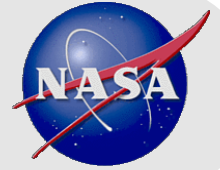# OBLATE-EARTH EFFECTS ON THE CALCULATION OF E_C DURING SPACECRAFT RE-ENTRY
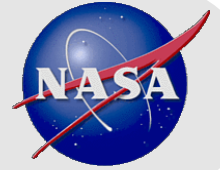
John B. Bacon, Ph.D. P.E.
Mark Matney, Ph.D.
NASA Orbital Debris Program Office

The problem being addressed

- **Casualty calculations for any given orbit ASSUME that an object enters with equal probability over all possible locations along an orbit arc. (Equally distributed in Argument of Latitude ("ArgLat" or Θ)**

- **Simulations and qualitative physics show that the equatorial bulge of the Earth pushes a "wall of air" into the path of the object at the equator, equal to several times the density scale height, and potentially up to a 50x local density increase at the equator relative to the density at the poles.**

- **Periodic geometric and gravitational variations are <u>both</u> distorting the orbit**

- **THEREFORE:**
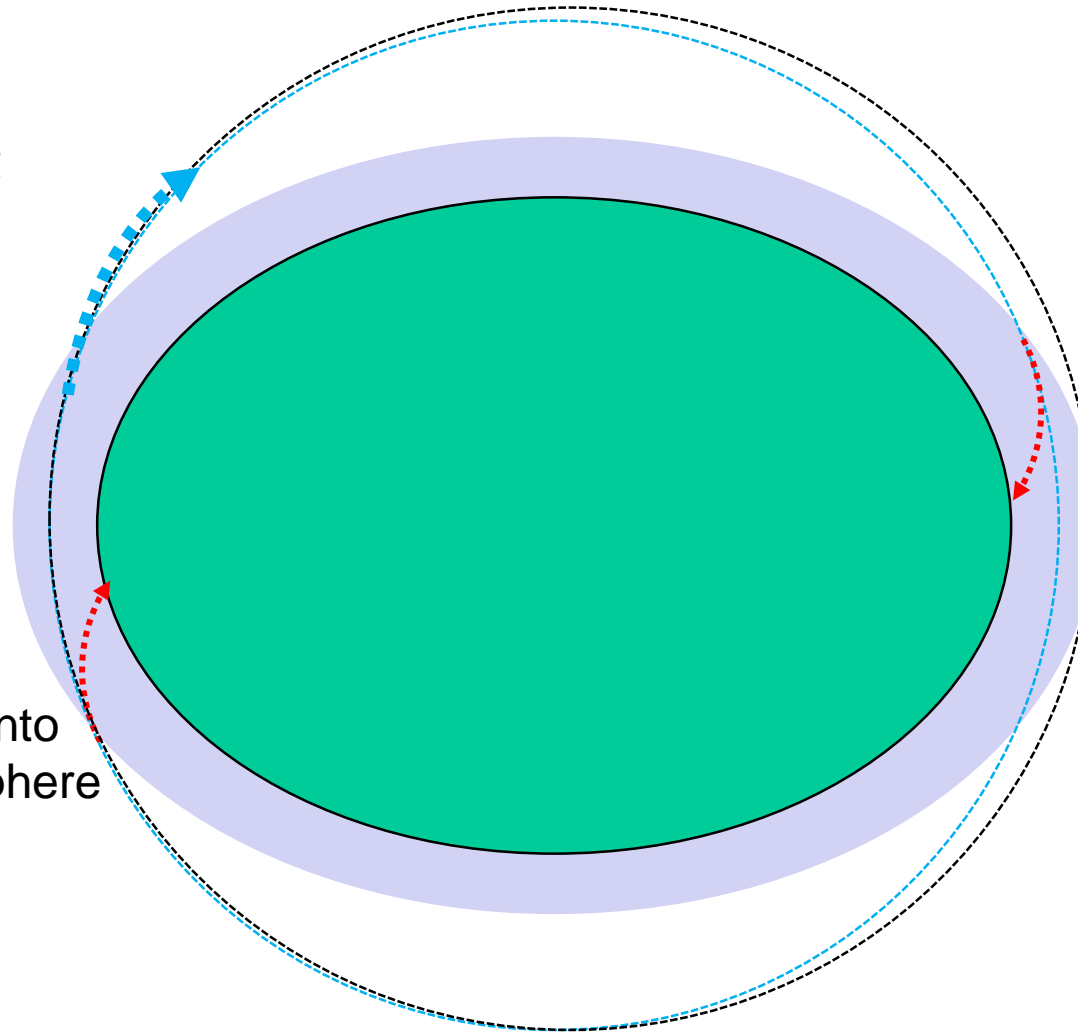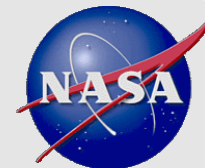 - **there should be periodicity!**
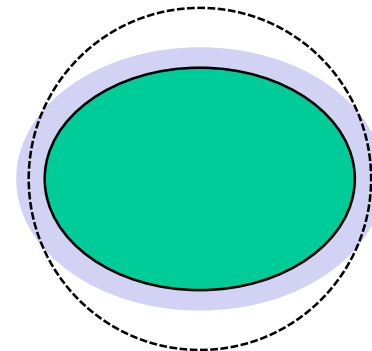
Why it is Cyclic
(and why it precedes the nodal crossing)

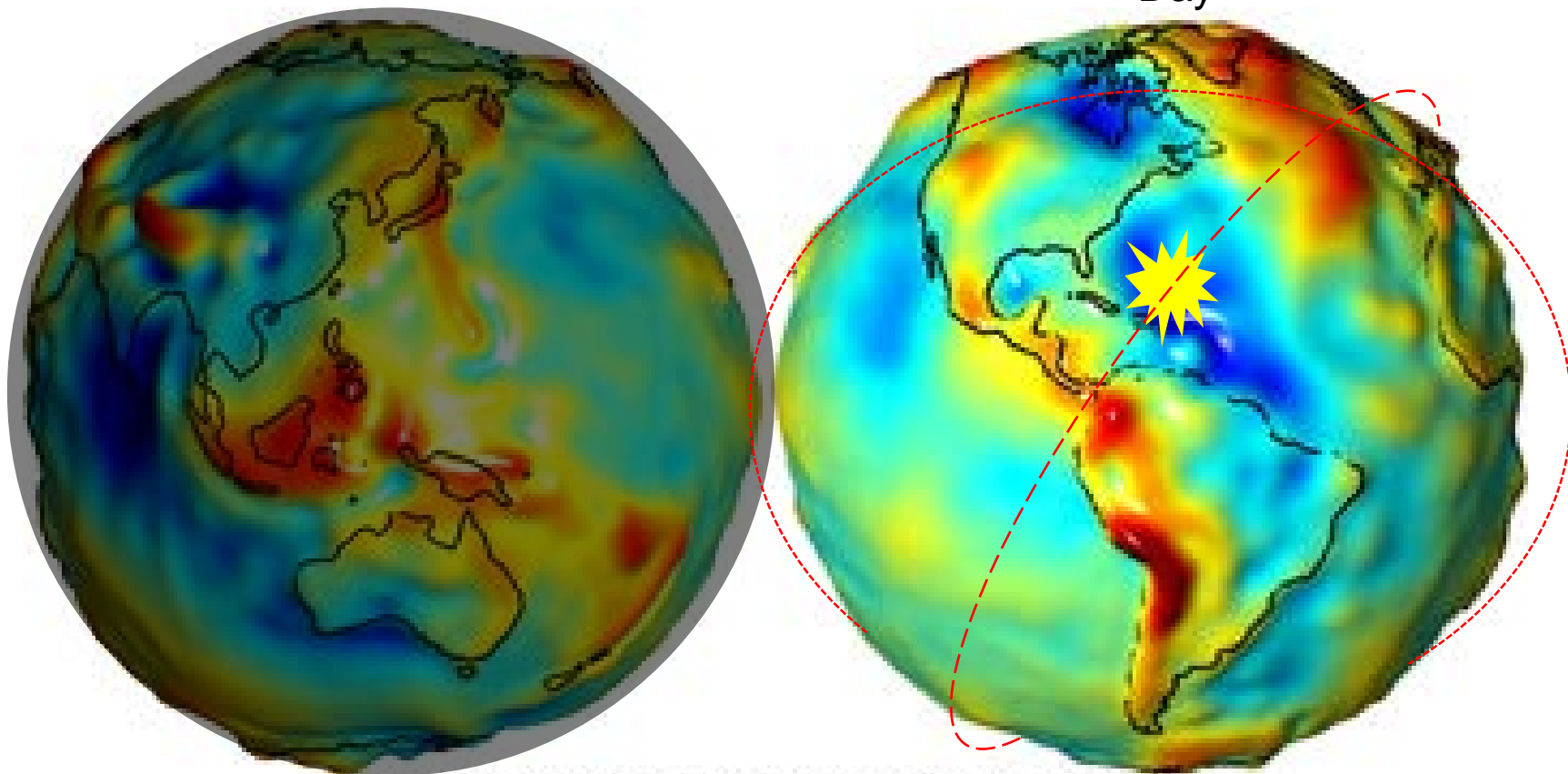Atmosphere "falls away" faster than decay rate for most objects that survive passage through the "wall of air". Unlikely decay for next ¼ orbit

Objects that survive the previous pass have perigee near equator, and rapidly rising density on the approach

Dives into atmosphere

JBB

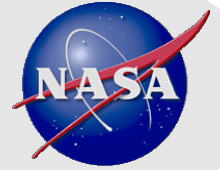# Why it is not simple:

Night

Day

Earth's Gravity Field Anomalies (milligals)
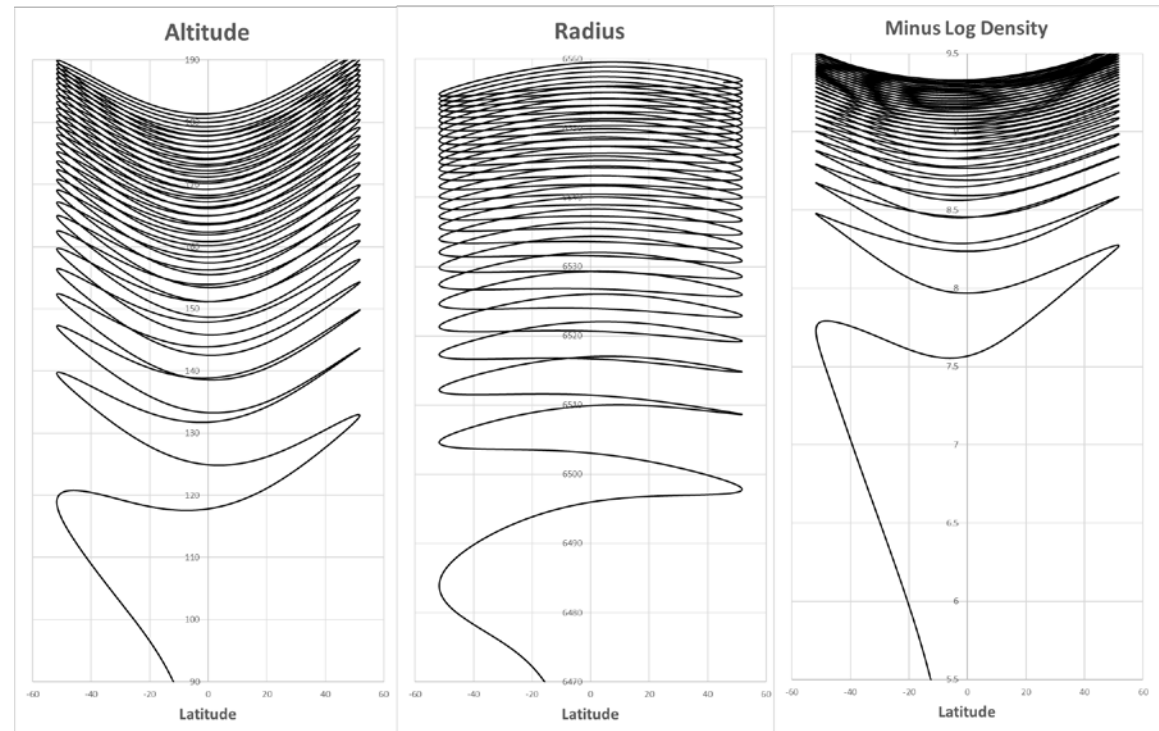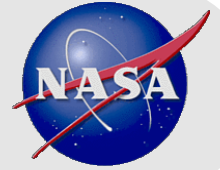
-50 -40 -30 -20 -10 0 10 20 30 40 50

Earth's gravity
Field as mapped by
the NASA GRACE
Mission

JBB

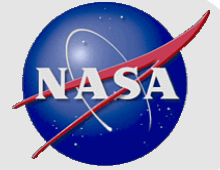# Final Decay Parameters vs. Latitude

- **The equatorial bulge affects drag.   Note the <u>concave</u> altitude and density plots vs. latitude.**

- **However, there are interesting (and countering) J3 <u>convex</u> effects going on in the radius plot**
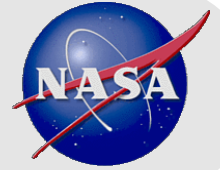
Prior Work

- **Matney and Bacon had independently hypothesized an effect due to the oblate Earth**
    - Matney, M. J., "Statistical Issues for Uncontrolled Reentry Hazards," 3rd IAASS Conference, Rome, Italy, 2008.
    - Bacon, J.B. "The Clustering of Natural Decays in Equatorial Zones" Internal ISS Whitepaper 8/17/2015
- **Matney published an early study of real polar decays, indicating a clustering of decays near the equator.**
    - Matney, M. J., "Statistical Issues for Uncontrolled Reentry Hazards: Empirical Tests of the Predicted Footprint for Uncontrolled Satellite Reentry Hazards", 4th IAASS Conference, Versailles-Paris, France, 2011.
- **Bacon and Matney collaborated on an IAASS workshop paper in May 2016 confirming that the effect was significant, demonstrating a sinusoidal compression and rarefaction of decays, and suggesting forward work.**
    - Bacon, J.B. & Matney, M.J. (2016). Statistical Issues for Calculating Reentry Hazards, IAASS 8th Workshop on Space Safety, May 21 2016.
- **Bacon at Darmstadt 2017 provided additional insight into the mechanisms and structure of the decay compression**
    - Bacon, J.B. (2017). *Minimum dV for Targeted Spacecraft Disposal* Proc. 7th European Conference on Space Debris, Darmstadt, Germany, 18–21 April 2017
- **Claire Fremeaux and her group at CNES suggested in Darmstadt 2017 that a J3 gravitational effect was also at work, evident in her decay studies of highly eccentric orbits.**
    - Cid Borobia, E., Fremeaux, C., and Goester J.F.: *"Fast Re-Entry Deorbitation with Acceptable Risk Level"* Proc. 7th European Conference on Space Debris, Darmstadt, Germany, 18–21 April 2017
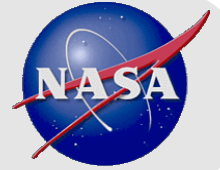
JBB

So…The New Approach

- **We go looking for periodicity (cyclic effects) in decay profiles**
  - **Both symmetric and asymmetric around the equator**
  - **Convert analysis space to <u>Argument of Latitude,</u> not <u>Latitude</u>**
    - **Normalizes to common reference curve**
    - **Directly models the concentration effect in ways meaningful to Ec**
    - **Latitude can be reconstructed: ArcSin[{Sin(Inclination)}*{Sin(Argument of Latitude)}]**

- **To be complete, we need to study a LOT of profiles**

- **To characterize periodicity, we formulate the clustering effect as a Fourier series, and look for the major periodic terms.**

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# First we simulate a <u>lot</u> of entries:

- **We ran the General Mission Analysis Tool (GMAT) on <u>over 30K</u> decay cases to 90 km:**
  - **18 inclinations from 5-90 in 5-degree steps**
  - **4 Ballistic numbers (50, 100, 150, 200 kg/m2)**
  - **4 Right Ascensions of Ascending Node at the Vernal Equinox**
  - **101 incremental BN changes to smoothly dither the 4-day decay**
    - **AND, there are about 18,000 propagations per run, for > 0.5E9 propagations!**

- **Several dozen follow-up cases were also run for comparison:**

- **101 dithered runs <u>each</u> for:**
  - **Entry on Summer solstice at 100 BN, Inclination =60, 24 RAANs**
  - **BN=600 kg/m2 all inclinations, Vernal Equinox RAAN = 90**
  - **BN= 300, 400, 500 at 90 inclination Vernal Equinox RAAN = 90**

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# 100 entries around Arg(Lat) for Every Case
…for every Inclination, BN, and RAAN

1. **First, find how many points (N points) lie along 360 degrees for Argument of Latitude**
   - **Typically not an integer: e.g., 57.632**

2. **The average expected gap in Argument of Latitude to the next point is Average = 360/N**
   - **In the example, Average = [360/(57.632)] = 6.247 degrees**

3. **Measure the actual gap in Argument of Latitude to the next point**

4. **Ratio the actual gap to the average gap**
   - **In regions of clustered decays, the number is <u>less</u> than one**
   - **In regions of rarified decays, the number is <u>greater</u> than one.**

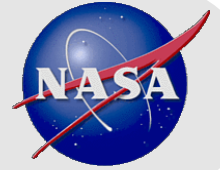- **PLOT the compression ratio vs Argument of Latitude**
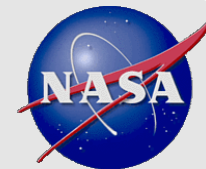
Sample Compression File

Compression Curve for 50 Degree Inclination
RAAN=180, BN=200

Note: nodes and peaks precede the equatorial crossings and peak latitudes
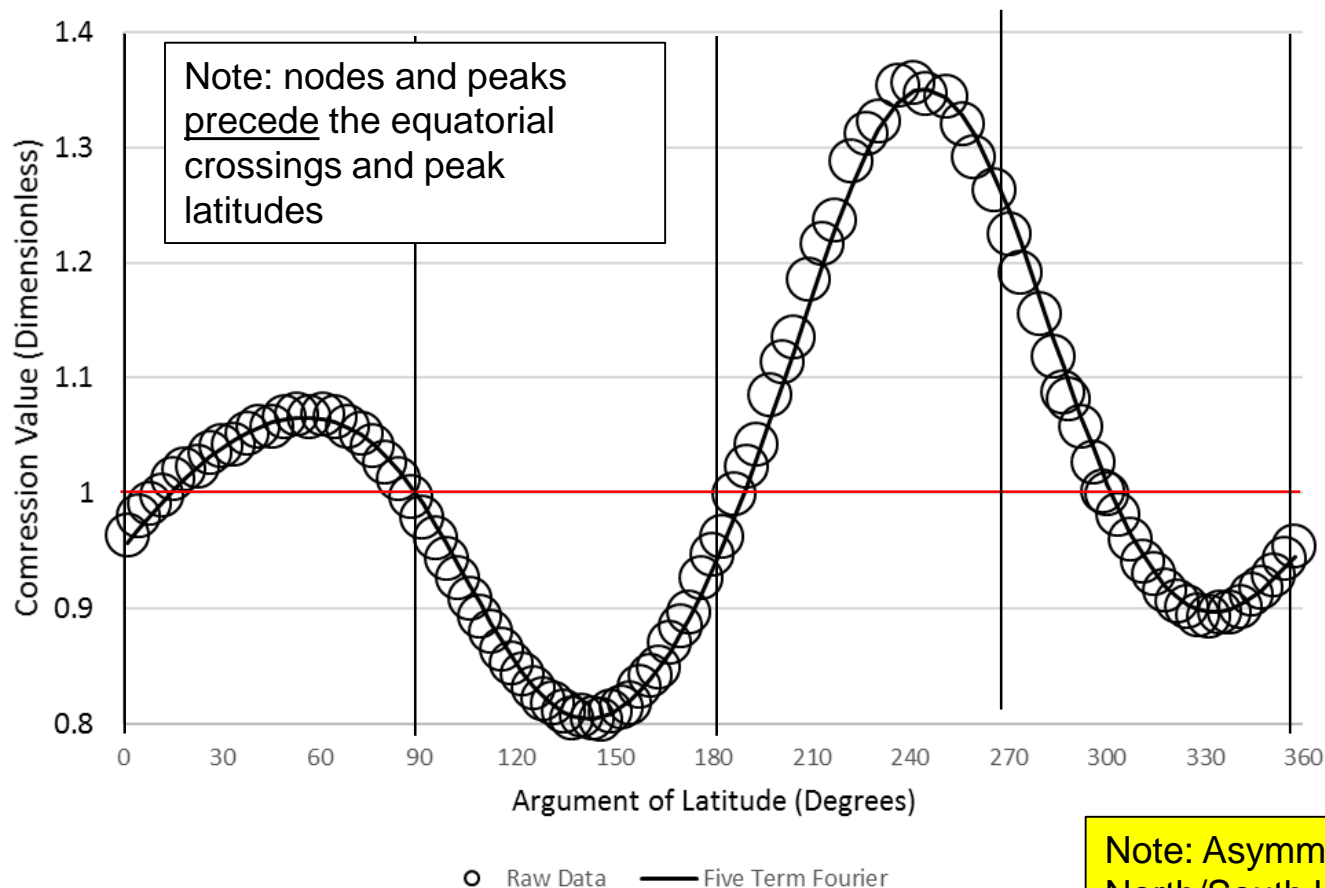
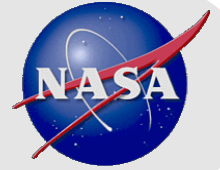Note: Asymmetry w.r.t. North/South latitudes

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Fourier Transform the Compression Curve

- **Divide Argument of Latitude into 256 even increments**

- **<u>Sort</u> the actual compression curve points by Argument of Latitude**

- **LOOK UP the closest Compression Curve value to each of the 256 Argument of Latitude sample bins**
  - **NOTE: we only have 100 points, and thus a chunky/noisy binned representation, BUT we only need the 1st 4 terms (long averages)**

- **Fourier transform the bin-sampled curve**
  - **(XCEL application)**

Raw Data   256 Fast Fourier Bins
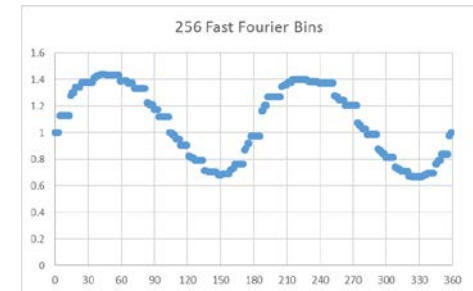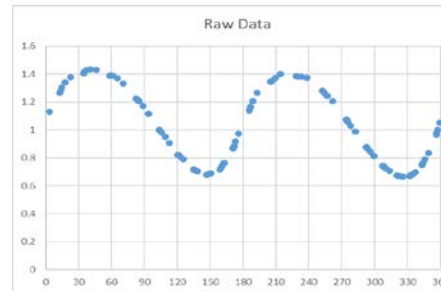
- **Collect amplitude and phase of <u>first four</u> Fourier terms**
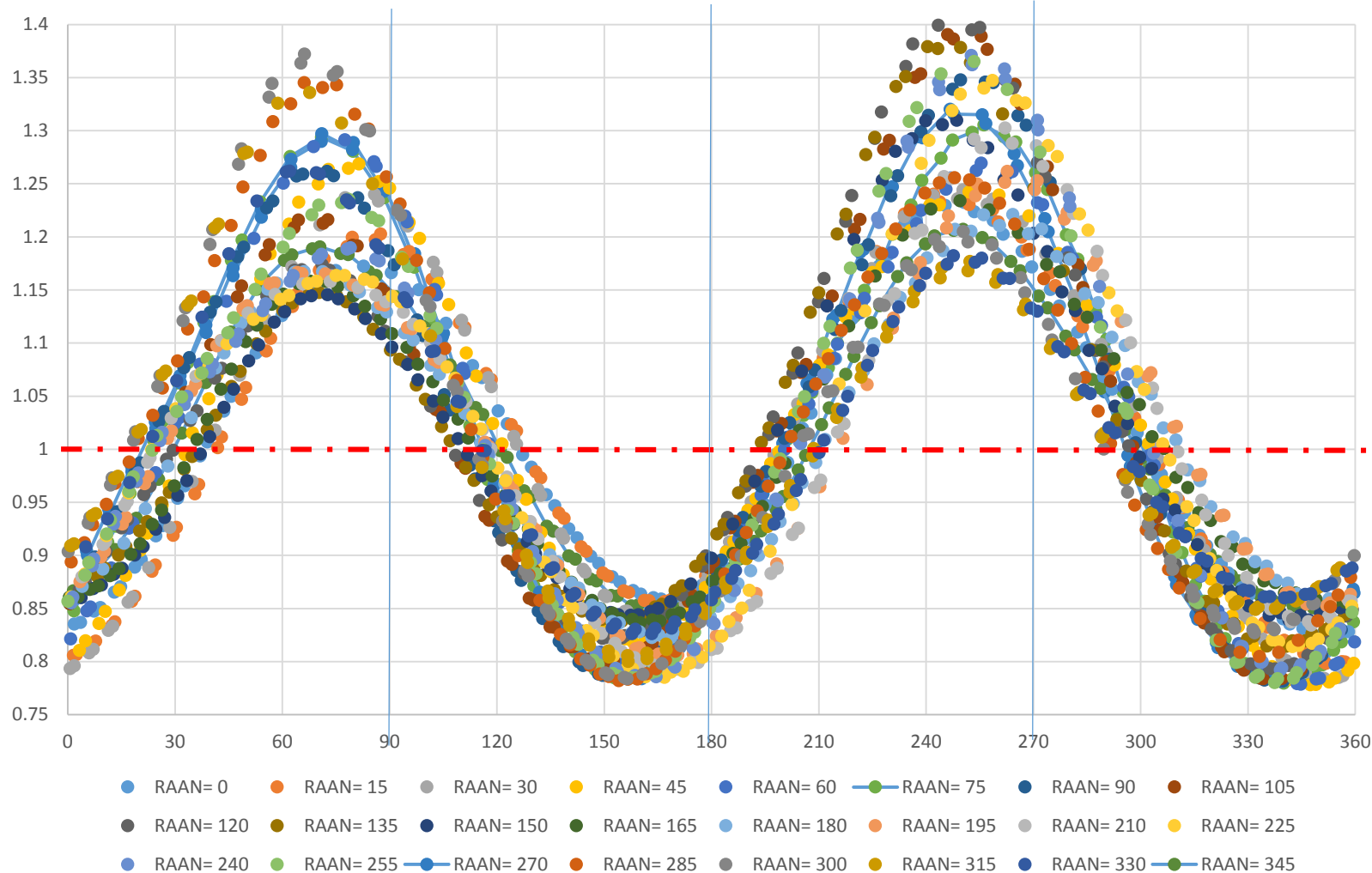 - **Constant (Zeroth term) is forced equal to ONE.**

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Compare Sizes and Phases of the Fourier Terms
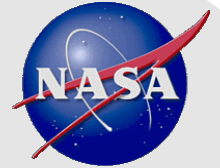
- **<u>Even-numbered terms</u> are symmetric North-South**
  - **(J2, J4)**
  - **The Cos(2Θ) term is dominant (30-40% compression factor)**
    - **It varies slightly with RAAN and in a sine-like way with inclination**
    - **This is due principally to the "wall of air", plus perhaps a J2 Gravitation effect**
    - **We don't yet understand the "S-curve" growth of amplitude with inclination***
  - **The Cos(4Θ) term is 1-3% compression factor**
    - **This generally rises with inclination, from near zero**

- **<u>Odd-numbered terms</u> show North-South biasing**
  - **The Cos(Θ) term is highly erratic and variable with inclination and RAAN, generally < 10% and >1% compression**
    - **We believe that this is solar heating dominating a small J3 effect**
  - **The Cos(3Θ) term is 1-3% compression and is very repeatable over all conditions.**
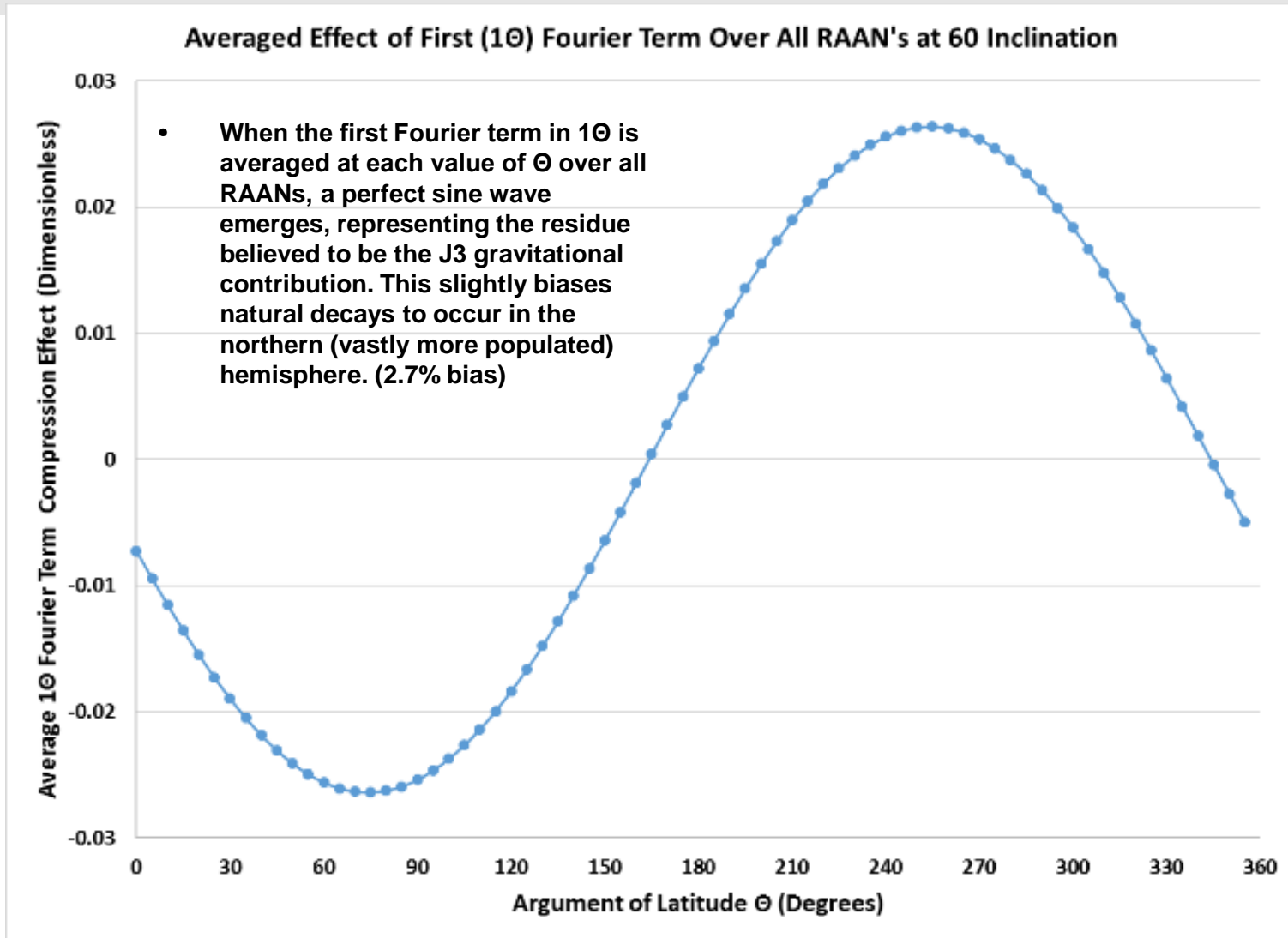 - **The Cos(5Θ) term is negligible, and not expected from J4 Gravity model**

JBB
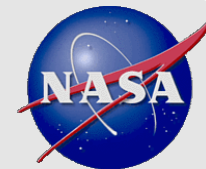
RAAN Family of Compression Curves:

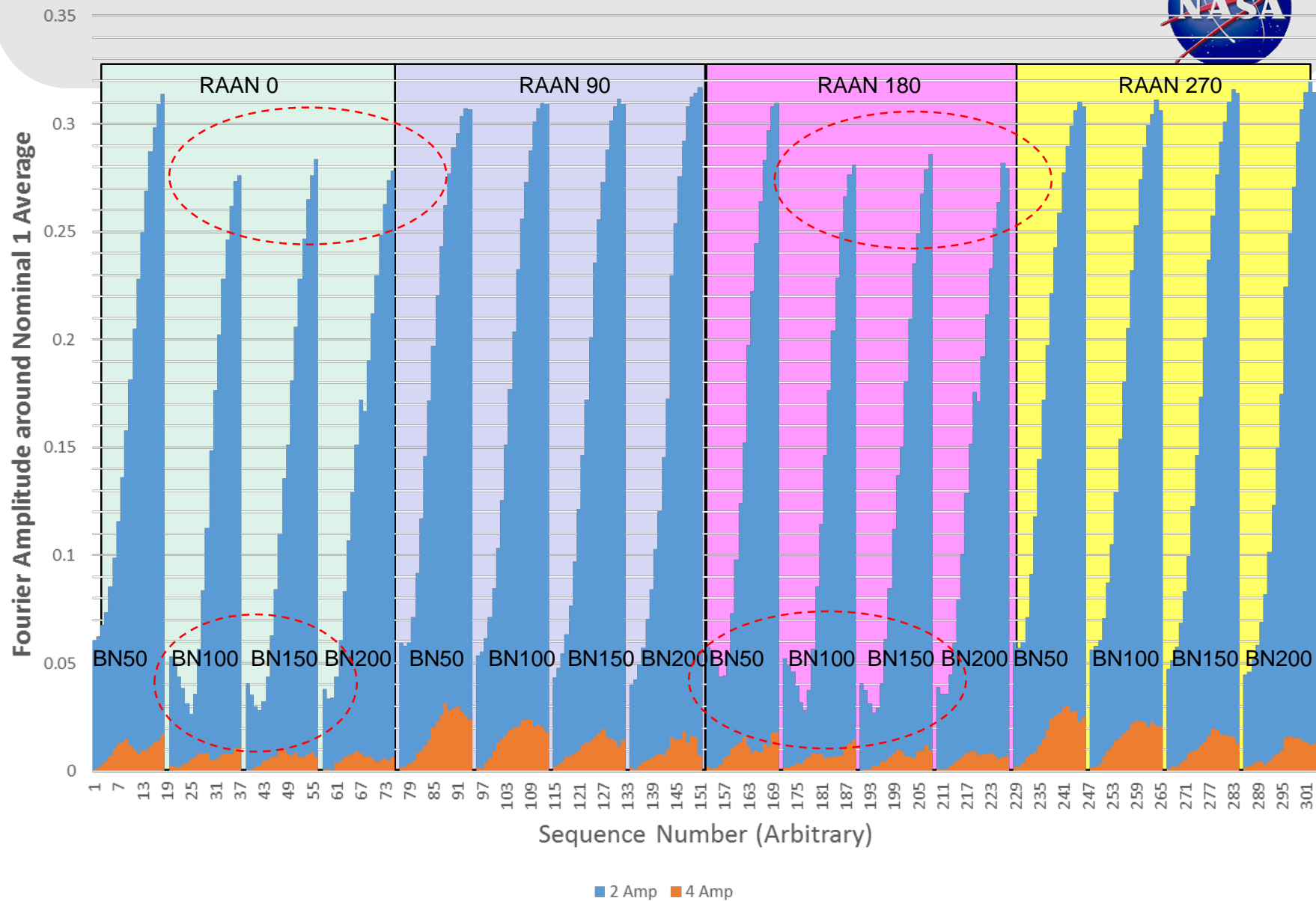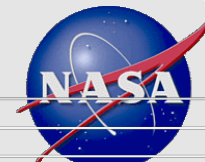

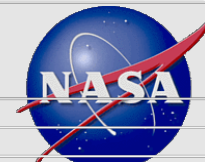Solstice/ Inc=60 / BN=100 / RAAN_Variable Compression Curves

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The J3 Term Falls Out When Averaged!

Averaged Effect of First (1Θ) Fourier Term Over All RAAN's at 60 Inclination

- **When the first Fourier term in 1Θ is averaged at each value of Θ over all RAANs, a perfect sine wave emerges, representing the residue believed to be the J3 gravitational contribution. This slightly biases natural decays to occur in the northern (vastly more populated) hemisphere. (2.7% bias)**

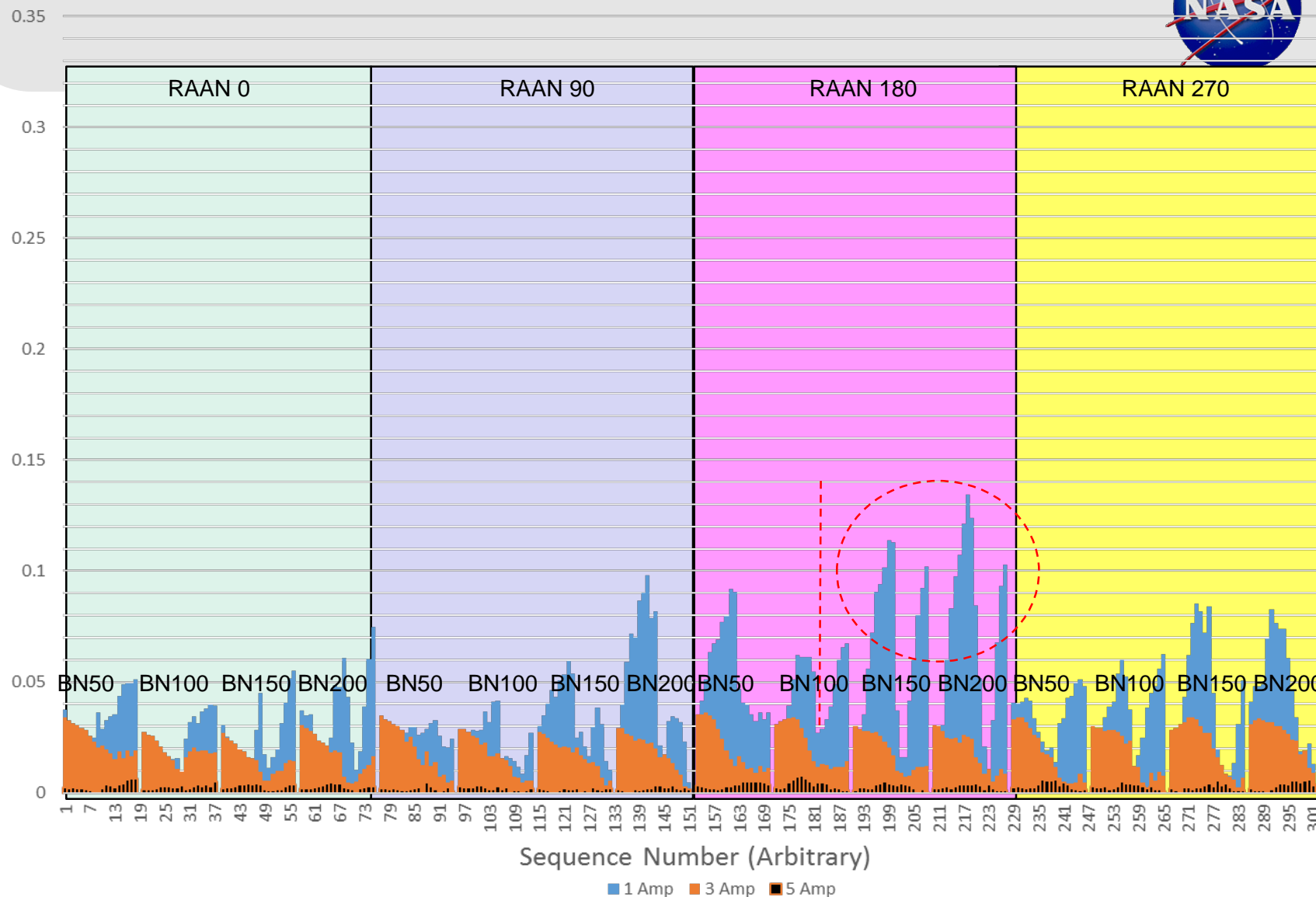14/32   **JBB**

Compression Curve Second and Fourth Fourier Amplitudes

Compression Curve First, Third, & Fifth Fourier Amplitudes

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Compression Curve Components at 90 Inclination

Combined-Even and -Odd Fourier Terms High Inclination

Terms 2 & 4

Terms 1&3

Legend: INC_90 BN_50 RA_90 — INC_90 BN_100 RA_90 — INC_90 BN_150 RA_90 — INC_90 BN_200 RA_90

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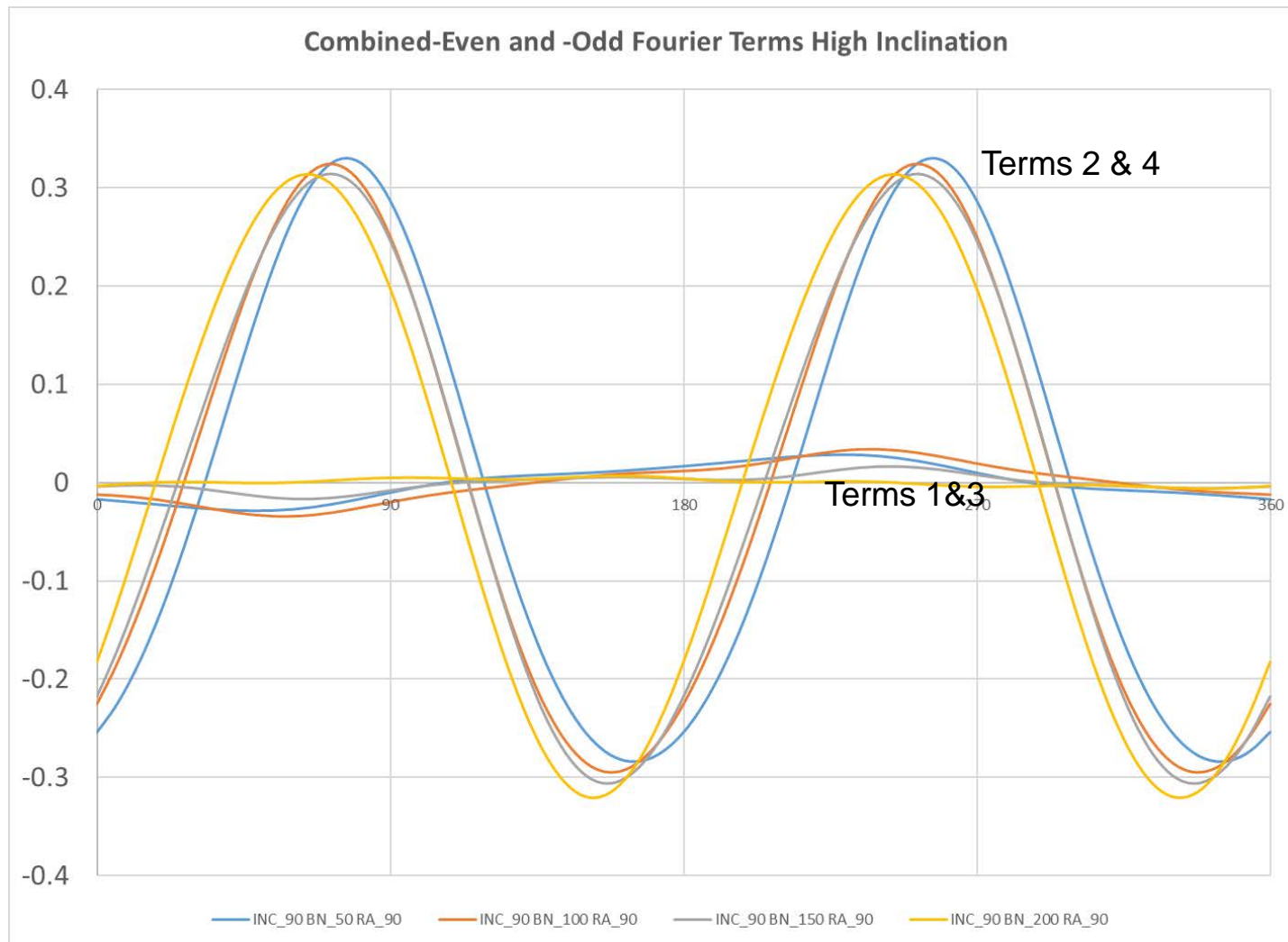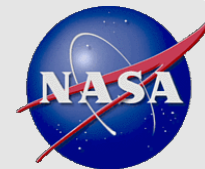
Compression Curve Components at 10 Inclination

Combined-Even and -Odd Fourier Terms Low Inclination

Terms 1&3   Terms 2 & 4

INC_10 BN_50 RA_90 ——— INC_10 BN_100 RA_90 ——— INC_10 BN_150 RA_90 ——— INC_10 BN_200 RA_90

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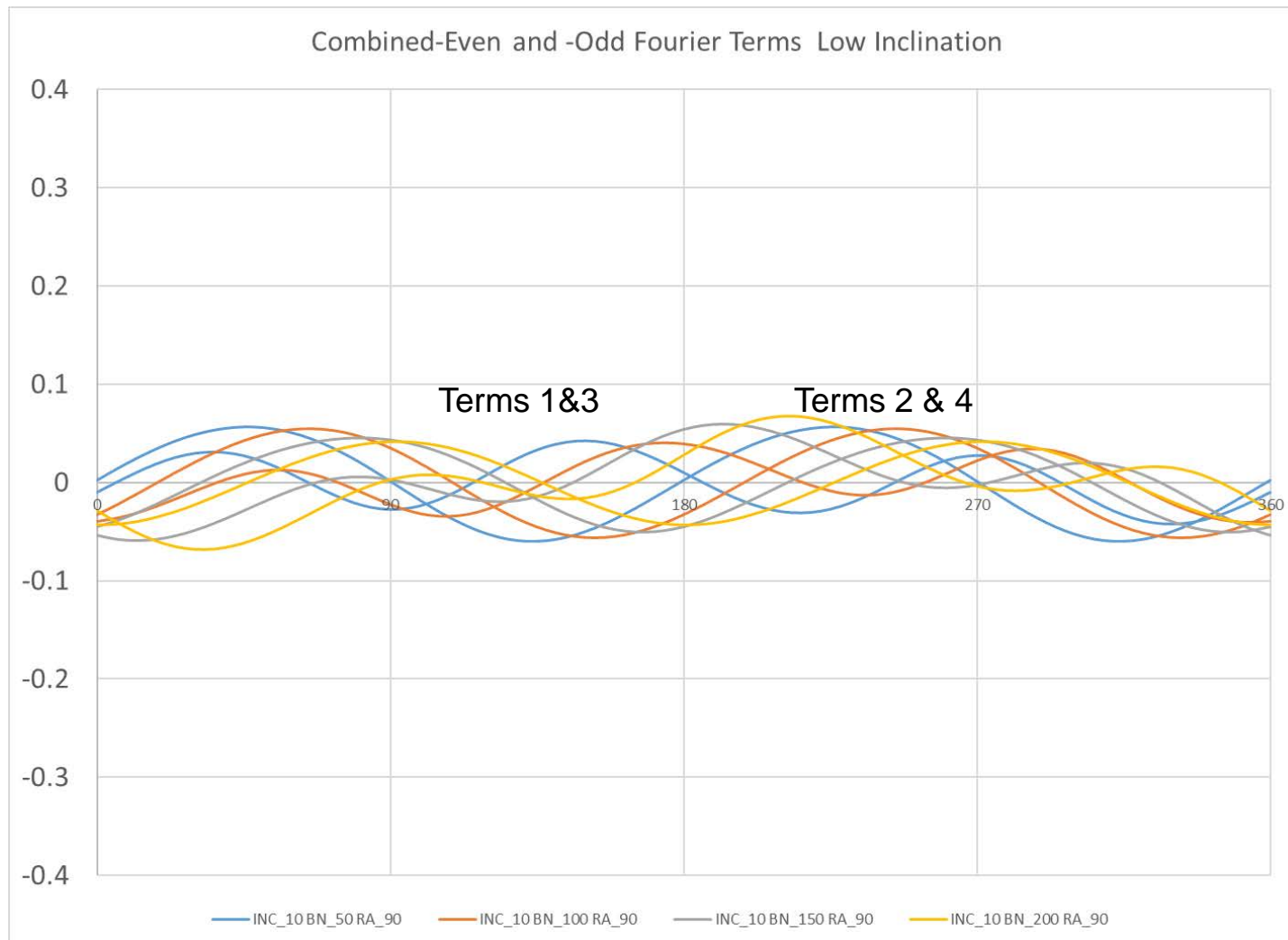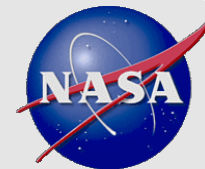
Amplitude and Phase of Fourier Terms vs. BN
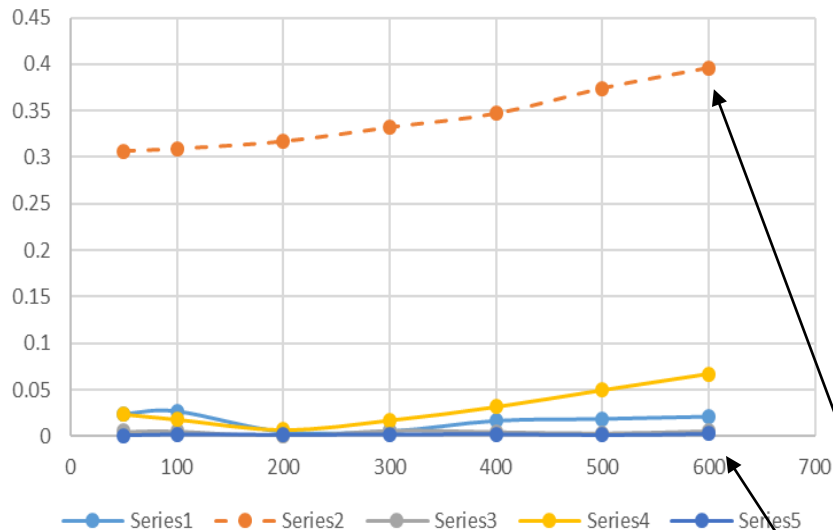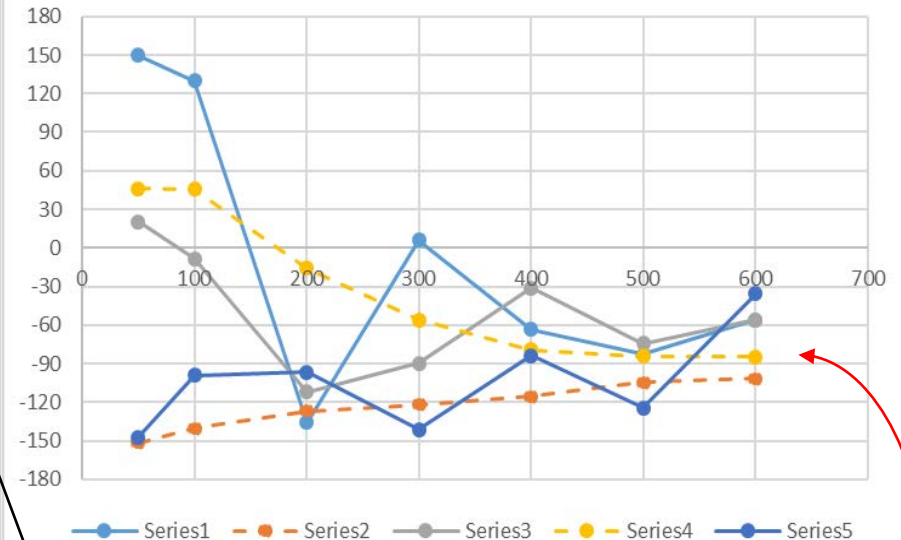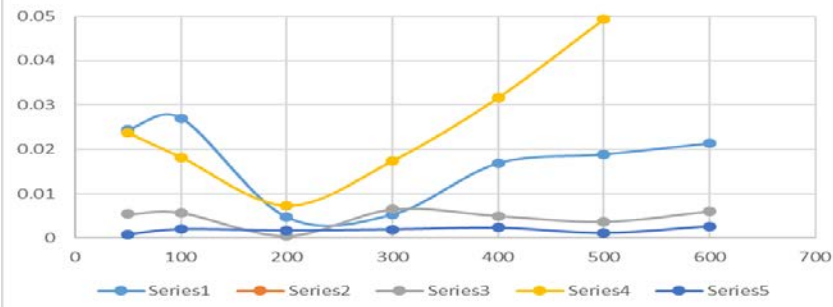
2Θ term dominates amplitude

All other terms a <5% effect

Only the <u>even</u> terms show monotonic phase behavior as BN varies

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To Apply Decay Compression Factor to E_c:

- **We now have an analytic curve of how concentrated or rarified the entries are around any given orbit, vs. Argument of Latitude**
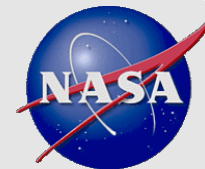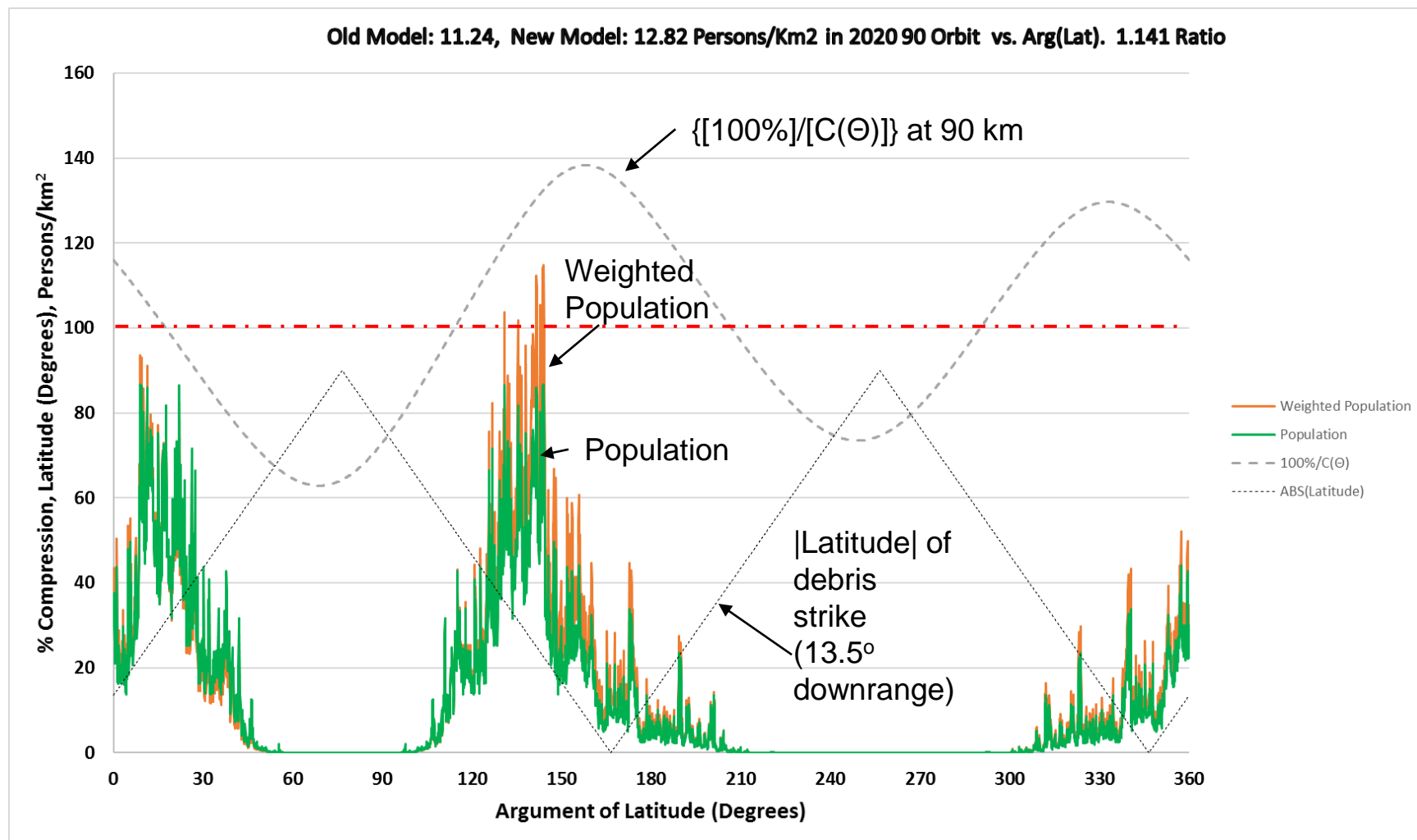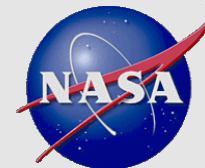
- **We incrementally step N (>>1) steps in Argument of Latitude to make a 360 arc.**

- **From our analytic representation, we have the concentration factor (C(Θ)) for that portion of the arc**

    Remember: C(Θ) is < 1 for locally concentrated decays!

- **We calculate the latitude at that particular step _i_;**

    Latitude = ArcSin[{Sin(Inclination)}*{Sin(Argument of Latitude$_i$)}]

- **We look up the average population in that latitude band**

    – **Note: we get to the same latitude twice, with different compression factors at each passage!)**

- **We DIVIDE the average population per km^2 by the compression factor C(Θ)**

    - **(Note! A binomial correction is needed!)**

- **We sum the result in each step, and divide by N**

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# Population Effects (90º Orbit, 200BN)

Old Model: 11.24, New Model: 12.82 Persons/Km2 in 2020 90 Orbit vs. Arg(Lat). 1.141 Ratio

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Net E_c Ratio vs. BN and Inclination

Ratio of Population Densities under Different Orbits by Ballistic Number

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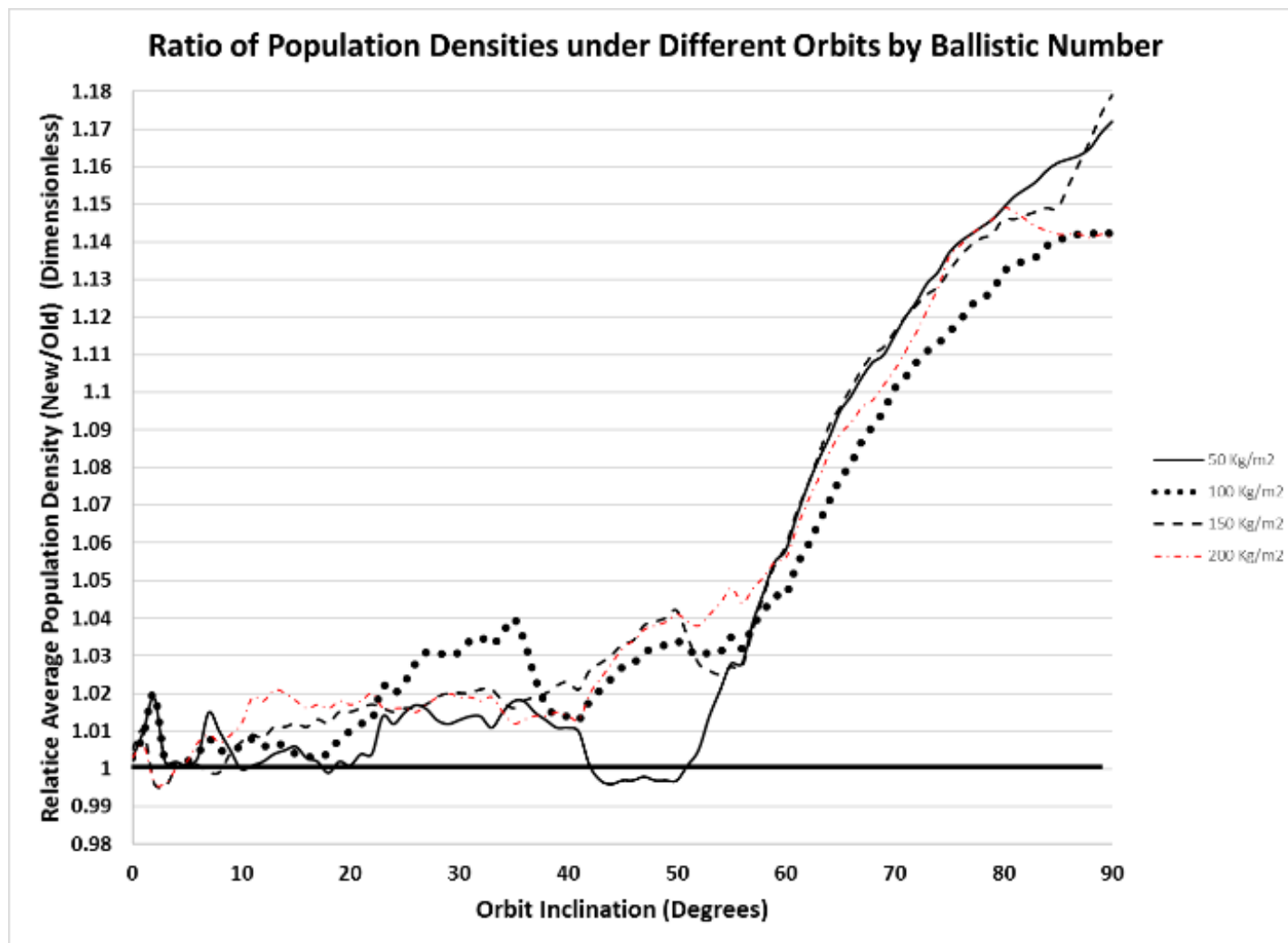
So: What we do:

- ## We have a dumb model:
  - **Find the <u>net</u> risk increase from a summary risk chart of the already-calculated cases.**
    - **Because of the huge computation effort, needs the assumption of reasonably uniformly-distributed net population growth by latitude band (not true..).**

- ## Or a <u>slightly</u> better model
  - **(parametric fit published in the paper…)**

- ## Or an even better model:
  - **Linearly interpolate phases and amplitudes between those of the bracketing 2 inclinations and BNs (a big spreadsheet…), and apply this best-estimate curve to exact latitude band population forecast.**

- ## Or a best model:
  - **Just run an automated GMAT series at all RAANs for the BN and inclination desired, and then apply the averaged Fourier curve to the best population model**
    - **ESPECIALLY if we know the decay season or date.**

JBB
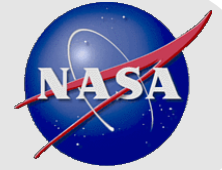
Crude Model (in Paper)

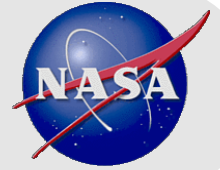- **We set out to make a universal, <u>simple</u> model (orange curves) of the averaged Fourier terms (blue curves)**

- **4 cosine amplitudes, 4 phases**

- **Some terms were easy to fit simply, some weren't**

- **A lot of interpolation may miss fine details**

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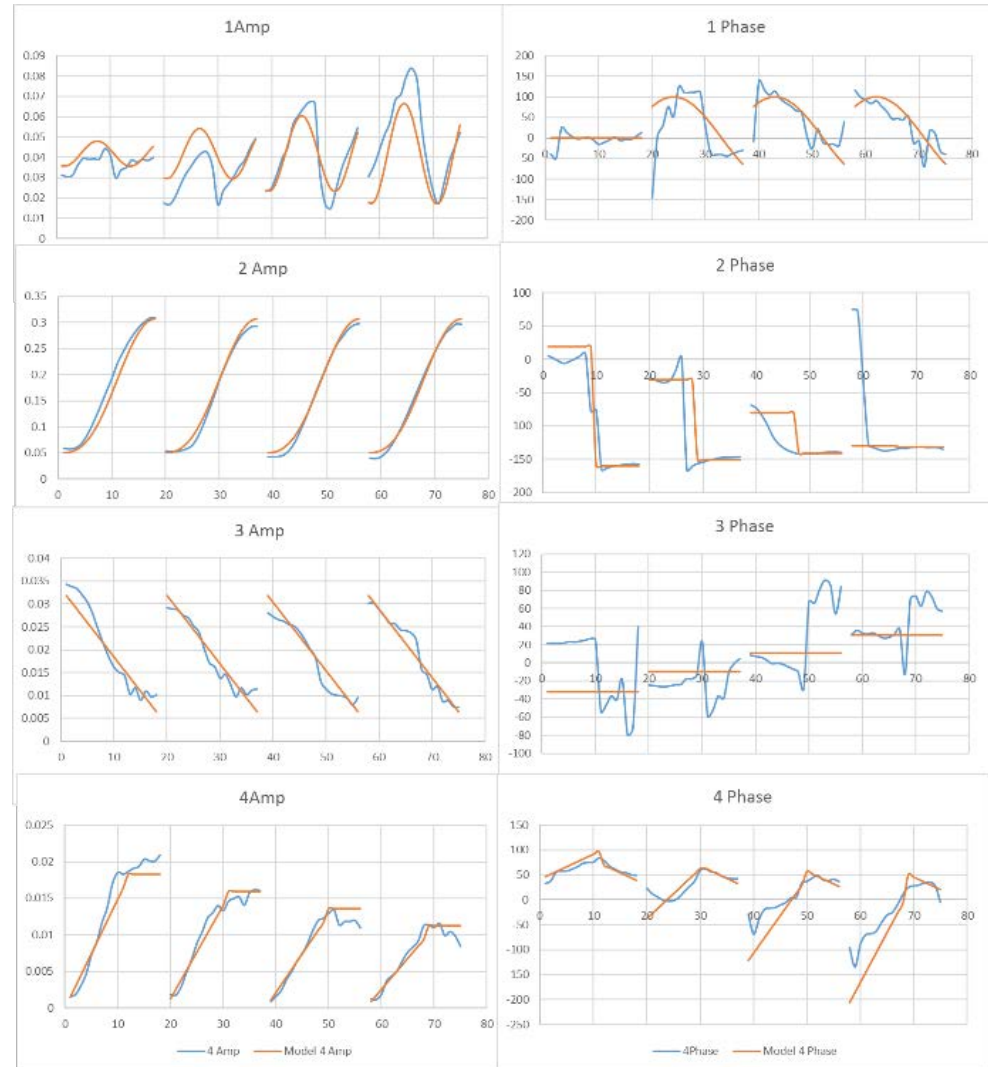
Conclusions:

- **The wall-of-air is a real effect that grows in importance as inclination and BN increase**

- **Diurnal (1Θ) effects are real and large, <u>but average out</u> over the seasons and orbit precessions.**
  - **These may be important if one knows exact entry date**

- **The higher-order symmetric (4Θ) and asymmetric (3Θ) terms are purely gravitational, and about 1/6 as important as the atmospheric terms**
  - **Although small, because the factors are highly repeatable over all BNs and RAANs they can be added as real biases to the compression curves**

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And most importantly:

- **We recommend that analysts use a Fourier model with <u>only</u> the 2Θ, 3Θ, and 4Θ derived terms of a <u>specific</u> GMAT entry simulation* that will represent the average entry concentration.**

- **The compression curve for the specific case should then be applied to the population bands as per slide 21.**

  – **(*Simulation conditions include specific inclination and BN of the spacecraft to <u>80 km</u> + estimated footprint length downrange as Argument of Latitude, at the Vernal equinox, 270 RAAN)**

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BACKUP

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True confessions

- **These curves don't take us to the ground. We need to get there analytically before calculating Ec.**

- **The method described does not account for lateral spread of the footprint**

- **We have not proven yet that the 1Θ term averages to zero**
  - **In work, see hot-off-the-presses chart**

- **We don't have physics explanations (yet) for some of the interesting knees/inflections in the Fourier constant trends**

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Potential Issues (so far…)

1. **There's no such thing as a circular orbit or perfect initial conditions**

   1. BUT: Minimum dV entry study and current runs show that the entry is driven mostly by nonlinear growth of deceleration in the last 3 orbits, and is thus very insensitive to initial conditions three days earlier. Chose to make orbits <u>approximately</u> equal altitude at ascending and descending node.

2. **Decay to 90 km speeds up the runs by nearly a factor of four vs. propagation to 80.**

   - Bacon study in April 2017 showed little change in remaining downrange gap from 90 to 80km over all BNs and inclinations, but this adds some fuzz
   - For E_c we need to calculate to the GROUND, and must add in a correction factor

3. **Running intentionally at (near) the Vernal equinox <u>eliminates</u> a suspected diurnal bulge which can put the sub-solar point up to 23.5 degrees off the equator as a major density perturbation**

   - We are just <u>starting</u> to see if this subsolar location effect averages out over a year.
   - See Hot-off-the-presses chart

4. **The spread over 1, 2, or often 3 orbits of dither shows MINOR changes in the compression curves. These are smoothed and averaged, but not characterized.**

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Potential Issues (so far…) Part II

5. **The Fourier binning of the data adds some scatter**
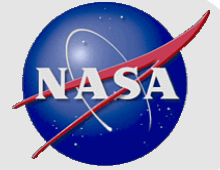

    - **The premise is that this does not matter in the first few terms, whose period is much greater than the raw data gap**

6. **Differing precession rates and decay times mean that there was some small variation in the RAAN (and solar vector) location of the entry: it only STARTED at 90-degree increment RAANs**
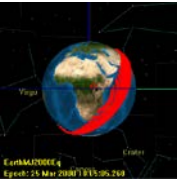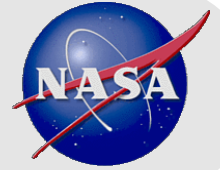
7. **The compression curve algorithm only generates gaps to the next point forward. To be mathematically correct, one should average with the gap backwards too.**

8. **Sampling using VLOOKUP in EXCEL gets the closest value BELOW the one you are looking up. There is no telling whether the next higher one is 1000x closer, without complicating every equation.**

    - **This affects the Fourier binning process.**
   - **Both this and the prior issue affect the <u>phase</u> of the compression curve slightly**

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Potential Issues (so far…) Part III

9. **The latitude equation ArcSin(Sin(Inclination)*Sin(ArgLat)) does not account for the elliptical geoid effects.**

10. **The future population in any given latitude band is still a bit of a guess, despite Chris Ostrom's recent excellent work!**

11. **The compression curves are inverted to some people's liking**

   - It made intuitive mathematical sense to normalize each gap value to some local constant (the average gap width) rather than the inverse.   This gets a curve in <u>normalized gap</u>, whose inverse at any local point is the "compression"
   - Unfortunately, the AVG(1/X) is not 1/AVG(X), and the resulting curves need a small tweak to renormalize the ultimate PDF to equal 1 over all latitudes.

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J3 Effect

- ## From Fremeaux et al 2017:

  – *Indeed, due to the Earth potential term J3, re-entry orbits with arguments of perigee in the North hemisphere are more prone to a faster descent than the orbits with South perigee arguments, as shown in Figure 3.*
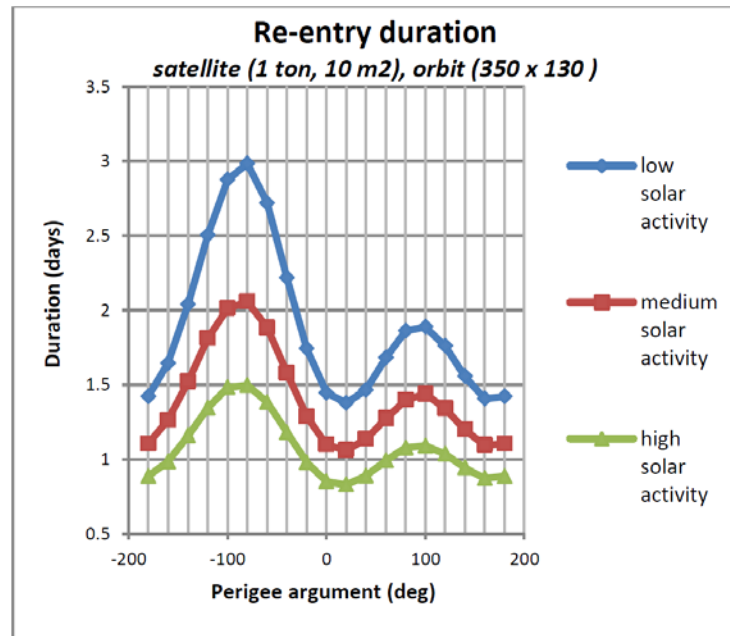
  – *Figure 3. Re-entry duration depending on perigee argument (ω) and solar activity.*