Color Algebras

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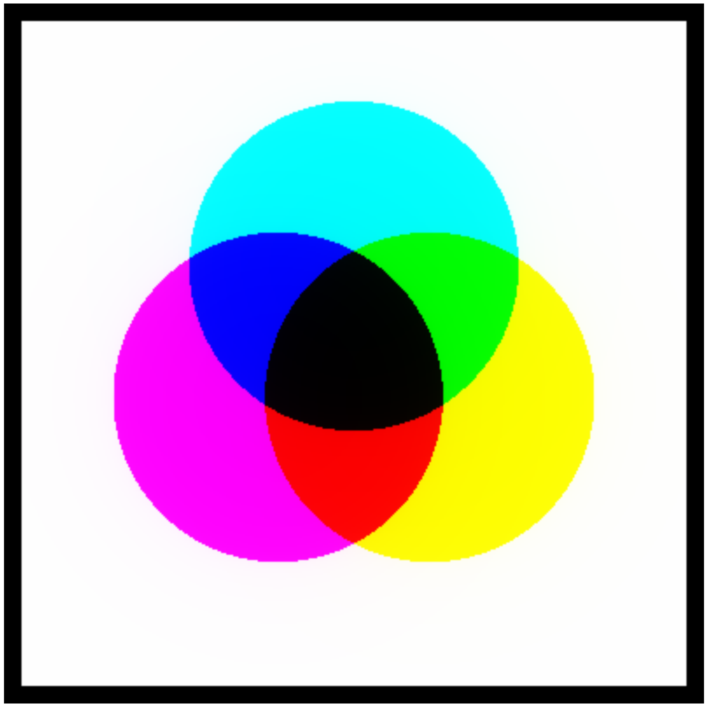
Two kinds of color mixture

Additive



Subtractive (multiplicative)

Color algebra

- Additive color mixture perfectly described

  by vector addition of colors

$$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a},$$

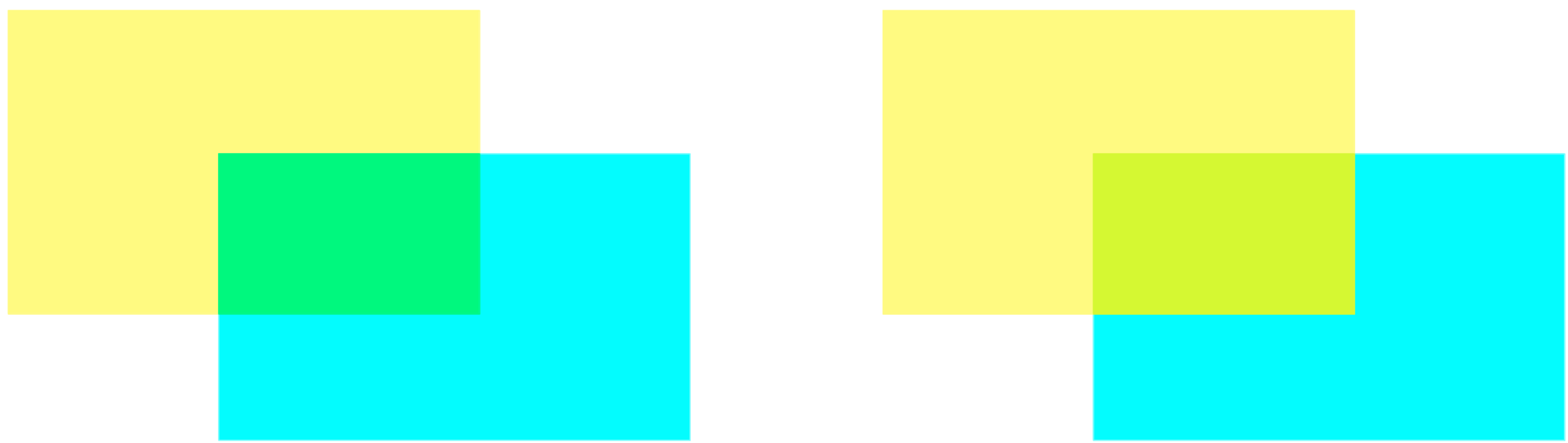
$$(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$$

- Subtractive color mixture NOT well-defined from input colors, so

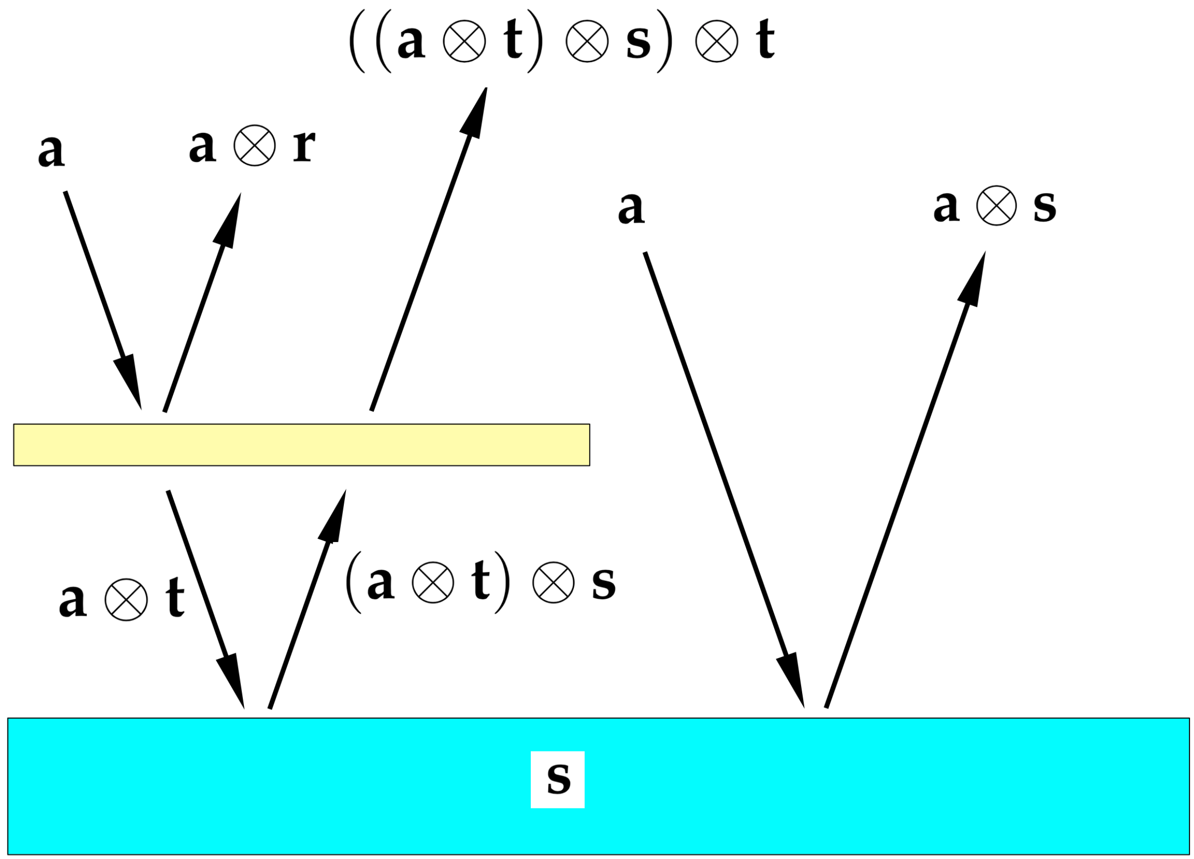
  we must invent the color product operator!

$$\mathbf{a} \otimes \mathbf{b}$$

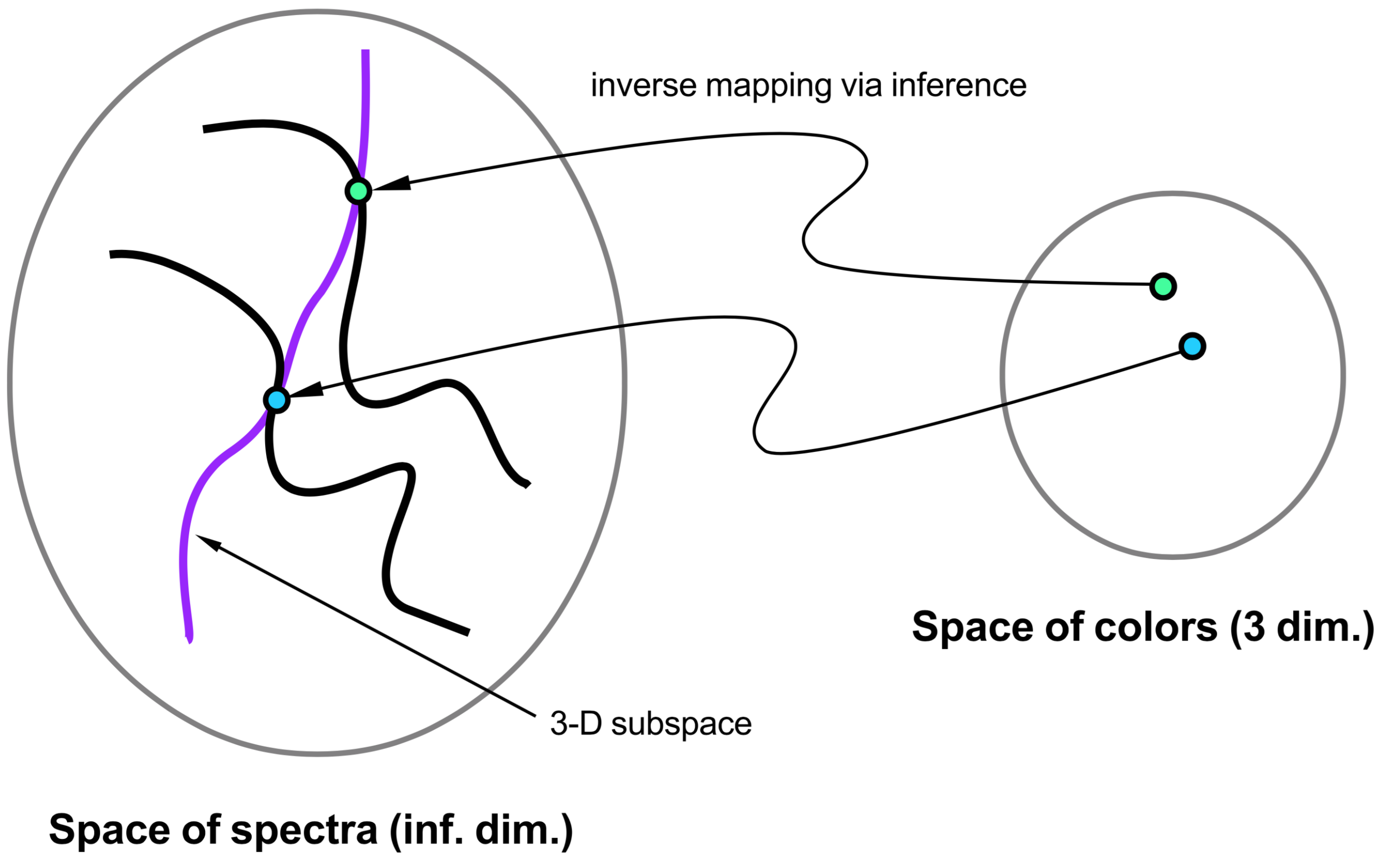
Which looks correct?



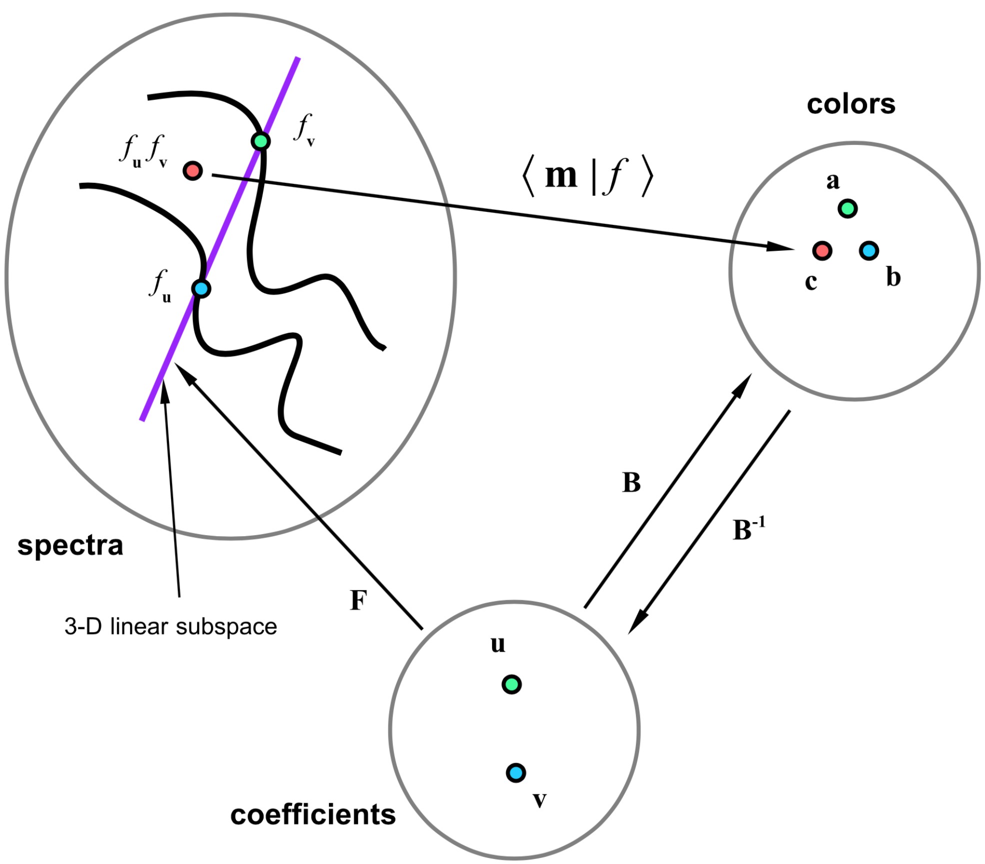
The algebra of transparency



General approach: spectral model

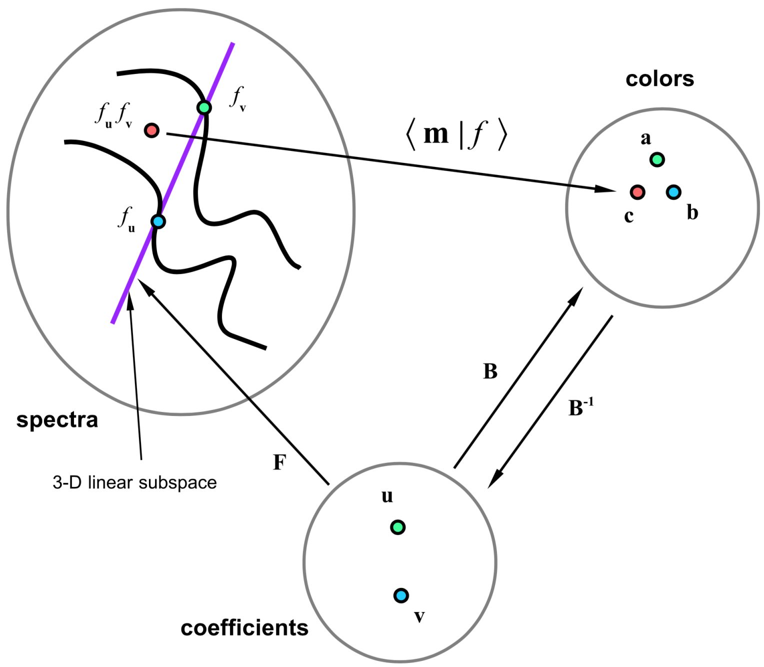


Example: linear spectral models



$$f_{\mathbf{u}} = \sum_{i=1}^{3} u^i f_i(\lambda)$$

Bilinear color product



$$\begin{aligned}\mathbf{a} \otimes \mathbf{b} &= \mathbf{c} \\ &= \langle \mathbf{m} | f_{\mathbf{u}} f_{\mathbf{v}} \rangle \\ &= \mathbf{u}^\top \mathbf{P} \mathbf{v} \\ &= (\mathbf{B}^{-1}\mathbf{a})^\top \mathbf{P} \mathbf{B}^{-1} \mathbf{b} \\ &= \mathbf{a}^\top (\mathbf{B}^{-1})^\top \mathbf{P} \mathbf{B}^{-1} \mathbf{b}\end{aligned}$$

$$p_{ij}^k = \langle m^k | f_{ij} \rangle$$

Bilinear color division

$$\mathbf{c} = \mathbf{a} \otimes \mathbf{b} = \mathbf{T_a b}$$

$$\mathbf{T_a} = \mathbf{a}^\top (\mathbf{B}^{-1})^\top \mathbf{P B}^{-1}$$

$$\mathbf{b} = \mathbf{T_a}^{-1} \mathbf{c}$$

$$\mathbf{b} = \mathbf{c} \oslash \mathbf{a}$$

- division = "discounting the illuminant"

Associativity

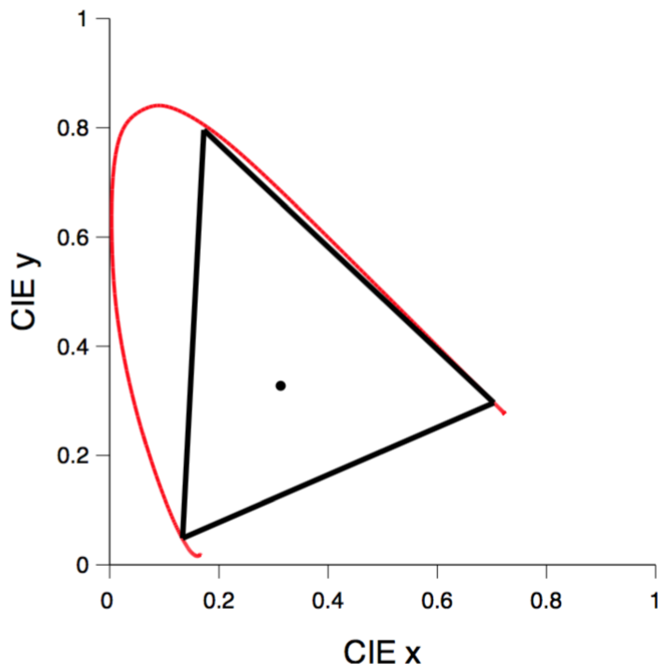
$$(\mathbf{a} \otimes \mathbf{b}) \otimes \mathbf{c} \stackrel{?}{=} \mathbf{a} \otimes (\mathbf{b} \otimes \mathbf{c})$$

- Associative law can fail for the linear model
- Approximation required when product spectra lie outside the model space
- Closure under multiplication guarantees associativity

A problem with the linear model

- Not all combinations are legal spectra
- Spectral values cannot be negative
- Choice of basis functions determines valid gamut

Example: RGB gamut



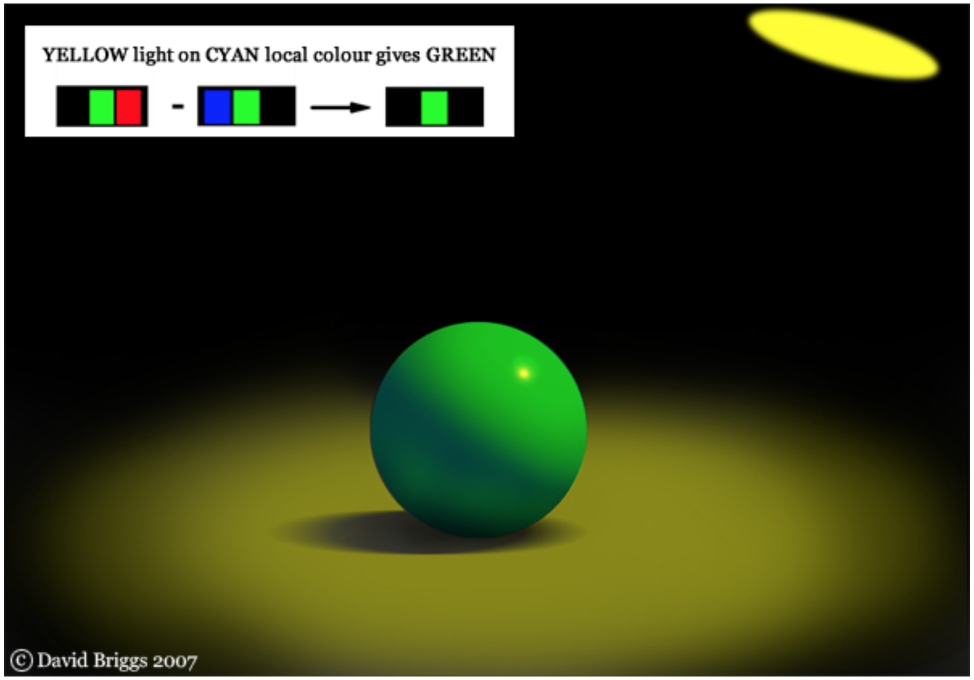
approximate ITU Rec. 2020

$$f_i(\lambda) = \begin{cases} \alpha_i & |\lambda - \lambda_i| \leq \Delta\lambda_i \\ 0 & \text{otherwise} \end{cases}$$

$$f_{ii}(\lambda) = \alpha_i f_i(\lambda),$$
$$f_{ij}(\lambda) = 0 \qquad i \neq j$$

$$\alpha_i = 1$$

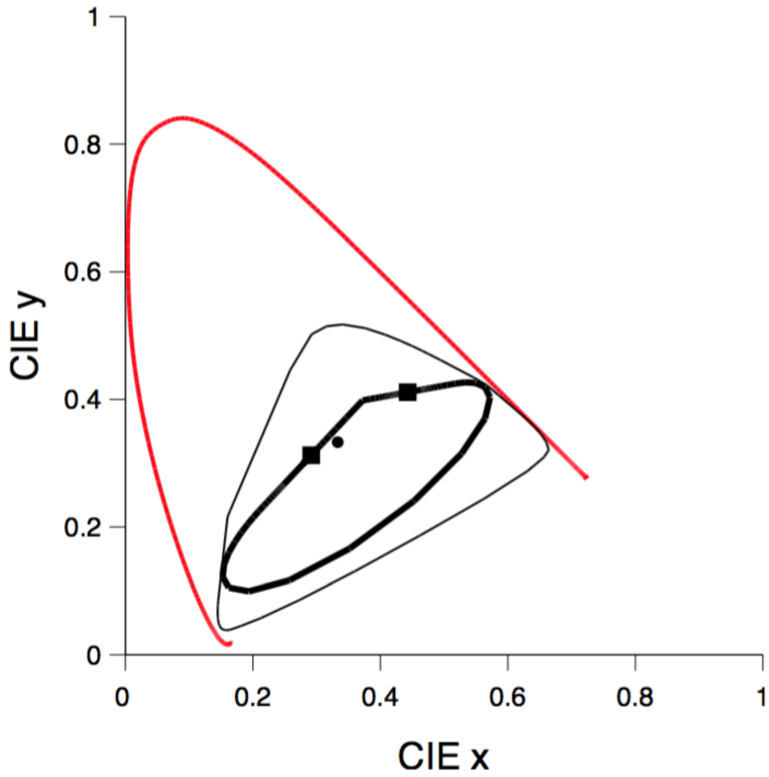
RGB Spectral Model

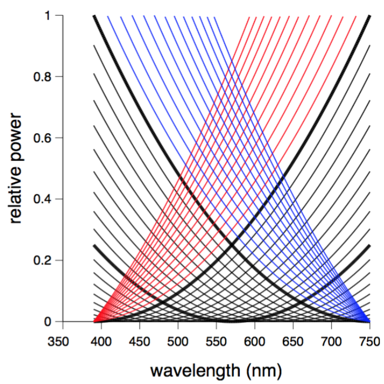


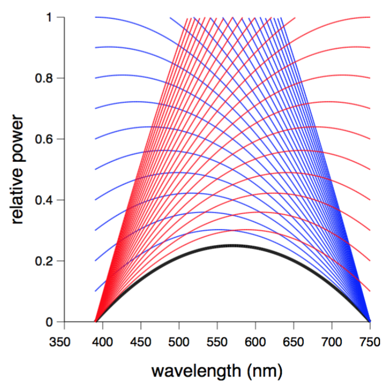
from http://www.huevaluechroma.com/051.php

- OpenGL spec. does not specify!

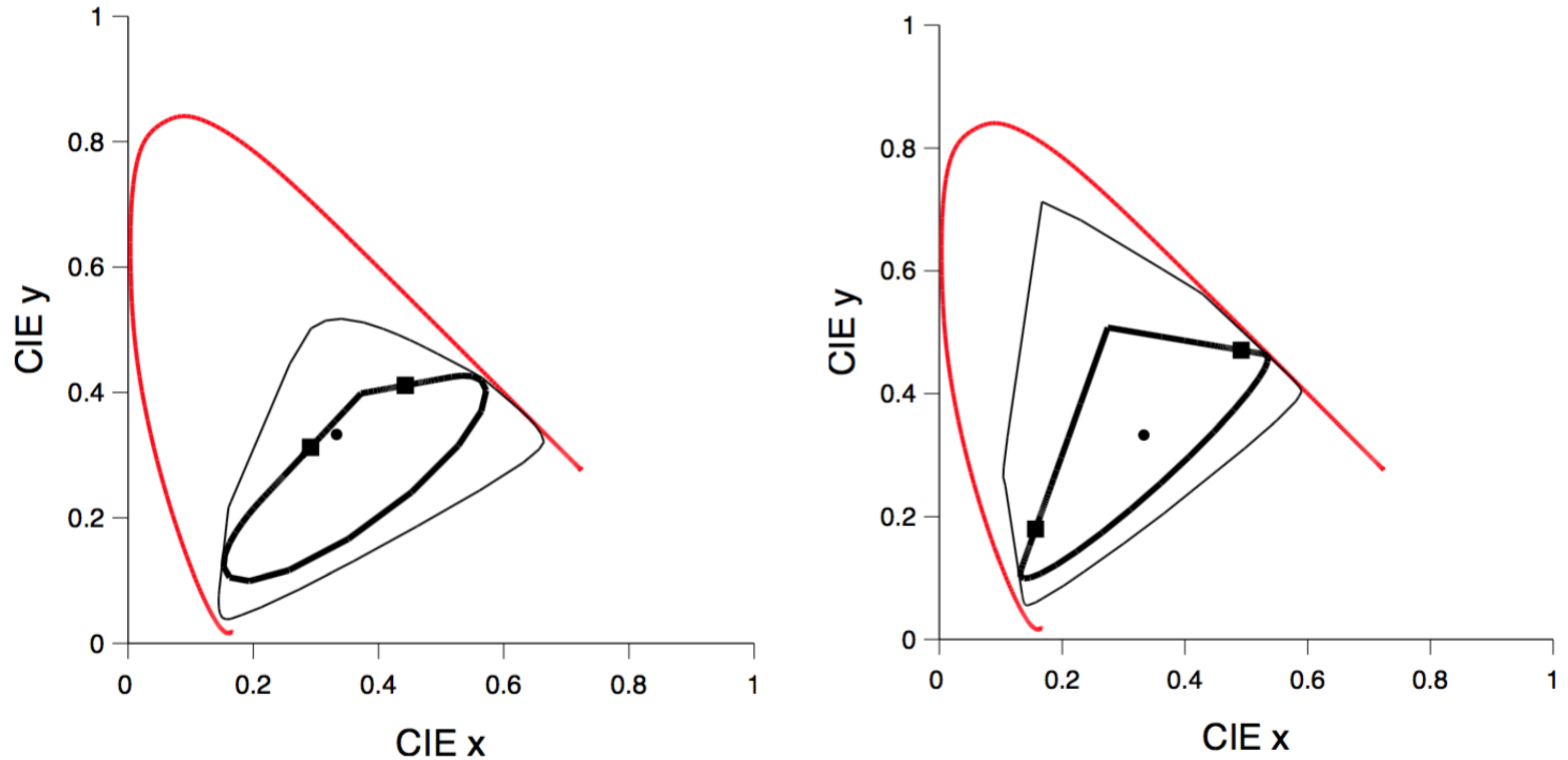
Example: Quadratic gamut



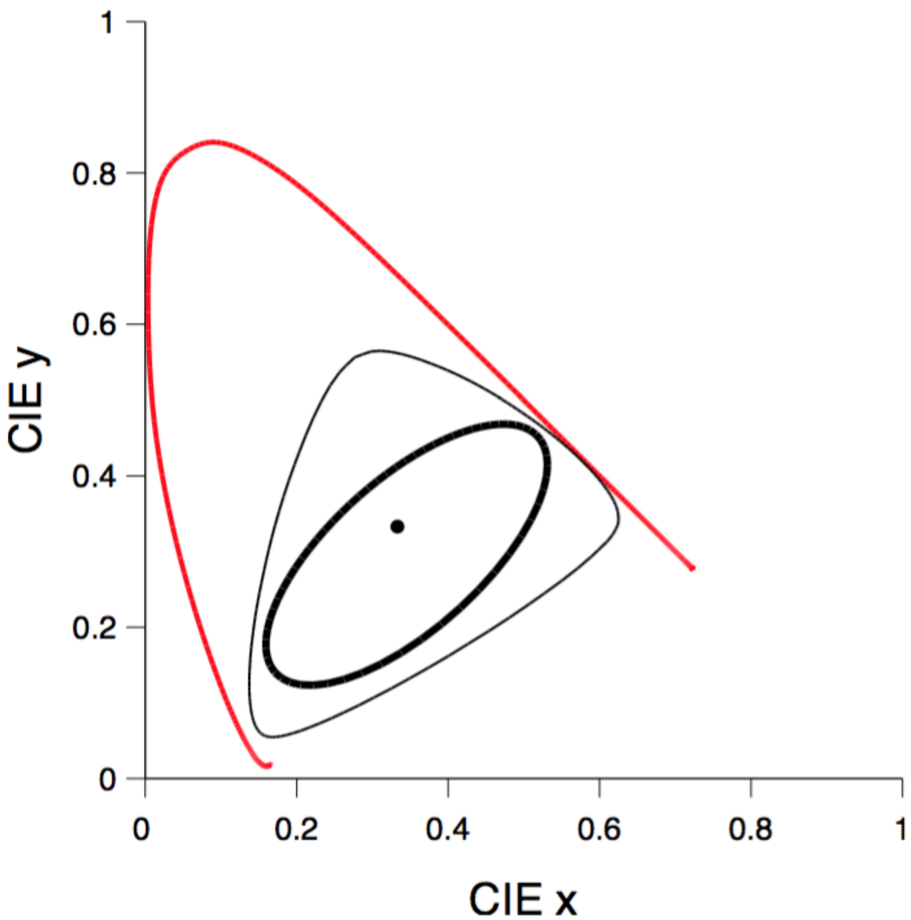




Example: Quadratic gamut



Example: Sinusoidal gamut

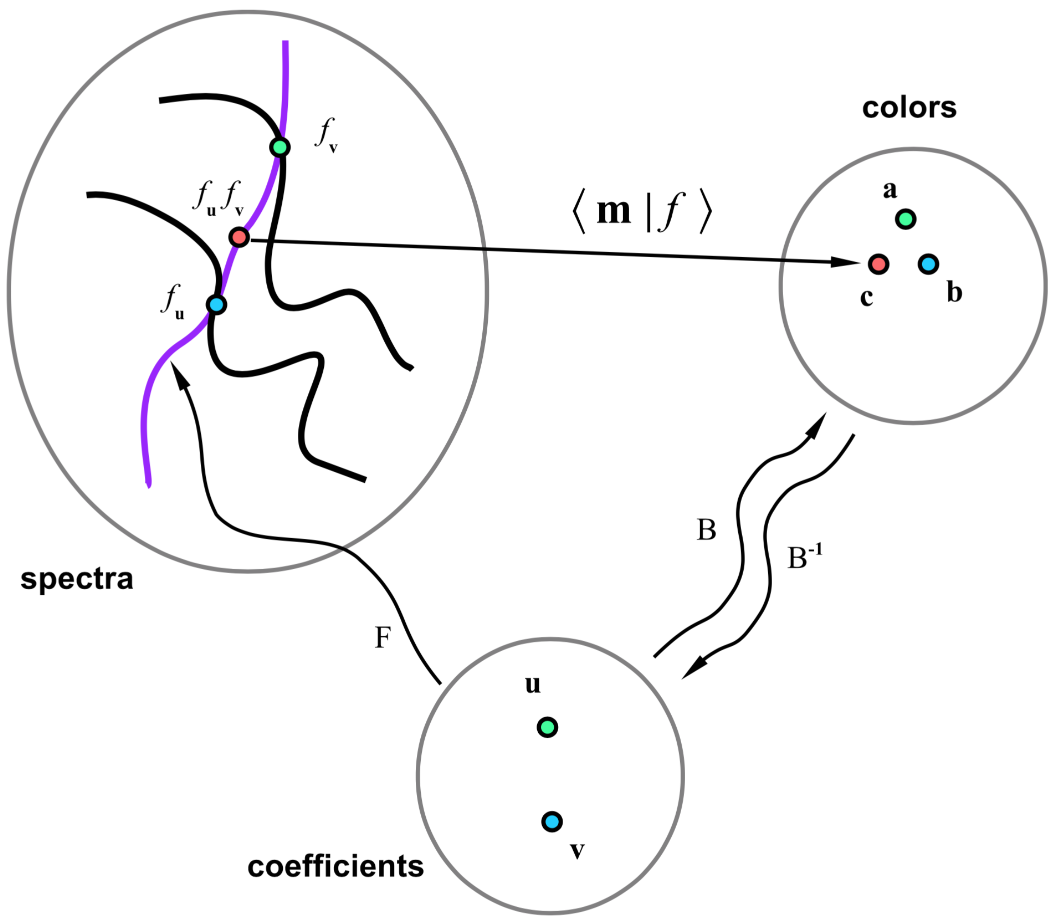


Log-linear spectral models

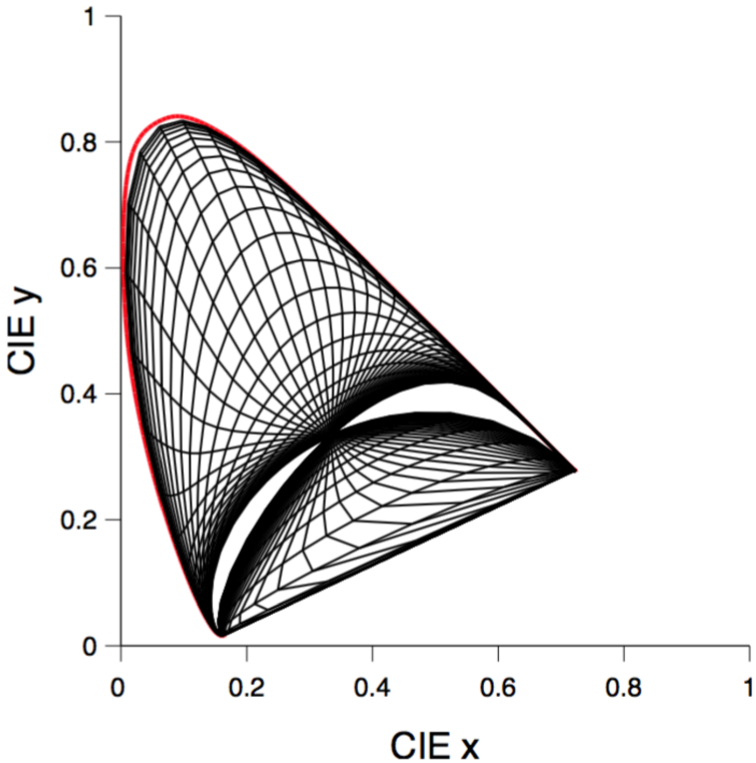
$$\log(f_{\mathbf{u}}) = \sum_{i=1}^{3} u^i f_i \qquad f_{\mathbf{u}} = \prod_{i=1}^{3} e^{u^i f_i}$$

- Suggested by Golz and MacLeod (2002), MacLeod and Golz (2003)
- Linear function space in log energy
- Closed under multiplication - associative law holds
- quadratic → Gaussian (and inverse-Gaussian)
- sinusoidal → Von Mises

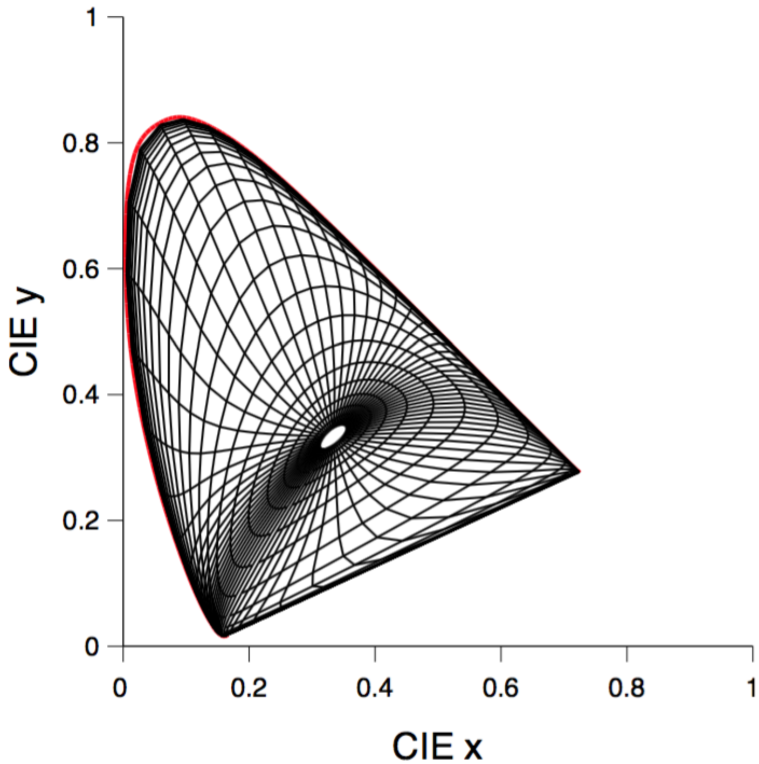
Log-linear spectral models (cont.)



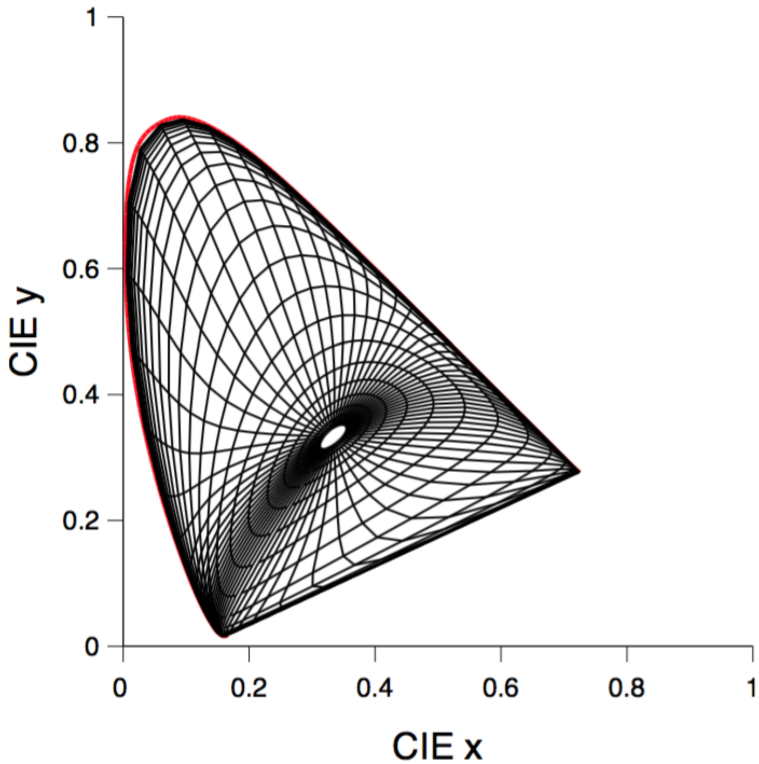
Example: Gaussian gamut

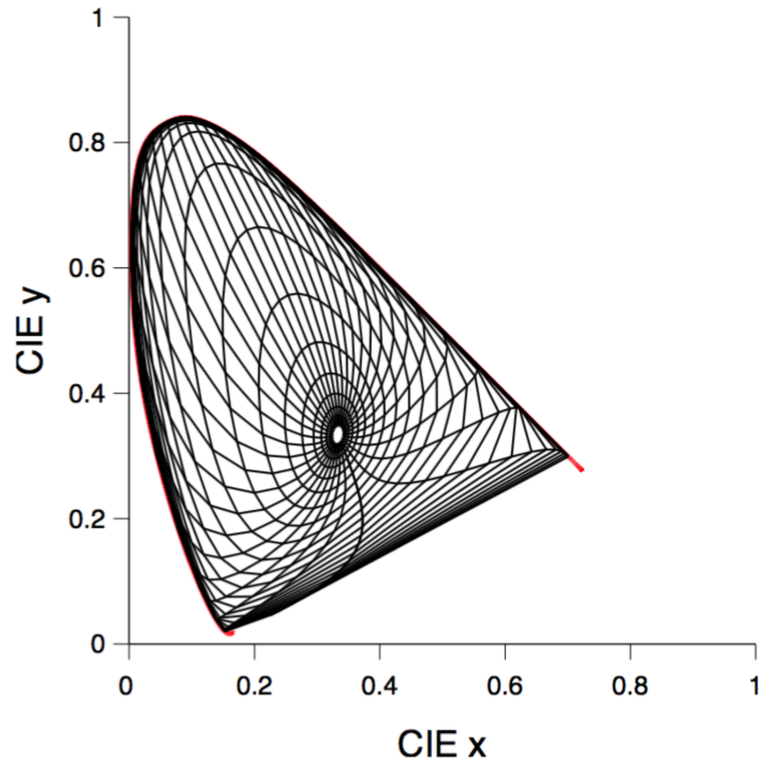


Example: Von Mises gamut



Example: Von Mises gamut

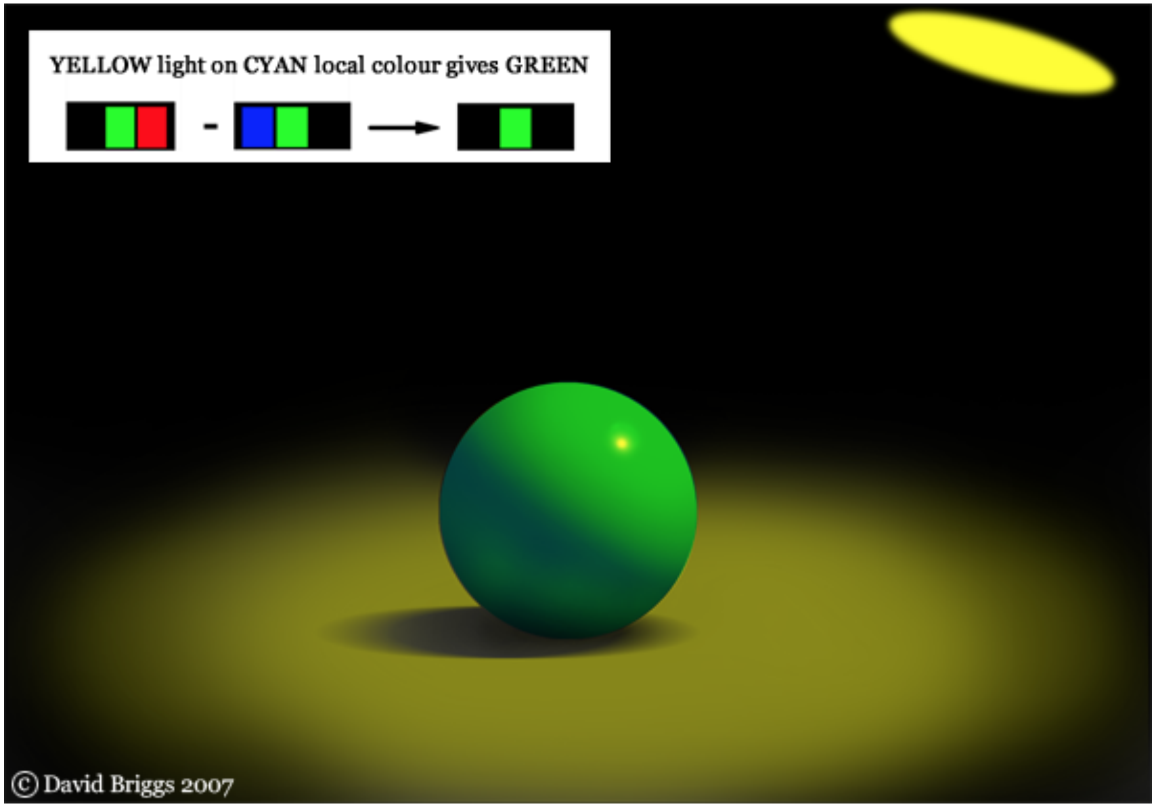




Advantages of the Von Mises model

- More physically plausible than RGB
- Can represent all colors
- Nonlinear wavelength transformation can generate individual differences (Abney effect, unique hue settings)
- Computational issues not solved - neural network?

Rendering reconsidered



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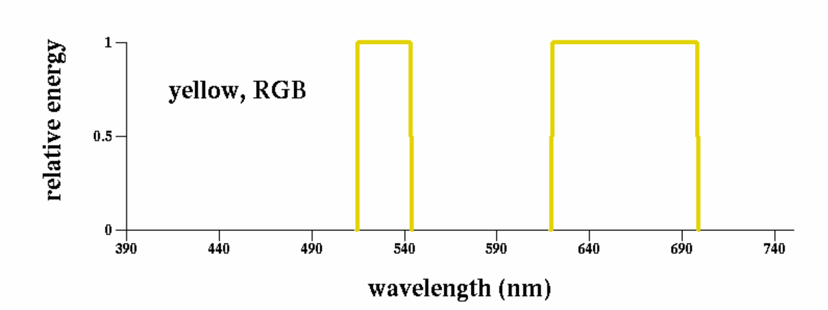
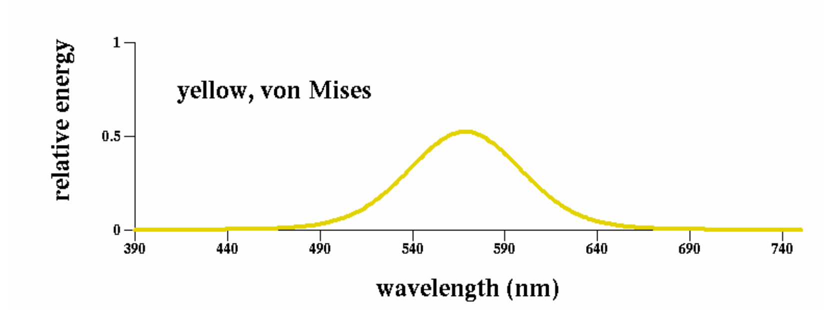
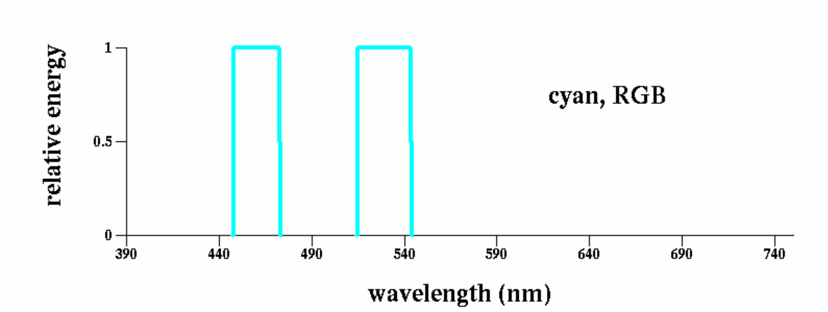
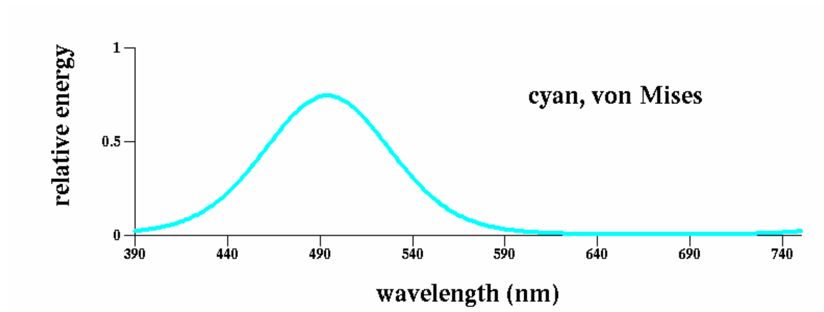
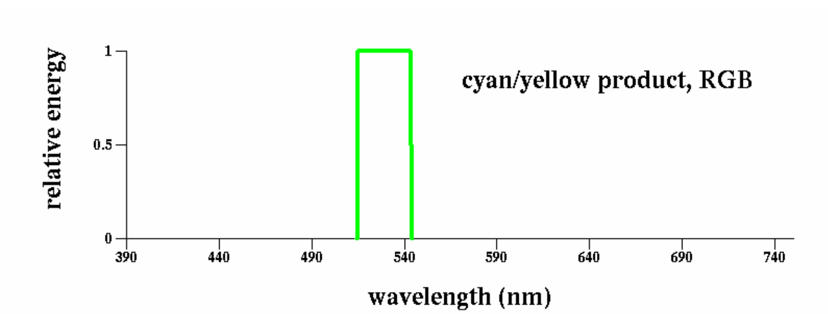
Rendering reconsidered (cont.)

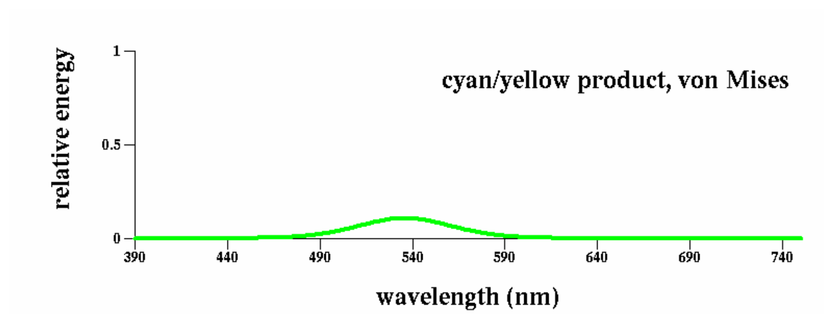




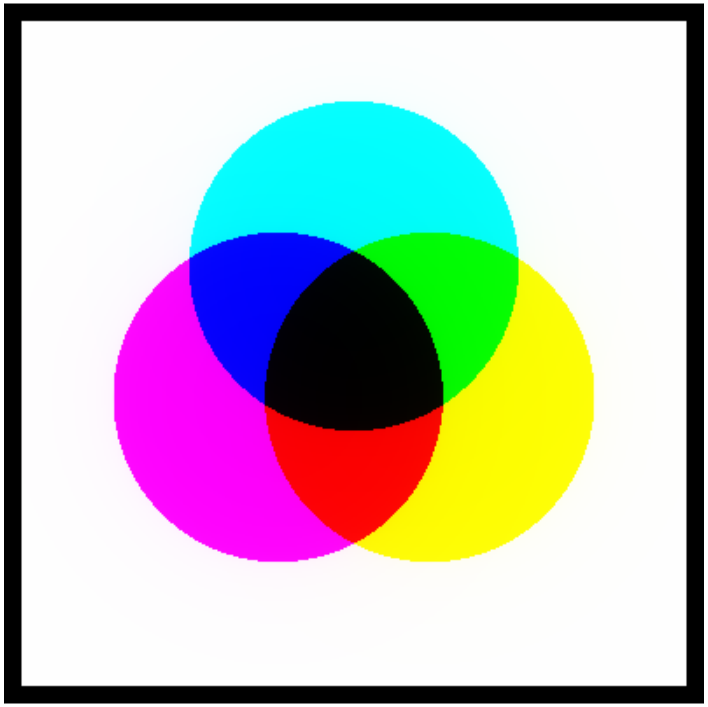




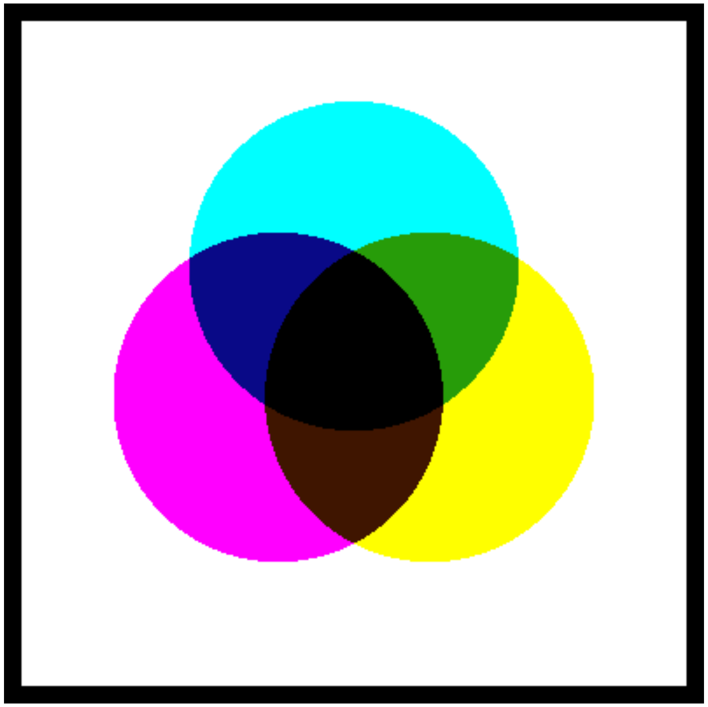




Rendering reconsidered (cont.)



RGB spectral model (standard)



Von Mises spectral model

Summary

- A computational framework for an algebra of colors to predict additive and subtractive color mixture
- Applications to graphics and perception
- Log-linear spectral models provide best performance