

# GRAVITY-ASSIST TRAJECTORIES TO THE ICE GIANTS: AN AUTOMATED METHOD TO CATALOG MASS- OR TIME-OPTIMAL SOLUTIONS

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This work presents an automated method of calculating mass (or time) optimal gravity-assist trajectories without a priori knowledge of the flyby-body combination. Since gravity assists are particularly crucial for reaching the outer Solar System, we use the Ice Giants, Uranus and Neptune, as example destinations for this work. Catalogs are also provided that list the most attractive trajectories found over launch dates ranging from 2024 to 2038. The tool developed to implement this method, called the Python EMTG Automated Trade Study Application (PEATSA), iteratively runs the Evolutionary Mission Trajectory Generator (EMTG), a NASA Goddard Space Flight Center in-house trajectory optimization tool. EMTG finds gravity-assist trajectories with impulsive maneuvers using a multiple-shooting structure along with stochastic methods (such as monotonic basin hopping) and may be run with or without an initial guess provided. PEATSA runs instances of EMTG in parallel over a grid of launch dates. After each set of runs completes, the best results within a neighborhood of launch dates are used to seed all other cases in that neighborhood—allowing the solutions across the range of launch dates to improve over each iteration. The results here are compared against trajectories found using a grid-search technique, and PEATSA is found to outperform the grid-search results for most launch years considered.

## INTRODUCTION

An orbiter and atmospheric probe mission to Uranus (and possibly Neptune pending feasibility) is NASA's next highest priority flagship-class mission.<sup>1</sup> Recent work by Hughes,<sup>2</sup> JPL (for the Ice Giants Study directed to JPL by NASA headquarters),<sup>3,4</sup> and Mansel et al.<sup>5</sup> has characterized the design space for such a mission to the Ice Giants. The processes used to complete such work, however, require lots of human-in-the-loop hours. Additionally, much of the work involves grid-search methods that do not produce optimal solutions (be it with respect to delivered mass, flight time, etc.). The method proposed in this paper will remove much of the human-in-the-loop time required to conduct such a large-scale investigation.

Other studies that have cataloged trajectories to the Outer Planets include: low-thrust trajectories to the Ice Giants by Landau et al.,<sup>6</sup> low-thrust trajectories to Uranus by Dankanich and McAdams,<sup>7</sup> chemical-propulsion trajectories to Uranus by Spreen et al.<sup>8</sup> and to Neptune by Hughes et al.,<sup>9</sup> chemical and low-thrust trajectories to Neptune by Campagnola et al.,<sup>10</sup> and trajectories to Jupiter via chemical propulsion by Petropoulos et al.<sup>11</sup>

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## METHODS

The Python EMTG Automated Trade Study Application (PEATSA) used to implement the automated method presented in this work is a wrapper tool for the NASA Goddard Space Flight Center (GSFC) trajectory optimization tool, the Evolutionary Mission Trajectory Generator (EMTG). EMTG can be used to design gravity-assist trajectories that use either low-thrust<sup>12</sup> or chemical-propulsion.<sup>13</sup> For chemical propulsion trajectories (which are the focus of this work), the maneuvers are modeled as impulsive. EMTG uses a multiple-shooting architecture and direct-optimization techniques to find the launch date (and flyby encounter dates) as well as maneuver locations to maximize the final mass (or minimize flight time) given a set of constraints. For this work, the focus is to maximize the final mass given a flight-time constraint. Various launch vehicles and propellant tank capacities can also be considered; however, in this simplified study, only the Delta IV Heavy launch vehicle is used, with an unconstrained spacecraft propellant tank. These search parameters are consistent with a subset of the solutions presented in a set of Uranus and Neptune trajectory catalogs created by Hughes,<sup>2</sup> who used a grid-search approach via the Satellite Tour Design Program (STOUR)<sup>14,15</sup> developed by the Jet Propulsion Laboratory (JPL) and Purdue University. The results from Hughes' grid-search are used as a basis for comparison of the approach taken in this paper.

EMTG can be operated both with and without providing an initial guess for the optimal solution. If no initial guess is provided, EMTG randomly selects the decision variable values using a Pareto probability distribution.<sup>16</sup> With given (or randomly generated) decision variables, EMTG then uses calculus-based direct-optimization techniques to find the optimal solution (a local optimum) via the Nonlinear Programming (NLP) solver SNOPT.<sup>17</sup> EMTG then uses a stochastic method called monotonic basin hopping (MBH) to "hop" to a new set of decision variables,<sup>16,18,19</sup> which, if run for a sufficient amount of time, will eventually find the global optimum.

In practice however, we must run EMTG for a finite amount of time, and we never really know how much runtime is needed to reach the global optimum. Because PEATSA runs many EMTG cases across a range of launch dates simultaneously, and seeds all neighboring solutions with the best cases, it is able to not only find the global optimum more quickly, but also provides the user some insight as to the behavior of the dynamics over the launch-date range considered.

In addition to allowing neighboring cases to share information to improve the subsequent EMTG runs, PEATSA is also capable of randomly modifying the design parameters of EMTG which may not have been known by the designer a priori. For mission designers, a typical challenge is determining the most favorable gravity-assist combination, which is often not known without a broad-search investigation of trajectories. For this work, many gravity-assist combinations are run in PEATSA, which are listed in Table 1. This set of flyby combinations is taken from Hughes;<sup>2</sup> however, if more time (or more processors) would have been available, PEATSA could be set to randomly try different gravity-assist combinations, to automatically determine the best gravity-assist combinations over the launch dates considered. The only user input required for PEATSA to do this random selection is a list of the bodies to consider for gravity assist and the total number of gravity assists to consider. Again, for this study, the set of flyby combinations provided in Table 1 is used.

Regardless of how PEATSA is given the set of flyby combinations to run, it proceeds to run all combinations over the span of specified launch dates. Once a feasible, or improved, solution is found, PEATSA will then use that solution as an initial guess for the neighboring solutions about that launch date (with the same flyby combination)—eventually populating the solution space for the range of launch dates for which that solution remains beneficial. As each iteration is computed,

**Table 1. Gravity-Assist Combinations Considered**

Uranus				Neptune		
0	*2,2,5	*4,3,5	*2,3,2,3,5	0	*2,3,5	2,3,3,6
2	2,2,6	4,3,6	2,3,2,3,6	3	2,3,6	2,3,4,3
3	2,3,2	4,4,2	2,3,3,3,5	4	2,4,2	*2,3,4,5
4	*2,3,3	4,4,3	2,3,3,3,6	*5	2,4,3	2,4,2,5
5	2,3,4	4,4,4	*2,3,3,4,5	6	*3,3,5	4,3,3,5
6	*2,3,5	4,4,5	2,3,3,4,6	2,2	*3,4,5	*2,2,3,3,5
2,2	2,3,6	4,4,6	2,3,4,3,5	2,3	3,4,6	*2,3,2,3,5
2,3	2,4,2	4,5,6	2,3,4,3,6	3,2	3,5,6	*2,3,3,3,5
2,4	2,4,3	*2,2,2,5	2,4,2,3,5	3,3	4,2,3	*2,3,3,4,5
2,5	2,4,4	2,2,2,6	*2,4,3,3,5	*3,4	4,2,5	2,3,4,3,5
2,6	2,4,5	*2,2,3,5	2,4,3,3,6	*3,5	4,2,6	2,4,2,3,5
3,2	2,4,6	*2,2,3,6	4,3,2,3,5	3,6	4,3,3	2,4,3,3,5
3,3	2,5,6	2,2,4,2	4,3,4,3,5	4,3	4,3,4	4,3,2,3,5
*3,4	*3,3,5	2,2,4,5	4,3,4,3,6	4,4	*4,3,5	4,3,4,3,5
*3,5	3,4,5	2,2,4,6		*4,5	4,3,6	
3,6	3,4,6	2,3,3,3		4,6	4,4,3	
4,2	3,5,6	*2,3,3,4		5,6	4,4,4	
4,3	4,2,2	*2,3,3,5		2,2,2	4,4,5	
4,4	*4,2,3	2,3,3,6		*2,2,3	4,4,6	
4,5	4,2,4	*2,3,4,3		2,2,4	4,5,6	
4,6	4,2,5	*2,3,4,5		2,2,5	*2,2,2,5	
5,6	4,2,6	*2,4,2,5		2,2,6	*2,2,3,5	
2,2,2	4,3,2	*4,3,3,5		2,3,2	2,2,4,5	
*2,2,3	*4,3,3	4,3,3,6		*2,3,3	2,3,3,4	
2,2,4	4,3,4	*2,2,3,3,5		2,3,4	*2,3,3,5	

it is possible that, due to MBH, an even better solutions will be found, which will then propagate to the neighboring solutions. As PEATSA continues to iterate, the mission designer simply has to check-in on its progress, and stop the program once the solutions no longer appear to improve. A more detailed description of PEATSA and its applications is provided by Knittel et al.<sup>20</sup> The set of search parameters considered for this study are provided in Table 2. For all cases, the optimization objective is to maximize the final mass, given a flight-time constraint. The “seed distance” is the neighborhood of launch dates for which PEATSA will consider to seed (i.e. use as an initial guess) for subsequent PEATSA iterations. The EMTG run time is the time that MBH is allowed to “hop” per PEATSA iteration.

**Table 2. Parameters for broad search of trajectories to Uranus and Neptune**

Parameter	Value
Launch Date Range	1-1-2024 to 12-31-2038
Launch Vehicle	Delta IV Heavy
Maximum Flight Times	12 & 13 yrs (Uranus), 14.5 yrs (Neptune)
Capture Orbit Periapsis	3000 km (altitude)
Capture Orbit Period	20 days
Spacecraft $I_{sp}$	323 s
Propellant Mass Margin	12%
Seed Distance	10 days
EMTG Run Time	2 min

Due to time constraints, the number of flyby combinations considered was successively reduced

\*Flyby sequence kept throughout analysis.

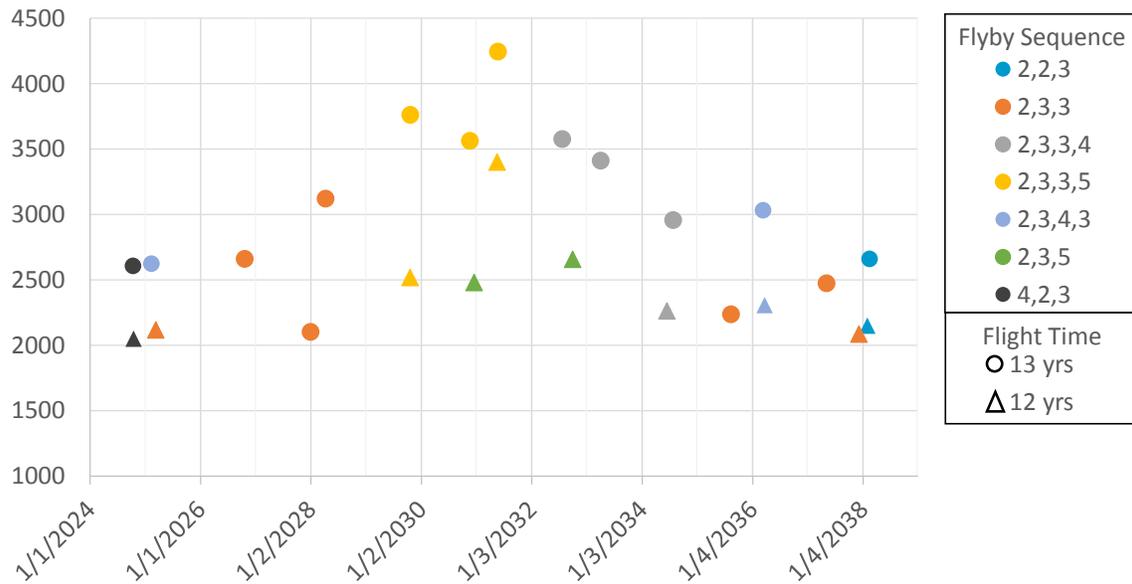
after determining that such combinations did not produce competitive cases (compared to other paths) after several PEATSA iterations had occurred. Any flyby combination kept throughout the analysis (i.e. that is never truncated from the list of sequences considered) is indicated by a \* in Table 1. For the case of trajectories to Uranus, flyby sequences are kept in the analysis if they provide a maximum-mass solution (over the 15-year launch period) that meet or exceed 2000 kg. Flyby sequences are also kept if they provide the maximum-mass solution for any particular launch year. This logic is applied twice throughout the computation process, ultimately reducing the number of Uranus flyby combinations to 23. Similarly, flyby combinations are truncated for the computations of trajectories to Neptune, but with slightly different logic. For the first round of truncations, combinations to Neptune are kept if their maximum-mass solution delivers at least 500 kg, or provides the maximum-mass solution for any particular launch year. For the second round of truncations, all combinations whose maximum-mass solution delivers at least 1000 kg are kept, as well as any combination that delivers the most mass for any particular launch year. This ultimately reduces the number of flyby combinations to Neptune to 18.

## RESULTS AND DISCUSSION

The maximum mass solutions per launch year are shown in Fig. 1 and Table 3, for trajectories to Uranus, and Fig. 2 and Table 4, for trajectories to Neptune. In the case of Uranus, because both 12- and 13-year flight-time constraints are used, a “best case” is chosen for both flight time constraints and for each launch year. An exception to this selection criteria is for any launch year where a 12-year solution does not exist that can deliver a mass greater than 2000 kg into the 20-day orbit at Uranus. Since the Neptune trajectory analysis generally resulted in solutions that delivered less mass, all maximum mass solutions (per launch year) are included in Fig. 2 (i.e. including those with a delivered mass less than 2000 kg). Many of the trajectories to Uranus did not have flight times that reached the 12- or 13-year constraint. Since these flight time constraints are less than what is required for successive Hohmann transfers between each flyby body to either Uranus or Neptune (the most  $\Delta V$ -efficient transfers), the mass-optimal solutions (which are expected to be very similar to the  $\Delta V$ -optimal solutions) are expected to maximize the available flight time once the global optimum is met. The fact that this did not occur for many of the Uranus solutions suggests that PEATSA was not given sufficient time in this short study to find solutions. Due to the increased complexity of the Uranus analysis over that of Neptune, the Uranus analysis completed a total of 12 iterations, whereas the Neptune analysis completed 32 iterations. In both cases the analyses were allowed to run for approximately 5 weeks on a 64- and 60-core server for each Uranus and Neptune analysis, respectively. In general, the best cases presented in this paper are simply the maximum-mass solution for either the 12- or 13-year analyses; however, for the launch year 2029, one of the “12-year solutions” was found that delivered comparable mass to the 2029 maximum-mass solution, but in only 11.9 years, and was selected as the best case.

Inspection of Figs. 1 and 2, provides further evidence of the Uranus analysis needing more time to complete, as the trend in solutions for the Neptune results is much smoother than those for Uranus. Since the Uranus plot includes a best case for both 12- and 13-year flight times, two sets of smooth curves (each similar to that of the Neptune results) is expected.

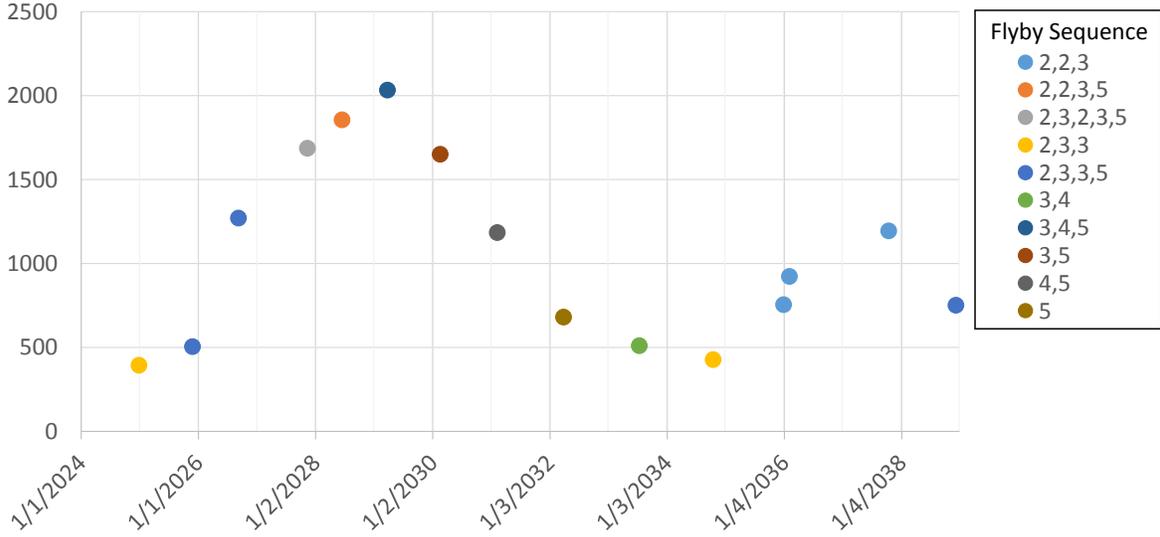
For reference, the trajectory catalogs from the STOUR grid-search approach by Hughes<sup>2</sup> are provided in Tables 5 and 6 for Uranus and Neptune, respectively. Since Hughes’ selection criteria was to identify the minimum-time solution (of the results found in the grid search) that could deliver at least 2000 kg, the shorter flight-time solutions for Uranus presented in Table 3 are favored (when



**Figure 1. Best Uranus trajectories for 12- and 13-year flight times.**

**Table 3. Trajectories to Uranus**

Launch Date	Flyby Sequence	Flight Time, years	Final Mass, kg
10/12/2024	4,2,3	13.0	2607
10/15/2024	4,2,3	12.0	2050
2/11/2025	2,3,4,3	13.0	2624
3/12/2025	2,3,3	12.0	2116
10/22/2026	2,3,3	13.0	2661
12/31/2027	2,3,3	13.0	2102
4/9/2028	2,3,3	13.0	3121
10/20/2029	2,3,3,5	11.9	2517
10/21/2029	2,3,3,5	13.0	3762
11/20/2030	2,3,3,5	13.0	3563
12/17/2030	2,3,5	12.0	2480
5/19/2031	2,3,3,5	12.0	3401
5/25/2031	2,3,3,5	13.0	4245
7/23/2032	2,3,3,4	13.0	3577
9/30/2032	2,3,5	12.0	2657
4/4/2033	2,3,3,4	13.0	3411
6/16/2034	2,3,3,4	12.0	2262
7/28/2034	2,3,3,4	13.0	2957
8/15/2035	2,3,3	13.0	2238
3/13/2036	2,3,4,3	13.0	3033
3/23/2036	2,3,4,3	12.0	2306
5/7/2037	2,3,3	13.0	2474
12/8/2037	2,3,3	12.0	2085
2/1/2038	2,2,3	12.0	2149
2/16/2038	2,2,3	13.0	2660



**Figure 2. Best Neptune trajectories, all with flight times of 14.5 years.**

available). We note that the delivered mass values reported by Hugues are only to two significant digits.

A more direct comparison of the PEATSA trajectory search to the STOUR grid-search results is provided in Tables 7 and 8 for Uranus and Neptune, respectively. For a more direct comparison of the Uranus results when multiple solutions are present in a given launch year, solutions are favored that have the shortest flight time with a delivered mass of at least 2000 kg. For comparison of the Neptune results, only PEATSA solutions that deliver at least 1000 kg are considered. Additionally, mass values from the PEATSA results are only reported to two significant digits to be consistent with those from the STOUR grid search.

The comparison shows that overall, the PEATSA results outperformed the STOUR results. Although the STOUR solutions are found from a grid-search approach and are not optimized, this result is not necessarily obvious since the stochastic methods used by PEATSA provide no guarantee that the breadth of the solution space will be explored. Additionally, the PEATSA solutions for Uranus likely require several more iterations before the global optimum is approached. Another surprising result is for the launch year 2031 of the Neptune results, where both PEATSA and STOUR found similar solutions with the same flyby combination. Yet, in this case, the STOUR solution is significantly better. Since the Neptune results were expected to be near the global optimum, this is evidence that the Neptune solutions still require more iterations. This also illustrates the sensitivity of these solutions using this technique, and how it is easy to "get stuck" on a local optimum, and miss the global optimum.

Thus, the results suggest that, although PEATSA was able to outperform STOUR in most cases, there are a few years where STOUR was better. Thus, there is still a benefit to using grid search methods, as they guarantee a large breadth of the solution space will be explored. Furthermore,

**Table 4. Trajectories to Neptune**

Launch Date	Flyby Sequence	Flight Time, years	Final Mass, kg
12/27/2024	2,3,3	14.5	393
11/26/2025	2,3,3,5	14.5	504
9/8/2026	2,3,3,5	14.5	1270
11/12/2027	2,3,2,3,5	14.5	1686
6/15/2028	2,2,3,5	14.5	1856
3/25/2029	3,4,5	14.5	2032
2/18/2030	3,5	14.5	1651
2/8/2031	4,5	14.5	1184
3/28/2032	5	14.5	680
7/13/2033	3,4	14.5	510
10/16/2034	2,3,3	14.5	427
12/30/2035	2,2,3	14.5	754
2/4/2036	2,2,3	14.5	923
10/15/2037	2,2,3	14.5	1194
12/9/2038	2,3,3,5	14.5	751

**Table 5. Trajectories to Uranus from Hughes Grid-Search<sup>2</sup>**

Launch Date	Flyby Sequence	Flight Time, years	Final Mass, kg
12/06/2024	2,3,3	13.3	2.1
01/05/2025	2,3,3,3	14.4	2.0
10/02/2026	2,3,3	12.7	1.9
09/17/2027	2,2,3,3,5	13.9	2.1
04/19/2028	2,3,5	13.5	2.1
12/30/2029	2,3,3,5	11.7	2.0
11/05/2030	4,3,3,5	11.7	1.8
12/10/2030	2,2,3,5	12.1	2.1
07/18/2031	2,3,5	10.9	2.1
11/19/2032	2,3,3,4	12.4	1.9
12/16/2032	2,2,5	13.2	2.3
01/05/2033	2,3,3	13.4	2.0
04/08/2033	2,3,3,4	12.2	1.8
03/26/2034	2,2,3	13.0	2.0
12/21/2035	2,3,3	13.1	1.8
02/09/2036	2,2,3	13.2	2.1
11/10/2037	2,3,3	12.6	2.0
02/08/2038	2,3,3	12.8	2.1

**Table 6. Trajectories to Neptune from Hughes Grid-Search<sup>2</sup>**

Launch Date	Flyby Sequence	Flight Time, years	Final Mass, kg
7/13/2026	2,3,3,5	14.8	1300
10/31/2027	2,3,3,5	14.9	1200
1/24/2028	2,2,3,5	14.7	1300
9/20/2029	2,3,5	14.3	1000
2/12/2031	4,5	14.1	1300

**Table 7. PEATSA-STOUR comparison of trajectories to Uranus**

Launch Year	Flyby Sequence		Flight Time, years		Final Mass, kg	
	PEATSA	STOUR	PEATSA	STOUR	PEATSA	STOUR
2024	4,2,3	2,3,3	12.0	13.3	2100	2100
2025	2,3,3	2,3,3,3	12.0	14.4	2100	2000
2026	2,3,3	2,3,3	13.0	12.7	2700	1900
2027	2,3,3	2,2,3,3,5	13.0	13.9	2100	2100
2028	2,3,3	2,3,5	13.0	13.5	3100	2100
2029	2,3,3,5	2,3,3,5	11.9	11.7	2500	2000
2030	2,3,5	2,2,3,5	12.0	12.1	2500	2100
2031	2,3,3,5	2,3,5	12.0	10.9	3400	2100
2032	2,3,5	2,2,5	12.0	13.2	2700	2300
2033	2,3,3,4	2,3,3	13.0	13.4	3400	2000
2034	2,3,3,4	2,2,3	12.0	13.0	2300	2000
2035	2,3,3	2,3,3	13.0	13.1	2200	1800
2036	2,3,4,3	2,2,3	12.0	13.2	2300	2100
2037	2,3,3	2,3,3	12.0	12.6	2100	2000
2038	2,2,3	2,3,3	12.0	12.8	2100	2100

**Table 8. PEATSA-STOUR comparison of trajectories to Neptune**

Launch Year	Flyby Sequence		Flight Time, years		Final Mass, kg	
	PEATSA	STOUR	PEATSA	STOUR	PEATSA	STOUR
2026	2,3,3,5	2,3,3,5	14.5	14.8	1300	1300
2027	2,3,2,3,5	2,3,3,5	14.5	14.9	1700	1200
2028	2,2,3,5	2,2,3,5	14.5	14.7	1900	1300
2029	3,4,5	2,3,5	14.5	14.3	2000	1000
2030	3,5	NA	14.5	NA	1700	NA
2031	4,5	4,5	14.5	14.1	1200	1300
2037	2,2,3	NA	14.5	NA	1200	NA

grid-search techniques that employ algorithms such as Lambert solvers (such as STOUR) require significantly fewer computations to explore this breadth of the solution space than optimization techniques. The grid-search solutions could then be used as initial guesses for PEATSA to help it reach the global optimum.

That being said, using grid-search tools as an initial guess provides its own set of challenges. One significant challenge arises by the way that “best” cases are typically identified. For a set of design objectives that one may want to optimize for a particular mission, the best cases are often found by identifying the non-dominated (or Pareto) set of solutions. These are simply the solutions that cannot be bested by any other solution in every category. However, since there is no way of knowing a priori how far a solution from a grid search may be from the optimum, there is no guarantee that a Pareto set identified from a grid search will actually produce the Pareto front of optimized solutions, if used as initial guesses for an optimization tool. For example, the comparison of best cases in Tables 7 and 8 show that for many of the launch years, the best cases for PEATSA and STOUR used a different flyby combination. Thus, if we were to take the STOUR solution as an initial guess with a 2,2,3,3,5 flyby sequence (as for the year 2027 to Uranus), the optimizer will not be able to take that trajectory and find PEATSA’s solution with the path 2,3,3. This issue may be best resolved using an approach similar to PEATSA, which may start with the grid-search solutions, but can then iteratively try other cases and propagate the best solutions found to the neighboring cases. Such an approach could provide the best of both tools, since the final set of trajectories would be optimized and have stochastically searched the solution space, and, additionally, we are guaranteed that a large breadth of the solution space has been searched since the optimized results would have been informed by the seeds provided by the grid-search.

## CONCLUSIONS

The iterative, stochastic, optimization techniques implemented in PEATSA were successful in constructing a catalog of attractive trajectories to Uranus and Neptune, with very little involvement by a mission designer after the initial setup.

For most of the launch years considered in this study, the results found by PEATSA outperformed the grid-search solutions found by Hughes<sup>2</sup> using STOUR. However, not all solutions found by PEATSA were better than the grid-search solutions, which suggests that a hybrid approach would be the most beneficial. Such an approach would use the grid-search solutions as an initial guess for a tool such as PEATSA, which should reduce the number of iterations required for PEATSA to approach the global optimal solution set; and, additionally, provide a guarantee that the large breadth of the solution space spanned by the grid search has been considered in the otherwise stochastic optimization technique used by PEATSA.

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