Genetic Algorithm-based Optimization to Match Asteroid Energy Deposition Curves

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Abstract

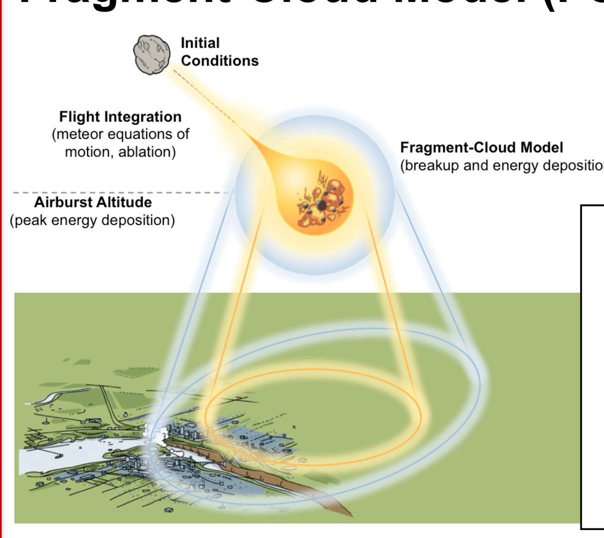
An asteroid entering Earth's atmosphere deposits energy along its path due to thermal ablation and dissipative forces that can be measured by ground-based and space-borne instruments. Inference of pre-entry asteroid properties and characterization of the atmospheric breakup is facilitated by using an analytic fragment-cloud model (FCM) in conjunction with a Genetic Algorithm (GA). This optimization technique is used to inversely solve for the asteroid's entry properties, such as diameter, density, strength, velocity, entry angle, and strength scaling, from simulations using FCM. The previous parameters' fitness evaluation involves minimizing error to ascertain the best match between the physics-based calculated energy deposition and the observed meteors. This steady-state GA provided sets of solutions agreeing with literature, such as the meteor from Chelyabinsk, Russia in 2013 and Tagish Lake, Canada in 2000, which were used as case studies in order to validate the optimization routine. The assisted exploration and exploitation of this multi-dimensional search space enables inference and uncertainty analysis that can inform studies of near-Earth asteroids and consequently improve risk assessment.

Objectives

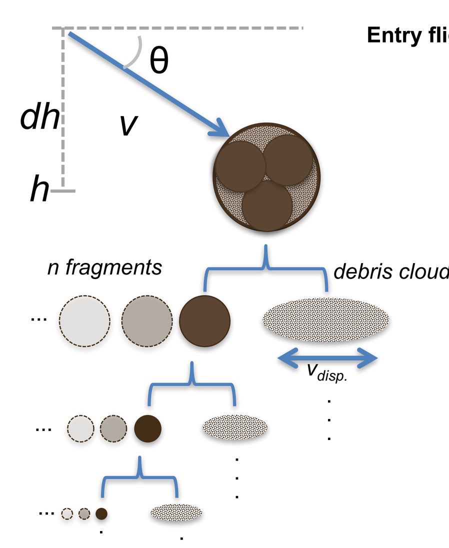
- Infer asteroid properties (diameter, density, strength, and velocity) from good matches for Chelyabinsk and Tagish Lake meteors, paying particular attention to diameter.
- Automate the matching of measured asteroid energy deposition curves to simulated ones.
- Identify the best performing objective function, genetic operators, and genome representation.
- Validate and verify the optimization routine's solutions using an artificial FCM-generated curve.

Methodology

Fragment-Cloud Model (FCM) [1]



Initial Conditions
- Diameter
- Density
- Strength
- Velocity
- Entry angle
- Strength scaling α
- Ablation coefficient σ
- Number of fragments
- Fragment mass distribution



Entry flight: integrates meteor equations of motion and ablation

$dm/dt = -0.5\rho_{air}v^3A\sigma$

$dv/dt = \rho_{air}v^2AC_D/m - g\sin\theta$

$d\theta/dt = (v/(R_E+h) - g/v)\cos\theta$

$dh/dt = v\sin\theta$

Fragmentation when pressure > strength

$\rho_{air}v^2 > strength$

Each break yields:
- Multiple independent, identical fragments
- Debris cloud of specified mass fraction

Fragment strengths increase with decreased size

$S_2 = S_1(m_1/m_2)^\alpha$

Clouds broaden and slow under common bow shock

$v_{dispersion} = v_{cloud}(3.5\rho_{air}A/\rho_{cloud})^{1/2}$

Energy deposition computed as change in total KE of all fragments/clouds as a function of altitude.

Genetic Algorithm (GA)

Set up population → Select individuals for mating → Mate individuals → Mutate offspring → Compute energy deposition → Evaluate fit → Keep best individuals → Evolve the population

Genomic representation of continuous variables:

Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	Gene 6	Gene 7
• Entry velocity • 11-72 km/s	• Density • 1.5-5 g/cm³	• Strength • 1 kPa – 10 MPa	• Initial diameter • 1-100 m	• Entry angle • 0.1 - 90°	• Cloud fraction • 0.1 – 0.9	• Strength scaling • 0.1-0.9

Mutation (prob. 0.5):

Selected genome

Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	Gene 6	Gene 7	Objective Value
• Entry velocity • 15 km/s	• Density • 1.5 g/cm³	• Strength • 1 MPa	• Initial diameter • 30 m	• Entry angle • 20 °	• Cloud fraction • 0.5	• Strength scaling • 0.3	• 8.7

Mutated genome

Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	Gene 6	Gene 7	Objective Value
• Entry velocity • 15.6 km/s	• Density • 1.49 g/cm³	• Strength • 999 kPa	• Initial diameter • 30 m	• Entry angle • 20 °	• Cloud fraction • 0.5	• Strength scaling • 0.3	• 7.8

Objective Function:

$$\min 10\sqrt{\sum_{i=1}^{n}\varepsilon_i^2} + \max(\varepsilon)$$

Mating using Even-Odd Crossover (prob. 0.65):

Parent 1

Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	Gene 6	Gene 7	Objective Value
• Entry velocity • 15 km/s	• Density • 3 g/cm³	• Strength • 1 MPa	• Initial diameter • 20 m	• Entry angle • 20 °	• Cloud fraction • 0.5	• Strength scaling • 0.3	• 10.6

Parent 2

Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	Gene 6	Gene 7	Objective Value
• Entry velocity • 48 km/s	• Density • 1.5 g/cm³	• Strength • 500 kPa	• Initial diameter • 30 m	• Entry angle • 65 °	• Cloud fraction • 0.5	• Strength scaling • 0.9	• 18.3

Child 1 (Even)

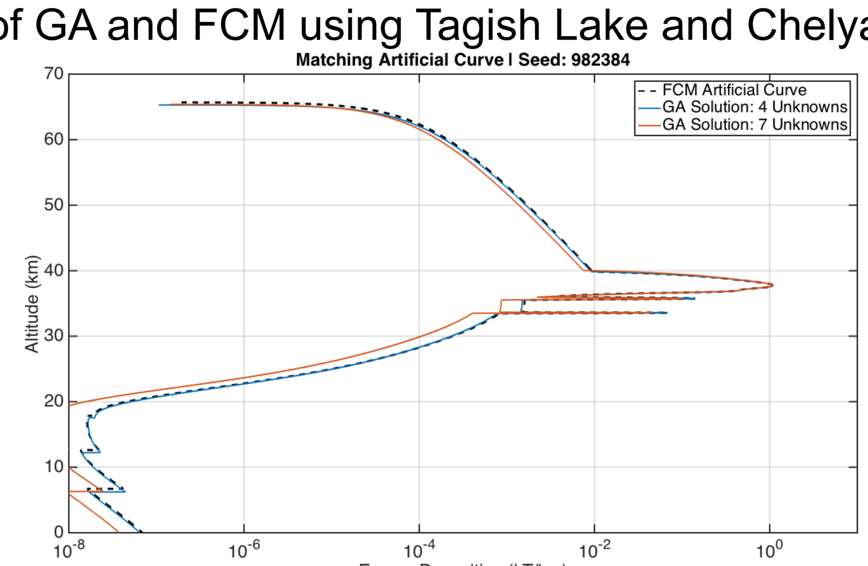
Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	Gene 6	Gene 7	Objective Value
• Entry velocity • 15 km/s	• Density • 1.5 g/cm³	• Strength • 1 MPa	• Initial diameter • 30 m	• Entry angle • 20 °	• Cloud fraction • 0.5	• Strength scaling • 0.3	• 8.7

Child 2 (Odd)

Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	Gene 6	Gene 7	Objective Value
• Entry velocity • 48 km/s	• Density • 3 g/cm³	• Strength • 500 kPa	• Initial diameter • 20 m	• Entry angle • 65 °	• Cloud fraction • 0.5	• Strength scaling • 0.9	• 37.8

Matching Synthetic FCM Curve

In order to determine the best genetic operators and objective function, we used a synthetic curve to test the GA without FCM's influence. Then, to establish the GA's sensitivity to population size, number of genes, and generation needed as stopping criteria, we show two GA solutions: 1) 7 genes allowed to vary freely and 2) 7 genes allowed to vary while 3 are restricted by published measurements of velocity, density, and entry angle. Furthermore, we tested the combination of GA and FCM using Tagish Lake and Chelyabinsk energy deposition curves using the same methods.



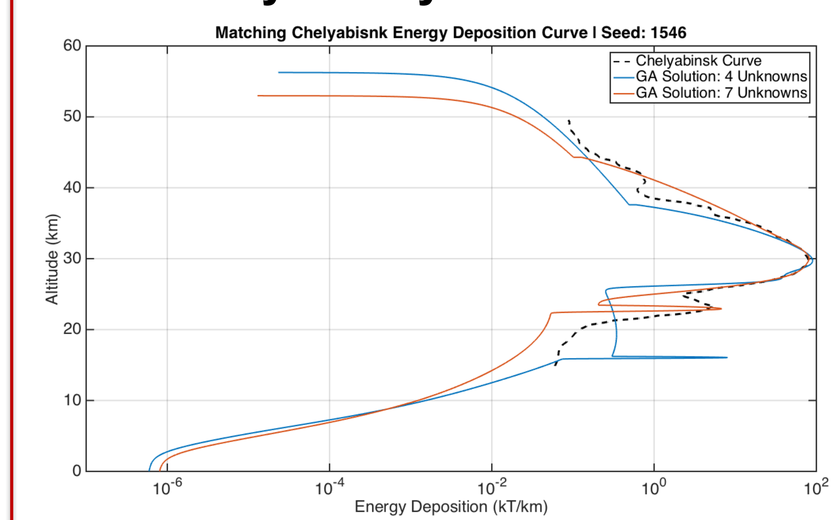
Curve Type	Objective Value	Velocity	Density	Strength	Diameter	Entry angle
Artificial curve	0.0	15.8 km/s	1.68 g/cm³	1 MPa	4 m	18 °
7 unknowns	0.0931	17.2 km/s	3.10 g/cm³	1.15 MPa	3.08 m	17.2 °
4 unknowns	0.0188	15.5 km/s	1.80 g/cm³	0.97 MPa	3.95 m	17.0 °

Curve Type for 4 Unknowns	Restricted Velocity (km/s)	Restricted Density (g/cm³)	Restricted Entry Angle (degrees)
Synthetic FCM	14-17	1.5-2	17-19
Chelyabinsk [3]	19.01-19.31	3.29-3.31	18.3-18.5
Tagish Lake [2], [4]	15.2-16.4	1.62-1.66	16.8-18.8

Results

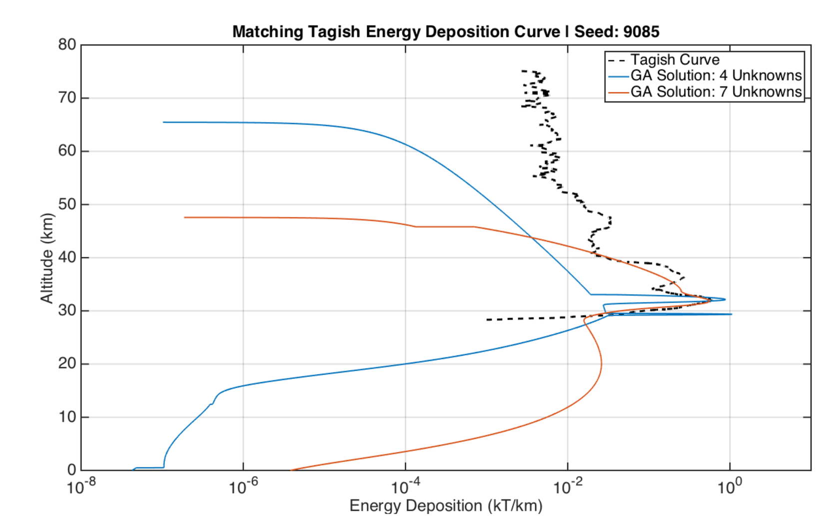
FCM and GA were combined to estimate the asteroids' initial diameter. First, all 7 genes varied freely. Then, using published values of velocity, entry angle, and density [2]-[4], we obtained diameter estimates that not vary significantly—list of restricted values are in the Methodology section. [1] also found that reducing the bulk density for Chelyabinsk led to a better fit. [2] couldn't match the Tagish Lake curve without the use of a porosity model.

Case Study: Chelyabinsk



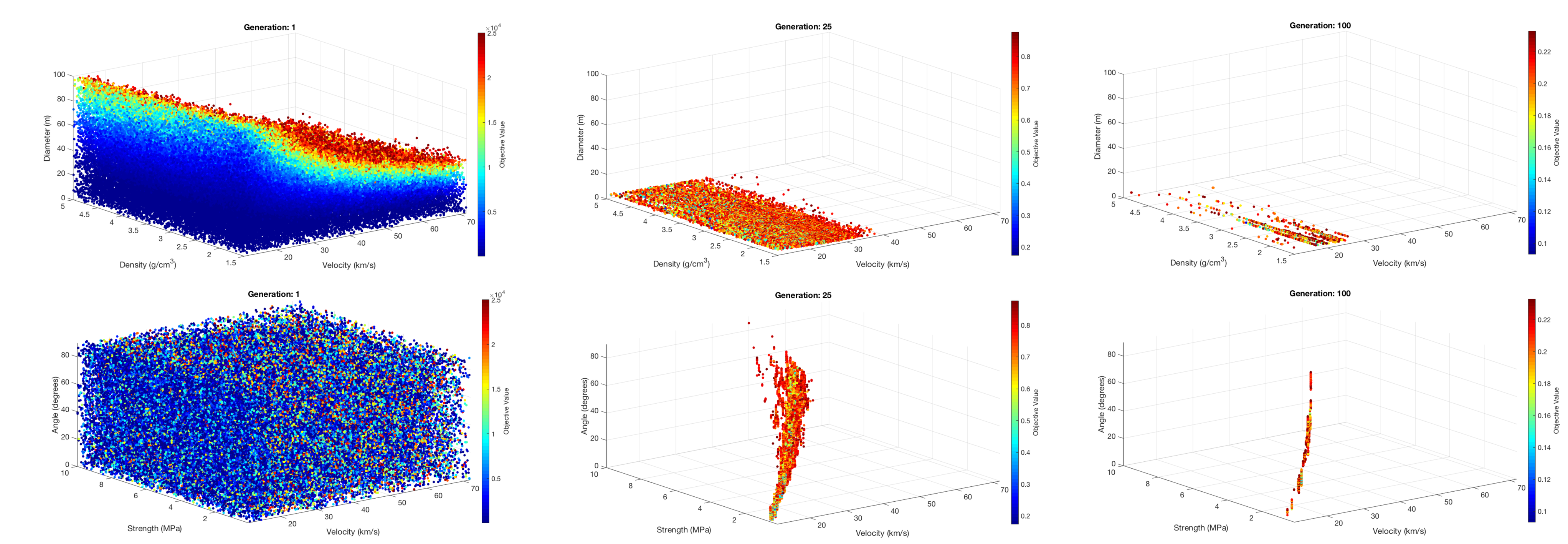
Curve Type	Objective Value	Velocity	Density	Strength	Diameter
Chelyabinsk	?	19.16 km/s	3.30 g/cm³	?	?
7 unknowns	14.3	17.30 km/s	2.29 g/cm³	0.58 MPa	22.3 m
4 unknowns	31.7	19.03 km/s	3.31 g/cm³	2.11 MPa	17.9 m

Case Study: Tagish Lake



Curve Type	Objective Value	Velocity	Density	Strength	Diameter
Tagish Lake	?	15.8 km/s	1.67 g/cm³	?	?
7 unknowns	0.415	11.2 km/s	4.13 g/cm³	0.19 MPa	4.27 m
4 unknowns	0.825	15.3 km/s	1.65 g/cm³	2.74 MPa	3.64 m

Gene Evolution for Synthetic Curve with 7 Unknowns:



The GA reveals the error space and how genes evolve. The figures above demonstrate that diameter denotes a surface where only few solutions are possible. This well demarcated surface is contrasted by the scatter plot of strength, angle, and velocity, where there is no recognizable surface.

Discussion and Future Work

- GA selects diameter, velocity, strength, density, and then entry angle, sequentially. Even though the other features vary, diameter is quickly selected and is usually within 25% of the published estimates.
- The root-mean-square error (RMSE) dominates at the beginning of evolution but then becomes secondary as the GA evolves. Minimizing the RMSE forces the GA to match the curves' main trends, especially focusing on the peaks, where main fragmentation occurs, since error is calculated in linear space. Minimizing the maximum error, known as runout, ensures that the main fragmentation events occur at the heights with the dominating peaks.
- Gaussian mutator allows for proper local exploration when steady-state GA starts converging on a solution.
- When parameters are restricted, the GA has more difficulty matching the curve because it has less freedom to vary parameters to produce better matches.

Future Work

- Relax FCM assumptions to include uneven mass distributions, porosity, and distinct ablation coefficients for cloud and fragments.
- Run on more cases to see how the GA performs with other energy deposition curves.
- Perform sensitivity analysis to ascertain dominating parameters.
- Use a gradient-based search after a given generation to refine results.

Conclusions

- Asteroid properties can be inferred, especially diameter at early stages of evolution.
- The GA shows that the equations of motions provide solutions that may not be unique.

Acknowledgements and References

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