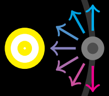
Forbidden mass ranges for shower meteoroids

Althea Moorhead

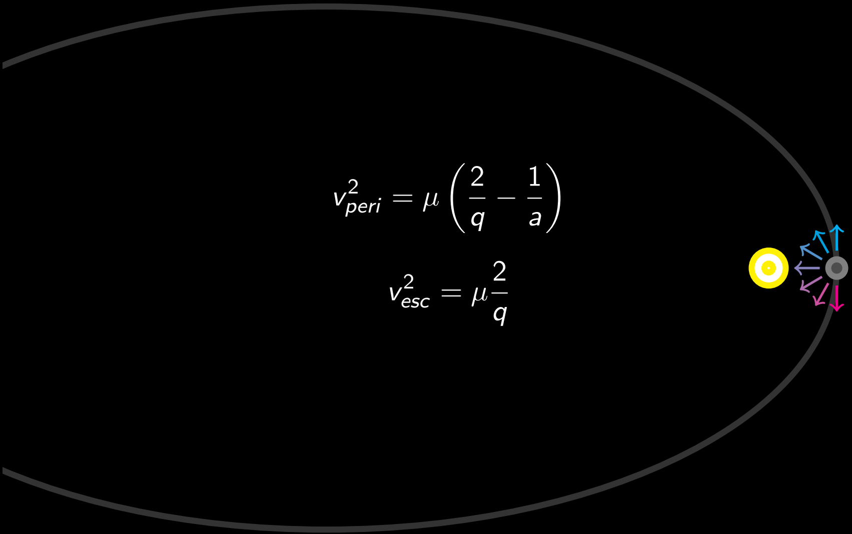
NASA Meteoroid Environment Office, MSFC



49th Annual Division for Planetary Sciences Meeting

October 16, 2017

Eccentric comets approach v_{esc} at perihelion



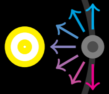
$$v_{peri}^2 = \mu \left(\frac{2}{q} - \frac{1}{a} \right)$$

$$v_{esc}^2 = \mu \frac{2}{q}$$

Small particles are subject to radiation pressure

- Radiation pressure follows inverse square law
- Reduces central potential by β:

$$\beta = \frac{F_r}{F_g}$$



- Effect is inversely proportional to size (and density):

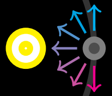
$$\beta \propto 1/s$$

Meteoroids can be ejected directly onto escape trajectories

$$v_{esc}^2 = \mu(1-\beta)\frac{2}{q}$$

- For $\beta \geq 1$, there are no bound orbits
- For $\beta < 1$, v_{esc} is reduced
- Comet's velocity alone exceeds v_{esc} for:

$$\beta > \frac{1-e}{2}$$



Burns, Lamy, & Soter (1979)

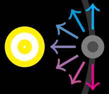
Ejection speed can give meteoroids a boost

- Meteoroids ejected in the direction of the comet's motion get a boost; trailing particles the opposite.
- For large particles:

$$\Delta v = v_0 \sqrt{\beta}$$

(Whipple, 1951; Jones, 1995; etc.)

- The value of β above which particles are unbound has an analytical solution. For leading particles:



$$y = \sin^{-1}\left(\frac{v_{peri}}{\sqrt{v_0^2 + v_{esc}^2}}\right) - \mathrm{atan2}(v_{esc}, v_0)$$

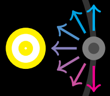
$$\beta_L = \sin^2 y$$

- A similar equation exists for trailing particles

Calculating β

- The only thing left to do is calculate β:

$$\beta = 5.7 \times 10^{-4}\ \mathrm{kg\ m^{-2}} \times (Q_{pr}/\rho s)$$



- Geometric optics: $Q_{pr} = 1$
- But there are some complications ...

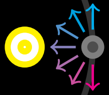
What about small particles?

- For small particles:

$$\Delta v \not\propto \sqrt{\beta}$$

- Instead, we must numerically integrate (see Jones, 1995):

$$\frac{d^2x}{dt^2} = \frac{A\Gamma}{2} m^{-1/3} \rho_d^{-2/3} \rho_{gas}(x) \left[v_{gas}(x) - \frac{dx}{dt} \right]^2$$



- Then:

$$\Delta v = \left.\frac{dx}{dt}\right|_{t \to \infty}$$

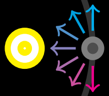
What about small particles?

- Δv has no analytic form, but is very close to:

$$\Delta v \simeq v_{gas,0} \left(0.38532 + 0.50341 \cdot \xi^{-1.054}\right)^{-0.949}$$

$$\xi = \frac{A\Gamma}{2} m^{-1/3} \rho_d^{-2/3} \rho_{gas,0}\; x_c$$

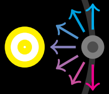
  Ugly, but easy to code up.



- Calculating β is another matter.

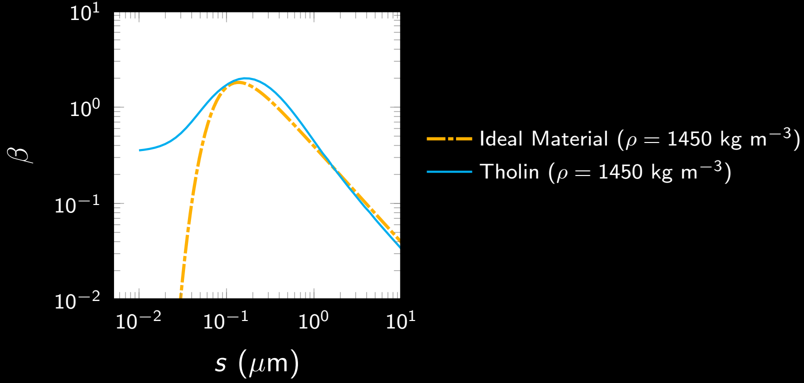
Calculating β for small particles and real materials

$$\beta = 5.7 \times 10^{-4} \text{ kg m}^{-2} \times (Q_{pr}/\rho s)$$



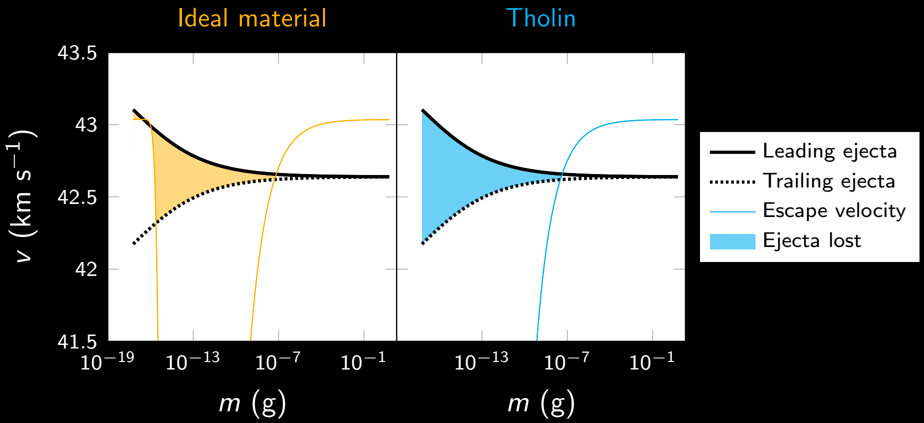
- Geometric optics: $Q_{pr} = 1$
- "Ideal material": $Q_{pr} = 1$ for $\lambda < 2\pi s$, 0 otherwise
- Real materials: Calculate Q_{pr} using Mie theory (Python code available from Navarro & Werts, 2012)

Calculating β for real materials



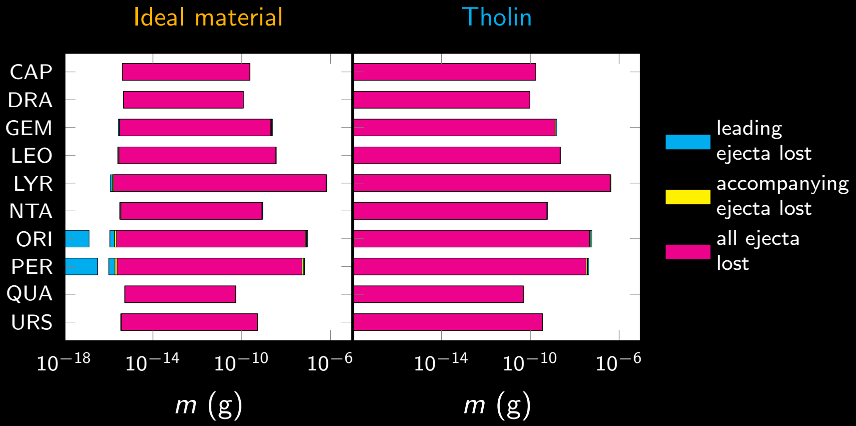
- I'll compare the "ideal material" case with one real material
- Tholins are a reddish brown polymer found on icy bodies

Perseids



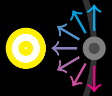
Ideal material
Tholin
43.5
43
42.5
42
41.5
v (km s^{-1})
10^{-19}
10^{-13}
10^{-7}
10^{-1}
m (g)
Leading ejecta
Trailing ejecta
Escape velocity
Ejecta lost

Forbidden mass ranges for 10 major showers



Summary

- Small meteoroids originating from eccentric comets may be on unbound orbits. We've extended this to handle the ejection velocity imparted by the sublimation process:
  - Analytic solution for β limit for large particles
  - Semi-numerical solution for Δv (and thus β limit) for all particles
  - New Δv equation also useful for stream modeling



- We've calculated β for small particles/real materials.
  - Ideal material: very small particles may remain in stream
  - Tholins: small particles do not remain in stream
- Large comets: some small particles can still be ejected
- Eccentric comets: excluded range can be large: no Lyrids smaller than 4×10^{-7} g