Fast optimization for aircraft descent and approach trajectory

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ABSTRACT

We address problem of on-line scheduling of the aircraft descent and approach trajectory. We formulate a general multiphase optimal control problem for optimization of the descent trajectory and review available methods of its solution. We develop a fast algorithm for solution of this problem using two key components: (i) fast inference of the dynamical and control variables of the descending trajectory from the low dimensional flight profile data and (ii) efficient local search for the resulting reduced dimensionality non-linear optimization problem. We compare the performance of the proposed algorithm with numerical solution obtained using optimal control toolbox General Pseudospectral Optimal Control Software. We present results of the solution of the scheduling problem for aircraft descent using novel fast algorithm and discuss its future applications.

1. INTRODUCTION

In the future air traffic management system, the trajectory becomes the fundamental element of a new set of operating procedures collectively referred to as trajectory-based operations (TBO) (Cate, 2013). The basis for TBO is that each aircraft's expected flight profile and time (or airspeed) information will be specified by a four-dimensional (4D) trajectory (K. H. Shish et al., 2015; Young et al., 2016; K. Shish et al., 2016).

One of the challenges in development of the future TBO is management of the airport congestion especially under convective weather conditions (M. Kamgarpour, W. Zhang, &

C.J. Tomlin, 2011). One of the key ingredients to the solution of this problem is fast online optimization of the aircraft descent trajectory. The difficulty in solving this problem stem from the fact that there are multiple phases (configurations) that have to be flown during descent while respecting the system dynamics and satisfying a large number of linear and nonlinear constraints.

The standard approach procedure required by the ATC implies starting a continuous steady descent from 6,000 ft, or higher, following a steep descent to a set of cleared altitudes and joining the $3°$ glide-slope approach from below. The speed during final approach is based on the reference speed, V_{ref}, which is calculated on the basis of reference speed for Flaps 30 (for a specific type of the aircraft) and depends on the mass of the aircraft.

In addition, the trajectory planning operation specifies a set of transitions to the given flaps and landing gear configurations with corresponding reference speed. The various phases flown by the aircraft during descent are controlled by the pilots and involve a set of nominal actions that are required to enable the designed vertical flight profile including e.g. capturing localizer and gliding slope.

Overall, it can be seen that the flight planning problem render itself as a complex multiphase trajectory optimization problem subject to dynamical, path, and control constraints (Betts & Cramer, 1995; Tomlin, Lygeros, & Sastry, 2000; Tomlin, Mitchell, Bayen, & Oishi, 2003; de Jong, 2014; de Jong et al., 2015, 2017).

The resulting trajectory optimization problem can be addressed using multiphase optimal control techniques. However, the duration of the final approach is only several minutes and changes in the flight plan may require fast online op-

timization. Current implementation of the optimization algorithm in an experimental Flight Management System (FMS) developed using General Pseudospectral Optimization Software (GPOPS) require more than 30 sec of optimization time from the top of descent to the runway (de Jong, 2014; de Jong et al., 2015, 2017).

Furthermore, future implementations of the FMS demand continuous online estimations of the feasible time bounds for each phase of the flight and fast rescheduling of the flight plan in e.g. convective weather conditions. These demands call for analysis of alternative approaches to the online solution of multiphase trajectory optimization problem.

Here we present a novel approach to the solution of this problem and compare performance of our algorithm with the performance of a conventional technique.

The paper is organized as follows. In the next section we provide formulation of the multiphase vertical trajectory optimization problem. In Sec. 3 we analyze performance of the solution of this problem using General Pseudospectral Optimization Software. In Sec. 4 we describe novel algorithm and its application to the optimization of the final approach to a runway in San Francisco airport. Finally, in Conclusions we summarize the obtained results and discuss future applications of the algorithm.

2. OPTIMIZATION OF THE AIRCRAFT DESCENDING TRAJECTORY

2.1. Model Equations

The system state in the vertical plane during the approach is defined as

$$x = \{V, \gamma, x, h\}, \tag{1}$$

where V is the speed, γ is the flight path angle, h is the altitude, and x is the distance to the runway.

In addition, the aircraft control is represented by two virtual control inputs - thrust T and angle of attack α

$$u = \{\alpha, T\}. \tag{2}$$

To simplify analysis without loss of generality we neglect mass change due to fuel burned and the wind. The resulting model takes the form

$$\begin{aligned} m\dot{V} &= T\cos\alpha - D - mg\sin\gamma, \\ mV\dot{\gamma} &= (T\sin\alpha + L) - mg\cos\gamma, \end{aligned} \tag{3}$$

$$\dot{x}_e = V\cos\gamma, \dot{h}_e = V\sin\gamma, \tag{4}$$

We note that actual control variable is pitch rate. However, due to time separation between slow and fast aircraft dynamics, the angle of attack α is considered to be virtual control input.

The angles and forces used in the model are illustrated (Miquel & Suboptimal, 2015) in Fig. 1
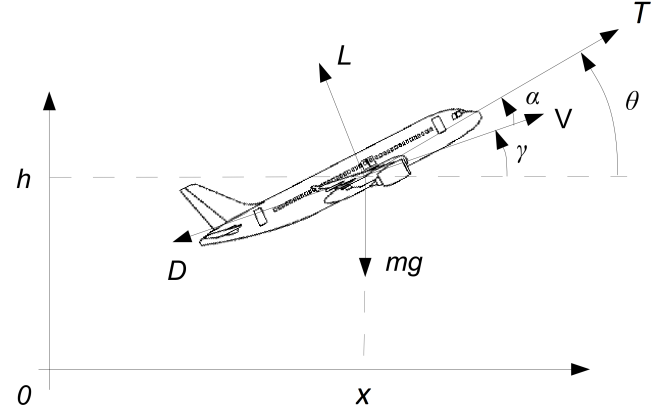

Figure 1. The aircraft forces and angles during vertical flight with nonzero climb rate.

The lift and drag coefficients in the model are

$$\begin{aligned} D &= \frac{1}{2}\rho V^2 S \left(C_{D_0} + C_{D_\alpha}\alpha + C_{D_{\alpha^2}}\alpha^2 + \right. \\ &\quad \left. C_{D_{\delta_{sp}}}\delta_e + C_{D_{fl}}\delta_{fl} + C_{D_{lg}}\delta_{lg} \right), \\ L &= \frac{1}{2}\rho V^2 S \left(C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta_{sp}}}\delta_e + \right. \\ &\quad \left. C_{L_{fl}}\delta_{fl} + C_{L_{lg}}\delta_{lg} \right). \end{aligned} \tag{5}$$

where ρ is the air density, S is the net wing surface area, δ_{sp} is the spoiler deflection, δ_{fl} is the flap deflection, δ_{lg} is the gear setting.

2.2. Problem formulation

In general, the descent trajectory optimization requires the following problem to be solved (Betts & Cramer, 1995; Tomlin et al., 2000, 2003; Becerra, 2010; de Jong, 2014; Patterson & Rao, 2015; de Jong et al., 2015, 2017). Find the control trajectories, $u^{(i)}(t), t \in [t_0^{(i)}, t_f^{(i)}]$, state trajectories $x^{(i)}(t), t \in [t_0^{(i)}, t_f^{(i)}]$, static parameters $p^{(i)}$, and times $t_0^{(i)}, t_f^{(i)}$ where $i = 1, ..., N_p$ is the number of phases, to minimize the following performance index (Becerra, 2010):

$$\begin{aligned} J = \sum_{i=1}^{N_p} &\left(\varphi^{(i)} \left[x^{(i)}(t_f^{(i)}), p^{(i)}, t_f^{(i)} \right] + \right. \\ &\left. \int_{t_0^{(i)}}^{t_f^{(i)}} L^{(i)} \left[x^{(i)}(t), u^{(i)}(t), p^{(i)}, t \right] dt \right) \end{aligned} \tag{6}$$

subject to the dynamical constraints:

$$\dot{x}^{(i)}(t) = f^{(i)} \left[x^{(i)}(t), u^{(i)}(t), p^{(i)}, t \right], t \in [t_0^{(i)}, t_f^{(i)}], \tag{7}$$

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the path constraints

$$h_L^{(i)} \leq h^{(i)} \left[x^{(i)}(t), u^{(i)}(t), p^{(i)}, t \right] \leq h_U^{(i)}, \qquad (8)$$

for $t \in [t_0^{(i)}, t_f^{(i)}]$, the event constraints:

$$e_L^{(i)} \leq e^{(i)} \left[x_0^{(i)}, u_0^{(i)}, x_f^{(i)}, u_f^{(i)}, p^{(i)}, t_0^{(i)}, t_f^{(i)} \right] \leq e_U^{(i)}, \quad (9)$$

the phase linkage constraints:

$$\Psi_l \leq \Psi[x_0^{(1)}, u_0^{(1)}, x_f^{(1)}, u_f^{(1)}, p^{(1)}, t_0^{(1)}, t_f^{(1)}$$
$$\vdots \qquad\qquad (10)$$
$$x_0^{(N_p)}, u_0^{(N_p)}, x_f^{N_p}, u_f^{(N_p)}, p^{(N_p)}, t_0^{(N_p)}, t_f^{(N_p)}] \leq \Psi_u$$

where $x_{0,f}^{(i)} x^{(i)}(t_{0,f}^{(i)})$ and $u_{0,f}^{(i)} u^{(i)}(t_{0,f}^{(i)})$

the bound constraints:

$$u_L^{(i)} \leq u^{(i)}(t) \leq u_U^{(i)}, \quad t \in [t_0^{(i)}, t_f^{(i)}]$$
$$x_L^{(i)} \leq x^{(i)}(t) \leq x_U^{(i)}, \quad t \in [t_0^{(i)}, t_f^{(i)}]$$
$$p_L^{(i)} \leq p^{(i)} \leq p_U^{(i)}, \qquad\qquad (11)$$
$$\underline{t}_0^{(i)} \leq t_0^{(i)} \leq \bar{t}_0^{(i)},$$
$$\underline{t}_f^{(i)} \leq t_f^{(i)} \leq \bar{t}_f^{(i)},$$

and the time constraints:

$$t_f^{(i)} - t_0^{(i)} \geq 0. \qquad\qquad (12)$$

In each phase functions and variables in (6)-(11) have appropriate dimensions that may change from phase to phase.

2.3. Flight phases

The following problem will be considered. Optimize the final approach trajectory using objective function (6) for the descending flight between CEPIN and the stabilized approach fix (500 ft) of the RWY 28R in SFO. The phases included into the analysis are shown in the Table 1.

This simplified schedule was developed by taking into account general requirements for the descent trajectory, see e.g. (Prats et al., 2014). Some additional limitations were also included. E.g. it was assumed that the LOC signal extends 18 nm a way from and 4500 ft above the antenna site (see e.g. FAA-S-8081-9B, June 2001). It was also assumed that the glide slope capture is initiated $2 \div 0.75$ nm away from DUMBA in a level flight. In addition, acceleration and altitude increase during descent were excluded from this optimization test.

A number of optimization problems can be formulated within this framework. The critical parameters of interest during airport congestions are earliest and latest time of arrival. A pa-

rameter of common interest is fuel consumption. In this work we were primarily interested in scheduling transitions times between various descent phases and estimation of the earliest and latest transition time for each phase.

Table 1. Phases included into the first test

#	Name	ALT (ft)	DST (nm)	SPD (kn)
1	initial state	10000	29	230
2	Descent to ARCHI	$8000 \leq h \leq 10000$	$29 \leq x \leq$ ARCHI	$175 \leq V \leq 230$
3	Descent to ZILED	$6000 \leq h \leq 10000$	ARCHI $\leq x \leq$ ZILED	$175 \leq V \leq 230$
4	Flaps5	$5000 \leq h \leq 10000$	ZILED $\leq x \leq$ GIRRR	$175 \leq V \leq 230$
5	LOC capture	$4000 \leq h \leq 4500$	GIRRR $\leq x \leq$ DUMBA	$175 \leq V \leq 230$
6	Flaps15	$1800 \leq h \leq 4500$	DUMBA $\leq x \leq$ CEPIN	$165 \leq V \leq 215$
7	Flaps20	$1800 \leq h \leq 4500$	CEPIN $\leq x \leq$ AXMUL-2	$165 \leq V \leq 195$
8	Gear down	$1800 \leq h \leq 4500$	AXMUL-3 $\leq x \leq$ AXMUL	$165 \leq V \leq 195$
9	GS capture	$1800 \leq h \leq 2000$	AXMUL-2 $\leq x \leq$ AXMUL+1	$165 \leq V \leq 195$
10	Flaps25	DST $\times$ $tn(\gamma \pm \delta\gamma)$	AXMUL $\leq x \leq$ AXMUL+3	$155 \leq V \leq 185$
11	Flaps30	DST $\times$ $tn(\gamma \pm \delta\gamma)$	AXMUL+2 $\leq x \leq$ FIX	$150 \leq V \leq 170$
12	stabilized appr fix	446	1.4	150

We will now consider solution of this problem using freely available package Gauss Pseudospectral Optimization Software (GPOPS) (Rao et al., 2010)

3. EXAMPLE OF NUMERICAL SOLUTION USING GPOPS

The GPOPS package is easy to install and to use. It was shown to perform well for multiple aerospace applications including optimization of the descent aircraft trajectory at National Aerospace Laboratory (Netherlands). And we have chosen this package for initial evaluation for application to the scheduling problem. Because the convergence of the algorithm was quite slow we have initially chosen a subset of phases from the Table 1.

Here we provide an example of optimization for descending trajectory with the following five phases

1. flaps 15 gear up; $V_{max}^{(1)} = 215$ kn; $V_{min}^{(1)} = 95.3$ kn;

2. flaps 15 gear down; $V_{max}^{(2)} = 215$ kn; $V_{min}^{(2)} = 95.3$ kn;

3. flaps 20 gear down; $V_{max}^{(3)} = 195$ kn; $V_{min}^{(3)} = 91.1$ kn;

4. flaps 25 gear down; $V_{max}^{(4)} = 185$ kn; $V_{min}^{(4)} = 87.5$ kn;

Note, that the method allows for many different types of objective function (Rao et al., 2010). For example, the final altitude in each phase could be minimized or maximized using functions similar to (14) and (15) and substituting time t with altitude h.

Alternatively, to minimize the total mechanical energy to fly along a given path the objective function can be chosen as

$$J = \sum_i \int_{t_0^{(i)}}^{t_f^{(i)}} V(t) \cdot T(t) \, dt. \tag{13}$$

In the presented analysis the objective function of the optimization (the performance index in eq. (6)) was chosen to minimize

$$J = t_f^{(1)} + t_f^{(2)} + t_f^{(3)} + t_f^{(4)} + t_f^{(5)} \tag{14}$$

or maximize

$$J = -(t_f^{(1)} + t_f^{(2)} + t_f^{(3)} + t_f^{(4)} + t_f^{(5)}) \tag{15}$$

transition time between phases.

The dynamical constraints in each phase $t \in [t_0^{(i)}, t_f^{(i)}]$ with $i = 1, ..., 5$ (cf eq. (7)) were given by

$$\begin{aligned}
m\dot{V} &= T \cos\alpha - D - mg \sin\gamma, \\
mV\dot{\gamma} &= (T \sin\alpha + L) - mg \cos\gamma, \\
\dot{x}_e &= V \cos\gamma, \\
\dot{h}_e &= V \sin\gamma, \\
\dot{T} &= (T_c - T)/\tau_T,
\end{aligned} \tag{16}$$

The bounds on the aircraft speed in each phase were given by $V_{max}^{(i)}$ and $V_{min}^{(i)}$ listed above for each flaps configuration.

The results of the minimization of transition times for four phases are shown in Fig. 2. It can be seen from the figure that transitions to the configurations gear down, flaps 20 and 25 occur as soon as velocity of the aircraft approaches the corresponding limiting value. The flight path angle γ stays close to the lower bound corresponding to the fast descent. The convergence time was found to be very sensitive to the parameters of the problem and varies between 10 and 50 sec.

The convergence time for maximization problem can vary between 50 and 300 sec and the convergence is not robust. We note that the best reported performance of the optimized implementation of the multiphase pseudospectral algorithm (de Jong, 2014) in C++ using PSOPT package was 30 sec for the whole descent and final approach trajectory.

We therefore conclude that although GPOSP package is potentially very useful for trajectory optimization at present it can only be used for off-line applications. To enable fast on-
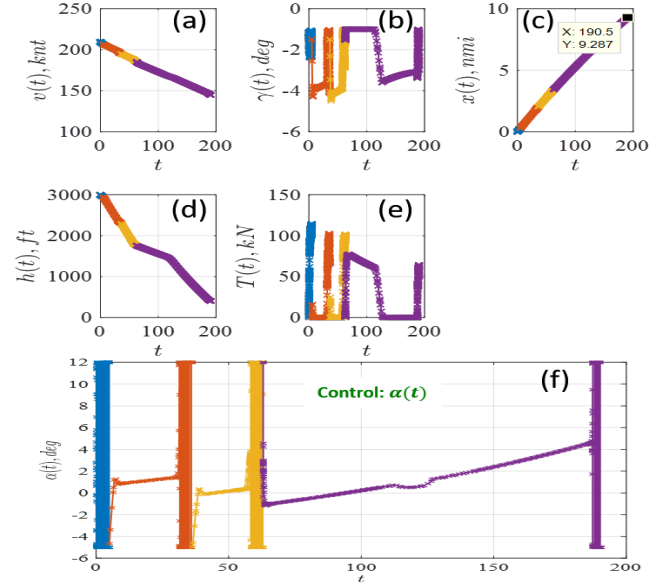

Figure 2. Minimization of transition times for descending trajectory with 4 phases. The figures show the dependence of dynamical variables of the system on time: (a) Velocity; (b) γ; (c) distance; (d) Altitude; (e) Thrust; and (f) Angle of attack. Different colors indicate different phases of the flight and correspond (left to right) to phases (i) to (iv).

line optimization of the multiphase descending trajectory we propose a novel algorithm, which is considered in the next section.

4. FAST MULTIPHASE OPTIMIZATION ALGORITHM

Before we introduce the algorithm, let us provide some estimations for the final ILS approach to RWY 28R at SFO. The starting point for our analysis is the transition from fast to slow deceleration, which may normally happen at the flight level FL100 and speed $V_{CAS} \leq 250$ kn.

According to the "3:1 rule of descent" this transition point is located ∼33 nm from the runway. At the point of stabilized approach the aircraft speed should be ∼140 kn. So during descent the altitude and speed are reduced by approximately 330 ft and 3.3 kn per each nautical mile.

In addition, the aircraft has to capture the $3°$ gliding slope from below at the altitude approximately 1800 ft and distance ∼5.5 m from the runway, which translates into ∼740 ftm vertical speed at the ground speed 140 kn.

The flight plan normally should also accommodate a number of actions including: (i) localizer intercept; (ii) setting a sequence of flaps configurations; (iii) deploying landing gear; (iv) capturing gliding slope; (v) initiating flare; (vi) changing to touchdown phase. All this actions set additional constraints on the flight profile. Note that there can also be mul-
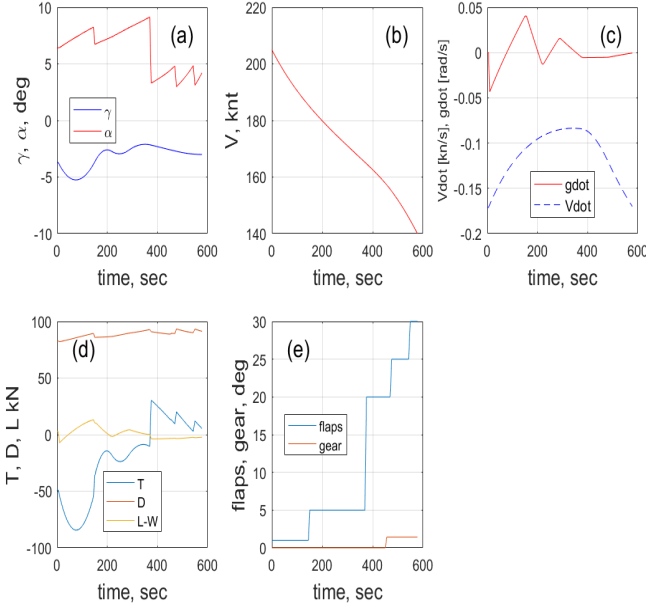
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Figure 3. Aircraft dynamics reconstructed from the flight profile defined as $h(x)$ and $V(x)$: (a) $\alpha(t)$ and $\gamma(t)$; (b) $\dot{V}(t)$; (c) $\dot{\gamma}(t)$ and $\dot{V}(t)$ (d) $T(t)$ - thrust, $D(t)$ - drag, and $L(t) - W$ - lift minus weight; (e) flaps and landing gear configuration.

Figure 4. Bounds on the location of the events obtained using Table 1 are shown by colored transparent parallelepipeds in 3D space (V - SPD, x - DST, h - ALT). The initial location of events is shown by open gray green squares (connected by green dashed line) located at the centers of the parallelepipeds. The optimal solution is show n by the open blue circles connected by the solid blue line.

tiple ATC corrections (constraints) to the normal approach procedure that have to be included into the flight profile.

The large number of phases (aircraft configurations), dynamical constraints, and bounds on the control and dynamical variables render trajectory optimization a complex multiphase optimization problem, see Table 1, cf. with 14 phases defined in (de Jong, 2014; de Jong et al., 2015, 2017).

On the other hand, we see that a large number of phases must be accommodated on relatively short distance along the descending path. This fact can be used to substantially reduce the dimensionality of the problem by approximating segments connecting neighboring transition points with low degree polynomials (in particular, straight lines). This assumption also agrees with the results of the optimization obtained GPOPS code, see Fig. 2 and cf. (de Jong, 2014; de Jong et al., 2015, 2017).

Using polynomial approximation of the flight trajectories connecting phase transition points the dimension of the optimization problem is reduced to

$$(N_{tr} - 1) \times P_{tr} \times D_{tr}. \qquad (17)$$

Here N_{tr} is the number of transition points between the phases of the flight, D_{tr} is the dimension of each transition point (e.g. distance, altitude, speed), and P_{tr} is the degree of the polynomial connecting two neighboring transition points. In this formulation the decision variables are the coefficients
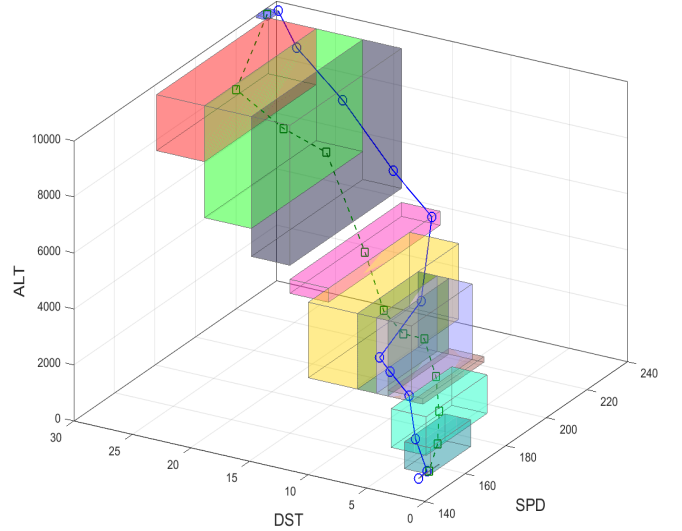
of the polynomials and the locations of the phase transition points.

Furthermore, using this approximation one can avoid dynamical constraints all together by reconstructing all the dynamical and control variables from the piece-wise polynomial approximations of the flight profile.

4.1. Inferring descending trajectory from vertical profile

To infer the model dynamics from the flight profile we notice that eqs. (3) and (4) can be rewritten in the form

$$u' = \frac{1}{m\cos\gamma}\left(T\cos\alpha - \rho S u C_D - D - W\sin\gamma\right)$$
$$\gamma' = \frac{1}{2mu\cos\gamma}\left(T\sin\alpha + \rho S u C_L - W\cos\gamma\right) \qquad (18)$$
$$h' = \tan\gamma\,,$$

using transformation

$$dx = V\cos(\gamma)dt \qquad (19)$$

and introducing distance x as a new independent variable along the flight path (Vinh, 1981). In eqs. (18) $W = mg$, u is the specific kinetic energy $\frac{V^2}{2}$ and prime refers to derivative with respect to x, e.g. $h' = dh/dx$.

Since V and h are known along a given flight profile, thrust T, angle of attack α, and flight path angle γ can be found as functions of distance using eqs. (18) and then as functions of time using eq. (19). The results of calculations are shown in Fig. 3. In this example the flight profile corresponds to the spline approximation of the initial guess of the trajectory

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corresponding to the location of transition points shown by the open gray circles in Fig. 4.

It can be seen from the figure that all the dynamical and control variables can be successfully inferred from the given flight profile. The obtained results allow one to avoid collocation methods (C.R.Hargraves & S.W.Paris, 1987) of dynamical trajectory optimization and obtain online solutions to the problems of primary interest for aircraft descent operations including minimization of additional drag and thrust during descent, and enforcing required time of arrival.

To solve these problems we have to combine the algorithm, outlined above, with the optimization algorithm for the location of the transition points.

4.2. Fast optimization algorithm

Using piece-wise linear approximation to the flight profile and location of phase transitions points as decision variables, we reduce the problem of multiphase optimization of the descent trajectory to the following standard NLP:

$$\begin{aligned}
\underset{x}{\text{minimize}} \quad & f(x) \\
\text{subject to} \quad & g_i(x) \leq 0, \ i = 1, \ldots, m, \\
& h_j(x) = 0, \ j = 1, \ldots, n, \\
& A \cdot x \leq b, \\
& A_{eq}(x) \leq b_{eq}, \\
& lb \leq x \leq ub.
\end{aligned} \quad (20)$$

Here vector of decision variables x has dimension $N_{tr} \times D_{tr}$ number of phase transition pints times dimension of each point.

The performance index (cost function) $f(x)$ can have many different objectives. In the context of the descent trajectory optimization the most common objectives are minimization of the fuel use ($\sim$ minimum thrust) and required time of arrival (RTA) for maximum throughput of a given runway/airport.

In this work we do not consider the change of the phases order. The phases order will be fixed as shown in the Table 1. To enforce the phase order and no-climb, no-acceleration conditions we use inequality constraints in the form

$$A_{eq} \cdot x \leq b_{eq}, \quad (21)$$

where x is the vector of decision variables $\{x_1, \ldots, x_{tr}, h_1, \ldots, h_{tr}, V_1, \ldots, V_{tr}\}$, and the block-bidiagonal matrix A_{eq} is

$$A_{eq} = \begin{pmatrix} -1 & 1 & 0 & \ldots & 0 \\ 0 & -1 & 1 & \ldots & 0 \\ & & \ddots & & \\ 0 & \ldots & 0 & -1 & 1 \end{pmatrix}.$$
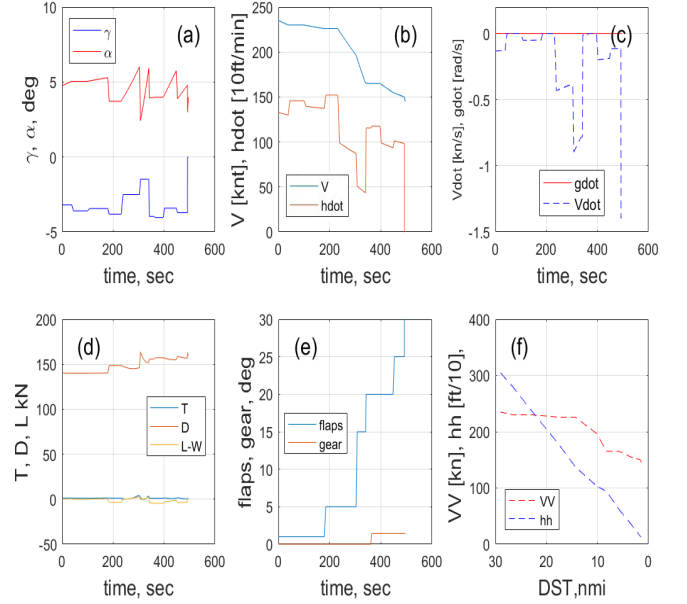

Figure 5. Aircraft dynamics obtained as the result of optimization of the descending trajectory using local solver fmincon (MATLAB): (a) $\alpha(t)$ and $\gamma(t)$; (b) $V(t)$ and $\dot{h}(t)$; (c) $\dot{\gamma}(t)$ and $\dot{V}(t)$ (d) $T(t)$ - thrust, $D(t)$ - drag, and $L(t) - W$ - lift minus weight; (e) flaps and landing gear configuration; (f) speed and altitude as functions of the distance.

By setting vector b_{eq} to zero we ensure that values of the distance x, altitude h, and speed V at a given step are no larger than the corresponding values at the preceding step.

Bounds and the initial guess of the trajectory are shown in Fig. 4. It can be seen from the figure that bounds on the speed and altitude of the phase transition points can be both disconnected and overlapping. It can also be inferred from the figure that the initial guess of the descending trajectory was obtained as a set of locations of the centers of the bounding boxes.

In the first test we choose minimization of thrust as the objective of the problem:

$$f(x) = \int_{x_0}^{x_f} T^2(x) \, dx. \quad (22)$$

The solution of the NLP problem (20) - (22) was obtained using a number of optimization solvers including local search with of fmincon (MATLAB), and global solvers such as genetic algorithm (MATLAB) and pattern search (nomad, OPTI) (Currie & Wilson, 2012).

The best results were obtained using with local search fmincon (MATLAB) as shown in Fig. 5. It can be seen from the figure that optimized values of the thrust are indeed very small, cf. Fig. 3. The corresponding variations of the flight path angle are also small, see Fig. 5(a). In addition, it can

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be noticed from the figure (f) that the optimal vertical descent path is a smooth trajectory respecting the constraints. All these features are expected to be the main features of the descent trajectory with minimum thrust.

We note that the vertical velocity is above 1000 ft/min for most time during descent. This is a quite high value and additional nonlinear constraints

$$| V \cdot \sin(\gamma) | \leq 700\text{ft/min} \tag{23}$$

may have to be imposed to keep vertical speed within predefined limits.

Importantly, the convergence of the local search algorithm was consistently less than 5 sec. This result may suggest that there exist some strong, albeit hidden, convex properties of the objective function.

For comparison, the convergence of the global search algorithms (genetic algorithm and pattern search) was slow. Despite long convergence time the thrust found by the global search algorithms was much larger than the one described above. This fact also indicates that the cost function has strong convex property.

Some insight into the properties of the cost function can be gained by rewriting it in the form (keeping terms $\sim \alpha^2, \gamma^2$)

$$
\begin{aligned}
\int_{t_0}^{t_f} T^2(t)\, dt &= \sum_{i=1}^{N_{tr}-1} \Delta t_i \left[\left(m\dot{V} + W\gamma \right) \right. \\
&- \rho g S \frac{(V_i + \delta V)^2}{2} + \left(\left(\frac{\tilde{C}_L}{2} - C_{D_\alpha} \right) \alpha \right. \\
&+ \left. \left(\frac{C_{L_\alpha}}{2} - C_{D_{\alpha^2}} \right) \alpha^2 \right) - \tilde{C}_D + W\frac{\alpha}{2} \Big]^2 ,
\end{aligned}
\tag{24}
$$

where V_i are the mean value of speed in each phase, δV are the speed variations around the mean value, $\tilde{C}_L = C_{D_0} + C_{D_{fl}} \delta_{fl} + C_{D_{lg}} \delta_{lg}$, and $\tilde{C}_D = C_{L_0} + C_{L_{fl}} \delta_{fl} + C_{L_{lg}} \delta_{lg}$.

From the eq. (24) one can conjecture that the cost function is convex with respect to "hidden" optimization variables α and δV during the time interval Δt_i corresponding to each phase. In addition, it is expected that there is a smooth continuous dependence of the thrust on the location of the boundaries of each phase. However, the vector of decision variables has 36 components and the full analysis of the convex properties of the cost function is beyond the scope of this work and will be considered elsewhere.

In this work we were primarily concerned with estimation of the earliest and latest transition times for each phase of the flight and accordingly the earliest and latest arrival times. To find these estimations the cost function was chosen as
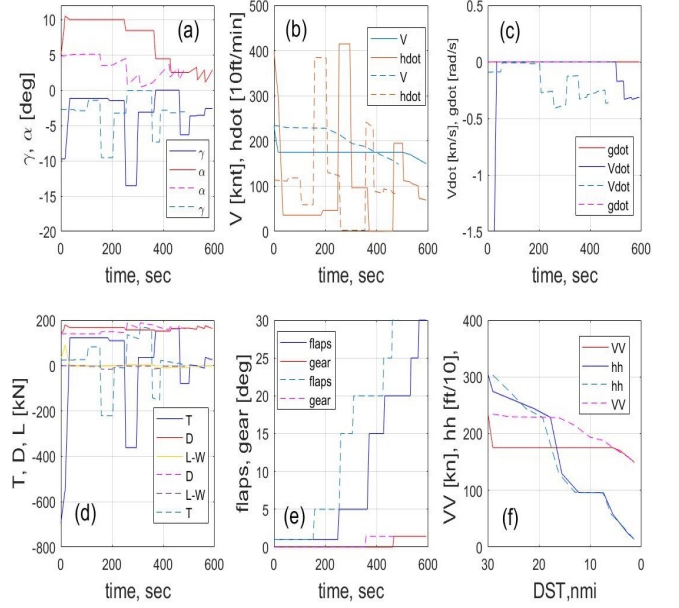
$$Cost = \sum_{i=1}^{N_{tr}-1} \Delta t_i$$

Figure 6. The results of the minimization (dashed lines) and maximization (solid lines) of the required time of arrival. The notations are the some as in Fig. 5.

to minimize arrival time and in the form

$$Cost = - \sum_{i=1}^{N_{tr}-1} \Delta t_i$$

to maximize it.

The solutions of the corresponding optimization problems is shown in Fig. 6. The obtained results show that the arrival times for the descent flight satisfying all the constraints are in the interval $484 \div 600$ sec. The time windows for the pilot actions during the descent are most clearly seen in the Fig. 6 (e) for the flaps and landing gear configurations. The corresponding changes in the flight profile can be observed in the Fig. 6 (f). It can be seen that the shortest time corresponds to a more steep descent with larger speed along the whole profile. These features are expected intuitively. They also confirm and quantify the intuitive idea that the aircraft speed during descent is the key scaling factor for the time windows allowed for the pilot actions.

We note that these results were obtained without constraints on the vertical speed of the type (23). It is expected that constraints will reduce the allowed time windows for pilot actions and for the arrival time.

5. CONCLUSIONS

We analyzed the problem of scheduling of the descent and approach of commercial aircraft. The problem was formulated as a general multiphase optimal control problem.

We developed solution of this problem using Matlab package GPOPS. To evaluate the performance of this package for specific applications to the scheduling problem we considered problem with 4 phases due to different flaps and landing gear configurations. It was shown that the package can solve complex multiphase problems in general form. However, its online applications are at present limited by slow convergence time.

We proposed novel fast algorithm of scheduling and optimization of the descent trajectory that reduces the original multiphase optimal control problem to the standard NLP problem of low dimension. It was shown that the dimension of the optimization problem can be reduced by

- using algorithm of reconstruction of the full set of dynamical and control variables along the descent path from a low-dimensional vertical and speed profile;

- avoiding collocation methods of dynamical trajectory optimization;

- choosing the location of the phase transition points as the decision variables;

- approximating flight paths connecting transition points by low-dimensional polynomials.

An additional advantage of the proposed algorithm is the ability to include pilot actions during descent (such as capturing LOC and glide slop, changing flaps and gear configuration, establishing stabilized approach configuration, initializing FLARE and touchdown etc.) directly into the flight plan.

Preliminary testing of the algorithm shows promising results for the future online applications. In particular, the proposed approach paves the way to development of a general optimization algorithm that combines optimization of arrival time during descent of individual aircraft with optimization of runway scheduling problem with multiple aircrafts, including situation with convective weather conditions.

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