National Aeronautics and Space Administration

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SPACE LAUNCH SYSTEM

Practical Methodology for the Inclusion of Nonlinear Slosh Damping in the Stability Analysis of Liquid Propelled Launch Vehicles

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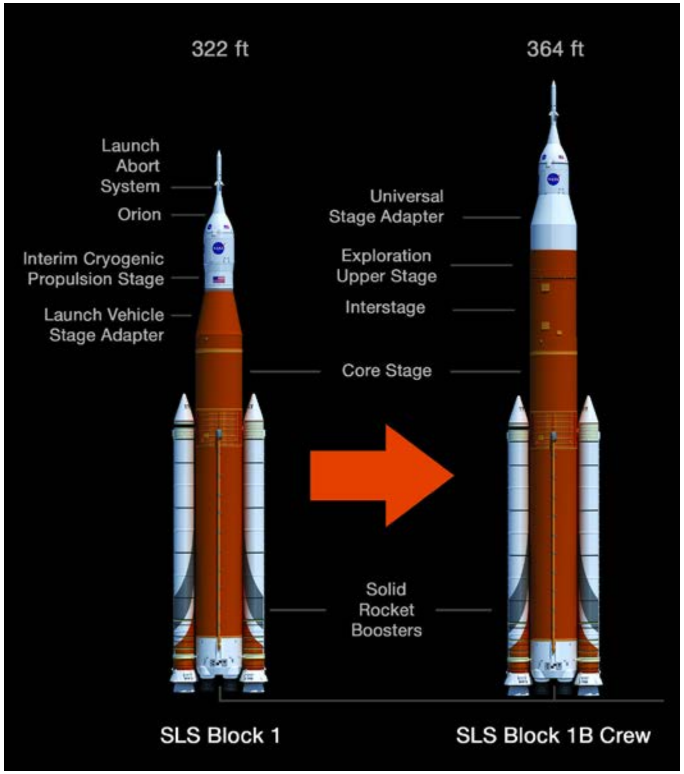
Introduction

- **Work performed as part of the development of the Space Launch System Block 1 and Block 1B configurations.**

- **Presenting the following:**
  - What is slosh and why is it a challenge to attitude control of liquid powered spacecraft and launch vehicles
  - A new derivation for slosh locations which are destabilizing
  - Describing function analysis of non-linear slosh damping
  - Method for predicting of amplitude of limit cycle of slosh-control interaction during degraded margin conditions



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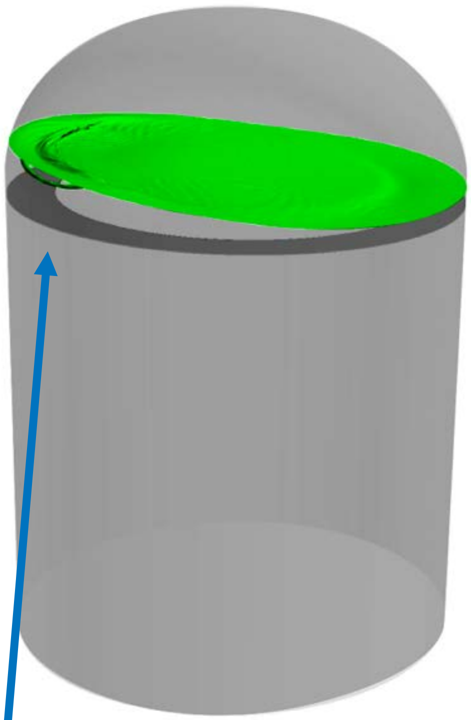
Liquid Slosh Introduction

- **In the context of launch vehicle/spacecraft attitude control, the primary concern is the lateral modes of the fluid dynamics inside the propellant tanks.**
- **Slosh dynamics are modeled for flight mechanics or flight control analysis with mechanical analogue models of either the pendulum model or the spring-mass model.**
  - These models are approximation of the non-linear fluid dynamics, with the rule of thumb that they are valid up to displacement amplitudes of 10% - 15% of the tank radius.
- **In presence of baffles, the effective damping coefficient is not constant.**
  - Increases as a function of slosh amplitude.
  - However, common engineering practice it to select a single wave height (often from heritage) and use the associated effective linear damping for all slosh flight mechanics analysis.



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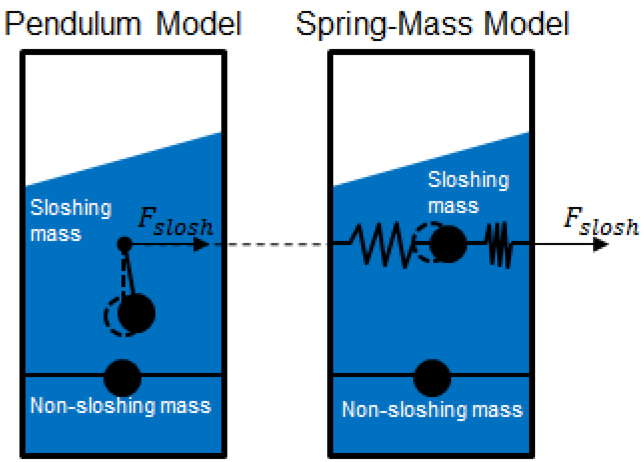
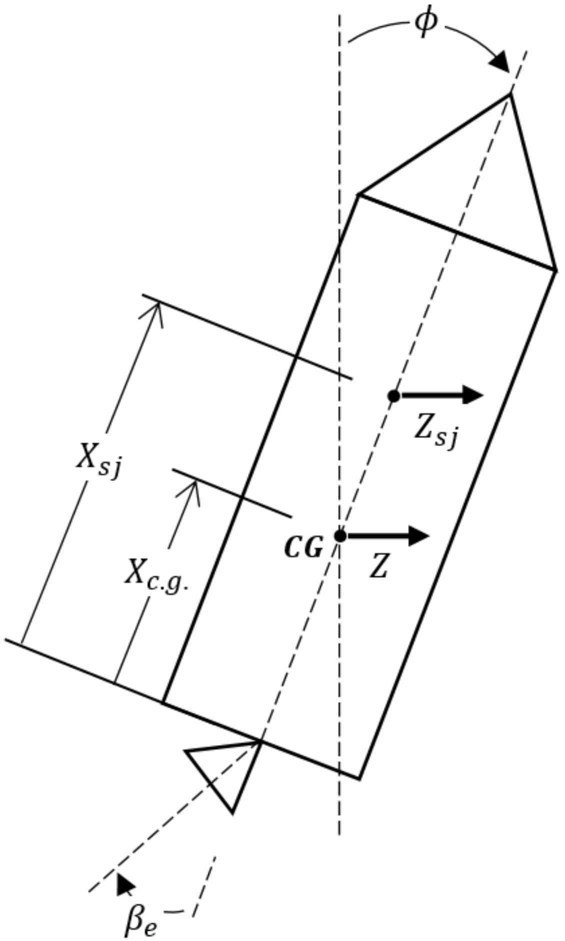


Devices such as baffles are added to tanks to increase the energy dissipation (damping) of the fluid, but due to weight penalty, they are often a source of conflict during the design phase of the vehicle.

Equation of Motion

- **The equations of motions are from Frosch and Vallely[7], and describe a powered vehicle's dynamics linearized about a gravity turn trajectory.**



Z	displacement of vehicle c.g. normal (m)
Z_{zj}	sloshing fluid displacement in jth tank (m)
β_E	engine angle (rad)
ϕ	angle of vehicle centerline (rad)
a_0	attitude control gain (-)
a_1	attitude-rate control gain (-)
c_2	$R'X_{c.g.}/I$ $(1/s^2)$
D	drag force (N)
F	total engine thrust (N)
I	pitch-yaw vehicle moment of inertia with engines and sloshing fluid (kg-m2)
k_3	F/M (m/rad-s2)
k_4	R'/M (m/rad-s2)
l_{sj}	c.g.-to-slosh mass distance $= X_{c.g.} - X_{sj}$ (m)
$l_{c.p.}$	center of percussion $= I/(MX_{c.g.})$ (m)
M	vehicle mass with engines and sloshing fluid (kg)
m_{sj}	slosh mass, jth tank (kg)
R'	vectored engine thrust (N)
$X_{c.a.}$	center of gravity measured from gimbal (m)
X_{sj}	slosh mass location measured from gimbal (m)
ξ_{sj}	slosh equivalent linear damping, jth tank (-)
ω_{sj}	slosh natural frequency, jth tank (rad/s)
α_{sj}	slosh damping slope, jth tank (1/m)
s	complex Laplace variable

Equations of Motions (2)

- **The dynamics are reduced to coupled rotational, translation, and slosh dynamics.**

$$\phi s^2 = -c_2\beta_e - \frac{1}{I}\sum_{j=1}^{n} m_{sj}(l_{sj}s^2 + k_3)Z_{sj}$$ Rotational Dynamics

$$Zs^2 = k_4\beta + k_3\phi - \frac{1}{M}\sum_{j=1}^{n} m_{sj}Z_{sj}s^2$$ Translational Dynamics

$$(s^2 + 2\xi_{sj}\omega_{sj} + \omega_{sj}^2)Z_{sj} = -Zs^2 + (l_{sj}s^2 + k_3)\phi$$ Slosh Dynamics (spring-mass-damper)

- **The dynamics can be converted to state space form for easier analysis (here assuming a single slosh mass, n=1).**

$$x = \begin{bmatrix} z \\ \phi \\ z_{s1} \\ \dot{z} \\ \dot{\phi} \\ \dot{z}_{s1} \end{bmatrix}$$

$$u = \beta_e$$

$$E\dot{x} = A'x + B'u$$

$$\dot{x} = (E^{-1}A')x + (E^{-1}B')u$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \frac{m_{s1}}{M} \\ 0 & 0 & 0 & 0 & 1 & -\frac{m_{s1}l_{sj}}{I} \\ 0 & 0 & 0 & 1 & -ls1 & 1 \end{bmatrix}$$

$$4' = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & k_3 & 0 & 1 & 0 & 0 \\ 0 & 0 & m_{s1}k_3/I & 0 & 0 & 0 \\ 0 & k_3 & \omega_{s1}^2 & 0 & 0 & -2\xi_{s1}\omega_{s1} \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_4 \\ -C_2 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Example System Parameters

- **An example configuration was created in order to illustrate results.**
- **Configuration represents a large upper stage with a diameter on the order of 10 m**
  - Somewhat larger than SLS Exploration Upper Stage or the Saturn S-IVB, but demonstrates the same issues faced on real-world stages.
- **Control gains result in bandwidth similar to previously flown upper stages.**

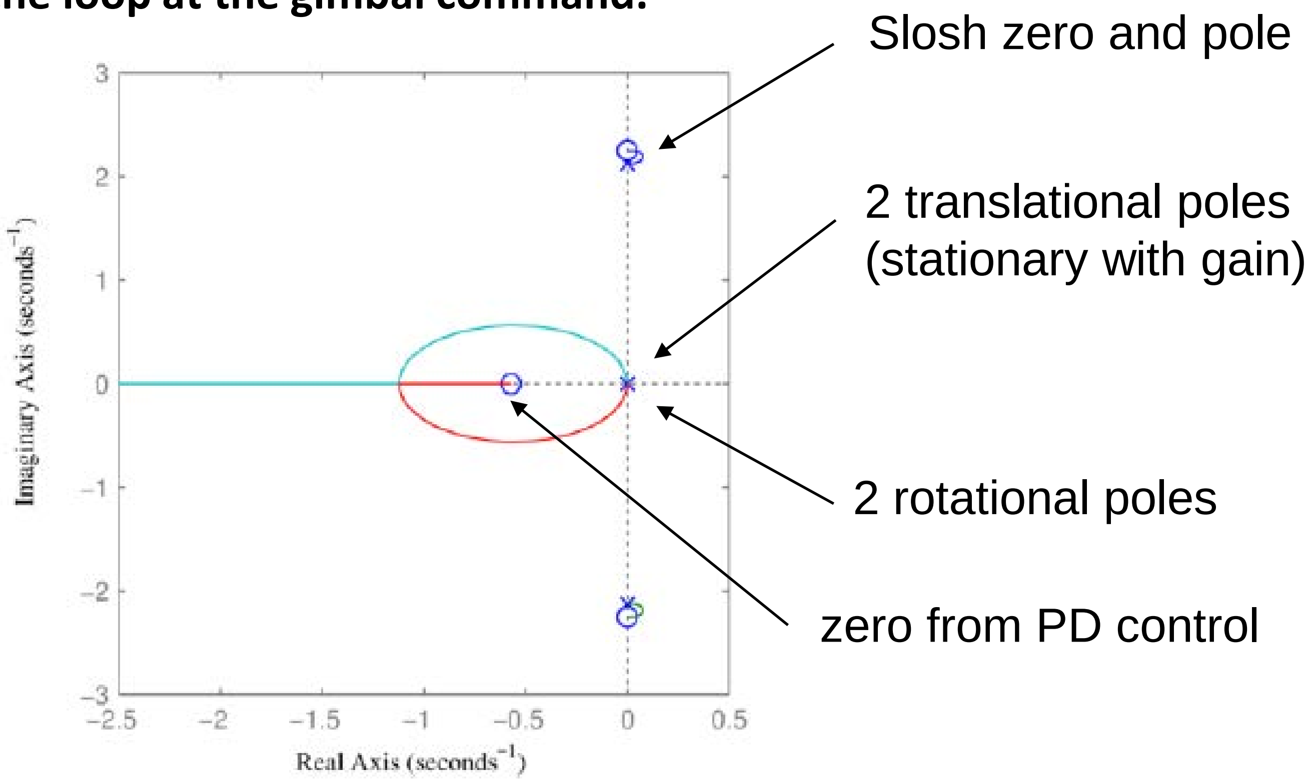
Name	Value	Units
$F(=R')$	$9x10^5$	N
M	$1.6x10^4$	kg
I	$9x10^5$	kg-m^2
$X_{c.g.}$	6	m
X_{s1}	9	m
m_{s1}	$1x10^3$	kg
ω_{s1}	2	rad/s
α_{s1}	0.1	1/m
a_0	0.08	rad/rad
a_1	0.14	rad/(rad/s)

Why Can Slosh be Destabilizing?

- **With PD attitude control,**

$$\beta_e = -a_0\phi - a_1\dot{\phi}$$

- **For zero slosh damping, the following open-loop root locus will result when breaking the loop at the gimbal command.**



- **From the root locus angle of departure rule, if the slosh zero is above the slosh pole, then the angle of departure is towards the right-half plane (unstable) and intrinsic slosh damping is needed to stabilize the system.**

Where is slosh destabilizing?

- **Looking at the gimbal angle to vehicle attitude angle transfer function,**

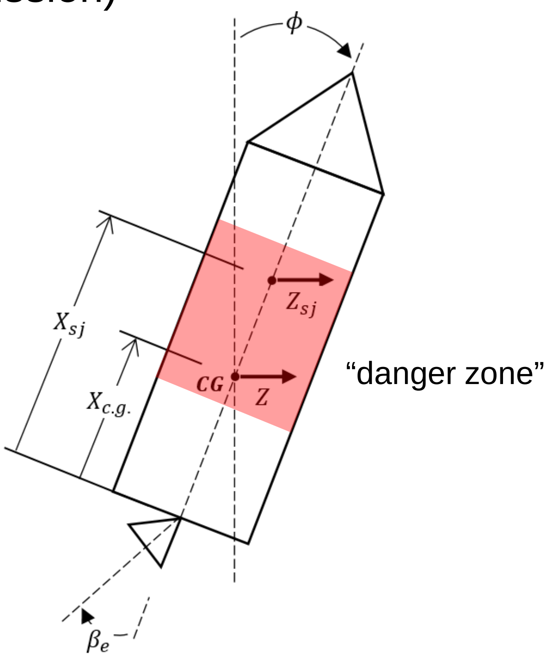
$$\frac{\phi(s)}{\beta_e(s)} = \frac{(c_2 - (c_2 m_{s1})/M + (k_4 l_{s1} m_{s1})/I)s^2 + c_2\omega^2 + (k_3 k_4 m_{s1})/I}{(m_{s1}/M + (l_{s1}^2 m_{s1})/I - 1)s^4 + ((k_3 l_{s1} * m_{s1})/I - \omega^2)s^2}$$

- **Can determine at what slosh mass location the slosh zero is above the slosh pole by assuming k3=k4 (all thrust is gimballed, drag is negligible).**

$$l_{s1} > \frac{-I}{M x_{c.g.}}$$ (Center of Percussion)

$$l_{s1} < \frac{F(M - m_{s1})}{M^2 \omega_{s1}^2}$$

l_{sj} c.g.-to-slosh mass distance $= X_{c.g.} - X_{sj}$ (m)

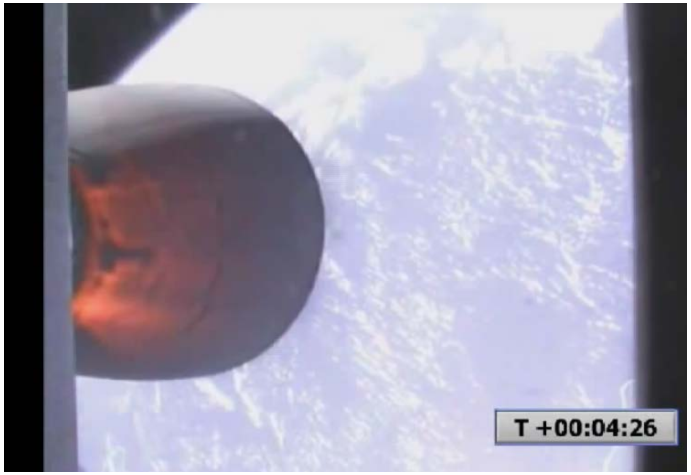


- **This is the so called "danger zone" for slosh described by Bauer[8] and Greensite[9].**
- **This zone extends from the "center of percussion" ahead of the CG to a point behind the CG.**

Traditional Slosh-Control Instability Mitigation

- **Lack of sufficient slosh damping can result in unacceptably large motion.**
  - See Falcon 1 Flight 2.
- **The general spacecraft/launch vehicle flight control design guideline is to achieve a 6dB/30deg of gain and phase margin (Dennehy[13]).**
  - However, due to mass penalty of slosh baffles required to achieve full margins, many stages (including manned flights) have gone below these values.
- **Justification of reduced/negative margins is often time domain analysis which showed period of instability could be "flown through."**
  - **Does not capture margins or frequency domain sensitivity.**
- **Margin can be quantified by taking into account the slosh damping to slosh amplitude relationship.**

SpaceX Falcon 1 Flight 2



https://www.youtube.com/watch?v=YMvQsmLv44o#t=16m15s

Non-linear Damping Describing Function

- **Real world fluid non-linear damping is complex and often does not fit a simple mathematical description.**
- **Assume non-linear induced acceleration due to damping is odd square law of slosh velocity (like drag)**
  - Convenient approximation of complex real world behavior

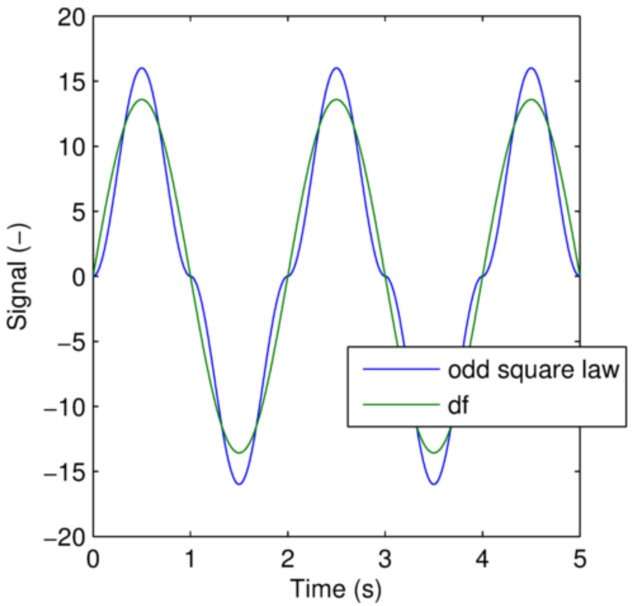
$$Acc_{d,nonlinear} = -b_{sj}\dot{z}_{sj}|\dot{z}_{sj}|$$

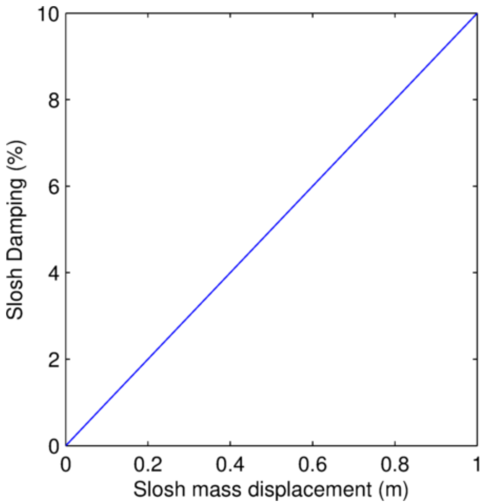
- **Known from Gelb[15],**

Function	*With Input*	*Describing Function*
$y = x\lvert x\rvert$	$x = A_x sin(\Omega t)$	$y = x\frac{8}{3\pi}A_x$

- **For the odd square law for damping acceleration, the effective linear damping coefficient for a spring-mass-damper model can be shown to be:**

$$\xi_{sj} = \left(\frac{b_{sj}}{2}\frac{8}{3\pi}\right)A_{z_{sj}}$$





Aside: Non-linear Damping in Traditional 6DOF Simulation

- **Traditional flight mechanics non-linear 6DOF simulations are often already configured for the slosh pendulum or spring-mass-damper analogue using a damping coefficient.**

- **If damping data exists as a function of slosh mass amplitude:**

$$\xi_{sj} = f_{\xi_{sj}}(A_{z_{sj}})$$

- **Then using the describing function method, the equivalent time varying damping coefficient to use can be found as:**

$$\xi_{sj} = \frac{3\pi}{8} f_{\xi_{sj}} \left(\frac{\dot{z}_{sj}}{\omega_{sj}} \right)$$

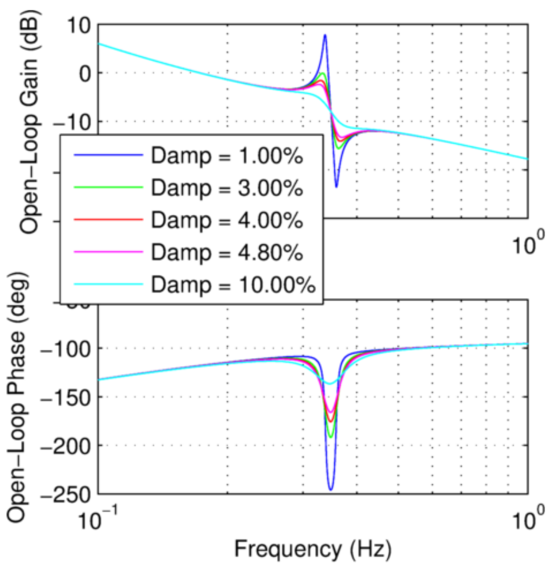
- **In practice, for non-linear damping following the odd square law, this method provided a perfect match in time domain results.**
  - For other types of damping (e.g., where the damping coefficient is a function of the square root of the slosh amplitude as seen in Miles[4] equation predicted damping) this method is still a good approximation as long as the damping curve can be locally approximated as linear.

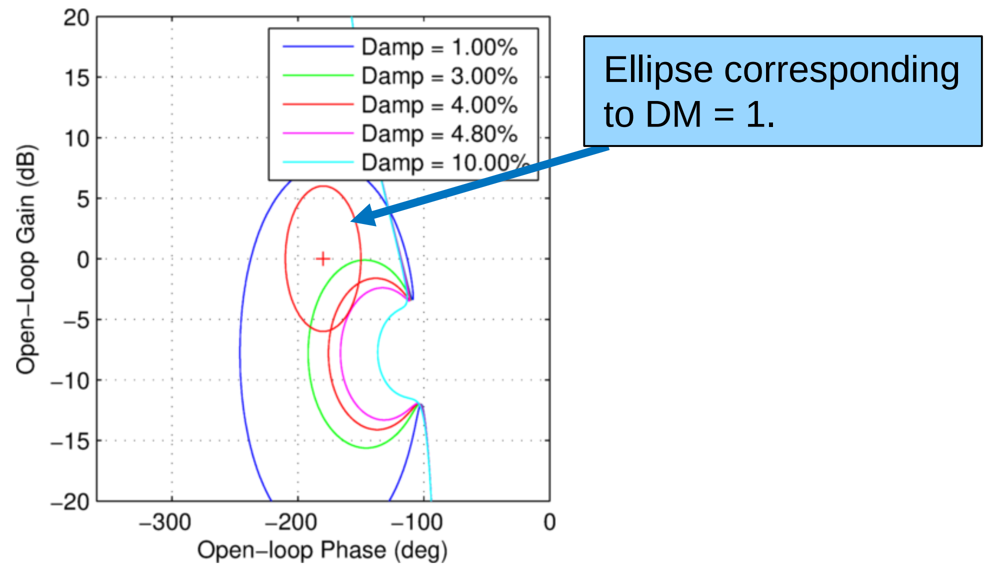
Impact of Slosh Damping on Stability Margins

- **Using disc margin which is the composite of phase and gain margin, normalized to the desired gain/phase margin of 6db/30deg,**

$$DM = \sqrt{\left(\frac{20log_{10}(|G(\omega)|)}{GMd}\right)^2 + \left(\frac{(\angle G(\omega) mod 2\pi) - \pi}{PMd}\right)^2}$$

- **Control margins exhibit strong sensitivity to slosh damping for unfavorably phased modes.**
  - Example system with 10% damping per meter of slosh mass lateral displacement.





- **Note that system is unstable at small slosh mass displacement and stable with margin at 0.48 m.**
  - Results in limit cycle existing at the amplitude corresponding to neutral stability.
- **These types of slosh limit cycles oscillations have been observed in flight in previous systems**

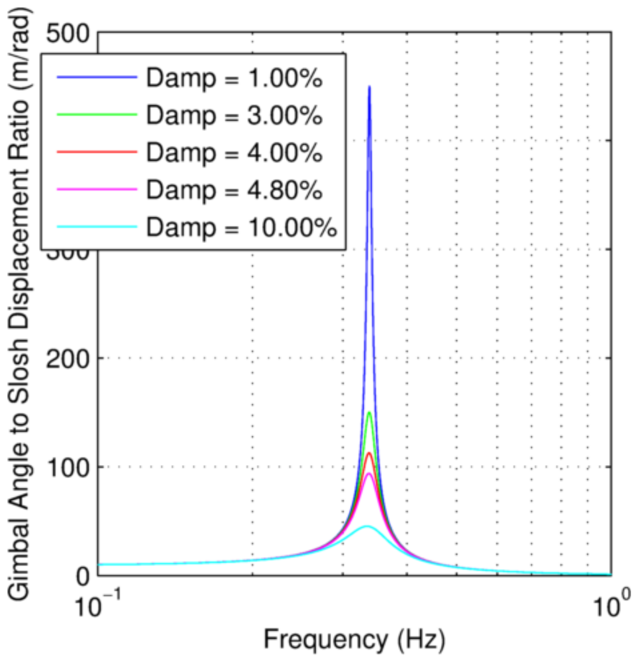
A More Useful Metric – TVC Amplitude

- **If control margins are so dependent on the assumed amplitude, how does one pick the amplitude?**
  - No agreed upon method.
  - Often heritage values are used whose rationale are no longer known.

- **In this paradigm, the amplitude of the limit cycle is arguably more valuable.**
  - Too large of a TVC gimbal command attitude could violate angle, rate, and duty cycle limits of the TVC system.
  - Too large of a slosh amplitude could uncover the sump causing engine starvation, and splashing could cause excess heat transfer to ullage gas, resulting in ullage collapse.

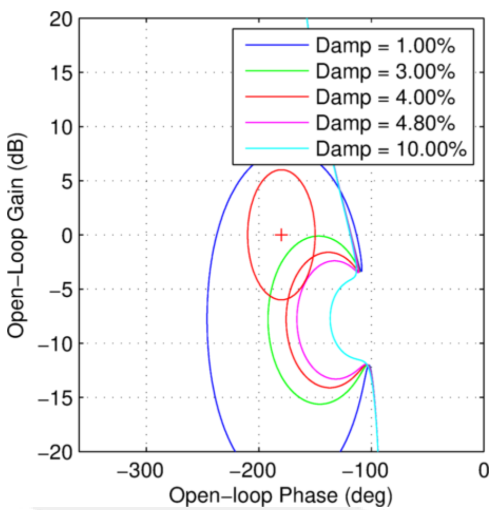
- **The slosh amplitude is readily available via the mechanical analogue slosh model.**

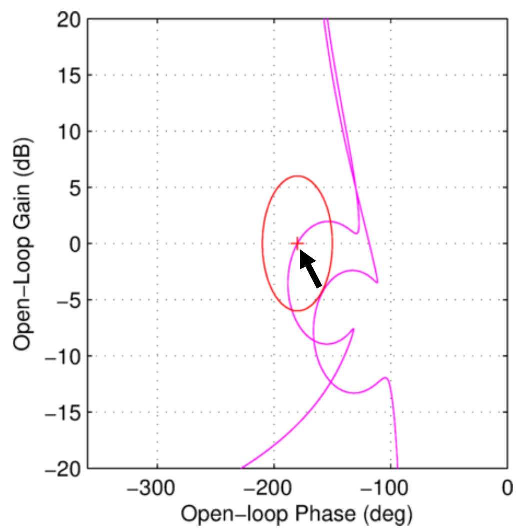
- **The TVC angle can be related to the slosh amplitude using the linear models employed for frequency domain analysis.**
  - Can produce the transfer function from gimbal angle to slosh displacement, which can be reversed.



Discussion of Margins

- **One could simply calculate the amplitude of the limit cycle of the system for the damping at neutral stability.**
  - However, this would only reflect the limit cycle at nominal condition, and there is no connection to the traditional linear stability margins.
- **A more robust way is to find the damping and slosh amplitude corresponding to meeting a desired disc margin (e.g., DM = 1) and then find the TVC amplitude of oscillation assuming the system has an oscillation at this amplitude.**
  - This protects for various system degradations which could degrade margins (slow actuators, extra time delay, mass property differences, etc).
- **This gives the designer two margins against which to assess stability for given slosh damping profile (e.g., baffle design)**
  - Extent of system margin degradation.
  - TVC amplitude LCO tolerance threshold.





Predicting Limit Cycle Amplitude During Degraded Margin

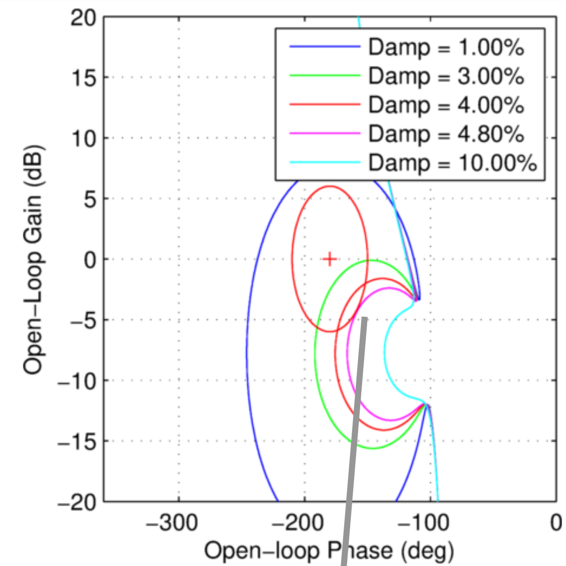
Given the effective linear slosh damping as a 1-1 function of slosh mass displacement amplitude, then:

1. **Pick a TVC limit cycle limit threshold (such as 10% of the capability)**
2. **Choose the amount of disc margin (phase/gain margin) degradation to protect for ($DM_{damping-threshold}$).**
   - Compute the damping needed to meet disc margin threshold, and compute the associated slosh mass displacement.
3. **Choose $DM_{limit-cycle-threshold}$ and find all frequencies in the response that have a margin of less than this value in order to isolate slosh frequencies.**
4. **Compute the predicted TVC amplitude by dividing the slosh mass displacement by the gain of z_{s1}/β_e over the frequencies isolated in step 3.**
5. **Check the maximum limit cycle amplitude over the frequency range against the chosen TVC threshold.**

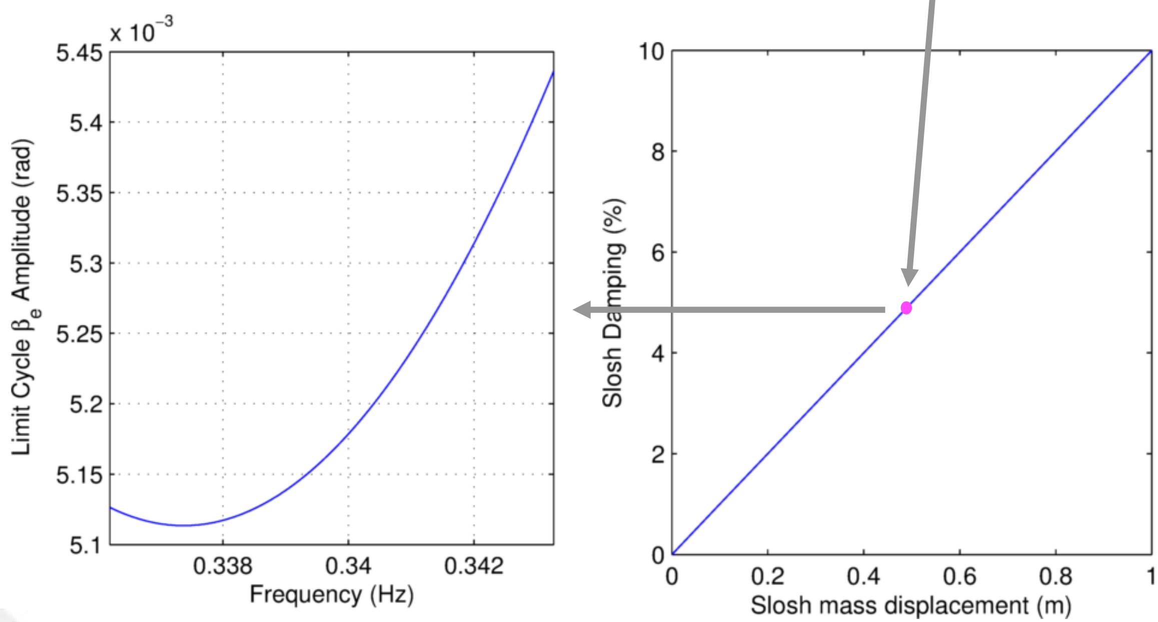
- **For the example problem we're using:**
  - TVC limit cycle limit threshold 0.5 degrees.
  - $DM_{damping-threshold}$ of 1.0 (corresponding to 6dB/30deg of gain/phase margin)
  - $DM_{limit-cycle-threshold}$ of 1.1

Example System Results

Parameter	Value
Damping to achieve DM=1	4.8%
Slosh amplitude for needed damping value	0.48 m
Frequency range for where DM<1.1	0.336 Hz to 0.343 Hz
Max TVC limit cycle amplitude over range	5.45 mrad (0.29 deg)

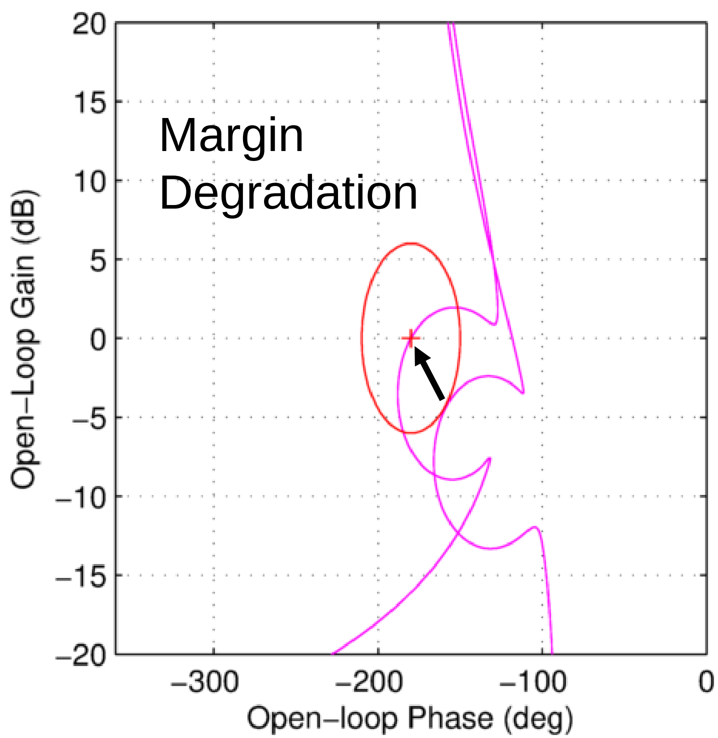


- **Prediction is less than the TVC amplitude threshold of 0.5 degrees.**

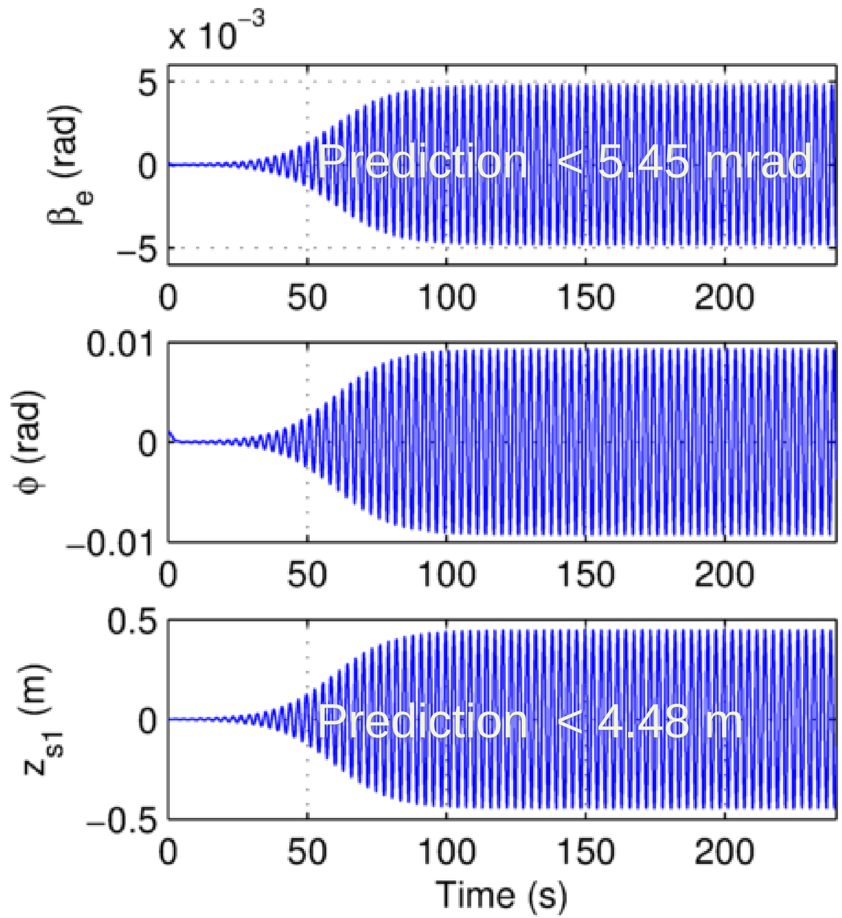


Time Domain Check of Degraded Case

- **Check the results in time domain with the linear equations of motion augmented with non-linear damping.**
- **Adding gain and time delay associated with a loss of a full 6dB/30deg disc margin to degrade system.**



With Odd Square Law Damping



- **Non-zero initial condition time response results in limit cycle within prediction from previous slide!**

Discussion/Conclusions

- **The limit cycle method is general and can be used with any linear dynamics model of the vehicle dynamics, from which control margin can be derived, and the plant gimbal angle to slosh amplitude gain can be determined.**
  - For SLS, the method is used with a 3-axis dynamic formulation which includes fully coupled flex, slosh, and nozzle dynamics, and a more complex attitude control formulation (FRACTAL).

- **The limit cycle method is not dependent on odd square law damping. Any damping profile which is monotonically increasing as a function of amplitude can be used.**

- **The method herein allows the selection of two metrics against which appropriate margins can be selected.**
  - Extent of system margin degradation.
  - TVC amplitude LCO tolerance threshold.

Discussion/Conclusions

- **The limit cycle prediction captures vehicle intrinsic stability, but the amplitude can go higher due to external forcing functions.**
  - It is still necessary to run dispersed mass-varying, time domain analysis with expected forcing from guidance, etc. to verify satisfactory performance.

- **The limit cycle prediction is a tool which complements existing methods of analysis of slosh-control interaction, but does not replace them.**

- **The SLS program has successfully used this method as part of the story of justifying reduced slosh baffle requirements (less mass) than would be required to meet full gain/phase margins at heritage wave heights.**

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