



## Smoothing-Based Relative Navigation & Coded Aperture Imaging

### Efficient Smoothing Algorithm and Optical Ranging for Improved Multi-Satellite Relative Position and Velocity Determination

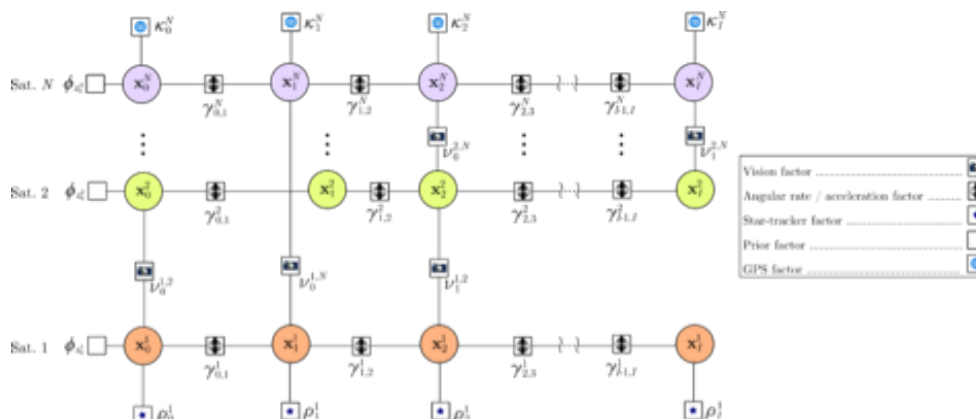
This project will develop an efficient smoothing software for incremental estimation of the relative poses and velocities between multiple, small spacecraft in a formation, and a small, long range depth sensor based on coded aperture imaging that is capable of identifying other spacecraft in the formation. The smoothing algorithm will obtain the maximum a posteriori estimate of the relative poses between the spacecraft by using all available sensor information in the spacecraft formation. This algorithm will be portable between different satellite platforms that possess different sensor suites and computational capabilities, and will be adaptable in the case that one or more satellites in the formation become inoperable. It will obtain a solution that will approach an exact solution, as opposed to one with linearization approximation that is typical of filtering algorithms. Thus, the algorithms developed and demonstrated as part of this program will enhance the applicability of small spacecraft to multi-platform operations, such as precisely aligned constellations and fractionated satellite systems.

“Depth from Coded Aperture” is an imaging technique that allows a depth map to be computed. It operates using a typical passive monocular camera whose circular aperture is replaced by a patterned, or coded, aperture that has high frequency components. By setting the scene out of focus, the size of the imaged aperture code is a function of the depth of the scene.

Smoothing is an estimation approach in a nonlinear optimization problem that looks for a solution that will maximize the a posteriori probability of all estimated variables  $\Theta$  given the observations  $Z$ , and control inputs  $U$  and measurement model  $h(\theta_i)$ . The model problem can be stated as a nonlinear optimization problem.

$$\begin{aligned}\Theta^* &= \underset{\Theta}{\operatorname{argmax}} P(\Theta, Z, U) \\ &= \underset{\Theta}{\operatorname{argmin}} \frac{1}{2} \sum_i \| h(\theta_i) - z_i \|^2_{\Sigma_i}\end{aligned}$$

The probability distribution  $P(\Theta, Z, U)$  is separated into the product of several independent factors, allowing the problem



*An example factor graph representation of the joint probability distribution for multi-satellite pose and velocity determination.*

to be visually represented as a factor graph (see Figure) and solved efficiently.  $N$  small satellites make occasional visual observations of one another (vision factors  $v$ ). Long sequences of high frequency angular rate and acceleration data are lumped into individual factors  $\gamma$ . The positions of the satellites can be further constrained by star-tracker measurement factors  $\rho$  and GPS measurement factors  $\kappa$ , which constrain the absolute rotation and position, respectively, of the satellite to which they are applied. A sliding time window can be used to limit the number of variables in the factor graph and bound the processing and memory requirements associated with obtaining a solution. Each satellite with adequate computational power solves the full factor graph. A communications link is used to send measurements from one satellite to another. The smoothing algorithm will be implemented in C++ and make use of the publicly available GTSAM software library that provides an extensive set of modular tools for creating and solving factor graphs. It will be verified experimentally using SPHERES-VERTIGO on the International Space Station.

The coded aperture imaging technique computes a detailed map of the surface distances (depth) of the spacecraft visible in a scene. It operates using a typical passive monocular camera whose circular aperture is replaced by a coded aperture that provides a refined image. By setting the scene out of focus, the size of the imaged aperture code is a function of the depth(s) of the image(s) in the scene.

The coded aperture imaging technique relies on the use of a long-range camera that includes a mask at its aperture instead of a traditional circular aperture. This mask enables an intentionally out-of-focus scene to be refined by use of high frequency components that add upon the image. The size of the resulting imaged aperture code provides a direct measure of the depth of the objects within the scene. The depth accuracy using this technique is linearly related to the depth resolution of the camera. This project develops a set of coded aperture depth cameras that can be integrated with the smoothing algorithms to perform relative navigation maneuvers to provide multi-satellite relative pose and velocity estimation rapidly and autonomously.

The project's computationally efficient smoothing algorithm for determining relative pose and velocity is applicable to a wide variety of NASA missions. This algorithm could enable constellations of small spacecraft with less sophisticated instruments to fly in close formation by making better use of the available sensor information. For satellite servicing mis-

sions, this technology could be used to enhance the accuracy and precision with which relative pose and velocity estimation is performed, improving the probability of mission success. Additionally, inspection of space debris or defunct satellites prior to their deorbit or servicing is a potential commercial application of the proposed technology. All software developed by this program will be made publicly available under an open source license to simplify and encourage its adoption by other missions. Coded aperture cameras that can provide long range depth sensing would be applicable to a number of NASA missions including formation flight, planetary entry, descent and landing and UAV flights (specifically on Mars and other planets and in GPS denied environments).

The Smoothing-Based Relative Navigation & Coded Aperture Imaging project is managed and funded by the Small Spacecraft Technology Program (SSTP) within the Space Technology Mission Directorate. The SSTP expands U.S. capability to execute unique missions through rapid development and in space demonstration of capabilities for small spacecraft applicable to exploration, science, and the commercial space sector. The SSTP will enable new mission architectures through the use of small spacecraft with goals to expand their reach to new destinations, and challenging new environments.

**For more information about the SSTP, visit:**

[www.nasa.gov/directorates/spacetech/small\\_spacecraft/](http://www.nasa.gov/directorates/spacetech/small_spacecraft/)

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