AAS 17-624

PRELIMINARY ANALYSIS OF GROUND-BASED ORBIT DETERMINATION ACCURACY FOR THE WIDE FIELD INFRARED SURVEY TELESCOPE (WFIRST)

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The Wide Field Infrared Survey Telescope is a 2.4-meter telescope planned for launch to Sun-Earth L_2 in 2026. This paper details a preliminary study of the achievable accuracy for WFIRST from ground-based orbit determination routines. The analysis here is divided into two segments. First, a linear covariance analysis of early mission and routine operations provides an estimate of the tracking schedule required to meet mission requirements. Second, a "simulated operations" scenario gives insight into the expected behavior of a daily Extended Kalman Filter orbit estimate over the first mission year given a variety of potential momentum unloading schemes.

INTRODUCTION

The Wide Field Infrared Survey Telescope (WFIRST) is a planned 2.4-meter space telescope currently slated for launch in 2026. WFIRST will operate in a libration orbit around the Sun-Earth L_2 point. The planned telescope will be capable of resolving details at the level of the Hubble Space Telescope, but with a field of view 100 times larger. The current goal of WFIRST is to study dark matter, exo-planets, and the structure of galaxies.[1,2]

In support of the WFIRST mission, the NASA Goddard Space Flight Center's Flight Dynamics Facility (FDF) is performing preliminary analyses to determine the navigational needs of the planned spacecraft. This paper details the analysis that specifically aims to develop requirements on the ground-based tracking configuration in order to meet the higher-level mission goals. A secondary goal of the studies in this paper is to characterize the achievable orbit solution accuracy when accounting for aspects of the current operational mission plan.

Two individual analyses make up the overall approach discussed here for determining orbit determination (OD) accuracy. The first analysis is a linear covariance-based look at a variety of scenarios for early orbit and nominal operations with the goal of deriving anticipated orbit uncertainties. The second study examines nominal operations in further detail to characterize the behavior of an operational orbit estimate derived from an Extended Kalman Filter. A key concern of both analyses is the scientific requirement for frequent momentum unloads to prevent excessive vibration in the imaging equipment. In the current design, momentum unload intervals may range anywhere from 15 to 200 hours while the residual delta-v ranges from 1 to 13 mm/s. Such frequent thruster firings

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pose a unique navigational challenge especially given the unpredictable nature of the momentum unload orbit perturbations. Each thrusting event introduces an additional source of error to the orbit estimate. For the purposes of this paper, each momentum unload thrust event is modeled with a random direction and a fixed magnitude.

The goal of the studies contained in this paper is to determine appropriate tracking schedules for the lifetime of the mission and characterize the expected performance. Though mission requirements for position and velocity accuracy are not currently defined, this study targets a position error of less than 10 km and a velocity error less than 10 cm/s. At this stage of development, the physical characteristics of the spacecraft are in flux and the needs of particular instruments on-board are still in discussion. As such, the results here are not a final determination of operational strategies or expected behavior. Rather, these studies are a first look into the behavior of a future spacecraft and an attempt at building an analytical foundation to guide further development.

GROUND STATION CHARACTERIZATION

To derive characteristics of ground station tracking performance, this study incorporates results from the FDF's Metric Tracking Data Evaluation (MTDE) effort. MTDE is a daily process run within the FDF that aims to monitor and characterize the accuracy of tracking data flowing into the facility. Automated jobs run daily solutions for 38 spacecraft from over 50 tracking stations. These jobs analyze quality of the tracking data and determine which sets may be "anomalous", or poor quality. Each week, the FDF summarizes the MTDE data and delivers a Network Tracking Data Status Report to its network of ground stations.

The basic principle of the MTDE process revolves around daily batch solutions for all FDF spacecraft. Each batch solution is seeded with the previous day's solution and incorporates tracking data up to a fixed time span before the current day. The Goddard Trajectory Determination System (GTDS) software performs each batch solution.[3] Since the MTDE process is interested exclusively in the quality of the tracking data, the orbit solution itself is typically discarded. The key metrics of interest in each batch solution are the measurement residuals. The residuals from a batch solution, denoted by $\boldsymbol{e}\left(t\right)$ are

$$\boldsymbol{e}\left(t\right) = \boldsymbol{y}\left(t\right) - \boldsymbol{h}\left(\boldsymbol{x}_0, t\right) \tag{1}$$

where $\boldsymbol{y}\left(t\right)$ is an observed measurement, $\boldsymbol{h}\left(\cdot\right)$ denotes a combined measurement and force model that predicts the current measurement, and $\boldsymbol{x}_0$ is the initial state of the spacecraft. The residuals from each daily batch solution give insight into the performance of the corresponding tracking stations. Measurement noise, biases, and tracking anomalies are all apparent in the resulting data set. The MTDE database informs both the covariance analysis and filter study discussed in this paper.

COVARIANCE ANALYSIS

The Orbit Determination Error Analysis System (ODEAS) is a tool that provides linear covariance analysis for batch orbit estimates. The system accepts inputs from the user in the form of an initial spacecraft state, a tracking schedule, force modeling options, and 3-sigma error values for other variables of interest. ODEAS, using these inputs, simulates measurements over the desired span and produces a position and velocity error profile.[4]

This section details two covariance analysis studies. The first study examines ground-based orbit determination accuracy of various combinations of ground stations and tracking intervals. The goal of the first study is to narrow down a collection of feasible tracking configurations. After determining a tracking schedule, the second study examines the expected navigation performance of this configuration in the early mission phases between launch and L_2 orbit insertion. This study aims to both determine the necessary volume of tracking data required to meet mission requirements and to characterize the expected performance.

The two sub-sections that follow detail the general configuration properties of the analyses. The two remaining sub-sections examine the results of each study.

Momentum Unload Modeling

The ODEAS tool has a maximum limit of 10 finite maneuvers modeled in an analysis span. Under the assumption that WFIRST will require momentum unloads frequently enough to violate this constraint, the ODEAS scenarios in this analysis make an approximation of the momentum unload frequency. This approximation combines momentum unloads as necessary to reduce the number of modeled maneuvers to within the capabilities of the tool. If the frequency of the momentum unloads is denoted by f, then the number of required momentum unloads is

$$n_{\text{req}} = (t_1 - t_0)\, f \tag{2}$$

where t_0 and t_1 are the initial and final times of the simulation, respectively. If n_{req} exceeds the maximum, then the number of maneuvers must be reduced. This reduction is possible by changing the spacing of the momentum unloads to

$$\Delta t = \frac{t_1 - t_0}{n_{\max} + 1} \tag{3}$$

where $n_{\max}$ is the maximum number of maneuvers. Adding 1 to the denominator ensures that no momentum unload will fall directly on the end of the analysis span. By spacing momentum unloads at intervals of Δt, the required number of maneuvers is guaranteed to not violate the ODEAS constraints. To capture an equivalent velocity perturbation, the velocity change, or Δv, of each maneuver must be scaled up appropriately. The scaling parameter, α, is

$$\alpha = \frac{n_{\text{req}}}{n_{\max}} \tag{4}$$

This approximation of the momentum unloads maintains the required number of maneuvers within the software limit but applies an equivalent velocity disturbance to the modeled spacecraft.

The perturbation from momentum unload is expected to be small – typically on the order of mm/s – but with largely uncertain orientation. To capture this behavior, each ODEAS scenario models the maneuver with 100 % uncertainty in the total Δv of the thrusting event. The uncertainty is spread evenly in all directions to simulate a perturbation from the momentum unload in any direction. The uncertainty along each axis in inertial space, denoted by Δv_{err} is

$$\Delta \boldsymbol{v}_{\text{err}} = \alpha \frac{\|\Delta \boldsymbol{v}_{\text{M}}\|}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{5}$$

where $\Delta \boldsymbol{v}_{\text{M}}$ is the expected average magnitude of the momentum unload perturbation. Note that the scaling parameter, α, scales the uncertainty to correspond with the magnitude of the maneuver perturbations. With this definition, the magnitude of the uncertainty vector is

$$\|\Delta \boldsymbol{v}_{\text{err}}\| = \alpha \, \|\Delta \boldsymbol{v}_{\text{M}}\| \tag{6}$$

Note that the ODEAS analyses discussed here include station keeping maneuvers in the simulation without any approximation. Rather than lumping station keeping maneuvers together with the momentum unload events, the simulation models each one as an independent event with some uncertainty. When a substantial thrusting event falls within the analysis span, the maximum, $n_{\max}$, in the equations decreases by one. This way the momentum unloads are spread across fewer modeled burns, but the more significant thrusting events are modeled fully.

Other Error Parameters

ODEAS allows the user to set a number of optional error sources.[4] This section summarizes the key uncertainty sources considered in both of the following analyses. These parameters are largely derived from the standard best practices document,[5] which details default uncertainty assumptions for most of the major parameters.

Each scenario uses a 4×4 subset of the Joint Gravity Model 2 (JGM-2) with corresponding error coefficients.[6] Table 1 shows the 3-sigma position and gravitational parameter errors for all perturbing bodies included in the scenario. These error parameters are standard values developed at the NASA GSFC.[5]

Table 1. List of third-body 3-sigma uncertainty parameters.

Sun Position	5 km
Moon Position	30 m
Earth Gravitational Parameter	0.03 ppm
Sun Gravitational Parameter	0.023 ppm
Moon Gravitational Parameter	0.734 ppm

Table 2 shows the 3-sigma error values associated with ground station tracking of WFIRST. The simulation assumes a flat 3 meter, 3-sigma uncertainty for the position of all ground stations.

Table 2. List of ground station 3-sigma uncertainty parameters.

Station X, Y, Z Locations	3 m
Tropospheric Refraction	45 %
Ionospheric Refraction	100 %

These error parameters are common to all of the covariance analysis studies in the following sections.

Tracking Schedule Study

This covariance analysis, covering a portion of the mission orbit of the spacecraft, focuses on narrowing down the appropriate tracking stations and schedules to meet the WFIRST mission requirements. This study specifically considers six possible ground stations capable of supporting WFIRST. Table 3 lists the location, four-letter station ID, and managing organization for all six stations. The station list includes three Deep Space Network (DSN) stations,[7] two Near Earth Network (NEN) sites,[8] and one European Space Agency (ESA) station.

Table 3. List of stations considered in the analysis.

Station Location	Station ID	Organization
Goldstone, California	DS24	DSN
Canberra, Australia	DS34	DSN
Madrid, Spain	DS54	DSN
Santiago, Chile	AGOS	NEN
White Sands, New Mexico	WS1S	NEN
New Norcia Station, Australia	NN1D	ESA

The covariance analysis incorporates ground station uncertainties derived from an aggregate of mission data in the MTDE database. These uncertainty values originate from analysis of numerous spacecraft and measurements over the course of a year of daily batch orbit solutions. Table 4 details the 3-sigma uncertainties implemented for each station. Note that the FDF does not currently receive data from the ESA station in New Norcia, therefore this analysis assumes uncertainties equivalent to the DSN Madrid station, DS54.

Table 4. Ground station 3-sigma uncertainty values for the mission orbit analysis.

	Range Bias (m)	Range Noise (m)	Range-Rate Noise (cm/s)
DS24	13.1	5.5	0.3
DS34	29.8	14.6	1.5
DS54	15.0	8.6	0.6
AGOS	7.8	4.1	2.7
WS1S	15.9	10.1	2.1

Overall, this study includes results from 192 scenarios with different variations of active ground stations, daily tracking schedules, solution spans, and momentum unload delta-v magnitudes. This paper covers only a subset of those results corresponding to a 21-day solution span and momentum unload perturbations of 1 mm/s, however the remaining results do not change the final conclusions.

Figures 1 and 2 summarize the results of this analysis. The figures separate ground stations into 4 categories: nen, dsn, esa, and mdrd. The "nen" and "esa" categories represent the NEN and ESA stations from Table 3. The DSN stations have been split into two categories: "dsn" and "mdrd" to allow for additional nuance in the station combinations. The tag "mdrd" refers to DS54, the Madrid DSN station, and "dsn" refers to the remaining DSN stations in Table 3. The title of each error bar in Figures 1 and 2 includes the name of the stations considered and the tracking time, in minutes, per day for each station. For example, "nen, 90min" indicates that the tracking configuration included both NEN stations and scheduled 90 minutes of tracking for each station per day.

The figures summarize the results in terms of maximum 3-sigma error over the entire 21-day span of the mission orbit. Blue segments indicate the solution error without measurement noise and the red portions show the aggregate error with the contribution from measurement noise. As a rule of thumb, a contribution of 10% or more from measurement noise is expected to lead to difficulty converging on a solution. Figure 1 contains the RSS position error for each case and Figure 2 shows the corresponding RSS velocity errors.

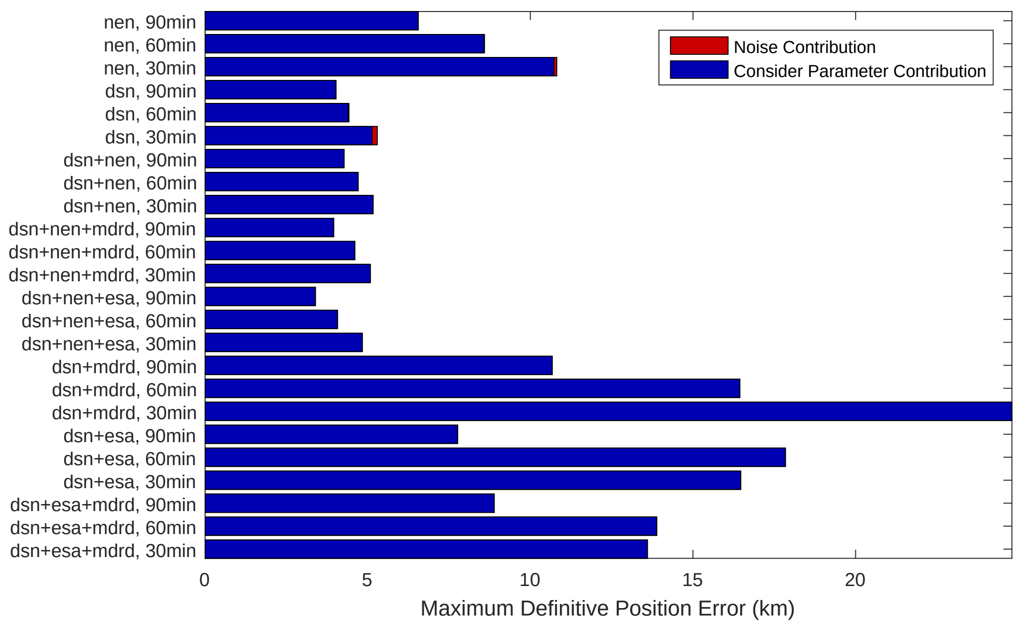

Figure 1. Maximum position error for a batch orbit solution over a 21 day observation span.

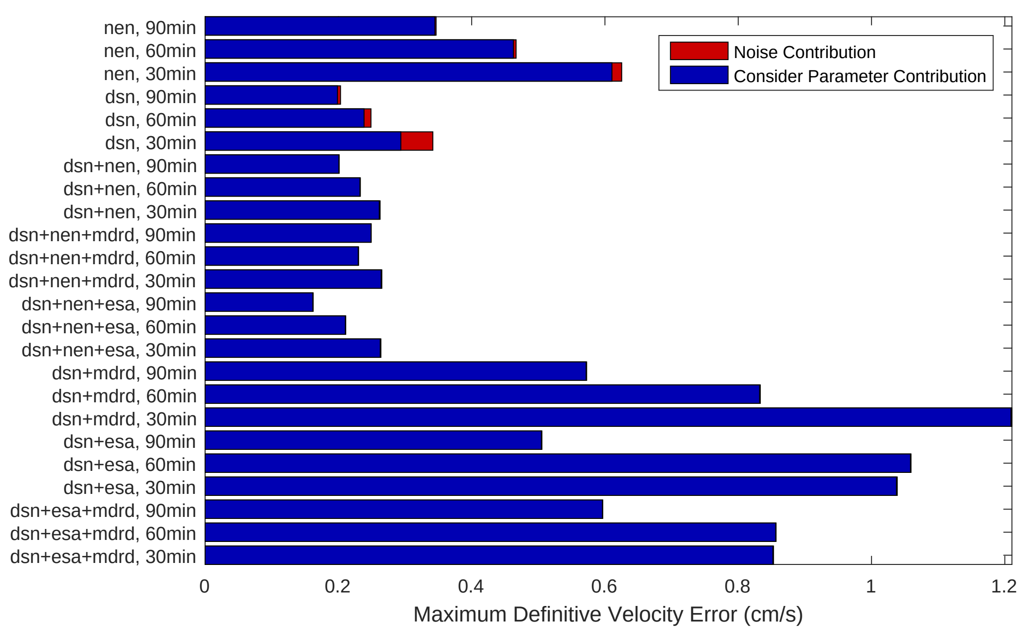

Figure 2. Maximum velocity error for a batch orbit solution over a 21 day observation span.

Of the single-network results, the DSN-only cases provided the lowest error over the span. Error for the DSN station pair is between 4-5 km in position and 2-4 mm/s in velocity. A caveat of using the DSN station pair only, however, is that the measurement noise may lead to convergence issues. The noise error is most prominent in the 30 minute tracking schedule but remains a concern in the 60 and 90 minute cases. The NEN station pair shows higher overall error in position and velocity but reduces the measurement noise concerns. Performance of the NEN station pair is between 6-12 km in position and 3-6 mm/s in velocity.

Additional tracking beyond the DSN and NEN station pairs, in general, reduces the influence of measurement noise while maintaining low position and velocity errors. Combinations of DSN, NEN, and ESA, show the lowest error in Figures 1 and 2 with the least error contributions from measurement noise. The expected accuracies for these configurations are approximately between 3-5 km and 1.5-3 mm/s. The ideal configuration in this study is the combination of the DSN station pair, the NEN station pair, and the ESA station tracking for 1.5 hours per day per station.

Due to nuances in the tracking geometry and batch solution profiles, more time tracking time or the addition of another station does not necessarily lead to lower position error as one might expect. In some cases, the noise and biases introduced by additional data changes the error profile in a way that leads to a higher peak. For example, the addition of the Madrid station (DS54) to the pair of DSN stations, DS24 and DS34, appears to increase the maximum orbit error substantially in Figure 1. A possible explanation for this behavior is the fact that both Madrid (DS54) and Goldstone (DS24) are located at similar latitudes, so the additional tracking does not provide a substantial benefit to the overall tracking geometry. In fact, the addition of Madrid tracking biases the overall data set toward the northern hemisphere.

This study serves as a road map for the remaining work to choose a tracking configuration capable of meeting the target accuracy requirements of 10 km position and 10 cm/s velocity. Based on this analysis, the tracking schedule for the remaining work narrows to three stations with a fixed tracking schedule. From the stations in Table 3, two ground stations remain: White Sands (WS1S) and Madrid (DS54). The remaining studies also incorporate another NEN station, located in Dongara, Australia and referenced by the four-letter acronym USPS. This station is geometrically similar to the ESA station NN1D.

Transfer Orbit Study

The ODEAS transfer orbit analysis aims to determine the span of tracking data required to meet mission requirements after the first mid-course correction (MCC-1) and prior to the Libration orbit insertion burn (LOI). The total time from MCC-1 to LOI is planned to be 106 days. This study focuses specifically on 30 day segments at the beginning and end of the total span.

The ground tracking schedule for this segment of the mission splits into two distinct segments: early-orbit high intensity support and routine mission orbit tracking. Prior to and just after the the Mid-Course Correction maneuver (MCC-1), WFIRST is in the high intensity support segment and requires near-continuous contacts. For this early portion, the simulation maintains constant communication with a single ground station at a time with priority given to stations beginning contact. After MCC-1 and once the orbit solution settles to pre-maneuver error levels, the routine tracking schedule for the mission orbit begins. In this tracking scheme, the White Sands station (WS1S) maintains contact with the spacecraft for its maximum availability to maintain telemetry and command data flow but also produces Doppler measurements. For one hour per day WS1S also collects coherent range data. The NEN Dongara station (USPS) collects range and Doppler data for one hour per day near the midpoint of its availability. The Madrid DSN station (DS54) collects one hour of range and Doppler per week. Figure 3 illustrates this tracking schedule.

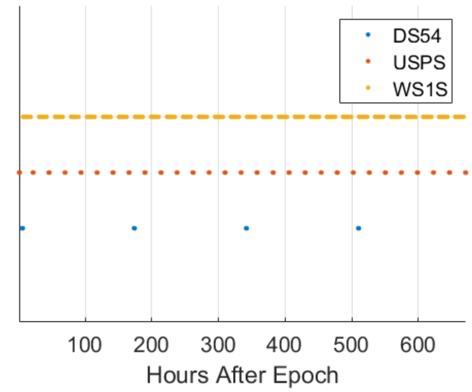

Figure 3. Tracking schedule for WFIRST during the coast phase, prior to L_2 orbit insertion planning.

This analysis and the filter study in the next section focuses the measurement uncertainty parameters to values derived from a specific spacecraft rather than the aggregate MTDE data. The numbers in Table 6 are based on actual tracking performance for the Deep Space Climate Observatory (DSCOVR). The spacecraft DSCOVR's orbit regime and tracking profile are more analogous to that of WFIRST than many of the other missions for which the FDF maintains statistics.

Table 5. Ground station 3-sigma uncertainty values for the transfer orbit analysis.

	Range Bias (m)	Range Noise (m)	Range-Rate Noise (cm/s)	Range-Rate Bias (cm/s)
DS54	15.00	3.00	0.10	0.00
USPS	2.01	11.40	0.60	1.44
WS1S	1.02	9.90	1.20	0.00

The results in Figures 4 - 6 detail the orbit solution accuracy for the three phases of interest in the transfer orbit. These figures present the orbit estimate accuracy as an expected error level at a specific point in time.

The first set of results corresponds to the pre-MCC-1 mission phase. Figure 4 illustrates the expected orbit solution error at the maneuver time versus the total span of tracking data included in the batch estimate. In the current mission plan, the MCC-1 maneuver occurs 25 hours after spacecraft separation. The x-axis in Figure 4 begins at separation and ends 24 hours later. The one hour buffer between the longest tracking span and the maneuver provides the minimum time for the ground crew to perform final maneuver planning prior to MCC-1.

Similar to the results in the previous section, the black line in Figure 4 indicates the error contribution without noise and the red line shows the total error with the contribution of noise. A noise contribution of 10% or greater is likely to cause convergence issues for a batch solution. The two graphics in Figure 4 show that a solution span of less than 12 hours is insufficient to guarantee convergence. Beyond 12 hours, the solution settles to minimum error values of 0.5 km and 6.5 mm/s with a 20 hour measurement span.

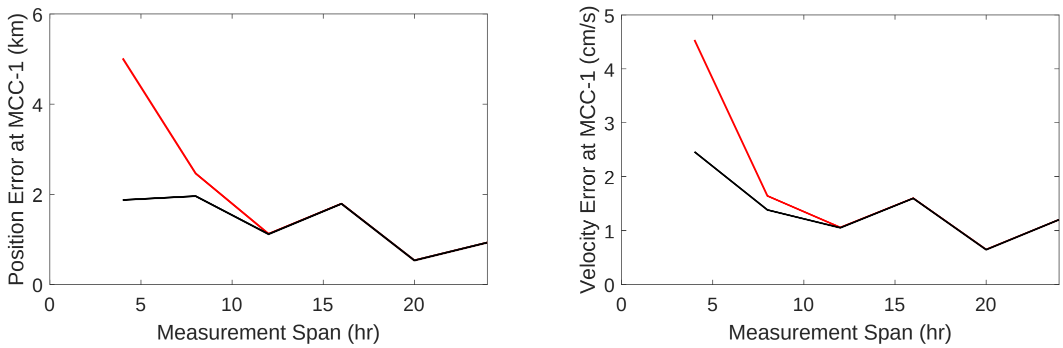

Figure 4. Position error at MCC-1 versus tracking data span. Red indicates the total error with the contribution from measurement noise.

Figure 5 contains results for the next phase of the early-orbit mission: the period of time just after MCC-1. This purpose of this portion of the analysis is to determine the necessary tracking span to successfully re-converge on a solution following the maneuver. The solution span begins just after the maneuver and includes no knowledge of the prior orbit or the applied delta-v. The tracking schedule for this mission segment is the same as the previous: continuous coverage by any available tracker with priority given to stations beginning their contact interval. The error reported in Figure 5 is the error after a 1-week prediction span.

The results in Figure 5 show that a short tracking span of 12 hours is insufficient to guarantee convergence. The 24 hour solution, though, sees much lower error contributions due to noise albeit the position error remains high at over 10 km. Ultimately the solution settles after 4 days of tracking to a minimum of approximately 0.8 km and 0.7 mm/s. Beyond a 4 day span the accuracy of the position solution begins to degrade while the velocity accuracy improves. This behavior is likely due to the circumstances of the trajectory that the spacecraft is following. As the spacecraft moves further from Earth, the larger distances lead to higher error along the unobservable axes, and the position knowledge degrades. The Doppler measurements see the opposite effect. From a ground station's point of view, the spacecraft is moving away from Earth almost entirely along the line of sight. This type of motion, combined with the fact that the Spacecraft is slowing with respect to Earth is ideal for Doppler measurements and leads to very precise velocity knowledge.

Based on these results, a minimum of 48 hours of tacking data is necessary to reach a position accuracy within the target accuracy levels of 10 km and 10 cm/s. Additional tracking data will further improve the solution up to the minimum around 4 days.

The final set of results, contained in Figure 6, illustrate the error profile for a batch solution prior to the L_2 orbit insertion maneuver. The batch solutions for this segment include a 7-day prediction to the insertion maneuver, so each scenario begins its solution span such that collection of tracking data ends exactly 7 days before the maneuver. At this point in its trajectory, WFIRST uses the tracking schedule depicted in Figure 3. Due to the decreased tracking intensity, the ground system requires additional time to collect sufficient tracking data. The scenarios in Figure 3 begin with a week long arc and extend to a maximum of four weeks (28 days).

In the results for this set of scenarios, it is clear that a tracking arc length of one week (168 hours)

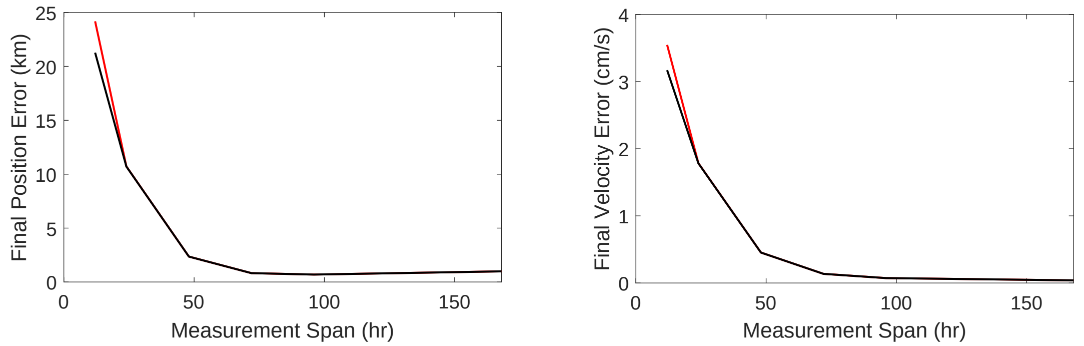

Figure 5. 7-day predictive position error after MCC-1 versus tracking data span. Red indicates the total error with the contribution from measurement noise.

is insufficient to guarantee convergence. The total solution error when accounting for measurement noise, indicated in red, is significantly different from the base error profile and likely to lead to problems for the batch solver. The 14-day interval (336 hours) reduces the contribution of measurement noise but may still produce a noisy and unreliable solution. The 21 and 28 day arcs provide the best performance. After a 28-day tracking arc, the error expected at the time of the Lagrange orbit insertion maneuver is approximately 1 km in position and 0.6 mm/s in velocity.

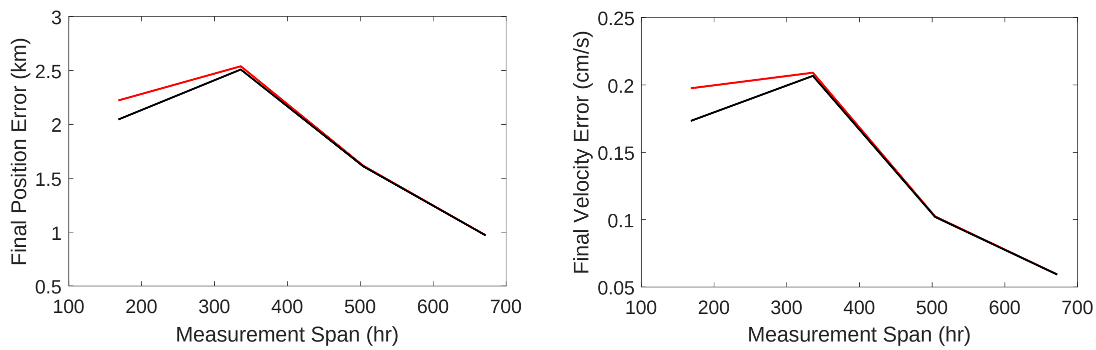

Figure 6. Position error at L_2 orbit insertion versus tracking data span. Red indicates the total error with the contribution from measurement noise.

Based on the results in Figure 6, a tracking arc of at least 21 days is desirable but a full four weeks provides the best overall performance.

SIMULATED OPERATIONS STUDY

The analysis performed in the previous section of this paper allowed for a more narrow focus on specific tracker combinations and tracking patterns in this study. The follow-up to the covariance analysis work uses an Extended Kalman Filter (EKF) to examine orbit accuracy for the first mission year in a simulated operational environment. This study is intended to provide a closer look at the anticipated day-to-day operations for WFIRST. Variables of interest are the frequency and magnitude of momentum unloads, tracking station configurations, and daily orbit solution accuracy.

Orbit modeling for the EKF scenarios is provided by ephemerides of the current reference trajectories for WFIRST. Among the various trajectories, the key difference is in the momentum unload modeling. Each trajectory captures a different unload interval and each unload event is randomly oriented. These trajectories retain fixed launch dates and orbit insertion maneuvers but vary station keeping delta-v vectors based on the needs of the spacecraft after undergoing the random perturbations due to the momentum unloads. In each case discussed below, the reference ephemerides represent the "truth" orbit.

The simulated EKF scenario closely matches the anticipated day-to-day operations for orbit determination in the FDF. In this environment, automated systems will run an EKF solution at most once per day. Depending on the operational constraints discussed in this study (e.g. - tracking schedule and momentum unload frequency), it is conceivable that solutions may be required less frequently than once per day. As an upper limit, a weekly EKF solution will be considered along with the intermediate steps of two day solutions, three day solutions etc. Operationally speaking, performing an OD solution every three days is advantageous as it would not require weekend support. Orbit determination jobs will gauge their accuracy through measures of self-consistency by comparing the current solution with predicted solutions from past runs. Figure 7 details comparison spans for up to three days. This study only considers the results from comparison with the truth ephemeris to compute absolute error values. For each run day, the simulation computes compares against truth for predictions of 1 to 7 days. This range of prediction spans captures the time necessary to plan upcoming station keeping maneuvers.

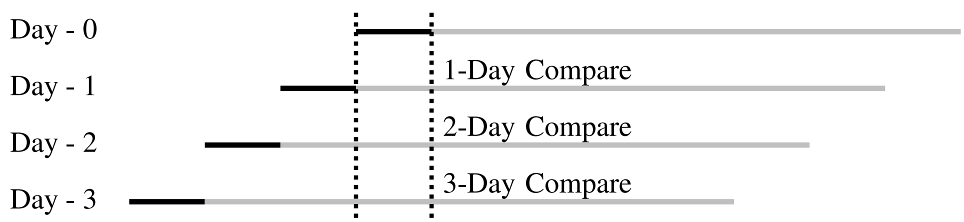

Figure 7. Depiction of daily truth ephemeris comparisons over 4 days. Dark and light lines indicate definitive and predictive solutions, respectively.

In order to closely model an operational environment, this study uses one year of simulated tracking data. The tracking data derives from the tracking schedule in Figure 3, the measurement uncertainty values from Table 6, and the WFIRST reference trajectories. In this case the noise parameters differ slightly from the previous simulations. The numbers in Table 6 are still based on real-world data from the MTDE database but the data has been down-selected further to remove outliers. The updated noise numbers also include a small contribution to the range-rate white noise introduced in the transponder.

Table 6. Ground station 1-sigma uncertainty values.

	Range Bias (m)	Range Noise (m)	Range-Rate Noise (cm/s)	Range-Rate Bias (cm/s)
DS54	1.00	1.00	0.05	0.00
USPS	0.67	3.80	0.41	0.48
WS1S	0.34	3.30	0.21	0.00

This study examines three possible momentum unloading profiles for WFIRST. Each momentum unloading configuration is designated by the interval between successive unload maneuvers. This paper examines results for unload intervals of 18, 40, and 200 hours. Other intervals are possible in the final mission plan, but the choice of these three cases is intended to capture the overall behavior of varying the unload interval in a way that will generalize to other alternatives. A small velocity perturbation accompanies each of these momentum unloading events. Figure 8 shows the assumed relationship between the momentum unload interval and the residual delta-v of the event. As the interval between unloads grows, the magnitude of the velocity perturbation grows at a rate of approximately 0.09 mm/s per hour. The three unload intervals in this study correspond to perturbations of 1.3, 2.7, and 13.3 mm/s.

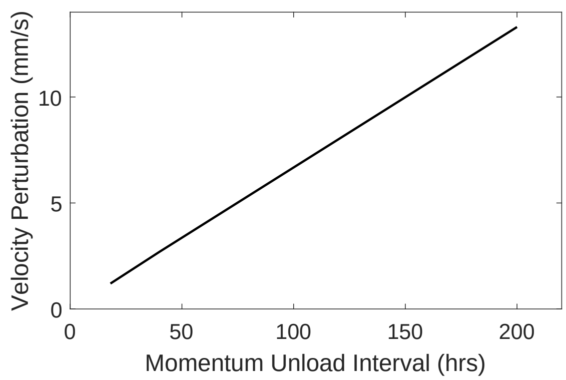

Figure 8. Projected values for velocity perturbations due to momentum unloading versus the time between unloads.

In the study, momentum unload events have fixed perturbation magnitudes but random directions. This configuration is intended to simulate the uncertainty in the way the thruster force balance ultimately settles due to misalignments among the attitude thrusters. In general there is no expectation of prior knowledge of the direction of the velocity perturbations and, depending on the way the momentum has accumulated in the spacecraft, the residual delta-v vector may not be consistent between events. In addition to momentum unloads, WFIRST also conducts a station keeping (SK) maneuver every 21 days. The necessary magnitude of each SK varies depending on the size of the momentum unload perturbations but the total delta-v is typically on the order of millimeters per second.

The filter handles these two types of maneuvers in different ways. Because of the sometimes unpredictable nature of momentum unloading events, the filter does not contain any knowledge of these maneuvers beforehand. As the momentum unloads are expected to occur autonomously, as needed, and with random direction, the EKF's process noise is tuned in such a way that the EKF can ”solve through” or ”process over” the momentum unloads without explicit knowledge of when they occur. Conversely, Station Keeping maneuvers will be planned several days before they are conducted, and it is expected that these maneuvers will be modeled in the EKF. To model real-world uncertainty in the actual performance of station keeping maneuvers, the filter's knowledge of the delta-v vectors includes a 10% 1-sigma Gaussian random perturbation in all directions. The filter uses this degraded knowledge as its “initial guess” of each station keeping maneuver.

The simulated operations scenario assumes a launch date of September 11, 2026. The orbit insertion maneuver occurs on December 27, 2026 and the daily EKF solutions begin on this date. Simulated filter operations run daily for 365 days and come to an end on December 27, 2027.

This simulation process is intended to generate a thorough examination of the behavior of WFIRST on-orbit. The daily comparisons produce extensive error metrics throughout an entire orbit that demonstrate the natural behavior of the end-to-end navigation system due to the tracking configuration, force modeling, and maneuver schedule. Through this approach, the effects of the momentum unload sizing and frequency on the accuracy of the orbital solutions should be effectively quantified.

Results

The operational scenario described in the previous section produces a daily ephemeris file containing the solution for that day with a 7-day prediction span. Upon completion of the run, the ephemeris files are compared to the truth ephemeris over a given prediction span and the maximum difference between the two is stored. This process repeats for prediction spans of 1 to 7 days in steps of 1. The collection of ephemeris files produces a set of predictive position and velocity errors that are then combined to generate these results. Figures 9 and 10 contain a summary of the re-sults from the three iterations of the EKF study. The figures reference each case by the momentum unload interval. Each point on the lines in the figures below represents an average peak prediction error, versus truth, over a prediction span between 1 and 7 days.

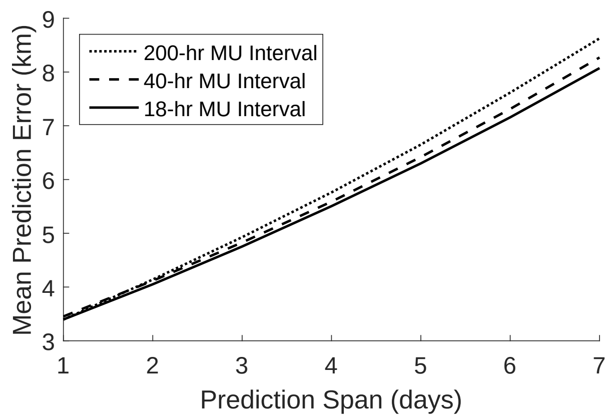

Figure 9. Average peak position prediction error versus prediction span.

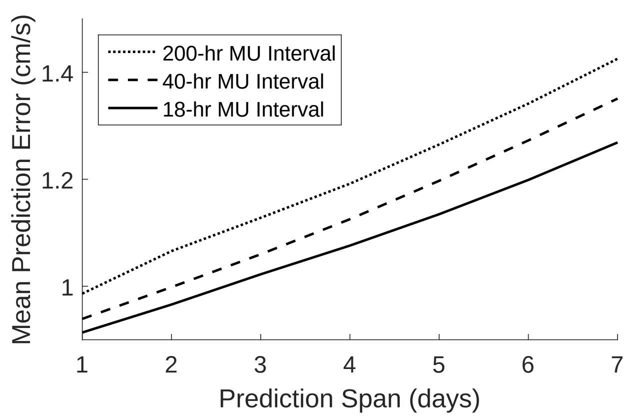

Figure 10. Average peak velocity prediction error versus prediction span.

Based on these results, more frequent momentum unloading with smaller velocity perturbations, on average, leads to lower position and velocity error. In both the position and velocity error plots, the 200-hour unload interval resulted in larger errors than the 18 and 40 hour cases. Likewise, the 40-hour interval case showed worse performance overall than the 18-hour case. For short-term predictions the three configurations are similar with position accuracy of approximately 3.5 km and velocity knowledge accurate to between 0.9 and 1 cm/s. With a prediction span of 7 days the three cases begin to diverge in position performance with errors between 8-9 km and 1.2-1.5 cm/s.

From these results it is apparent that the tradeoff between frequent momentum unloading and the orbital perturbation favors more frequent events. This effect is due to the fact that, as the unloads become more frequent, the perturbation becomes less significant to the overall solution. A perturbation on the order of 1 mm/s is easily captured in the process noise of the filter. A residual delta-v of 13 mm/s, on the other hand, is on the order of a station keeping maneuver and may significantly alter the orbit.

Note that all three configurations are within the target orbit solution accuracy of 10 km and 10 cm/s. This result suggests that the frequent momentum unloads expected for WFIRST will not significantly degrade ground-based orbit estimation capabilities except for momentum unload intervals beyond 200 hours.

CONCLUSIONS

The analysis discussed in this paper is intended to provide an early look into the expected tracking performance of WFIRST in an operational environment. From this analysis we are able to draw several conclusions about the appropriate tracking configurations to achieve missions requirements.

Prior to MCC-1, WFIRST requires at least 12 hours of continuous coverage to converge on a solution. More data reduces the error further, with a minimum corresponding to a 20-hour span. Expected error at the time of MCC-1 with the 20-hour configuration is approximately 0.5 km and 6.5 mm/s. Following MCC-1, convergence of a batch solution requires at least 24 hours of continuous coverage. Afterward, the solution settles to an accuracy of 0.8 km and 0.7 mm/s after 4 days of continuous coverage. Prior to the L_2 orbit insertion maneuver the spacecraft switches to a more sparse tracking schedule and therefore requires longer solution spans to meet requirements. Prior to the orbit insertion, WFIRST needs at least 21 days of tracking data to ensure a converged solution. A 28-day solution span is ideal with expected errors of 1 km and 0.6 mm/s.

Throughout the lifetime of the mission, an Extended Kalman Filter operational process provides a 1-day predictive accuracy of approximately 3.5 km and 1 cm/s. The 7-day predictive performance of the EKF setup is approximately 9 km and 1.5 cm/s. These values correspond to a 200-hour momentum unload interval. More frequent unloading will reduce these errors slightly, and less frequent unload events will increase the overall error.

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