Performance Optimizing Adaptive Control with Time-Varying Reference Model Modification

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***Abstract*—This paper presents a new adaptive control approach that involves a performance optimization objective. The control synthesis involves the design of a performance optimizing adaptive controller from a subset of control inputs. The resulting effect of the performance optimizing adaptive controller is to modify the initial reference model into a time-varying reference model which satisfies the performance optimization requirement obtained from an optimal control problem. The time-varying reference model modification is accomplished by the real-time solutions of the time-varying Riccati and Sylvester equations coupled with the least-squares parameter estimation of the sensitivities of the performance metric. The effectiveness of the proposed method is demonstrated by an application of maneuver load alleviation control for a flexible aircraft.**

I. Introduction

In this work, we develop a multi-objective performance-based adaptive optimal control with the goal of providing adaptation while seeking to optimize a performance metric. The notion of multi-objective optimization in the context of model reference adaptive control is considered to be novel. As such, research in this new area is quite limited. In the recent years, Nguyen investigates multi-objective adaptive control [1], [2]. In this work, bi-objective optimal control modification seeks to minimize both the tracking error and the predictor error which are used in the adaptation for systems with input uncertainty [1]. A further extension led to the development of multi-objective optimal control modification which seeks to minimize the effect of unmatched uncertainty [2]. Multi-objective optimal control has been developed by Nguyen to address drag minimization, flutter suppression, and maneuver and gust load alleviation for a flexible wing aircraft by taking advantage of the redundancy in multi-functional flight control surfaces [3]. With a similar objective, Wise et al. develop a direct adaptive reconfigurable flight control for a tailless aircraft [4] with maneuver load alleviation addressed in a control allocation scheme.

In the current work, we propose a new model-reference adaptive control that incorporates optimization of a performance metric of an uncertain plant. The plant is assumed to have control redundancy to provide multi-objective adaptive control tasks: 1) model-reference adaptive control for managing system uncertainties and 2) performance optimization. The performance metric is available from sensor measurements but its sensitivities with respect to the state and control variables are unknown but can be estimated by a least-squares parameter estimation method. The performance optimization is cast as a multi-objective optimization problem. This results in time-varying modified Riccati and Sylvester equations. The performance optimizing adaptive controller obtained from the solutions of the time-varying Riccati equation and Sylvester equation results in time-varying control gains. The closed-loop plant thus is time-varying. As a result, the linear time-invariant reference model must be modified accordingly to be time-varying so that asymptotic tracking can be achieved. The time-varying reference model modification is another novelty of the current work. Reference model modification in model-reference adaptive control has been investigated extensively by many researchers but time-varying modification is generally not considered in these investigations. Stepanyan and Lavretsky independently develop a closed-loop reference model modification that contains a tracking error feedback term in the reference model which is linear time-variant [5], [6]. Gibson investigates a closed-loop reference model modification originally developed for the observer-based output feedback adaptive control proposed by Lavretsky [7] for output feedback systems. The closed-loop reference model includes an output feedback term, but overall is a linear time-invariant reference model [8].

II. Performance Optimizing Adaptive Control

A. Problem Formulation

Consider a plant model

$$\dot{x} = Ax + B\left[u + \Theta^{*\top}\Phi(x)\right] \tag{1}$$

subject to $x(0) = x_0$, where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control vector, $\Theta^* \in \mathbb{R}^{l\times m}$ is

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an unknown constant matrix that represents a matched uncertainty, $\Phi(x) \in \mathbb{R}^l$ is a known regressor function, and $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are known matrices. The plant is considered to be a full-state system with all the available states for control. Therefore, the pair (A,B) is assumed to be controllable. The plant is associated with a performance metric

$$y = Cx + Du \tag{2}$$

where $y(t) \in \mathbb{R}^k$, where $k \leq n$, is a vector of performance metrics for the plant which is available from measurements, and $C \in \mathbb{R}^{k\times n}$ and $D \in \mathbb{R}^{k\times m}$ are sensitivity matrices which are assumed to be unknown unknown. The initial design of an adaptive controller without the performance optimization is to establish a linear time-invariant reference model

$$\dot{x}_m = A_m x_m + B_m r \tag{3}$$

based on the closed-loop plant without the matched uncertainty using a nominal controller $u_{nom}(t)$ given by

$$u_{nom} = K_x x + K_r r \tag{4}$$

where $A_m = A + BK_x \in \mathbb{R}^{n\times n}$ is a Hurwitz state transition matrix of the reference model, $B_m = BK_r \in \mathbb{R}^{n\times q}$ is a command transition vector of the reference model, and $r(t) \in \mathbb{R}^q$ is a bounded reference command signal with a bounded derivative, i.e., $r(t) \in \mathscr{L}_\infty$ and $\dot{r}(t) \in \mathscr{L}_\infty$ and the assumption that $q \leq m$. The reference command signal is generated by the following implementation

$$\dot{z} = A_r z + z_r \tag{5}$$

$$r = C_r z \tag{6}$$

subject to $z(0) = z_0$, where $z(t) \in \mathbb{R}^p$, $p \geq q$, $z_r \in \mathbb{R}^p$ is a constant command signal, $A_r \in \mathbb{R}^{p\times p}$ is a stable matrix, and $C_r \in \mathbb{R}^{q\times p}$. Without the performance optimization, an adaptive controller can be designed straightforwardly with

$$u = u_{nom} + u_{ad} \tag{7}$$

where $u_{ad}(t)$ is an augmented adaptive controller given by $u_{ad} = -\Theta^\top \Phi(x)$ with the adaptive law

$$\dot{\Theta} = -\Gamma \Phi(x) e^\top PB \tag{8}$$

where $e(t) = x_m(t) - x(t)$ is the tracking error and P solves the Lyapunov equation

$$PA_m + A_m^\top P = -Q \tag{9}$$

with $Q = Q^\top > 0$ a positive-definite weighting matrix. To address the performance optimization, we consider the following adaptive controller

$$u = u_{nom} + u_p + u_{ad} \tag{10}$$

where u_p is the performance optimizing augmentation controller. The plant with the adaptive controller now becomes

$$\dot{x} = A_m x + B_m r + Bu_p + Bu_{ad} + B\Theta^{*\top}\Phi(x) \tag{11}$$

We re-define $y(t)$ as an incremental performance metric due to the performance optimizing augmentation controller $u_p(t)$ with

$$y = Cx + Du_p \tag{12}$$

Let $\hat{y}(t)$ be the estimate of $y(t)$ where

$$\hat{y} = \hat{C}x + \hat{D}u_p \tag{13}$$

with $\hat{C}(t)$ and $\hat{D}(t)$ being the estimates of the sensitivity matrices C and D, respectively. Next, the performance metric estimation error is computed as

$$e_y = \hat{y} - y = \tilde{C}x + \tilde{D}u_p \tag{14}$$

where $\tilde{C}(t) = \hat{C}(t) - C$ and $\tilde{D}(t) = \hat{D}(t) - D$ are the estimation errors of the sensitivity matrices C and D, respectively. $\hat{C}(t)$ and $\hat{D}(t)$ can be estimated from a least-squares gradient method that minimizes the following cost function

$$J = \frac{1}{2} e_y^\top e_y \tag{15}$$

This results in the following least-squares gradient adaptive laws for estimating $\hat{C}(t)$ and $\hat{D}(t)$:

$$\dot{\hat{C}}^\top = -\Gamma_C \frac{\partial J}{\partial \tilde{C}} = -\Gamma_C x e_y^\top \tag{16}$$

$$\dot{\hat{D}}^\top = -\Gamma_D \frac{\partial J}{\partial \tilde{D}} = -\Gamma_D u_p e_y^\top \tag{17}$$

where $\Gamma_C = \Gamma_C^\top \in \mathbb{R}^{n\times n} > 0$ and $\Gamma_D = \Gamma_D^\top \in \mathbb{R}^{m\times m} > 0$ are positive definite adaptation rate matrices. To design a performance optimizing control, we consider the following ideal plant with $u_{ad}^*(t) = -\Theta^{*\top}\Phi(x)$ to achieve perfect tracking:

$$\dot{x} = A_m x + B_m r + Bu_p \tag{18}$$

We now cast this problem as a multi-objective optimization problem by using the following infinite-time-horizon cost function [3]:

$$J = \lim_{t_f \to \infty} \frac{1}{2} \int_0^{t_f} \left(\hat{y}^\top Q \hat{y} + u_p^\top R u_p \right) dt \tag{19}$$

where $Q = Q^\top \in \mathbb{R}^{k\times k} > 0$ and $R = R^\top \in \mathbb{R}^{m\times m} > 0$ are positive definite weighting matrices, subject to the plant dynamics in Eq. (18). We formulate an optimal control problem by establishing the following Hamiltonian function:

$$H = \frac{1}{2} \hat{y}^\top Q \hat{y} + \frac{1}{2} u_p^\top R u_p + \mu^\top (A_m x + B_m r + Bu_p) \tag{20}$$

where $\mu(t) \in \mathbb{R}^n$ is an adjoint vector which acts to enforce the dynamic constraint imposed by the plant.

Then, the necessary conditions of optimality are obtained as

$$\dot{\mu} = -\frac{\partial H}{\partial x}^\top = -\hat{C}^\top Q\left(\hat{C}x + \hat{D}u_p\right) - A_m^\top \mu \tag{21}$$

subject to the transversality condition $\mu\left(t_f\right) = 0$, and

$$\frac{\partial H}{\partial u_p}^\top = \hat{D}^\top Q\left(\hat{C}x + \hat{D}u_p\right) + Ru_p + B^\top \mu = 0 \tag{22}$$

The optimal control $u_p(t)$ is then obtained as

$$u_p = -\left(R + \hat{D}^\top Q\hat{D}\right)^{-1}\left(B^\top \mu + \hat{D}^\top Q\hat{C}x\right) \tag{23}$$

We proceed with the assumed solution of $\mu(t)$ having the form

$$\mu = Wx + Vz + Uz_r \tag{24}$$

Upon substitution, we obtain the following equations:

$$\dot{W} + W\bar{A} + \bar{A}^\top W - WB\bar{R}^{-1}B^\top W + \bar{Q} = 0 \tag{25}$$

$$\dot{V} + \left(\bar{A}^\top - WB\bar{R}^{-1}B^\top\right)V + VA_r + WB_mC_r = 0 \tag{26}$$

$$\dot{U} + \left(\bar{A}^\top - WB\bar{R}^{-1}B^\top\right)U + V = 0 \tag{27}$$

subject to the transversality conditions $W\left(t_f\right) = 0$, $V\left(t_f\right) = 0$, and $U\left(t_f\right) = 0$, where

$$\bar{A} = A_m - B\bar{R}^{-1}\hat{D}^\top Q\hat{C} \tag{28}$$

$$\bar{Q} = \hat{C}^\top Q\left(I - \hat{D}\bar{R}^{-1}\hat{D}^\top Q\right)\hat{C} \tag{29}$$

$$\bar{R} = R + \hat{D}^\top Q\hat{D} \tag{30}$$

Q is chosen such that $\bar{Q} > 0$ which implies

$$\hat{D}\bar{R}^{-1}\hat{D}^\top Q < I \tag{31}$$

Equation (25) is recognized as the time-varying differential Riccati equation with the time-varying matrix $\bar{A}(t)$ and time-varying positive definite weighting matrices $\bar{Q}(t)$ and $\bar{R}(t)$ which are updated at each time step as $\hat{C}(t)$ and $\hat{D}(t)$ are computed from the least-squares gradient adaptive laws. The existence of the solution of a time-varying differential Riccati equation depends on the properties of the time-varying matrices $\bar{A}(t)$, $\bar{Q}(t)$, and $\bar{R}(t)$ according to the following theorem:

Theorem 1: Let $\bar{A}(t) = \bar{A}^* + \delta_{\bar{A}}(t)$, $\bar{Q}(t) = \bar{Q}^* + \delta_{\bar{Q}}(t) > 0$, and $\bar{R}(t) = \bar{R}^* + \delta_{\bar{R}}(t) > 0$ where $\bar{A}^*$, $\bar{Q}^*$, and $\bar{R}^*$ are some constant matrices. If $\delta_{\bar{A}}(t)$, $\delta_{\bar{Q}}(t)$, and $\delta_{\bar{R}}(t)$ are piecewise continuous for all $t \in [0,\infty)$; $\lim_{t\to\infty}\delta_{\bar{A}}(t) = 0$, $\lim_{t\to\infty}\delta_{\bar{Q}}(t) = 0$, and $\lim_{t\to\infty}\delta_{\bar{R}}(t) = 0$; and furthermore, $\dot{\delta}_{\bar{A}}(t)$, $\dot{\delta}_{\bar{Q}}(t)$, and $\dot{\delta}_{\bar{R}}(t)$ are uniformly continuous; then $\bar{A}(t)$, $\bar{Q}(t)$, and $\bar{R}(t)$ tend to their constant solutions $\bar{A}^*$, $\bar{Q}^*$, and $\bar{R}^*$, respectively, as $t \to \infty$. Consequently, the solution of the time-varying differential Riccati equation exists and also tends to its constant solution in the limit as $t_f \to \infty$.

Proof: If $\delta_{\bar{A}}(t)$, $\delta_{\bar{Q}}(t)$, and $\delta_{\bar{R}}(t)$ satisfy the conditions in Theorem 1, then according to the Barbalat's lemma $\lim_{t\to\infty}\dot{\delta}_{\bar{A}}(t) = 0$, $\lim_{t\to\infty}\dot{\delta}_{\bar{Q}}(t) = 0$, and $\lim_{t\to\infty}\dot{\delta}_{\bar{R}}(t) = 0$. A function $f(t)$ that satisfies the conditions $\lim_{t\to\infty} f(t) = 0$ and $\lim_{t\to\infty}\dot{f}(t) = 0$ exhibit convergence properties such as an exponential decay function. Therefore, $\bar{A}(t) \to \bar{A}^*$, $\bar{Q}(t) \to \bar{Q}^*$, and $\bar{R}(t) \to \bar{R}^*$ as $t \to \infty$.

Let $W(t) = W^* + \delta_W(t)$ be the solution of the time-varying Riccati equation. Then,

$$\begin{aligned}\dot{\delta}_W + (W^* + \delta_W)\left(\bar{A}^* + \delta_{\bar{A}}\right) + \left(\bar{A}^* + \delta_{\bar{A}}\right)^\top (W^* + \delta_W) \\ - (W^* + \delta_W)B\left(\bar{R}^* + \delta_{\bar{R}}\right)^{-1}B^\top (W^* + \delta_W) \\ + \left(\bar{Q}^* + \delta_{\bar{Q}}\right) = 0\end{aligned} \tag{32}$$

Separating terms and taking the limit as $t \to \infty$ yield

$$W^*\bar{A}^* + \bar{A}^{*\top}W^* - W^*B\bar{R}^{*-1}B^\top W^* + \bar{Q}^* = 0 \tag{33}$$

$$\begin{aligned}\dot{\delta}_W + \delta_W\bar{A}^* + \bar{A}^{*\top}\delta_W - \delta_W B\bar{R}^{*-1}B^\top \delta_W \\ - \delta_W B\bar{R}^{*-1}B^\top W^* - W^*B\bar{R}^{*-1}B^\top \delta_W = 0\end{aligned} \tag{34}$$

Now transforming Eq. (34) into the time-to-go variable $\tau = t_f - t$ gives

$$\begin{aligned}-\frac{d\delta_W}{d\tau} + \delta_W\bar{A}^* + \bar{A}^{*\top}\delta_W - \delta_W B\bar{R}^{*-1}B^\top \delta_W \\ - \delta_W B\bar{R}^{*-1}B^\top W^* - W^*B\bar{R}^{*-1}B^\top \delta_W = 0\end{aligned} \tag{35}$$

subject to $\delta_W(\tau = 0) = 0$ since $W\left(t_f\right) = 0$. The solution of Eq. (34) yields $\delta_W = 0$ and $\dot{\delta}_W = 0$ as $t_f \to \infty$. Thus, $W(t) \to W^*$ as $t_f \to \infty$ which implies $\dot{W}(t) \to 0$.

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Equation (26) is recognized as the time-varying differential Sylvester equation. The existence of the solution of Eq. (26) can be established from Theorem 1. If $W(t) \to \bar{W}^*$, then the closed-loop state transition matrix $\bar{A}(t) - B\bar{R}^{-1}(t)B^\top W(t)$ tends to a constant Hurwitz matrix $A_c = \bar{A}^* - B\bar{R}^{*-1}B^\top W^*$. Equations (26) is transformed into the time-to-go variable as

$$\frac{dV^*}{d\tau} = A_c^\top V^* + V^*A_r + WB_mC_r \tag{36}$$

subject to $V^*(\tau = 0) = 0$. Since A_r is a stable matrix, and A_r can be chosen such that $\lambda_i(A_c) + \lambda_j(A_r) \neq 0$ for all i and j, then it follows that $V^*(t)$ is bounded as $t_f \to \infty$ and tends to a constant solution of the following algebraic Sylvester equation:

$$A_c^\top V^* + V^*A_r + W^*B_mC_r = 0 \tag{37}$$

Equation (27) is transformed into the time-to-go variable as

$$\frac{dU^*}{d\tau} = A_c^\top U^* + V^* \tag{38}$$

subject to $U^*(\tau = 0) = 0$. Since $V^*(t)$ exists, $U^*(t)$ is bounded as $t_f \to \infty$ and tends to the following solution:

$$U^* = -A_c^{-\top} V^* \tag{39}$$

Thus, the solutions of Eqs. (25), (26), and (27) are computed from their corresponding time-varying algebraic equations. The performance optimizing controller u_p is then expressed as

$$u_p = \bar{K}_x(t)\, x + \bar{K}_z(t)\, z + \bar{K}_{z_r}(t)\, z_r \tag{40}$$

where

$$\bar{K}_x = -\bar{R}^{-1}\left(B^\top W + \hat{D}^\top Q \hat{C}\right) \tag{41}$$

$$\bar{K}_z = -\bar{R}^{-1} B^\top V \tag{42}$$

$$\bar{K}_{z_r} = \bar{R}^{-1} B^\top A_c^{-\top} V \tag{43}$$

Note that the uniform continuity condition for $\dot{\delta}_{\bar{A}}(t)$, $\dot{\delta}_{\bar{Q}}(t)$, and $\dot{\delta}_{\bar{R}}(t)$ requires the parameter convergence of the estimates $\hat{C}(t)$ and $\hat{D}(t)$. This convergence can be stated in the following theorem:

Theorem 2: If the reference command signal $r(t)$ is persistently exciting, then $\hat{C}(t)$ and $\hat{D}(t)$ converge exponentially to their true values and the uniform continuity condition for $\dot{\delta}_{\bar{A}}(t)$, $\dot{\delta}_{\bar{Q}}(t)$, and $\dot{\delta}_{\bar{R}}(t)$ is thus satisfied.

Proof: Choose a Lyapunov candidate function

$$V\left(\tilde{C},\tilde{D}\right) = \text{trace}\left(\tilde{C}\Gamma_C^{-1}\tilde{C}^\top + \tilde{D}\Gamma_D^{-1}\tilde{D}^\top\right) \tag{44}$$

Then, $\dot{V}\left(\tilde{C},\tilde{D}\right)$ is evaluated as

$$\begin{aligned}\dot{V}\left(\tilde{C},\tilde{D}\right) &= -2e_y^\top \tilde{C}x - 2e_y^\top \tilde{D}u_p \\ &= -2\left(\tilde{C}x + \tilde{D}u_p\right)^\top \left(\tilde{C}x + \tilde{D}u_p\right) = -2e_y^\top e_y \le 0\end{aligned} \tag{45}$$

Therefore, $\tilde{C}(t) \in \mathscr{L}_2 \cap \mathscr{L}_\infty$ and $\tilde{D}(t) \in \mathscr{L}_2 \cap \mathscr{L}_\infty$, but $x(t) \in \mathscr{L}_\infty$ and $u_p(t) \in \mathscr{L}_\infty$. It follow that $\bar{K}_x(t) \in \mathscr{L}_\infty$, $\bar{K}_z(t) \in \mathscr{L}_\infty$, and $\bar{K}_{z_r}(t) \in \mathscr{L}_\infty$. Therefore, for the ideal plant, $\dot{x}(t) \in \mathscr{L}_\infty$.

We note that $\ddot{V}\left(\tilde{C},\tilde{D}\right)$ is bounded since $\dot{x}(t) \in \mathscr{L}_\infty$ and $\dot{u}_p(t) \in \mathscr{L}_\infty$ by the virtue of $\dot{z}(t) \in \mathscr{L}_\infty$, $\dot{\bar{K}}_x(t) \in \mathscr{L}_\infty$, $\dot{\bar{K}}_z(t) \in \mathscr{L}_\infty$, and $\dot{\bar{K}}_{z_r}(t) \in \mathscr{L}_\infty$. Therefore, $\dot{V}\left(\tilde{C},\tilde{D}\right)$ is uniformly continuous. Invoking the Barbalat's lemma, $\dot{V}\left(\tilde{C},\tilde{D}\right) \to 0$ as $t \to \infty$. This implies $e_y(t) \to 0$ as $t \to \infty$. However, this does not necessarily imply that $\tilde{C}(t)$ and $\tilde{D}(t)$ tend to zero since $x(t)$ and $u_p(t)$ can be zero signals after some finite time interval.

Since $x(t)$and $u_p(t)$ depend continuously on $r(t)$, then the persistent excitation condition is satisfied if the reference command signal $r(t)$ is persistently exciting:

$$\frac{1}{T}\int_t^{t+T} \begin{bmatrix} x \\ u_p \end{bmatrix} \begin{bmatrix} x^\top & u_p^\top \end{bmatrix} d\tau \ge \alpha I \tag{46}$$

for all $t \ge t_0$ and some $\alpha > 0$. Let $\tilde{\Delta}^\top = \begin{bmatrix} \tilde{C} & \tilde{D} \end{bmatrix}$, $\Gamma_\Delta = \text{diag}\left(\Gamma_c, \Gamma_D\right)$, and $\Psi(x,u_p) = \begin{bmatrix} x^\top & u_p^\top \end{bmatrix}^\top$. Then,

$$\begin{aligned}\dot{V}\left(\tilde{C},\tilde{D}\right) &= -2\Psi^\top(x,u_p)\,\tilde{\Delta}\tilde{\Delta}^\top \Psi^\top(x,u_p) \\ &\le -\frac{2V\left(\tilde{C},\tilde{D}\right)\left\|\Psi(x,u_p)\right\|^2}{\lambda_{max}\left(\Gamma_\Delta^{-1}\right)}\end{aligned} \tag{47}$$

Since $\left\|\Psi(x,u_p)\right\|^2 I = \Psi^\top(x,u_p)\Psi(x,u_p) I \ge \Psi(x,u_p)\Psi^\top(x,u_p)$, it follows that

$$\dot{V}\left(\tilde{C},\tilde{D}\right) \le -\frac{2\alpha V\left(\tilde{C},\tilde{D}\right)}{\lambda_{max}\left(\Gamma_\Delta^{-1}\right)} \tag{48}$$

This implies

$$\left\|\tilde{C}\right\| \le \sqrt{\frac{V_0}{\lambda_{min}\left(\Gamma_C^{-1}\right)}}\exp\left(-\frac{\alpha t}{\lambda_{max}\left(\Gamma_\Delta^{-1}\right)}\right) \tag{49}$$

$$\left\|\tilde{D}\right\| \le \sqrt{\frac{V_0}{\lambda_{min}\left(\Gamma_D^{-1}\right)}}\exp\left(-\frac{\alpha t}{\lambda_{max}\left(\Gamma_\Delta^{-1}\right)}\right) \tag{50}$$

where $V_0 = V\left(\tilde{C}(0), \tilde{D}(0)\right)$. Therefore, $\tilde{C}(t)$ and $\tilde{D}(t)$ converge exponentially to zero as $t \to \infty$. Note that $\tilde{C}(t)$ and $\tilde{D}(t)$ satisfy the uniform continuity condition. Then, it follows that $\delta_A(t)$, $\delta_{\bar{Q}}(t)$, and $\delta_{\bar{R}}(t)$ are also uniformly continuous.

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It follows from Theorem 1 that $\hat{C}(t) \to C$ and $\hat{D} \to D$ as $t \to \infty$. This implies that $\bar{A}(t)$, $\bar{Q}(t)$, $\bar{R}(t)$, $\bar{K}_x(t)$, $\bar{K}_z(t)$, and $\bar{K}_{z_r}$ all converge to their ideal values $\bar{A}^*$, $\bar{Q}^*$, $\bar{R}^*$, $\bar{K}_x^*$, $\bar{K}_z^*$, and $\bar{K}_{z_r}^*$, respectively.

The closed-loop plant with the performance optimizing controller $u_p(t)$ now becomes

$$\begin{aligned}\dot{x} = (A_m + B\bar{K}_x)\,x + B_m r + K_z z + K_{z_r} z_r + Bu_{ad} \\ + B\Theta^{*\top}\Phi(x)\end{aligned} \tag{51}$$

If the parameter convergence is achieved, then the ideal performance optimizing reference model as

$$\dot{x}_m^* = A_m^* x_m^* + B_m r + BK_z^* z + BK_{z_r}^* z_r \tag{52}$$

where $A_m^* = A_m + B\bar{K}_x^*$. Let $\hat{x}_m(t)$ be the estimate of $x_m^*(t)$. Then, the time-varying performance optimizing reference model is given by

$$\dot{\hat{x}}_m = (A_m + B\bar{K}_x)\,\hat{x}_m + B_m r + BK_z z + BK_{z_r} z_r \tag{53}$$

Thus, $\hat{x}_m(t) \to x_m^*(t)$ as $t \to \infty$. Let $e(t) = \hat{x}_m(t) - x(t)$ be re-defined as the tracking error based on the time-varying modified reference model. Then, the tracking error equation becomes

$$\dot{e} = (A_m + B_p\bar{K}_x)\,e + B\tilde{\Theta}^\top \Phi(x) \tag{54}$$

The MRAC update law for $\Theta(t)$ is based on the optimal control modification [9] and given by

$$\dot{\Theta} = -\Gamma\Phi(x)\left[e^\top W - \nu\Phi^\top(x)\Theta B^\top W\left(A_m + B\bar{K}_x\right)^{-1}\right]B \tag{55}$$

where $\nu > 0$ is a modification parameter for use in the design trade-off. By setting $\nu = 0$, we recover the new MRAC update law

$$\dot{\Theta} = -\Gamma\Phi(x)e^\top WB \tag{56}$$

where the time-varying matrix $W(t)$ replaces the constant matrix P in the standard MRAC update law in Eq. (8).

The stability of the optimal control modification adaptive law can be shown in the following proof:

Proof: Choose a Lyapunov candidate function

$$V\left(e,\tilde{\Theta}\right) = e^\top We + \operatorname{trace}\left(\tilde{\Theta}^\top\Gamma^{-1}\tilde{\Theta}\right) \tag{57}$$

Then, $\dot{V}\left(e,\tilde{\Theta}\right)$ is evaluated as

$$\begin{aligned}\dot{V}\left(e,\tilde{\Theta}\right) &= \\ &-e^\top\left(\dot{W} + WA_m + WB\bar{K}_x + A_m^\top W + \bar{K}_x^\top B^\top W\right)e \\ &-2\nu\Phi^\top(x)\Theta B^\top W\left(A_m + B\bar{K}_x\right)^{-1}B\tilde{\Theta}^\top\Phi(x)\end{aligned} \tag{58}$$

From Theorem 1, $\dot{W}(t) \to 0$ as $t_f \to \infty$. Then, substituting Eq. (41) into Eq. (58), we get

$$\begin{aligned}\dot{V}\left(e,\tilde{\Theta}\right) &= -e^\top\left(\bar{Q} + WB\bar{R}^{-1}B^\top W\right)e \\ -2\nu\Phi^\top(x)\Theta B^\top W&\left(A_m + B\bar{K}_x\right)^{-1}B\tilde{\Theta}^\top\Phi(x) \le -c_1\|e\|^2 \\ &-\nu c_2\|\Phi(x)\|^2\left(\|\tilde{\Theta}\| - c_3\right)^2 + \nu c_2 c_3^2\|\Phi(x)\|^2\end{aligned} \tag{59}$$

where $c_1 = \inf_t \lambda_{min}\left(\bar{Q}^* + W^*B\bar{R}^{*-1}B^\top W^*\right) > 0$, $c_2 = \inf_t \lambda_{min}\left(B^\top A_m^{*-\top}\left(\bar{Q}^* + W^*B\bar{R}^{*-1}B^\top W^*\right)A_m^{*-1}B\right) > 0$, and $c_3 = \frac{\sup_t\left\|B^\top W^* A_m^{*-1}B\right\|\|\Theta^*\|}{c_2} > 0$. Then, $\|\Phi(x)\| \le \Phi_0$ for some $0 < \nu < \nu_{max}$. Thus, $\dot{V}\left(e,\tilde{\Theta}\right) \le 0$ outside a compact set. Therefore, the closed-loop system is uniformly ultimately bounded.

If $\nu = 0$, then it can be shown that $\ddot{V}\left(e,\tilde{\Theta}\right)$ is bounded. Then, according to the Barbalat's lemma, $\dot{V}\left(e,\tilde{\Theta}\right) \to 0$ which implies $e(t) \to 0$ as $t \to \infty$. Thus, asymptotic tracking is achieved with the time-varying reference model modification by the performance optimizing controller for the new MRAC update law with $\nu = 0$.

III. Application

We implement a maneuver load alleviation adaptive control design for a flexible wing Generic Transport Model (GTM) equipped with the Variable Camber Continuous Trailing Edge Flap (VCCTEF) originally proposed by NASA [10], as shown in Fig. 1.

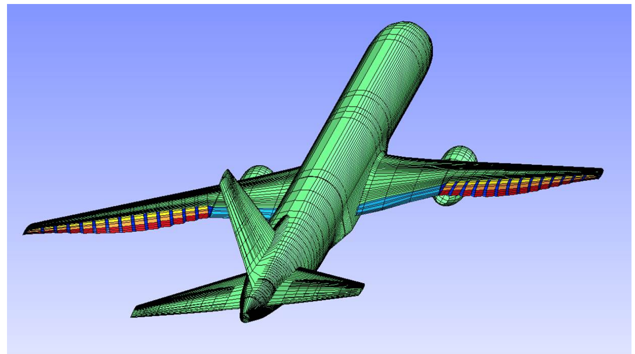

Figure 1. GeTM with Variable Camber Continuous Trailing Edge Flap

Consider a linearized reduced-order longitudinal model of a flexible aircraft with matched uncertainty in the rigid aircraft states

$$\dot{x}_r = A_r x_r + B_p u_p + B_a\left[u_a + \Theta^{*\top}\Phi(x_r)\right] + \Delta \tag{60}$$

where $x_r(t) \in \mathbb{R}^{n_r}$ is a rigid aircraft state vector, $u_p(t) \in \mathbb{R}^{m-1}$ is a control vector due to the VCCTEF, $u_a(t) \in \mathbb{R}$ is a control due to the elevator, $\Theta^* \in \mathbb{R}^{p\times m}$ is a constant and unknown matrix, $\Phi(x_r) \in \mathbb{R}^p$ is a vector of known regressors, and Δ represents the effect of unmodeled dynamics of the elastic wing modes. The reduced-order plant matrix A_r is assumed to be Hurwitz. For the maneuver load alleviation, the performance metric is taken to be the wing root bending moment which is measured from a strain gauge sensor. A virtual control concept is employed whereby the flap deflection is mapped into a mathematically smooth shape function whose coefficients are the virtual control variables [3]. Let $u_p(t) = \begin{bmatrix} c_0(t) & c_1(t) & c_2(t) & c_3(t)\end{bmatrix}^\top$ be a vector of the unknown coefficients of a cubic Chebyshev polynomial. We design a pitch attitude controller for the elevator using the reduced-order model to track the following second-order pitch attitude reference model:

$$\ddot{\theta}_m + 2\zeta\omega_n\dot{\theta}_m + \omega_n^2\theta_m = \omega_n^2 r \tag{61}$$

This reference model is designed to only track a pitch attitude command without any knowledge of the wing root bending moment to be minimized during a pitch maneuver. We initially choose a sinusoidal reference command signal $r(t) = \theta_0\sin\omega t$ where $\theta_0 = 20^\circ$ and $\omega = 2$ rad/sec to ensure the persistently exciting signal quality to demonstrate the parameter convergence. The simulation uses a full model with 65 states. The optimal control modification adaptive law is implemented with $\nu = 0.01$. Using the linear asymptotic property of the optimal control modification [11], the closed-loop plant in the limit is shown to be stable with $\nu = 0.01$. The aircraft response of $\theta(t)$ tracks the time-varying modified reference model very well and the amplitude of the performance metric $y(t)$ is reduced by 38.8% due to the performance optimizing adaptive controller, as shown in Fig. 2. Thus, the effectiveness of the performance optimizing adaptive controller is demonstrated. The estimated performance metric $\hat{y}(t)$ converges to the measured performance metric $y(t)$ extremely well. Figure 3 shows the time histories of the adaptive parameters $\Theta(t)$, $\hat{C}(t)$, and $\hat{D}(t)$. The elements of the estimated matrices $\hat{C}(t)$ and $\hat{D}(t)$ converge to their steady state values close to the true values. The elements of $\Theta(t)$ do not converge but fluctuate near their ideal values. The optimal control modification slightly reduces the initial high frequency oscillations in the signals.

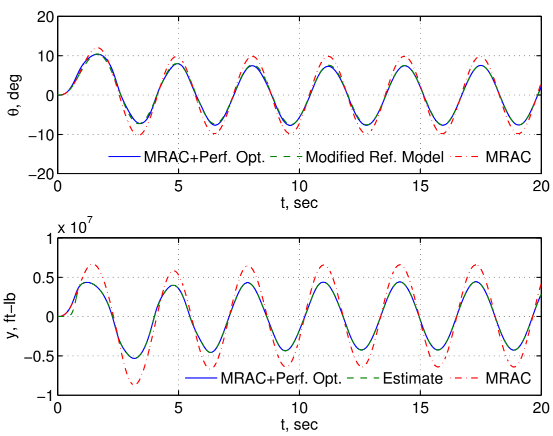

Figure 2. Aircraft Response to Performance Optimizing Adaptive Control with $r(t) = \theta_0 \sin \omega t$

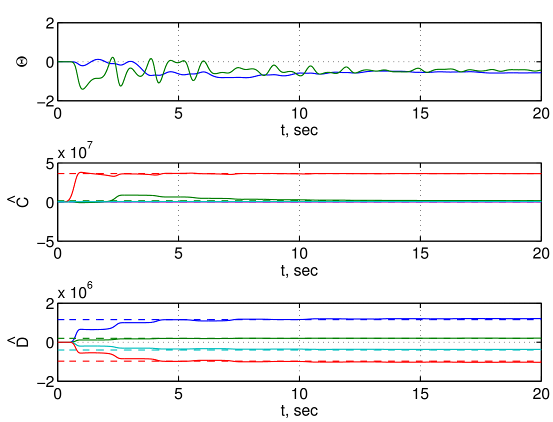

Figure 3. $\Theta(t)$, $\hat{C}(t)$, and $\hat{D}(t)$ with $r(t) = \theta_0 \sin \omega t$

IV. Conclusions

This paper presents a new method called performance optimizing adaptive control. The performance optimization is formulated as a multi-objective optimal control problem coupled with a least-squares adaptive law for estimating the unknown sensitivities of the performance metric needed. This results in time-varying Riccati and Sylvester equations whose solutions provide control gains for use in a time-varying modified reference model. The Lyapunov stability theory shows that the model-reference adaptive control with the time-varying modified reference model to achieve the performance optimization is stable and bounded. Simulations of maneuver load alleviation control for a flexible aircraft demonstrate the effectiveness of the performance optimizing adaptive control.

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