55th AIAA Aerospace Sciences Meeting

AIAA Paper 2017-XXXX

An ODE-based Wall Model for Turbulent Flow Simulations

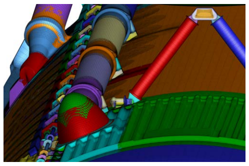
Marsha J. Berger
Courant Institute
New York University, NY, NY

Michael J. Aftosmis
Computational Aerosciences Branch
NASA Ames Research Center, Moffett Field, CA

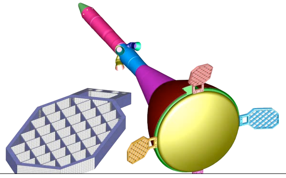
9 – 13 January, 2016, Grapevine, TX, USA

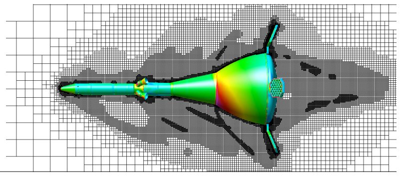
Motivation

Fully automated meshing for Reynolds-Averaged Navier-Stokes Simulations



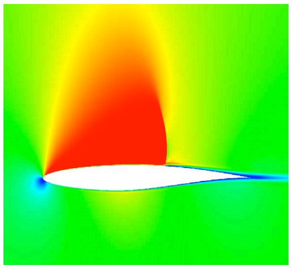
- Mesh generation for complex geometry continues to be the biggest bottleneck in the RANS simulation process
- Embedded boundary Cartesian methods routinely used for inviscid simulations about arbitrarily complex geometry
- These methods lack of an obvious & robust way to achieve near wall anisotropy
- **Goal:** Extend these methods for RANS simulation without sacrificing automation, at an affordable cost





Outline

- Previous work & analytic wall functions
- ODE-based wall models
  - A New ODE wall model
- Numerical examples
- Conclusions



Previous Work

Analytic wall functions

- Thin-layer form of streamwise momentum for RANS eqs.

$$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} + \rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]$$

Previous Work

Analytic wall functions

- Thin-layer form of streamwise momentum for RANS eqs.

$$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} + \rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]$$

- The diffusion model assumes that velocity is small and ZPG

$$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right) = 0$$

Previous Work

Analytic wall functions

- Thin-layer form of streamwise momentum for RANS eqs.

$$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} + \rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]$$

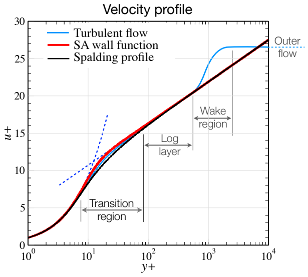
- The diffusion model assumes that velocity is small and ZPG

$$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right) = 0$$

- Assume mixing-length model for eddy viscosity: ν_t ~ distance to the wall

$$\mu_t = \rho\nu_t = \rho\kappa\nu y^+$$

- Gives a very good fit to experimental velocity data up through the log layer



Previous Work

Analytic wall functions

- Spalding model:

$$y^+(u^+) = u^+ + e^{-\kappa B}\left(e^{\kappa u^+} - 1 - \kappa u^+ - \frac{1}{2}(\kappa u^+)^2 - \frac{1}{6}(\kappa u^+)^3\right)$$

- SA wall function (2012):
  Derived, using a limiting form of SA turbulence model and integrating the diffusion model

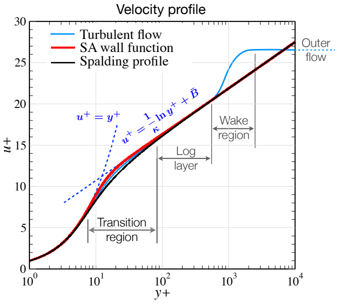
$$\begin{aligned} u^+(y^+) \;=\; \bar{B} \;+\; & c_1 \log((y^+ + a_1)^2 + b_1^2) \;-\; c_2 \log((y^+ + a_2)^2 + b_2^2) \\ -\; & c_3 \operatorname{arctan2}(y^+ + a_1, b_1) \;-\; c_4 \operatorname{arctan2}(y^+ + a_2, b_2) \end{aligned}$$

- Prefer SA wall function, since it gives direct relationship for velocity as a function of distance
- Knowing u at a point F, iterate to find u_τ, so that $u^+(y_F^+) = u_F^+ = u_\tau u_F$

Previous Work

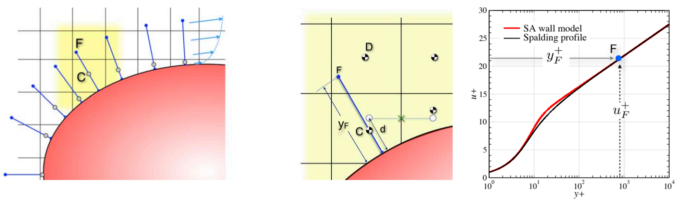
Analytic wall functions

- Spalding model: $y^+(u^+) = \ldots$
- SA wall function (2012): $u^+(y^+) \;= \ldots$
- Both:
  - Are good approximations and give accurate wall shear stress when anchored with F located out to the log-layer
  - Are inappropriate beyond the log layer (in the wake region)



Previous Work

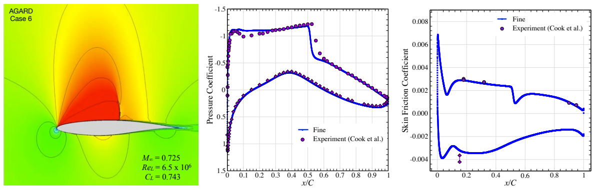
Constructing forcing points



- Construct forcing points at uniform distance from wall
- Interpolate data to point *F* from cell centered solution on outer grid
- With velocity an distance at forcing point, use wall function to find u_τ and wall shear $\tau_{\text{wall}} = \rho u_\tau^2$

Previous Work

Analytic wall functions



- Successfully coupled applied analytic wall functions with cut-cell Cartesian meshes in 2012*
- Introduced new SA wall function – worked well where wall functions were appropriate
- Results were comparable to body-fitted methods using wall functions
- Conclusion:

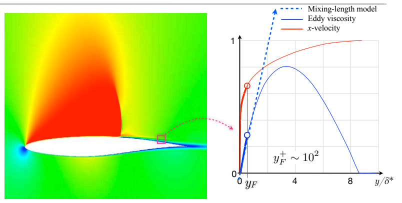
> Cartesian RANS is viable, but wall functions alone are probably not sufficient to make the approach cost competitive

** AIAA 2012-1301, "Progress Towards a Cartesian Cut-Cell Method for Viscous Compressible Flow", Berger, Aftosmis & Allmaras*

Good wall functions gone bad

Thick boundary layer

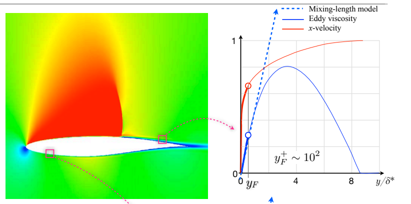
- $y_F^+ \approx 10^2$
- Forcing point in log layer
- Mixing length model gives good estimate eddy viscosity
- Analytic wall function is appropriate



Good wall functions gone bad

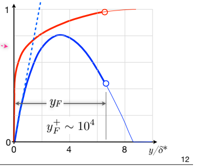
Thick boundary layer

- $y_F^+ \approx 10^2$
- Forcing point in log layer
- Mixing length model gives good estimate eddy viscosity
- Analytic wall function is appropriate



Thin boundary layer

- At the same distance from the wall, $y_F^+ \approx 10^4$
- Forcing point is now in the wake layer
- Eddy viscosity highly non-linear
- Mixing length model is a poor approximation, analytic wall function inappropriate



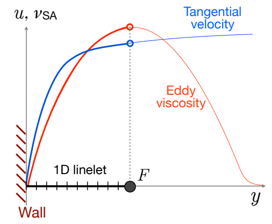
Outline

- Previous work & analytic wall functions
- ODE-based wall models
  - A New ODE wall model
- Numerical examples
- Conclusions

ODE-Based Wall Models

Proposed by several authors in last decade

- Solve ODE on 1D “linelet” normal to surface
- Solve:
  - Diffusion eq. for streamwise momentum
  - Turbulence model in wall-normal direction
- Produces a system of 2-point, 2nd-order BVPs
- Coupling: *Just like an analytic wall function*



see Kalitzin et al., J. Comp. Phys., **204**, 2005, Bond & Blottner, Intl. J. Num. Methods Fluids, **66**, 2011, or Capizzano, AIAA J. **54**(2), 2016

ODE-Based Wall Models

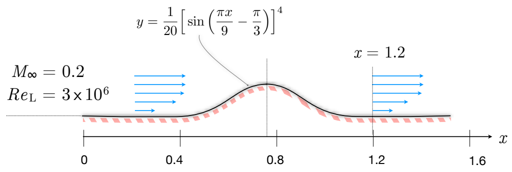
SA-BVP: Diffusion equation coupled with wall-normal SA turbulence model

x–momentum: (diffusion equation) $$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right)=0$$

SA model on linelet: $$\frac{\partial}{\partial y}\left((\nu+\tilde{\nu})\frac{\partial \tilde{\nu}}{\partial y}\right)=\underbrace{-c_{b2}\left(\frac{\partial \tilde{\nu}}{\partial y}\right)^2}_{\text{wall-normal diffusion}}+\text{Production}-\text{Destruction}$$

ODE-Based Wall Models

Compare SA-BVP with SA wall function on turbulent bump in channel

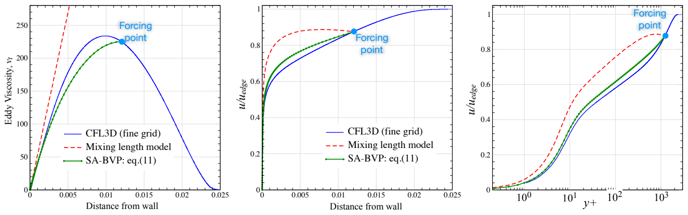


x–momentum: $$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right)=0$$

SA model on linelet: $$\frac{\partial}{\partial y}\left((\nu+\tilde{\nu})\frac{\partial \tilde{\nu}}{\partial y}\right)=-c_{b2}\left(\frac{\partial \tilde{\nu}}{\partial y}\right)^2+\text{Production}-\text{Destruction}$$

ODE-Based Wall Models

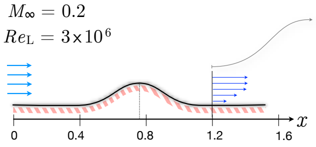
Compare SA-BVP with SA wall function on turbulent bump in channel @ $x = 1.2$

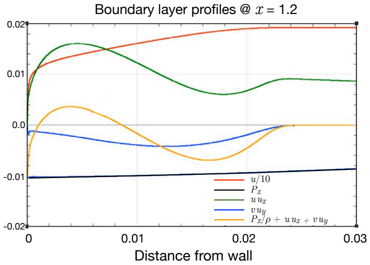


- Forcing point well out in wake layer, $y = 0.012$, $u = 0.85u_{\text{edge}}$
- Mixing length eddy viscosity inappropriate, so diffusion model alone does poorly
- Improved eddy viscosity makes a significant difference

ODE-Based Wall Models

Streamwise momentum in the wake





- Thin layer streamwise momentum: $((\mu+\mu_t)u_y)_y = p_x + \rho(uu_x + vu_y)$
- Examine relative magnitude of terms as we move away from the wall

ODE-Based Wall Models

Streamwise momentum in the wake

- At the wall, we have

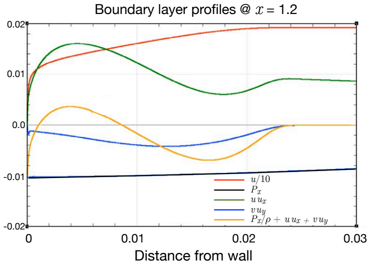
$$((\mu + \mu_t)u_y)_y = p_x$$

- Outside the boundary layer we approach:

$$0 = p_x + \rho(uu_x + vu_y)$$

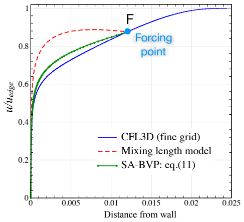
- In between we have the full streamwise momentum eq.

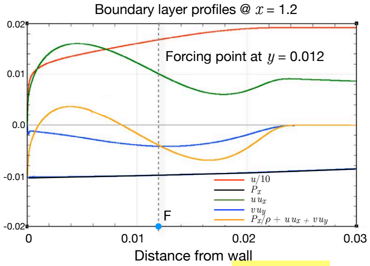
$$((\mu + \mu_t)u_y)_y = \underbrace{\underbrace{p_x} + \rho(\underbrace{uu_x} + \underbrace{vu_y})}$$


Boundary layer profiles @ x = 1.2

ODE-Based Wall Models

Streamwise momentum in the wake




Boundary layer profiles @ x = 1.2

- Forcing point @ $y = 0.012$ is in the wake. The convective balance has similar magnitude as p_x – Need to include!

$$((\mu + \mu_t)u_y)_y = \underbrace{\underbrace{p_x} + \rho(\underbrace{uu_x} + \underbrace{vu_y})}$$

ODE-Based Wall Models

Including the convective balance

- At the wall, velocity is zero, convective balance is zero

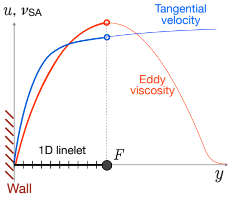
$$((\mu + \mu_t)u_y)_y = p_x$$

- But at the forcing point, it has the same magnitude as p_x

$$((\mu + \mu_t)u_y)_y = p_x + \rho(uu_x + vu_y)\big|_F$$

- Computing wall-normal variation of convective balance introduces streamwise coupling, and means computing the wall-normal velocity – *prefer not to do this*

- Introduce a cutoff function, $\psi(y)$, to turn it off as we approach the wall



ODE-Based Wall Models

Including the convective balance

- Introduce a cutoff function, $\psi(y)$, to turn it off as we approach the wall

$$\frac{\partial}{\partial y}\left((\mu + \mu_t)\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} + \psi(y)\rho\left[u_F \frac{\partial u}{\partial x}\bigg|_F + v_F \frac{\partial u}{\partial y}\bigg|_F\right]$$

$$\frac{\partial}{\partial y}\left((\nu + \tilde{\nu})\frac{\partial \tilde{\nu}}{\partial y}\right) = \text{wall-normal diffusion} + \text{Production} - \text{Destruction}$$

where $\psi(0) = 0$, $\psi(y_F) = 1$

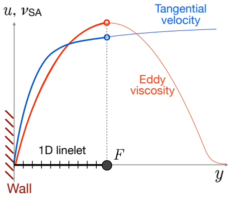
A New ODE-based Wall Model

bvp4 wall model

- Introduce a cutoff function, $\psi(y)$, to turn it off as we approach the wall

$$\psi(y) = \frac{u_{SA}(y)}{u_{SAF}} = \frac{u^+_{SA}(y)}{u^+_{SAF}}$$

so that $\psi(0) = 0,$ and $\psi(y_F) = 1$



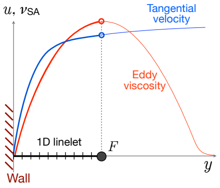
A New ODE-based Wall Model

bvp4 wall model

- Introduce a cutoff function, $\psi(y)$, to turn it off as we approach the wall

$$\psi(y) = \frac{u_{SA}(y)}{u_{SAF}} = \frac{u^+_{SA}(y)}{u^+_{SAF}}$$

so that $\psi(0) = 0,$ and $\psi(y_F) = 1$



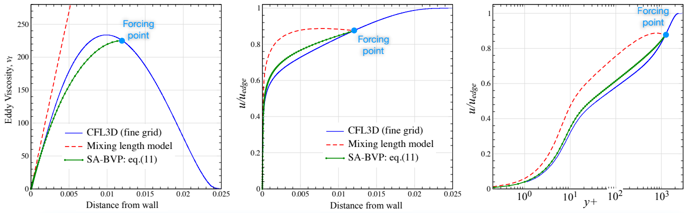
- The complete ***bvp4*** model becomes

$$\frac{\partial}{\partial y}\left((\mu + \mu_t)\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} + \psi(y)\rho\left[u_F \frac{\partial u}{\partial x}\bigg|_F + v_F \frac{\partial u}{\partial y}\bigg|_F\right]$$

$$\frac{\partial}{\partial y}\left((\nu + \tilde{\nu})\frac{\partial \tilde{\nu}}{\partial y}\right) = \text{wall-normal diffusion} + \text{Production} - \text{Destruction}$$

A New ODE-based Wall Model

bvp4 wall model: Include streamwise convective balance and pressure gradient



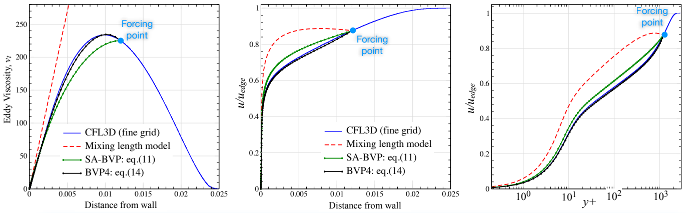
bvp4

x–momentum: $$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right)=\frac{\partial p}{\partial x}+\psi(y)\rho\left[u_F\frac{\partial u}{\partial x}\Big|_F+v_F\frac{\partial u}{\partial y}\Big|_F\right]$$

SA model on linelet: $$\frac{\partial}{\partial y}\left((\nu+\widetilde{\nu})\ \frac{\partial\widetilde{\nu}}{\partial y}\right)=\text{wall-normal diffusion}+\text{Production}-\text{Destruction}$$

A New ODE-based Wall Model

bvp4 wall model: Include streamwise convective balance and pressure gradient



bvp4

x–momentum: $$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right)=\frac{\partial p}{\partial x}+\psi(y)\rho\left[u_F\frac{\partial u}{\partial x}\Big|_F+v_F\frac{\partial u}{\partial y}\Big|_F\right]$$

SA model on linelet: $$\frac{\partial}{\partial y}\left((\nu+\widetilde{\nu})\ \frac{\partial\widetilde{\nu}}{\partial y}\right)=\text{wall-normal diffusion}+\text{Production}-\text{Destruction}$$

A New ODE-based Wall Model

ODE solver for *bvp4*

$$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right)=\frac{\partial p}{\partial x}+\psi(y)\rho\left[u_F\frac{\partial u}{\partial x}\Big|_F+v_F\frac{\partial u}{\partial y}\Big|_F\right]$$

$$\frac{\partial}{\partial y}\left((\nu+\widetilde{\nu})\ \frac{\partial\widetilde{\nu}}{\partial y}\right)=\text{wall-normal diffusion}+\text{Production}-\text{Destruction}$$

- Reformulate 2nd order equations as system of four 1st order BVPs
- Solve with 6th order adaptive ODE solver from Shampine and Muir
- Use warm starts on each linelet after initial solve ~2x cost of analytic WF
- Other details of implementation and coupling in paper

Numerical Results

Verification and Validation using examples from the NASA Turbulence Modeling Resource

- Computational Examples from TMR
  1. Turbulent bump in channel
  2. NACA 0012
  3. NACA 4412 with trailing edge separation

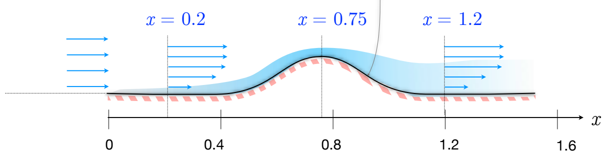
Turbulent Bump In Channel

TMR: “VERIF/2DB: 2D Bump-in-channel Verification Case”

$$y=\frac{1}{20}\left[\sin\left(\frac{\pi x}{9}-\frac{\pi}{3}\right)\right]^4$$

$M_\infty = 0.2$

$Re_L = 3\times10^6$



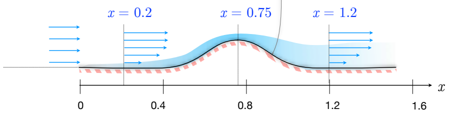
Turbulent Bump In Channel

TMR: “VERIF/2DB: 2D Bump-in-channel Verification Case”

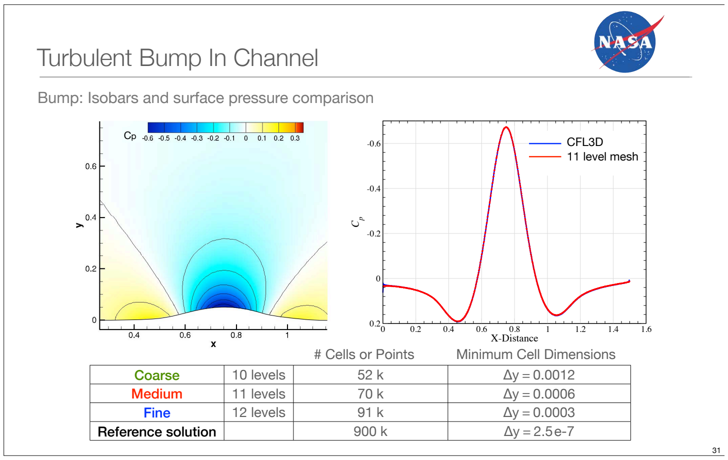
$$y=\frac{1}{20}\left[\sin\left(\frac{\pi x}{9}-\frac{\pi}{3}\right)\right]^4$$

$M_\infty = 0.2$

$Re_L = 3\times10^6$



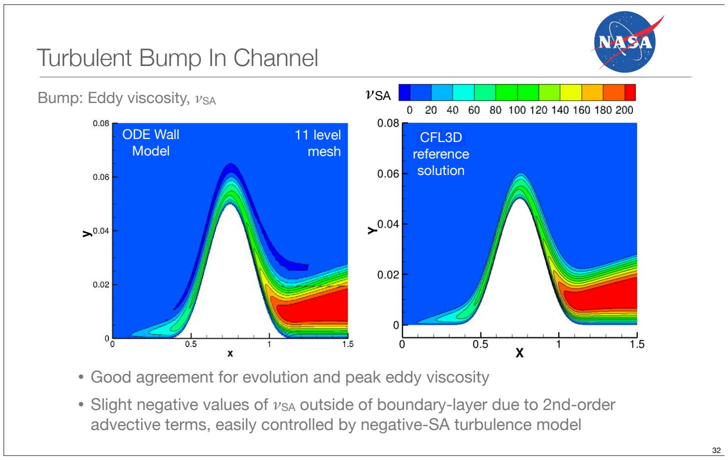
- Inlet & exit 25 units away, symmetry plane 5 units above
- Mesh-converged body-fitted results on 1409 x 641 mesh (~900k points)
- Compare results with CFL3D reference solution with SA turbulence model on finest mesh



Turbulent Bump In Channel

Bump: Isobars and surface pressure comparison

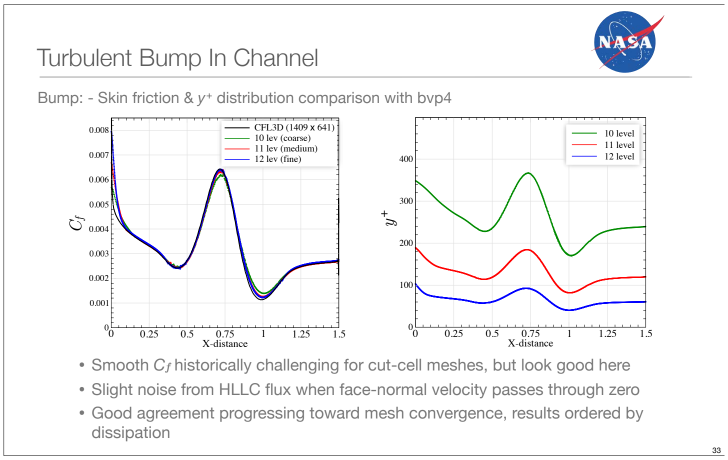
		# Cells or Points	Minimum Cell Dimensions
Coarse	10 levels	52 k	Δy = 0.0012
Medium	11 levels	70 k	Δy = 0.0006
Fine	12 levels	91 k	Δy = 0.0003
Reference solution		900 k	Δy = 2.5e-7



Turbulent Bump In Channel

Bump: Eddy viscosity, ν_{SA}

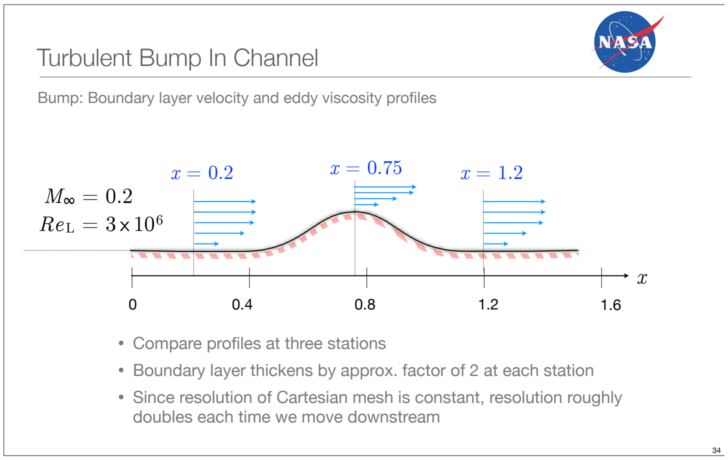
- Good agreement for evolution and peak eddy viscosity
- Slight negative values of ν_{SA} outside of boundary-layer due to 2nd-order advective terms, easily controlled by negative-SA turbulence model



Turbulent Bump In Channel

Bump: - Skin friction & y^+ distribution comparison with bvp4

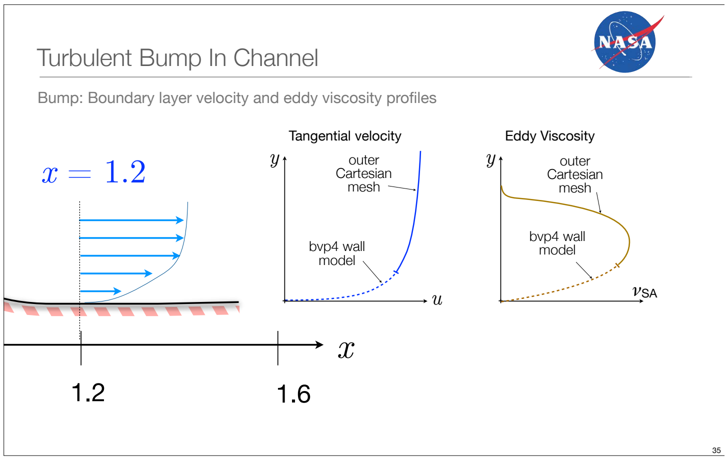
- Smooth C_f historically challenging for cut-cell meshes, but look good here
- Slight noise from HLLC flux when face-normal velocity passes through zero
- Good agreement progressing toward mesh convergence, results ordered by dissipation



Turbulent Bump In Channel

Bump: Boundary layer velocity and eddy viscosity profiles

- Compare profiles at three stations
- Boundary layer thickens by approx. factor of 2 at each station
- Since resolution of Cartesian mesh is constant, resolution roughly doubles each time we move downstream



Turbulent Bump In Channel

Bump: Boundary layer velocity and eddy viscosity profiles

Turbulent Bump In Channel

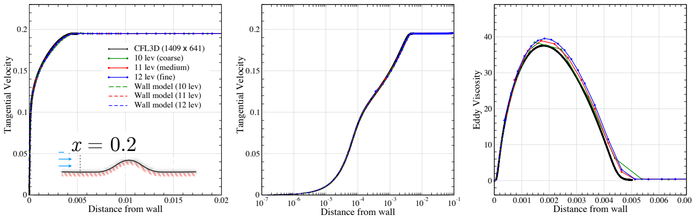
Bump: Boundary layer velocity and eddy viscosity profiles

Next comes SA vs BVP4 at coarsest station

Then comes coarse/medium/fine -- but need to mark pt F

Turbulent Bump In Channel

Bump: Boundary layer velocity and eddy viscosity profiles

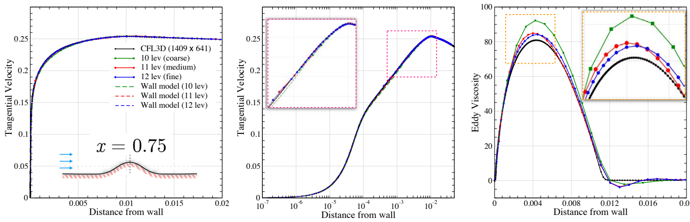


- 7 curves on each plot, (wall model & field solution) x (coarse, med, fine) + CFL3D
- Very good agreement for velocity, good agreement for eddy viscosity
- $x = 0.2$ is the most under resolved station, ~4-5 Cartesian cells in boundary layer

---

Turbulent Bump In Channel

Bump: Boundary layer velocity and eddy viscosity profiles

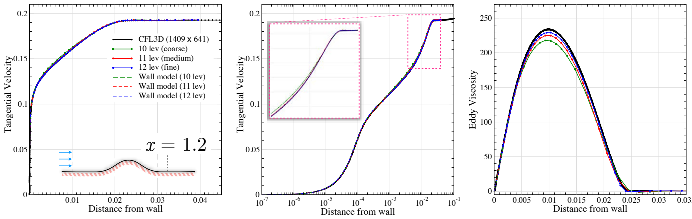


- About twice as much Cartesian resolution at $x = 0.75$
- Profile shows effects of moderate favorable pressure gradient on the front of the bump
- Data from the wall model collapses to reference solution regardless of outer resolution
- Eddy viscosity slightly overpredicted due to lack of resolution in outer mesh

---

Turbulent Bump In Channel

Bump: Boundary layer velocity and eddy viscosity profiles

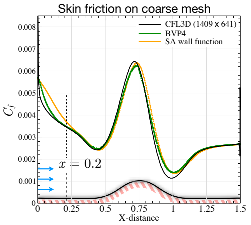


- Aft of bump, slight adverse pressure gradient, thick boundary layer
- Velocity profiles show very good agreement -- even on semi log scale
- Eddy viscosity peak being eroded slightly by dissipation on outer mesh

---

Turbulent Bump In Channel

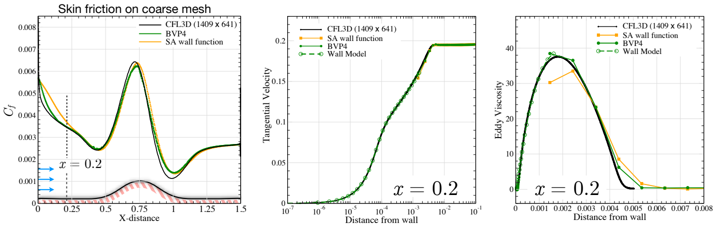
Bump: Compare bvp4 with analytic SA wall-function



- Even on coarsest grid, SA wall function does reasonably good job
- To see differences, look up front on coarse grid, $x = 0.2$, ($y^+ \approx 280$)

Turbulent Bump In Channel

Bump: Compare bvp4 with analytic SA wall-function

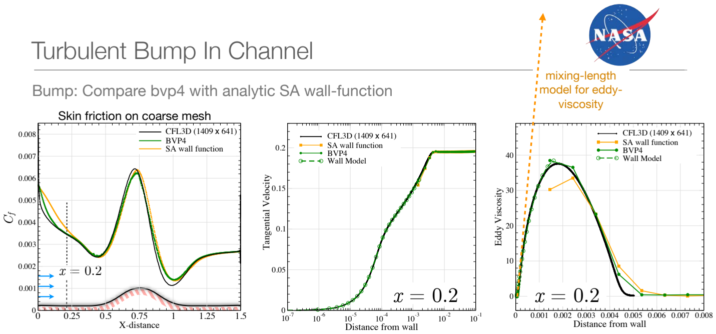


- Even on coarsest grid, SA wall function does reasonably good job
- To see differences, look up front on coarse grid, $x = 0.2$, ($y^+ \approx 280$)
- Skin friction discrepancy comes from misprediction of eddy viscosity by analytic wall function since it assumes a mixing-length model

---

Turbulent Bump In Channel

Bump: Compare bvp4 with analytic SA wall-function

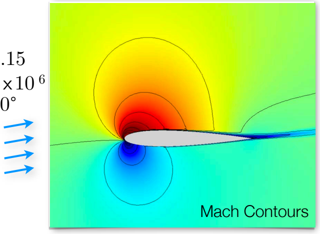


- Even on coarsest grid, SA wall function does pretty good job
- To see differences, look up front on coarse grid, $x = 0.2$, ($y^+ \approx 280$)
- Analytic wall function overpredicts eddy viscosity by about factor of 3,
  – is inconsistent with outer solution

NACA 0012

Modified NACA 0012 geometry with sharp trailing edge

$M_\infty = 0.15$
$Re_L = 6 \times 10^6$
$\alpha_\infty = 10°$



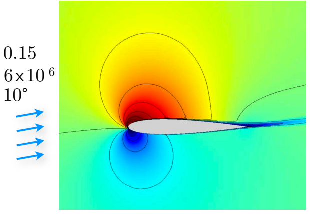
- Validation example "*2DN00: 2D NACA 0012 Airfoil Validation Case*" of TMR website
- Refinement studies on grids up to 14.7M points
- Compare with CFL3D, SA model with no circulation correction
- Mesh convergence sensitive to far-field boundary placement and LE & TE spacings

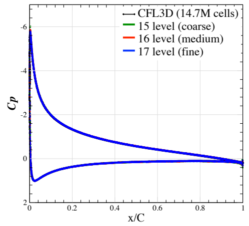
---

NACA 0012

Modified NACA 0012 - Surface pressure mesh convergence

$M_\infty = 0.15$
$Re_L = 6 \times 10^6$
$\alpha_\infty = 10°$

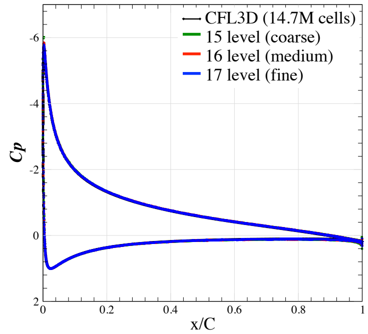




		# Cells or Points	Minimum Cell Dimensions
Coarse	15 levels	58 k	$\Delta x = \Delta y = 1.7\text{e-}3C$
Medium	16 levels	80 k	$\Delta x = \Delta y = 8.4\text{e-}4C$
Fine	17 levels	133 k	$\Delta x = \Delta y = 4.2\text{e-}4C$
Reference solution		14.7M	$1.\text{e-}7C \times 1.25\text{e-}5C$

NACA 0012

Modified NACA 0012 - Surface pressure mesh convergence



NACA 0012

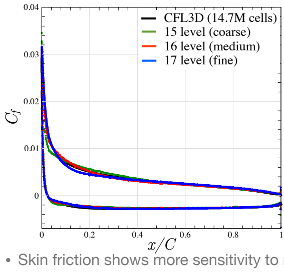
Modified NACA 0012 - Skin friction & y^+ distribution comparison

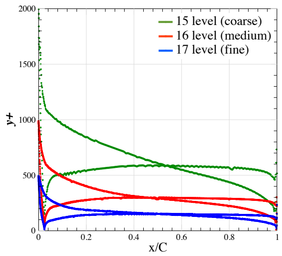
- 1. NOW COMES vs SA-BVP
- (slides 49/50, but first show skin friction, then show magnitude of terms)
- show what different levels of modeling are bying us

- 2. Then comes the cut at 1.001
- 3. Move adaptive to backup

NACA 0012

Modified NACA 0012 - Skin friction & y^+ distribution comparison



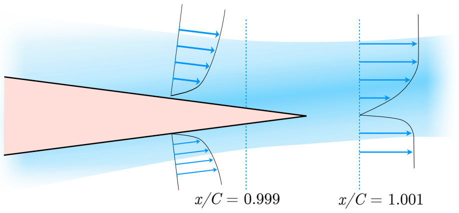


- Skin friction shows more sensitivity to resolution
- Finer meshes do better job near leading edge where physics is most under-resolved
- HLLC flux responsible for noise – disappears with refinement

---

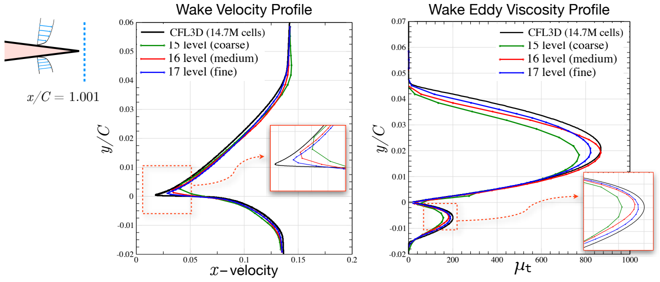
NACA 0012

Modified NACA 0012 - Wake surveys of velocity and eddy viscosity



- Experiment and simulation data exist for profiles at $x/C = 0.999$ and 1.001
- Examine data at $x/C = 1.001$, (similar results at $x/C = 0.999$)

NACA 0012

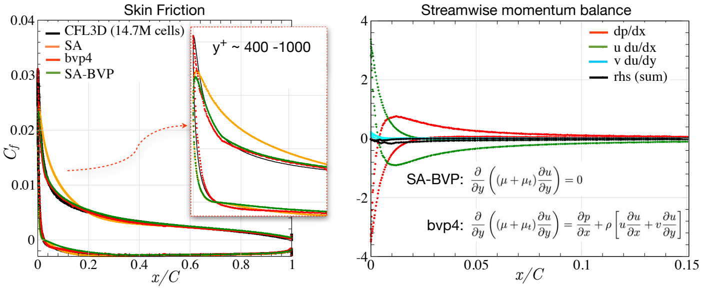


- Low dissipation inviscid flux helps resolution in cusp – good mesh convergence behavior
- Lower surface eddy viscosity shows good mesh convergence
- Upper surface eddy viscosity not yet mesh converged – literature shows slow convergence

---

NACA 0012

Modified NACA 0012 - Compare bvp4 with analytic SA-BVP & SA wall function



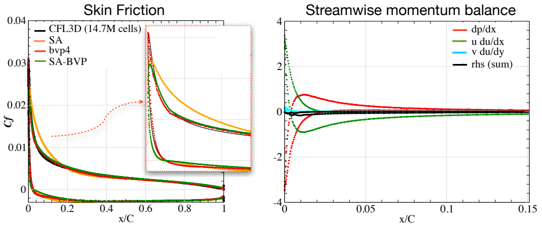
SA-BVP: $\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right) = 0$

bvp4: $\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} + \rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]$

(Terms measured at forcing point)

NACA 0012

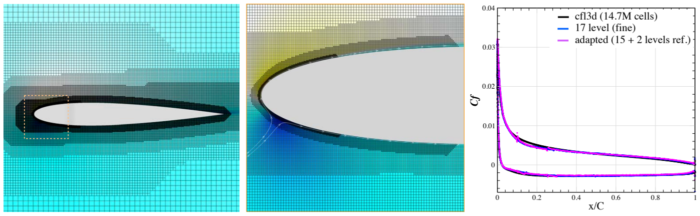
Modified NACA 0012 - Compare bvp4 with analytic SA-BVP & SA wall function



- SA-wall function lags both bvp4 model and SA-BVP skin friction
- SA-BVP does a good job, but misses peak and pressure-side near leading edge
- Note: RHS relatively small: p_x counterbalanced by $(u\,u_x + v\,u_y)$

NACA 0012

Modified NACA 0012 - Mesh refinement near leading edge

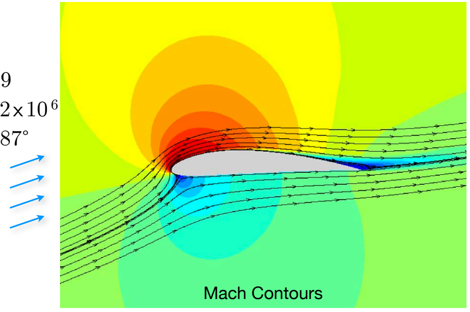


- Mesh refinement important tool for controlling cell counts
- 15 level mesh + 2 refinements near leading edge ~ 88 k cells
- C_f shows glitches near refinement interfaces at wall – *haven't dealt with interfaces yet*

NACA 4412

Modified NACA 4412 with trailing edge separation bubble

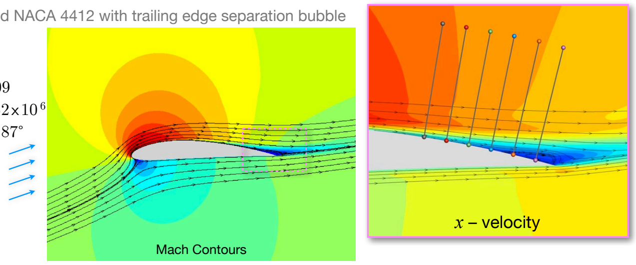
$M_\infty = 0.09$
$Re_L = 1.52\times10^6$
$\alpha_\infty = 13.87°$



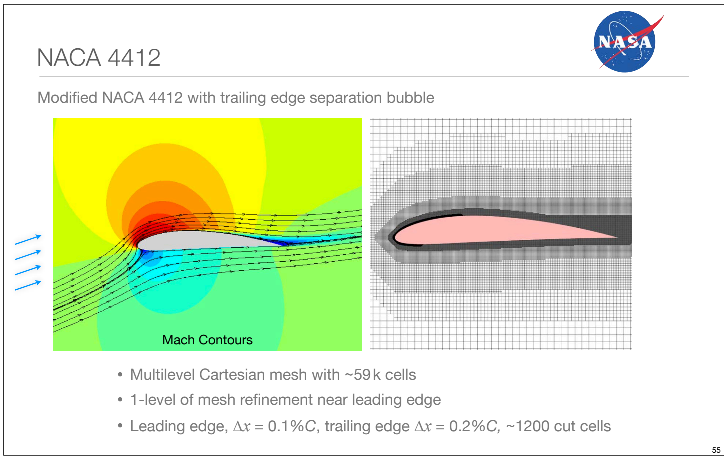
NACA 4412

Modified NACA 4412 with trailing edge separation bubble

$M_\infty = 0.09$
$Re_L = 1.52\times10^6$
$\alpha_\infty = 13.87°$



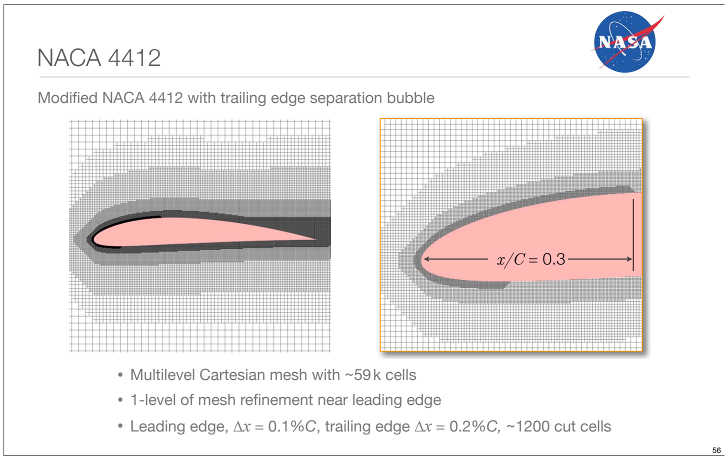
- Validation example in the “Extended cases” section of NASA TMR
- Smooth-body separation bubble near maximum lift conditions
- Experiment by Coles & Wadcock (1979) with hot-wire velocity profiles
- Reference data form CFL3D on 897 x 257 grid (≈230k points)



NACA 4412

Modified NACA 4412 with trailing edge separation bubble

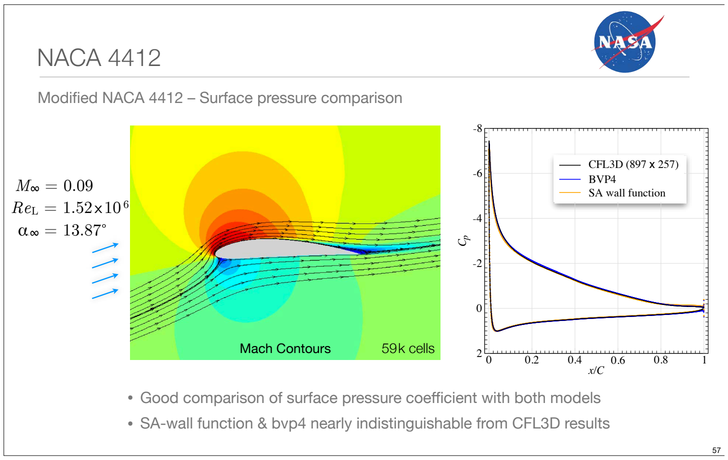
- Multilevel Cartesian mesh with ~59k cells
- 1-level of mesh refinement near leading edge
- Leading edge, Δx = 0.1%C, trailing edge Δx = 0.2%C, ~1200 cut cells



NACA 4412

Modified NACA 4412 with trailing edge separation bubble

- Multilevel Cartesian mesh with ~59k cells
- 1-level of mesh refinement near leading edge
- Leading edge, Δx = 0.1%C, trailing edge Δx = 0.2%C, ~1200 cut cells

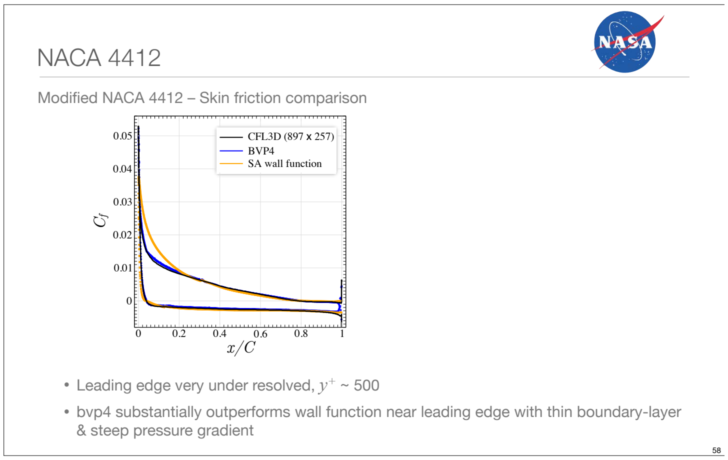


NACA 4412

Modified NACA 4412 – Surface pressure comparison

$$M_\infty = 0.09$$
$$Re_L = 1.52\times10^6$$
$$\alpha_\infty = 13.87°$$

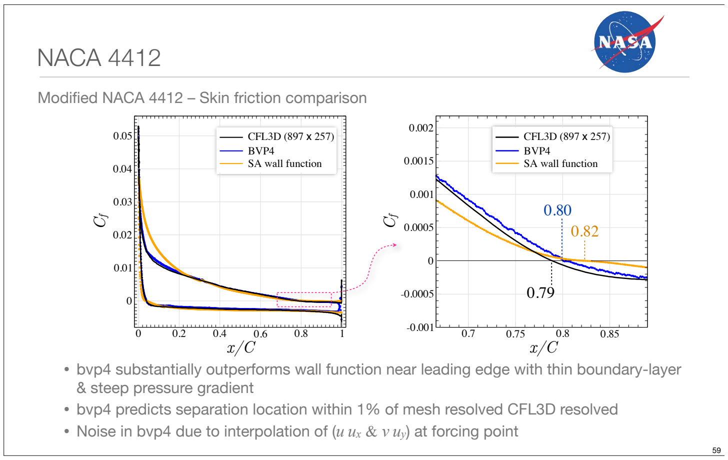
- Good comparison of surface pressure coefficient with both models
- SA-wall function & bvp4 nearly indistinguishable from CFL3D results



NACA 4412

Modified NACA 4412 – Skin friction comparison

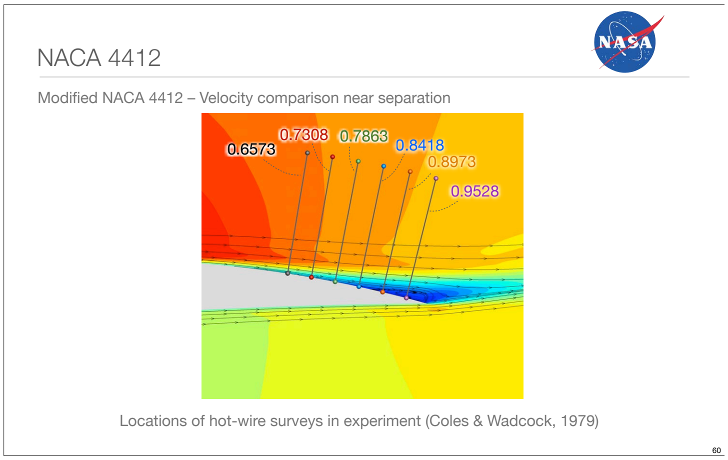
- Leading edge very under resolved, y^+ ~ 500
- bvp4 substantially outperforms wall function near leading edge with thin boundary-layer & steep pressure gradient



NACA 4412

Modified NACA 4412 – Skin friction comparison

- bvp4 substantially outperforms wall function near leading edge with thin boundary-layer & steep pressure gradient
- bvp4 predicts separation location within 1% of mesh resolved CFL3D resolved
- Noise in bvp4 due to interpolation of ($u\,u_x$ & $v\,u_y$) at forcing point



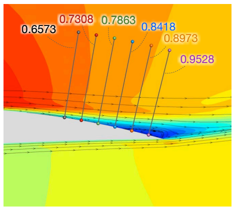
NACA 4412

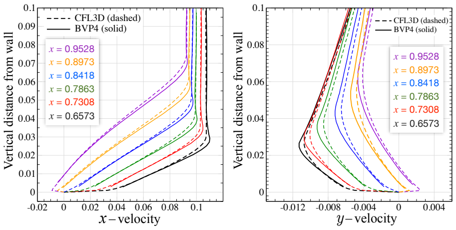
Modified NACA 4412 – Velocity comparison near separation

Locations of hot-wire surveys in experiment (Coles & Wadcock, 1979)

NACA 4412

Modified NACA 4412 – Velocity comparison near separation





- Good prediction of both x and y components of velocity through separation bubble
- Vertical velocity about an order of magnitude smaller than horizontal
- Slight "viscous overshoot" due to coarseness of Cartesian mesh, $\Delta x = \Delta y = 0.2\%C$

Summary

Atmospheric propagation and ground effects modeling

- presented V&V studies for a new ODE-based wall model for RANS equations
- demonstrated for several well-studied flows including smooth body separation
- bvp4 model:
  - Solves coupled solves a coupled set of ODEs posed as two-point boundary value problems for the streamwise velocity and the turbulent viscosity
- includes both the streamwise pressure gradient and the momentum balance valid farther from the wall

permits wall spacing on the Cartesian mesh 4 to 8 x coarser than with analytic wall functions,

order of magnitude farther out than analytic wall functions

- wall model itself about 2-3x the computational cost of analytic wf's on same mesh
- Can be applied in body-fitted or non-body fitted meshes

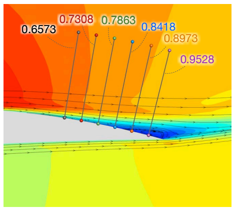
TODO

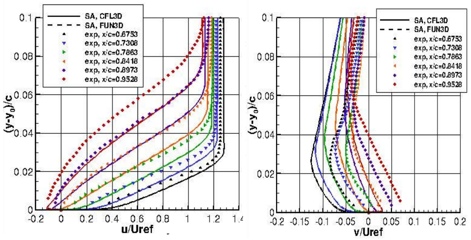
- Paper presents counting arguments for 3D

Questions?

NACA 4412

Modified NACA 4412 – SA model vs experiment data near separation



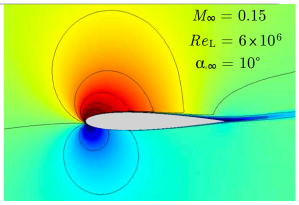


- Very sensitive example - near CL max
- SA makes some error in velocity profiles due to mis-prediction of separation location
  - Different turbulence models show up to 10%C variation in separation location
- Vertical velocity even more sensitive, since magnitudes ~10 x smaller

---

NACA 0012

Modified NACA 0012 - Integrated force coefficients



- TMR has loads data available for several codes
- Excellent prediction of viscous drag. Even coarsest mesh (15lev) is within 2 counts
- Net lift and drag not as good due to inviscid regions of flow
  - TMR documents sensitivity to far field boundary (100C vs 500C for reference)
  - Trailing edge spacing too large:
    ~500 x coarser than reference on adapted grid

	C_d Viscous	C_d Pressure	C_d Total	C_l Total
CFL3D[†] 14.7M point reference	0.00621	0.00607	0.01227	1.0908
17 Lev uniform wall spacing, 133k cells	0.00611	0.00767	0.01378	1.1120
Adapted 15 lev + 2 near LE, 88k cells	0.00607	0.00751	0.01358	1.1416

[†] Data from CFL3D with SA model on "family II" grid, no point vortex correction & 2nd-order turbulent advection.