in lieu of Equation 13a to allow the value of $\bar{Z}_0$ to change with temperature; if the differential form is used, $\bar{Z}$ is initially equal to $\bar{Z}_0$ at the *initial temperature*, but is never updated to reflect the current temperature. It should be noted that the dimension of the hardening state variable Z in the original model [5] is stress whereas now, after the introduction of the absolute temperature in Equation 12, the dimension is stress times absolute (Kelvin) temperature.

In order to determine the values of the state variables, the procedure outlined in Goldberg et al. [5] is utilized. The procedure is based on isothermal test data at different temperatures; once the temperature-dependent state variables are known at multiple temperatures they can be expressed as a function of temperature.

Conversion of Plastic Work to Heat

The heat energy equation, which expresses the relationship between mechanical deformation and spatial-temporal temperature change, is as follows

$$k\nabla^2 T - \alpha_M(3\lambda + 2\mu)T\dot{\varepsilon}_{kk}^e + \beta\boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}^{\boldsymbol{I}} = \rho C\dot{T}, \tag{16}$$

where k is the thermal conductivity, T is the temperature, α_M is the coefficient of thermal expansion, λ and μ are Lame's constants, ε_{kk}^{e} is the elastic volumetric strain, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\boldsymbol{\varepsilon}^{I}$ is the inelastic strain tensor, ρ is the density, C is the specific heat, and β is the inelastic heat fraction, which represents the proportion of plastic work converted to heat. The dot superscript denotes a time derivative. For adiabatic conditions, the thermoelastic and conduction terms in Equation 1 can be ignored [2, 10, 11] and the heat energy equation reduces to

$$\beta\boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}^{\boldsymbol{I}} = \rho C\dot{T}. \tag{17}$$

Assuming that the parameter β is known, either measured experimentally or assumed, Equation 2 can be integrated in time to compute the temperature change due to the conversion of plastic work to heat at each timestep in an incremental solution procedure.

Temperature Dependence of Elastic Properties

A novel time-temperature shifting methodology, similar to the Decompose-Shift-Reconstruct (DSR) method originally developed by Mulliken and Boyce [12], is utilized to compute the temperature and strain rate dependent shifts in elastic moduli based on dynamic mechanical analysis (DMA) tests conducted on a neat resin at various frequencies. Since the frequency at which a DMA test is conducted corresponds to a particular strain rate (depending on the specimen geometry), the rate-dependent shifts in the shear modulus can be obtained by conducting DMA tests at different strain rates, assuming that the shear modulus is equal to the shear storage modulus. A schematic of the shifting of the shear storage modulus versus temperature curve with strain rate is shown in Figure 1 for E862 DMA data [13]. The shifting of the DMA data allows the elastic properties to be determined at various strain rates (can also be extrapolated to higher/lower strain rate values than those at which the DMA tests were conducted) and temperatures. In this work, the DMA shear storage modulus versus temperature curve was taken to shift by 10K per decade strain rate for the Epon 862 resin. It should be noted that in this work, Poisson's ratio has been assumed to be independent of strain rate and temperature [10].

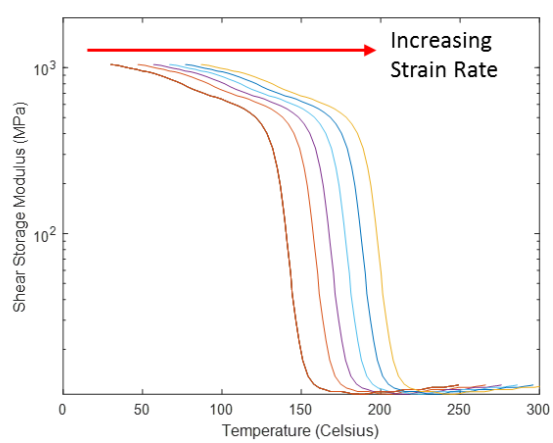


Figure 1: Illustration of shifting of DMA data with increasing strain rate

MICROMECHANICAL MODELING

To predict the effective behavior of a unidirectional composite, a MATLAB code has been developed in which the modified constitutive model is implemented within a strength of materials based micromechanics framework [14]. In the micromechanics model, the microscale repeating unit cell (RUC) consists of four subcells: three matrix and one fiber. A schematic of the microscale RUC is shown in Figure 2. By applying appropriate uniform stress and uniform strain assumptions, as well as displacement and traction continuity conditions between adjacent subcells and adjacent unit cells, the effective response of a unidirectional composite can be predicted based on the behavior of the individual constituents. The formulation used in this work is based on the reformulated Generalized Method of Cells (GMC) described in references [15] and [16].

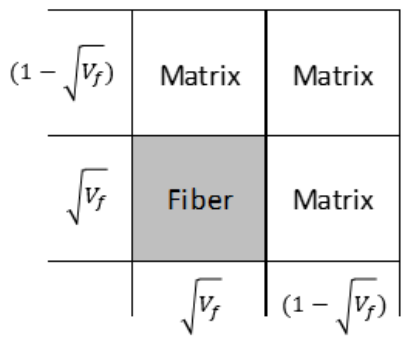


Figure 2: Schematic of four subcell repeating unit cell for micromechanics analysis

RESULTS

Monolithic Polymer Response

To demonstrate the capability of the modified constitutive model, the tensile and shear deformation response of the Epon 862 resin are simulated across a range of strain rates and temperatures. The resin material properties and model parameters used in the simulations are presented in Table I. The modified constitutive model is calibrated against available experimental data [13, 17]. Note that the inelastic heat fraction has been set to zero in these simulations (no adiabatic heating) due to the fact that many of the tests were conducted at low strain rates, where the adiabatic heating due to plastic straining can be assumed to be small, and the available tests at high strain rates failed at relatively low strains, where the material response displayed minimal nonlinearity, and thus likely limited adiabatic heating due to plastic deformation. Due to the minimal adiabatic heating that is likely present in these tests, the assumption can be made that the majority of the post-peak strain softening observed in the low strain rate test data is intrinsic strain softening, which is not currently modeled. Figure 3 shows the shear stress-strain response of Epon 862 epoxy resin at room temperature (30°C) for various strain rates whereas Figures 4a, 4b, and 4c show the tensile stress-strain response at room temperature, 50°C, and 80°C, respectively, for various strain rates. Reasonably

good correlations with experimental data are observed for all the strain rates and temperatures for both tensile and shear loading.

Table I: E862 Material Properties and Goldberg Model Parameters

Young's Modulus	*Taken from DMA curve (Figure 1)*
Poisson's Ratio	0.4
Density (g/m^3)	1.2e6
CTE (1/K)	5.4e-5
D_0	1e6
n	0.6454
q	75.4973
α_0	0.05
α_1	0.075
$Z_0(T)$ (Pa-K)	$-(1.526e9)T + 6.8897e11$
$Z_1(T)$ (Pa-K)	$-(2.7062)T + 1.3332e12$

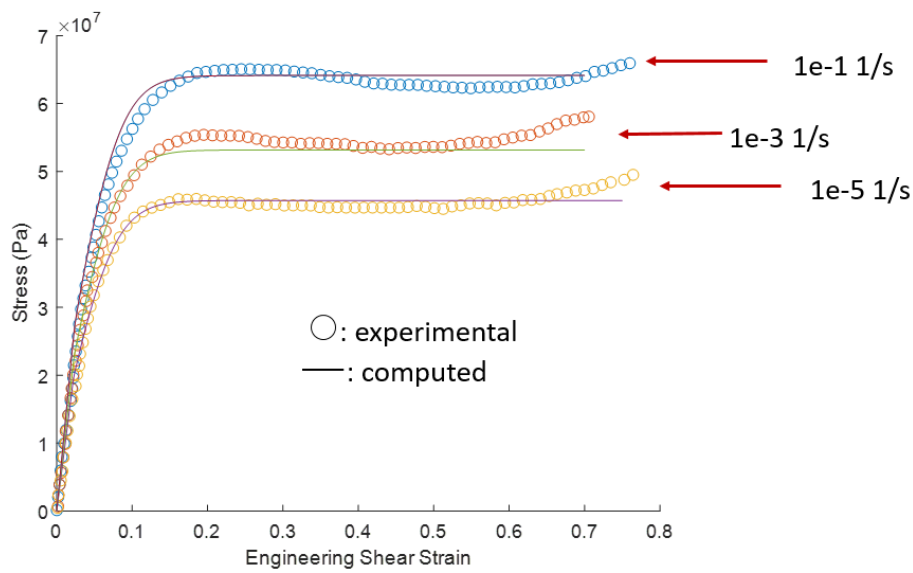
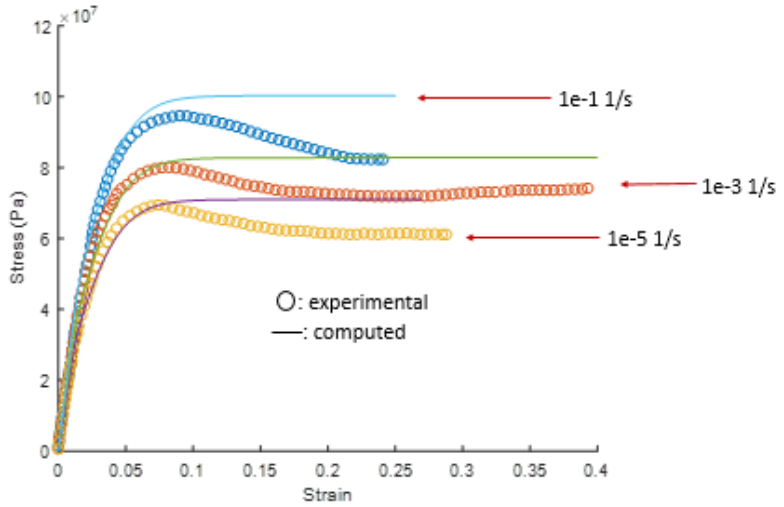


Figure 3: Shear stress-strain response of Epon 862 resin at various strain rates



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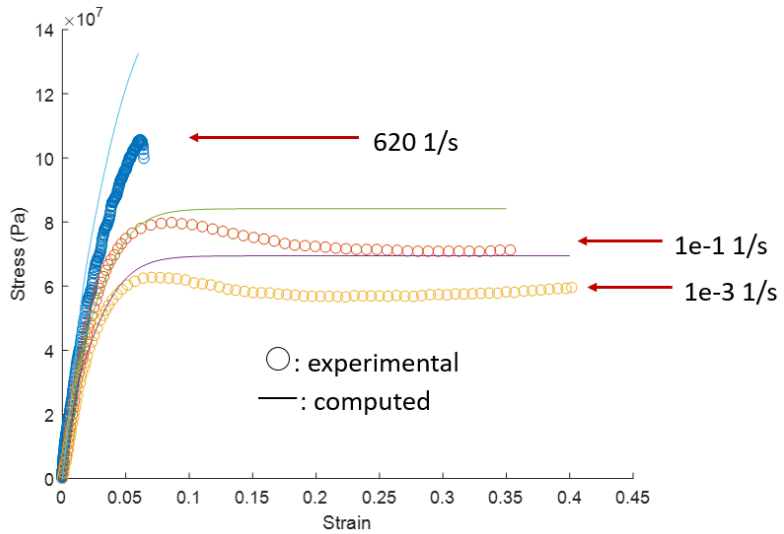

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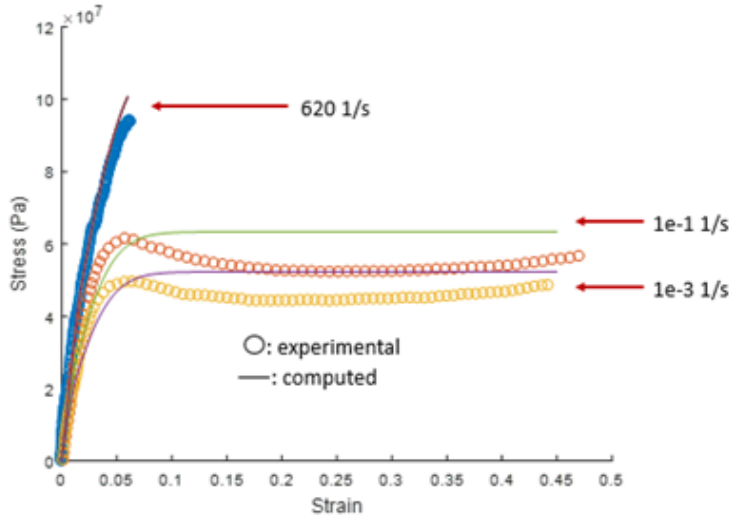


c)

Figure 4: Tensile stress-strain response of Epon 862 resin at a) room temperature, b) 50°C and c) 80°C at various strain rates

The effect of varying the inelastic heat fraction on the tensile stress-strain response of Epon 862 resin is investigated. Simulations conducted at room temperature at a strain rate of 1000 s^{-1}, and the corresponding temperature history, for various inelastic heat fractions are presented in Figure 5. More thermal softening is observed with increasing values of the inelastic heat fraction, β, as expected.

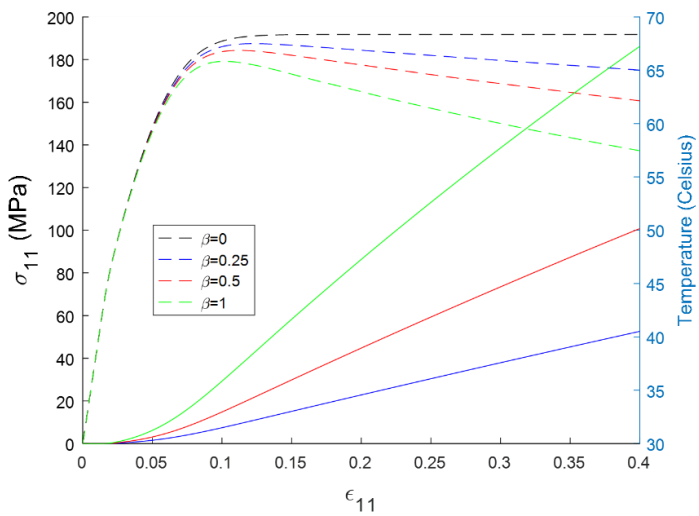


Figure 5: Tensile stress-strain response of Epon 862 resin at room temperature at a strain rate of 1000 s^{-1} and corresponding temperature history with various inelastic heat fractions

Unidirectional Composite Response

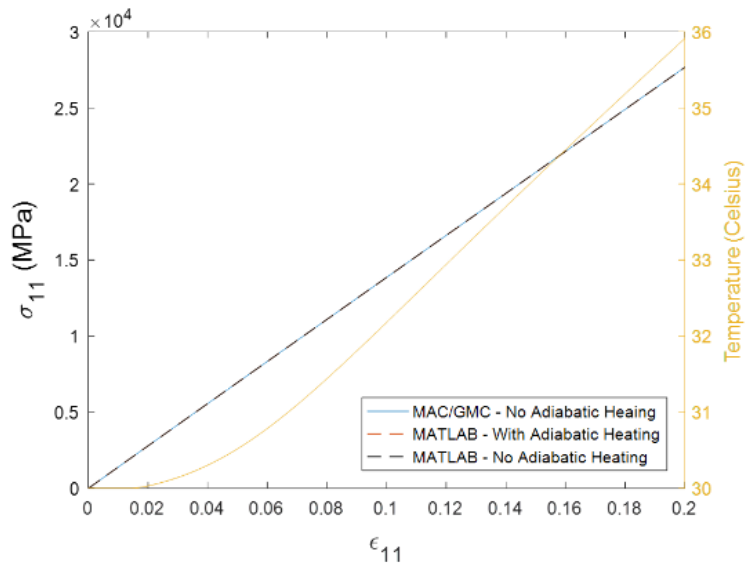
The effects of adiabatic heating on the axial tensile, transverse tensile, and in-plane shear response of a unidirectional composite are investigated next. Simulation results are presented for a T700/E862 unidirectional composite with a 60% fiber volume fraction at strain rates of 100 s^{-1} and 1000 s^{-1}. The T700 fiber properties used in the simulations are presented in Table II [18]. It should be noted that, currently, there is no unidirectional T700/E862 composite test data available; this material system was selected due to the availability of fiber and matrix material properties and experimental data. Because of the lack of experimental data for unidirectional T700/E862 composites, the micromechanics model predictions for the isothermal case are validated with those obtained using the MAC/GMC micromechanics code [19]; To examine qualitatively the effects of adiabatic heating, simulations were also conducted with a nonzero inelastic heat fraction. In the isothermal case, the results are identical to those of MAC/GMC. To demonstrate the model capability, the inelastic heat fraction is set to unity (i.e. all plastic work is assumed to be converted to heat) for all the nonisothermal results that follow, which means the temperature rises presented in this section represent an upper bound.

The response of the unidirectional composite subjected to axial tensile loading at strain rates of 100 s^{-1} and 1000 s^{-1} at room temperature (30°C) is presented in Figures 6a and 6b, respectively. Note that linear elastic response is observed for both strain rates. No significant thermal softening is observed due to adiabatic heating in the matrix although the average temperature rises in the RUC for 100 s^{-1} and 1000 s^{-1}

strain rate simulations were 5.91°C and 6.97°C, respectively. This is because the axial tensile response of unidirectional composites is fiber dominated; the fibers have been modeled as transversely isotropic and linear elastic.

Table II: T700 Fiber properties

Axial Young's Modulus	230 GPa
Transverse Young's Modulus	15 GPa
Axial Shear Modulus	0.2
Transverse Shear Modulus	0.3
In-Plane Shear Modulus	27 GPa



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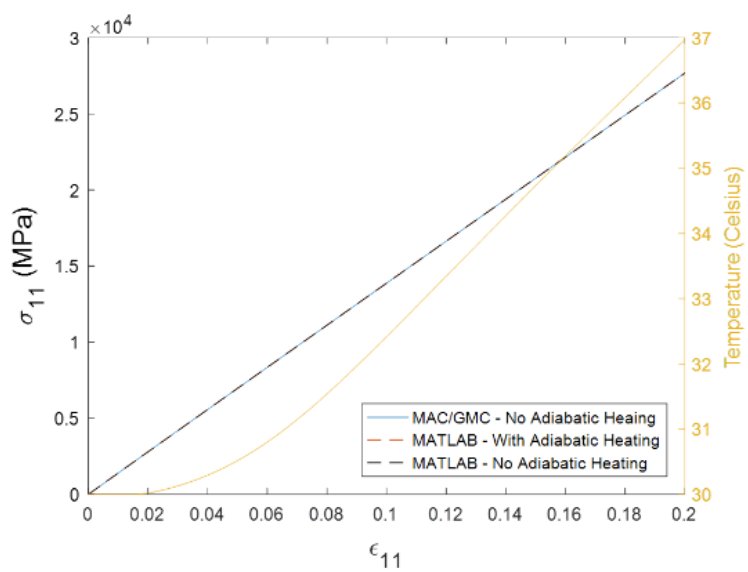

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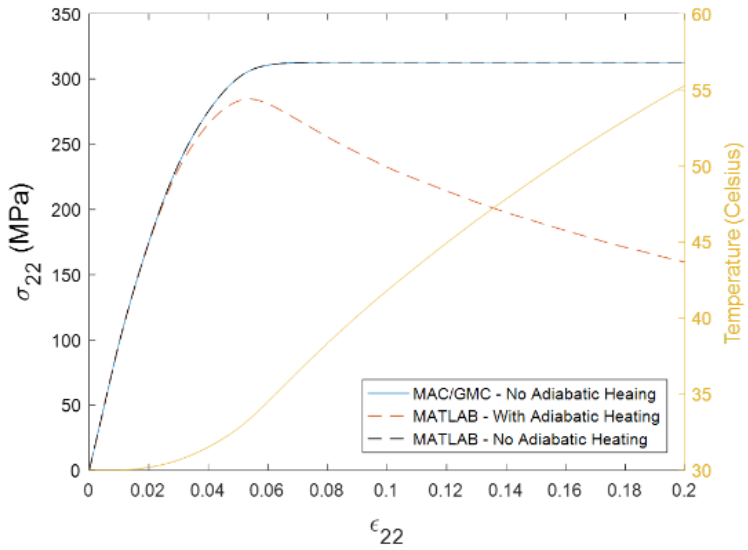
Figure 6: Axial tensile response of the unidirectional composite subjected to axial tensile loading at strain rates of a) 100 s^{-1} and b) 1000 s^{-1} at room temperature (°C)

The response of the unidirectional composite subjected to transverse tensile loading at strain rates of 100 s^{-1} and 1000 s^{-1} at room temperature are shown in Figures 7a and 7b.



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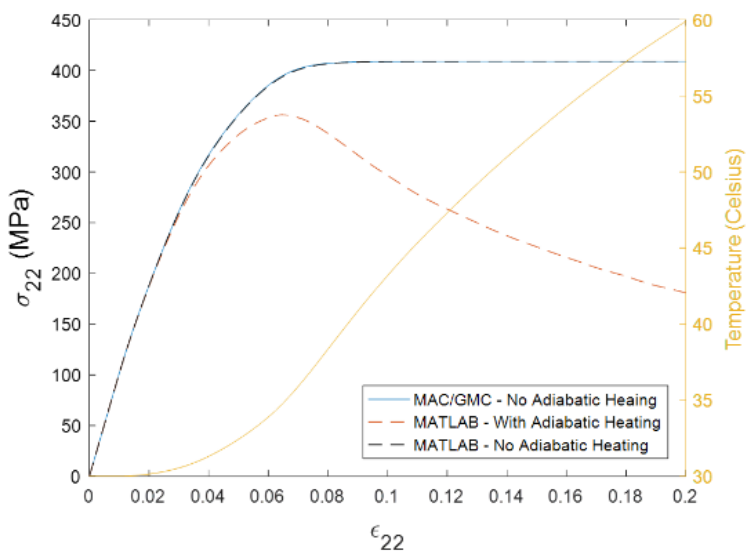

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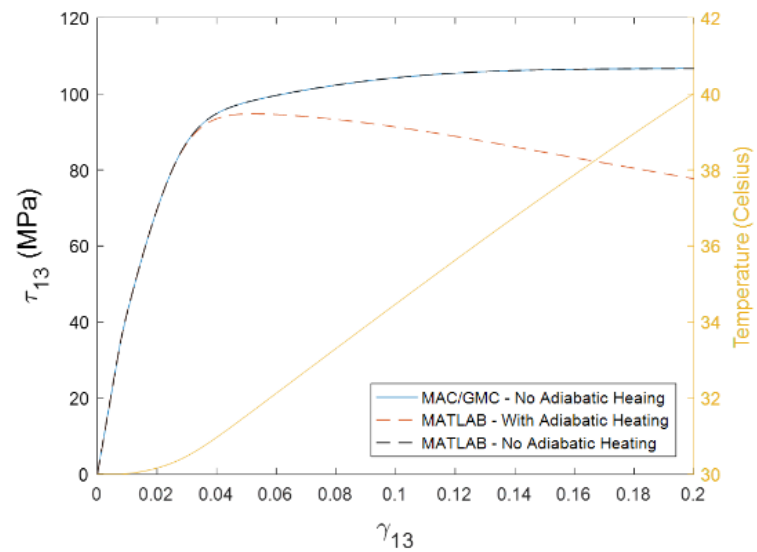
Figure 7: Transverse tensile response of a unidirectional composite subjected to transverse tensile loading at strain rates of a) 100 s^{-1} and b) 1000 s^{-1} at room temperature

Unlike the axial tensile response, the transverse tensile response is significantly affected by the thermal softening of the matrix. This is expected since the transverse tensile response of unidirectional composites is matrix dominated. The RUC average temperature rises for the 100 s^{-1} and 1000 s^{-1} strain rate simulations were 25.27°C and 29.89°C, respectively. However, the maximum local (subcell level) temperature rises for 100 s^{-1} and 1000 s^{-1} strain rate simulations were 103.92°C and 121.26°C,

respectively. Figures 8a and 8b show the response of the unidirectional composite subjected to in-plane shear loading at room temperature at engineering shear strain rates of 100 s^{-1} and 1000 s^{-1}, respectively. Significant thermal softening due to adiabatic heating in the matrix is observed for in-plane shear loading. The RUC average temperature rises for the 100 s^{-1} and 1000 s^{-1} engineering shear strain rate simulations were 10°C and 11.85°C, respectively. The maximum local (subcell level) temperature rises were 48.25°C and 58.32°C, respectively.



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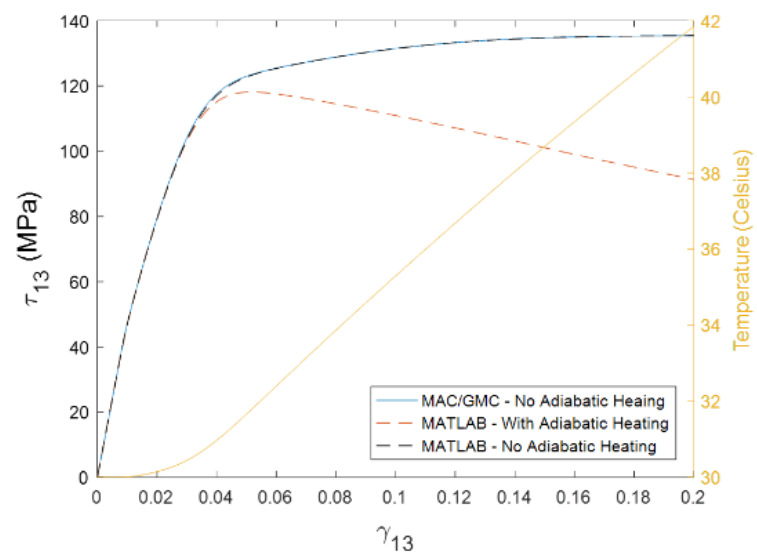

b)

Figure 8: In-plane shear response of unidirectional composite subjected to in-plane shear loading at engineering shear strain rates of a) 100 s^{-1} and b) 1000 s^{-1} at room temperature

CONCLUSION

In this work, preliminary efforts in modeling temperature rises due to matrix plastic deformation in PMCs subjected to dynamic loading conditions were presented. An existing viscoplastic polymer constitutive model was extended to nonisothermal conditions to model heat generation due to plastic deformation of the polymer matrix. A novel time-temperature shifting methodology, similar to the DSR method described by Mulliken and Boyce, was used to compute temperature and strain rate dependent shifts in elastic moduli based on DMA tests conducted on neat resin at various frequencies. Furthermore, the components of the inelastic strain rate tensor were modified to explicitly depend on temperature and the temperature dependence of state variables was assessed based on isothermal tensile and shear tests conducted on neat epoxy resin at various strain rates. The modified constitutive model was shown to be able to capture the tensile and shear behavior of Epon 862 epoxy resin across a range of temperatures and strain rates. The modified constitutive model was then embedded within a strength of materials based micromechanics framework to investigate the manifestation of matrix thermal softening, due to the conversion of plastic work to heat, on the high strain rate response of a T700/E862 unidirectional composite. Significant thermal softening was observed in high strain rate transverse tension and in-plane shear simulations of the unidirectional composite, whereas a nominal amount of thermal softening was observed in longitudinal tension. It has been shown in this work that adiabatic model predictions show a significant deviation from isothermal conditions and thus must be accounted for in modeling the high strain rate behavior of PMC structures.

ACKNOWLEDGEMENTS

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