Trilogy, a Planetary Geodesy Mission Concept for Measuring the Expansion of the Solar System

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Abstract
The scale of the solar system is slowly changing, likely increasing as a result of solar mass loss, with additional change possible if there is a secular variation of the gravitational constant, G. The measurement of the change of scale could provide insight into the past and the future of the solar system, and in addition a better understanding of planetary motion and fundamental physics. Estimates for the expansion of the scale of the solar system are of order 1.5 cm year^{-1} AU^{-1}, which over several years is an observable quantity with present-day laser ranging systems. This estimate suggests that laser measurements between planets could provide an accurate estimate of the solar system expansion rate. We examine distance measurements between three bodies in the inner solar system -- Earth's Moon, Mars and Venus -- and outline a mission concept for making the measurements. The concept involves placing spacecraft that carry laser ranging transponders in orbit around each body and measuring the distances between the three spacecraft over a period of several years. The analysis of these range measurements would allow the co-estimation of the spacecraft orbit, planetary ephemerides, other geophysical parameters related to the constitution and dynamics of the central bodies, and key geodetic parameters related to the solar system expansion, the Sun, and theoretical physics.

1. Introduction
The motions of the planets are a response to the totality of forces within the solar system and the fundamental laws of physics. The dominant force is that of gravity, principally of the Sun but also the gravitational interaction of each solar system body with every other body. Observations of the motions of the planets in the solar system over an extended period of time have embodied in them both the changes in our understanding of the laws of physics and the changes to our knowledge of the individual bodies within the solar system. (Folkner et al., 2014; Williams et al., 1996; Genova et al., 2017). Of major interest are the Sun's gravity field, which governs the scale of the solar system, and the laws of both Newtonian and relativistic physics that determine the dynamical evolution of solar system.

The mass of the Sun is believed to be slowly decreasing due to the conversion of hydrogen to helium within the deep solar interior, in which 2.9% of a proton mass is lost in the reaction (e.g., Sackmann et al., 1993; Noerdlinger, 2006). This lost mass is converted into energy and radiated from the sun as electromagnetic (E-M) and particle radiation in the solar wind and coronal mass ejections. Changes in solar radiation output and magnetic polarity, the former known to occur with periods of 11 and 22 years, collectively suggest that the loss of mass may not be constant. The decrease in the solar mass results in decreased gravitational attraction of planetary bodies in heliocentric orbit, resulting in an increase in orbital distances as the Sun progresses through the main sequence phase of its evolution. The measurement of changes in the orbits of the planets may thus provide insight into the nature of the Sun's deep interior processes. Other physical effects also affect the orbits of planets, and these must be understood in order to isolate the solar contribution.

2. Science background

Based upon the flux of radiation emitted by the Sun and the mass in the solar wind, the fraction of solar mass loss is of order (Zuber et al., 2017)

$$M\text{-}dot/M = -1 \times 10^{-13} \text{ yr}^{-1} \qquad\qquad [1]$$

where *M-dot* is the time derivative of the solar mass, *M*.

Despite the fact that the rate of the Sun's mass loss is small, its effect on the solar system is not insignificant. The major axis of a planetary orbit is inversely proportional to *GM*, the product of the solar mass, *M* and the gravitational constant, *G*. It has also been suggested that the gravitational constant could be changing, in which case it would also affect the scale of planetary orbits. Estimates of the rate of change of *G* have been derived from lunar laser ranging and analysis of pulsar timing. Some recent results are summarized in Table 1, but in all cases uncertainties are such that the results are consistent with a *G-dot* of zero. However, a change in *G* cannot be separated from a change in *M* in the motion of the planets about the sun but can be obtained independently, at least in principle, from changes in the Earth-Moon distance.

From Kepler's third law and conservation of angular momentum (for circular orbit) we see that

$$2\delta n/n + 3\delta a/a = \delta\mu/\mu, \text{ and } \delta n/n + 2\delta a/a = 0, \qquad\qquad [2]$$

where $\mu=GM$, *n* is the mean angular motion, *a* is the semi-major axis, δ is change, and

$$\delta\mu/\mu = -\delta a/a, \text{ and } \delta\mu/\mu = \tfrac{1}{2}.\delta n/n, \qquad\qquad [3]$$

which indicates that a decrease in *M* yields a linear increase in *a* and a linear decrease in *n*.

For *M-dot/M* = -1 x 10^{-13} yr^{-1}, the Earth's orbit increases by ~1.5 cm yr^{-1} and the orbital velocity changes by ~-9.4 cm yr^{-1}. This calculation assumes that angular momentum is conserved, but we recognize that an unknown small amount of momentum is lost to the solar wind and is ignored in this work.

Figure 1 shows the effect of a change in *GM*, on planetary orbit size and orbital velocity. Although both distance and angular velocity scale linearly, orbital velocity does not scale linearly with distance from the sun and thus shows a larger effect for the inner planets than the outer planets. Note that as the orbital radius increases the angular velocity decreases, so the planet falls behind its original path in addition to moving outward.

3. Changes in the distances between planets

We can estimate the magnitude of the change in a planet's position from a changing solar *GM* by propagating the positions of the planets. Figure 2 shows the direct effect in distance between the four inner planets, Mercury, Venus, Earth and Mars for a *GM-dot/GM* = 1 x 10^{-13} yr^{-1} over a four-year period.

The oscillations in the distances are at the synodic periods of planet pairs, and the relative distance anomaly steadily grows quadratically in amplitude over time as the two planets move further from their original trajectories (this is shown more clearly in Figure 5 where the time base is 3000 days). Mercury has the largest orbital velocity and causes the largest signal over the time span, with a total amplitude of more than 3-4 meters.

Measuring any one of the relative distances over time in Figure 2 can provide an estimate of the change in *GM* as has been demonstrated by Genova et al., (2017), who obtained (-6.13±1.41) x 10^{-14} from analysis of range and range-rate tracking data of the MESSENGER spacecraft in Mercury orbit over a 4-year period, 2011-1015. The ability to extract the signal of a changing solar *GM* is largely the result of the unique signature in the range measurements between Earth and Mercury, and the quadratic nature of the increase in magnitude seen in Figure 2 and more clearly in Figure 5. Correlations with other parameters, such as the state vectors of the spacecraft and planet, and the planet's *GM* were low (Genova et al, 2017) and did not adversely affect the recovery of *GM-dot*. Almost no other perturbing force increases the along-track position quadratically in time, indicating that the longer the data span the more separable a change in *GM* will be from other perturbing effects.

The accuracy can be substantially better if several baselines are measured over the same time span, particularly if the lines form a closed geometric shape, such as a triangle, providing angle information as well as range and not just as a result of the increase in the number of lines. A closed network of lines provides a

geometric constraint on the positions of the planets in at least 2 component directions such that any disturbing influence on one planet will effect the positions of the other planets through the observations. Eventually, the observational constraints will enable significant improvements in the positions of the planets, not just their geometrical relationships, remove ambiguities and weaknesses in the geometry, thereby improving the estimation of other parameters and forces affecting their motion. This is the primary measurement of the Trilogy concept.

In the following sections, we describe a conceptual planetary mission (Zuber et al., 2017) that we anticipate will be able to measure the expansion of the heliocentric orbits of 3 inner solar system bodies, Venus, Earth, and Mar

4. The Trilogy Concept

Trilogy is our concept for an interplanetary ranging constellation of geodetic satellites. Spacecraft are placed in stable orbits around Mars, Venus, and Earth's Moon (or around Earth). We suggest these planetary bodies for several reasons, including the unique signature of planetary orbit expansion happens more rapidly for the inner planets, that they can be reached with modest cruise times, and because ranging to the gas giants over much longer distances will require more capable instrumentation and more stringent pointing requirements than have been demonstrated in space. That said, we recognize the inclusion of other planets would clearly strengthen the overall science result. Once a 3-planet mission demonstrates the value of these inter-planet ranges, the addition of terminals at other planets would be relatively straightforward to implement, either as standalone spacecraft benefiting from rideshare or as additional payloads to planned missions.

Historically, planetary positions have been derived from directional information. But beginning with the space age the positions of planets have been generally derived from spacecraft orbiting the individual planets and resulted in more accurate planetary locations and ephemerides. For other bodies, such as asteroids, the directional method is still employed unless they have been visited by spacecraft, as for example asteroid 433 Eros, which hosted the NEAR Shoemaker spacecraft in 1998. However, all the observations of planets have been Earth-centric, i.e., they have been made from Earth. Trilogy would, for the first time, make metric measurements between spacecraft orbiting other planets and provide a second constraint in a quasi-orthogonal direction to Earth and form a closed but continuously changing triangle.

Figure 3 illustrates the Trilogy concept. The spacecraft that will orbit the host planets measure their distances from each other, nominally to an accuracy of a few centimeters using the asynchronous transponder approach discussed below. The resulting range and range-rate observations will be used to determine the spacecraft orbits about the host planets and the trajectories of the planet centers of mass.

The altitudes of the spacecraft need to be high enough to be unperturbed by the planet's atmosphere (where appropriate) and the higher harmonics of the gravity field to provide the highest accuracy determination of the spacecraft orbit. Such orbital altitudes would have the benefit of being able to monitor the time-variable, long-wavelength gravity field (less than spherical harmonic degree and order 4) and the orientation of the planet. The inclination of the orbit is less constrained, provided it does not restrict observations of the other host planets, although it may be preferable to avoid a true polar or sun-synchronous orbit as these orbits are geometrically frozen in inertial or solar orientation, despite great merit for other studies. The desired orbits are not atypical of some present and future planetary missions.

Our Trilogy concept is based upon orbiters at the host planets, although landers could be an alternative option and would be more stable. In that case, the measurements would need to be corrected for any site movement due to planetary rotation, or possible tidal or other dynamical effects and visibility may be restricted due to planetary rotation. The effects of an atmosphere on spacecraft orbital motion may complicate the analysis as well, especially seasonal effects.

Accurate measurements between the spacecraft are the key to the concept, and Trilogy would acquire range and range-rate between all three spacecraft in both directions. The preferred method would be to use laser ranging terminals on each of the spacecraft. Although radio systems provide high-accuracy data for planetary spacecraft, it is difficult to envisage radio links between small radio transponders over interplanetary distances able to provide the accuracy desired. Indeed, lasers are preferred over microwave systems because of their narrow beam divergence and because there is no requirement for large spacecraft-mounted antennae. Laser ranging systems can also provide data of greater accuracy (centimeter level) than radio systems by about 2 orders of magnitude. We also note that the laser ranging systems could be used as optical communications terminals to send and receive data.

Satellite Laser Ranging (SLR; Tapley et al., 1985; Pearlman et al., 2002) to retro-reflectors onboard Earth-orbiting spacecraft is one form of "two-way" laser ranging, but a viable link is only achievable over short ranges due to the rapid falloff ($1/R^4$) of received energy. Another form of two-way ranging is the asynchronous transponder approach (Degnan 2002), in which laser pulses are transmitted between both spacecraft, usually simultaneously. Given the large distances, at any one time, many laser pulses are in transit between spacecraft pairs. These observations are one-way measurements in both directions over all links between the spacecraft. Performed at regular intervals over a nominal period of five years, these observations would strengthen the planetary ephemerides substantially, and longer time spans will yield even more reliable results. We also recognize that synchronous transponder systems could also provide the desired measurements. The coherent processing of ranges received

at each terminal avoids the need to estimate some of the spacecraft clock parameters, reducing the range uncertainty, but also places stringent requirements on the laser systems. Others (e.g., Turyshev SG, Williams JG (2007) have discussed concepts for measuring planetary distances with lasers to estimate various relativistic parameters but in most cases the desire is for millimeter or better accuracy in contrast to the cm level suggested for Trilogy.

This method was successfully demonstrated by the Lunar Laser Communication Demonstration (LLCD) instrument (Boroson et al., 2009) on the Lunar Atmosphere and Dust Environment Explorer (LADEE) mission to the Moon (Blau 2014) at cm-level ranging and mm s^{-1} in range rate. Asynchronous two-way laser ranging was demonstrated between an Earth station and the MESSENGER laser altimeter (MLA) (Cavanaugh et al. 2007) to 20-cm precision during cruise to Mercury, at a range of 24 Mkm (Smith et al., 2006). In addition, the Laser Ranging system (Zuber et al., 2010) on LRO made one-way range measurements between a laser station on Earth and LRO in lunar orbit over a period of 5 years (Mao et al., 2017), with 10-cm single shot precision at 28 Hz limited by the laser altimeter electronics used for timing, demonstrating that the basic measurement of Trilogy can be achieved routinely.

The asynchronous transponder measurement of range requires knowledge of the time a laser pulse leaves one spacecraft and the time it arrives at the other spacecraft. It also requires knowledge of the precise behavior of the oscillators (clocks) on the two spacecraft. In general, the two clocks will have a timing bias and will drift from true time at different rates, both of which need to be estimated as part of the analysis. When range measurements are made in both directions concurrently, the combined system becomes an asynchronous transponder (Degnan 2002) and the true range and clock differences can be derived from the range measurements. Further, the time needs to be referenced to a stable timing system, such as an atomic clock on Earth, which is easily satisfied if the Earth or Moon, is one of the corners of the interplanetary constellation. Figure 4, which has been modified from Degnan (2002), shows the transponder measurement system between two moving spacecraft and the way the range, which is a function of time, can be derived.

5. Analysis of range measurements
The analysis of the range measurements between the three spacecraft will be, at a minimum, a simultaneous solution for the spacecraft orbits around the host planets and the heliocentric orbits of the three host planets about the Sun, but will most probably include the use of planetary tracking data to other planets as is done in the development of planetary ephemerides (Folkner et al., 2014).

Figure 1 shows that the major effect of a change in *GM* on the orbits of the inner planets is along-track rather than radial, and that the along-track perturbation is 5 to 10 times larger for the inner planets than the outer planets. Figure 5 compares the changes in the relative distances of the Trilogy host planets over 3000 days

due to a change in *GM* of 1×10^{-13} y^{-1}. It is important to recognize that Figures 1 and 5 show the predicted orbital perturbation not the orbital residual. There will be many other perturbing effects and modeling errors that the analyst will need to estimate or adequately model.

The Venus-to-Mars distance has a full variation in range of ~2.5 m over a period of 4 years (Figure 5), the Earth to Mars distance about 1 m, and Earth to Venus ~1.5 m, indicating that changes in the distances between each pair of planets in the range of 1-2 m over the 1500 days should be expected. Figure 5 also shows that there are times when the distance anomalies between the planets change rapidly, reaching a centimeter or more per day and that for longer time periods, say 3000 days, the full range variation will reach 5 to 10 meters on all lines as a result of the quadratic growth of the signal with time.

It is useful to note that the pattern of changes in distance between the planets is fully predictable for a given *GM-dot*. These three signals are embedded in the range observations along with other influences that are perturbing the orbits of each of the planets, but only a single adjustment of the scale of the *GM-dot* signal is required to satisfy all three baselines.

In addition, the present day fit to planetary range data is a few meters (Folkner et al.,, 2014) for the inner planets over periods of several years, comparable to the signal from a changing *GM* and suggests that the extraction of the expansion of the solar system from the range data is a reasonable objective provided it can be separated from other parameters. We believe the separation is made possible by the combination of the linear increase in the semi-major axes of the planetary orbits and the quadratic growth of the in-orbit position of the planets. The result of Genova et al (2017) supports the separability and also indicates low correlations with other parameters.

To fully understand the ability to recover the expansion of the solar system for Trilogy will require a comprehensive geodetic simulation which we are hoping to initiate in the near future, but it is valuable to note that in the development of planetary ephemerides the scale of the solar system is a parameter that is both accounted for and estimated (Folkner et al., 2014).

6. Additional science
Earlier we suggested that additional science results might be expected from Trilogy. Additional results are possible because spacecraft perturbations are sensitive to not only the changing solar mass but also other physical influences indicative of solar system structure or other physical parameters. Table 2 is a list of some of the influences on Trilogy that could conceivably be recoverable from the data, or to which Trilogy is expected to be sensitive. In practice it will be important to trade the benefits of choosing orbits that minimize these additional perturbations with their implications for measuring solar mass loss.

Analyses of decades-long time series of optical measurements from Lunar Laser Ranging (Dickey et al., 1994; Williams et al.,, 1996) and recent analyses of X-band radio tracking observations from the MESSENGER mission (Genova et al., 2017; Park et al., 2017) demonstrate the considerable promise of the latter observational strategy.

A parameter of special interest is the solar gravitational flattening, J_2, the second-degree zonal coefficient in the Sun's gravity field, because of its potential importance in understanding the structure of the solar interior, and like the solar mass, may not be a constant. Table 3 lists some of the recent estimations of solar J_2.

The solar gravitational flattening causes a long-term precession of the node of a planetary orbit and the advance of perihelion which are collectively manifest as an additional along-track motion of the planet. This motion has been observed in the orbit of Mercury (Verma et al., 2013; Folkner et al., 2014; Genova et al., 2017) and Figure 6 shows the total acceleration on the orbits of Mercury, Venus, Earth and Mars for $J_2 = 2.11 \times 10^{-7}$. Table 4 shows the magnitude of the advance of perihelion. Unlike a rate of change in GM, which scales distances in the solar system and is larger for the outer planets, the solar J_2 decreases with distance from the Sun and is consequently much smaller for Mars than Mercury. It is evident from Table 2 that the effect of solar J_2 is larger than the potential change in solar GM but does it not increase quadratically with time. Further, the solar J_2 is probably known to about 10-15% (Table 4), suggesting the signal uncertainty in the range measurements is of order a few meters, comparable to the expected change in GM and the present capability of planetary position determination (Folkner et al., 2014).

We also anticipate that the spacecraft will be able to make geophysical observations of the host planets. It is highly likely that a spacecraft in Mars orbit could measure the changing gravitational attraction of the seasonal caps, refine the knowledge of solar tide, etc., if the spacecraft orbit is suitably chosen. Similarly, we anticipate that the spacecraft in orbit about Venus would be able to observe the gravitational changes due to dynamical motions of the planet's dense atmosphere and solar tidal effects, and probably improve the accuracy of the low-degree gravity field, which will elucidate the structure of the deep solid planet interior as has been accomplished at Mercury (Smith et al., 2012; Hauk et al., 2013; Mazarico et al., 2014; Park et al., 2017; Genova et al., 2017).

For launch, there are two basic options for the Trilogy spacecraft, either a single launch of all three spacecraft, or as three separate launches in which the Trilogy spacecraft are either instruments on the science payload or carried as secondary payloads. In the latter case of a secondary payload the ability to influence the final choice of planetary orbit could well be limited.

An electric propulsion and a chemical propulsion assessment of the launch options by GSFC for Earth-Venus-Mars transfers confirmed the Trilogy mission concept is viable on a single launch vehicle. The study, assuming a AV401 launch vehicle with a C3=9 km^2s^{-2}, indicated payloads of at least 120 kg could be deployed into stable orbits at both Venus and Mars with a total travel time of order 2+ years.

In principle, therefore, the Trilogy spacecraft could not only make measurements of the scale of the solar system but also acquire important observations of the host planets. They could also carry other scientific instruments dedicated to non-geodetic objectives. In the near-future, we will conduct a comprehensive simulation to determine which parameters and forces can be estimated and how much Trilogy will be able to improve them.

7. Spacecraft and instrumentation concepts

The key element in the acquisition of the measurements is the laser transponder system that can provide accurate measurements of range and range-rate between two spacecraft over distances of several AU. We have presented a mission concept with laser transponders on three dedicated spacecraft, but it is feasible for the laser transponders to be part of a more general scientific payload of spacecraft orbiting each planet or Moon, provided that orbital constraints and operational requirements can be met. Indeed, there may be advantages in that most science spacecraft are tracked at microwave frequencies by the Deep Space Network (DSN) from Earth which will provide additional data for the estimation of the distances between the three spacecraft and provide the necessary communications link with Earth for the inter-satellite ranging data themselves for analysis.

Interplanetary laser ranging is an emerging technology in which laser beams can be much better collimated and pointed because of the much shorter wavelength. The transmitter and the receiver sizes are much smaller than a radio frequency (RF) tracking system and pulsed laser transponders directly measure the range between the two terminals. The ranging errors are independent random variables and do not accumulate over time, which is important for Trilogy. A two-way asynchronous laser ranging system over planetary distance has been proposed with sub-millimeter ranging precision and accuracy demonstrated in the laboratory and field (Chen et al., 2013). The time of flight measurement of the LADEE Lunar Communications Demonstration experiment (LLCD) on the Lunar Atmosphere Dust and Environment Explorer (LADEE) mission has demonstrated <4 cm rms ranging error at lunar distance (Boroson and Robinson 2014; Stevens 2016). Another accomplishment of LLCD was the successful demonstration of sub-arcsecond laser pointing, acquisition, and tracking. The size and mass of the LLCD flight terminal could be fit into a SmallSat.

NASA GSFC has been developing laser ranging technologies that are compatible with SmallSat and even CubeSat (Yang et al., 2016a). Recent progress in the

coherent laser communication by industry and the laser tacking of NASA's Laser Interferometer Space Antenna (LISA) has made it possible to track the phase of the transmitted and the received laser signals at optical frequency. Besides two-way range measurements, like LLCD, the Doppler shift can also be measured from the phase and frequency of the optical carrier. An instrument noise floor of 0.3 µ s^{-1} at 1 s integration time has been demonstrated in the lab with a link margin for 2.5 AU distance (Yang et al., 2017), although there is no requirement for this quality measurement for Trilogy. Technical challenges remain, but a laser transponder on a SmallSat is expected to be feasible.

8. Summary & conclusions

We have described a planetary mission concept that would be able to measure the expansion of the solar system resulting from the decrease in mass of the Sun and possible change in the gravitational constant. The concept involves orbiting spacecraft around each of the planets Mars and Venus, and around Earth's Moon, with each spacecraft able to make precise range measurements to the other two spacecraft over a period of several years. The orbits of the spacecraft would be at altitudes that minimize the perturbations from gravity and any atmosphere effects. The proposed range measurements would be made by laser ranging systems on each spacecraft considered to be preferable over microwave because of greater intrinsic accuracy and relatively small size of the required instrumentation. The range data would enable the orbits of the spacecraft around the host planets, and the orbits of the host planets around the Sun, to be determined to high accuracy, from which the rate of expansion of the planetary orbits and the solar system as a whole could be derived. The technology has been demonstrated on previous and existing planetary missions and the analysis of the data acquired over many years on several experiments indicates that the accuracy required is achievable.

The determination of the rate expansion of the solar system would be an important accomplishment that could have significant impact on our understanding of the Sun and the evolution of the solar system.

Acknowledgements

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Tables

Table 1
Recent estimates of the time derivative in the gravitational constant, *G*.

Data Set	*Gdot/G*	Reference
Pulsar timing	$(-0.6 \pm 1.1) \times 10^{-12} \, y^{-1}$	Zhu et al. (2015)
Lunar laser ranging	$(1 \pm 2.5) \times 10^{-13} \, y^{-1}$	Müller et al. (2014)
Lunar laser ranging	$(-0.7 \pm 3.8) \times 10^{-13} \, y^{-1}$	Hofmann et al. (2010)
MESSENGER mission	$< 2 \times 10^{-14} \, y^{-1}$	Genova et al. (2017)

Table 2
Some parameters that Trilogy may be able to estimate depending on spacecraft orbits.

Fundamental Physics
Test of equivalence principle
Lense-Thirring precession of reference frame
Relativistic PPN parameters, β, γ
Gravitational flattening of the sun, J_2
Host Planets
Precession, nutation & rotation
Obliquity, tides, moment of inertia
Low-degree gravity, seasonal changes
Heliocentric orbits

Table 3
Recent estimates of the solar gravitational flattening, J_2.

Source	Solar J_2, 10^{-7}	Reference
Dynamics	2.40 ± 0.2	Verma et al. (2013) (MESSENGER)
Dynamics	2.11 ± 0.7	Folkner et al. (2014) (DE430, DE431)
Dynamics	2.246 ± 0.022	Genova et al. (2017) (MESSENGER)
Helioseismology	2.20	Mercheri et al. (2004)
Helioseismology	2.206	Roxburgh (2001)

Table 4
Perihelion precession of the terrestrial planets due to the J_2 of the Sun. The perihelion shift in m/year was computed by multiplying the precession rate by the perihelion distance.

	Perihelion Precession	
	arcsec c^{-1}	*m y^{-1}*
Mercury	0.026213	58.460
Venus	0.002743	14.292
Earth-Moon barycenter	0.000888	6.333
Mars	0.000207	2.069

Figures

Fig. 1
The effect of a *GM*-dot of 1 x 10^{-13} y^{-1} on the scale and velocity of planetary orbits. The change in orbital velocity is more significant than the change in scale for terrestrial planets. The linear scaling of the solar system is ~1.5 cm y^{-1} AU^{-1}.
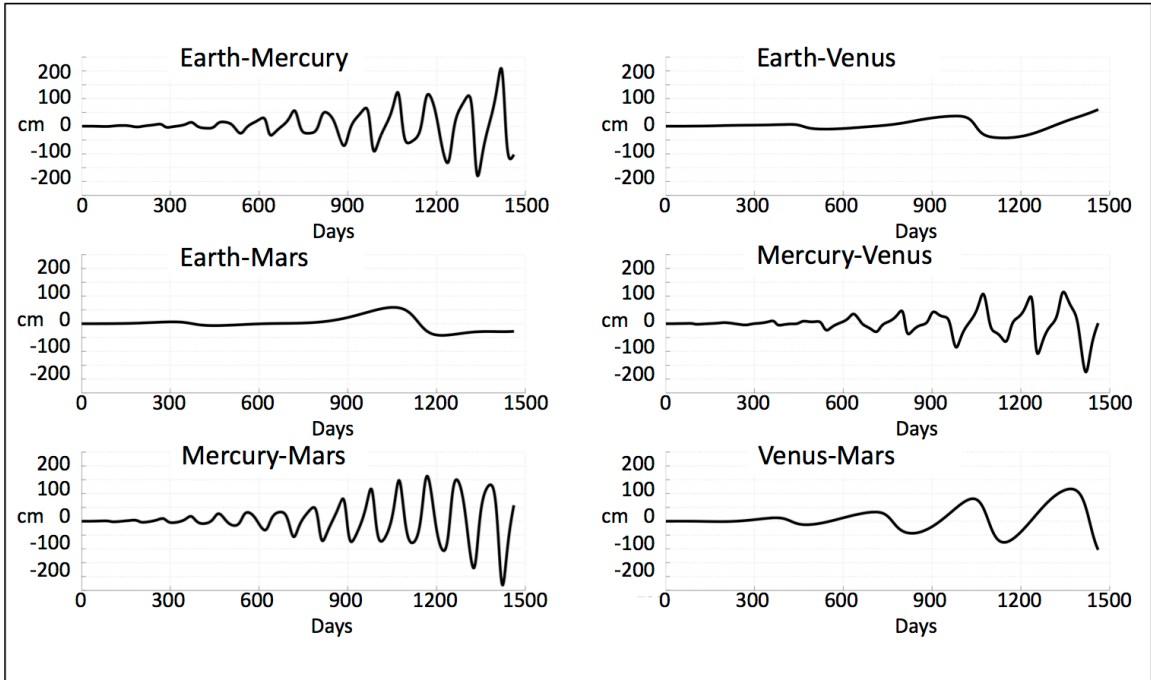
Fig. 2
Changes in distance between the four inner planets over a four-year period as a result of a fractional change in solar mass of 10^{-13}/yr. The oscillations are the synodic period of the two planets that change the balance between the along-track and radial perturbations. The time span covers Jan 1, 2008 to Jan 1, 2012.
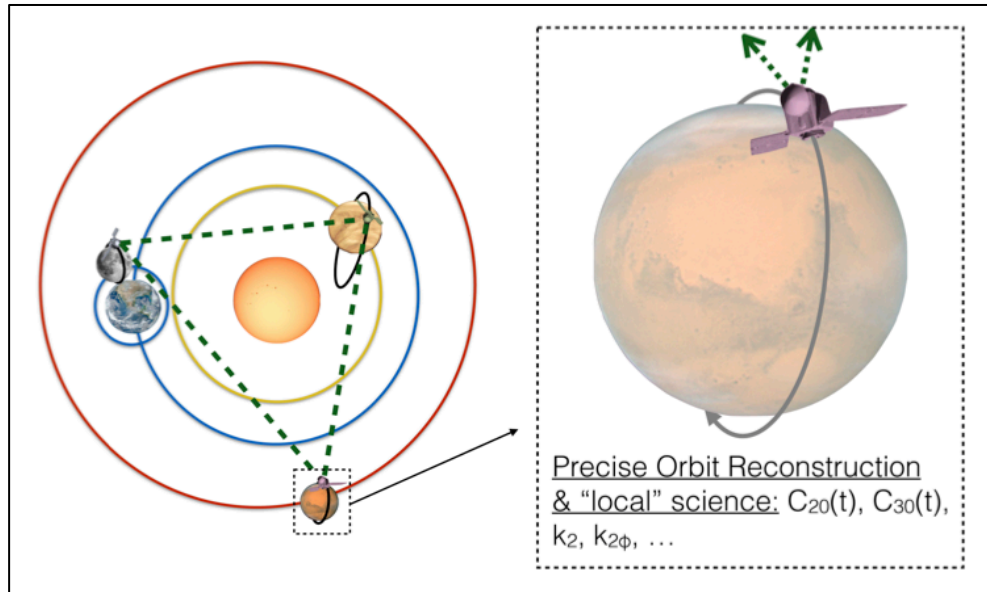
Fig. 3
Trilogy concept. Spacecraft are placed in orbit about Earth's moon, Mars and Venus. Range measurements are made between the three spacecraft for obtaining the orbits of the spacecraft and the positions of the centers of mass of the three host planets. Not to scale.
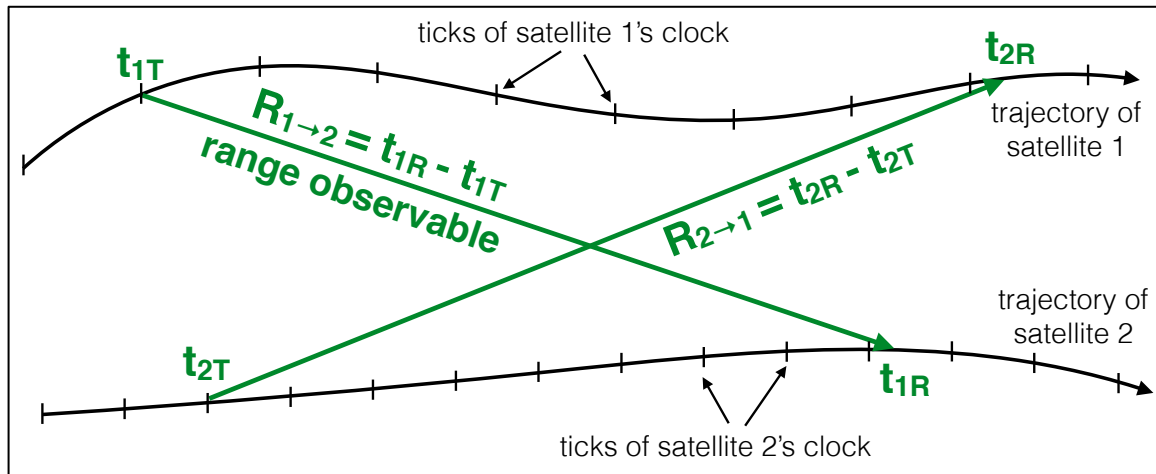
Fig. 4
Schematic of the concept of asynchronous two-way range measurements from optical transponders. The measured quantities are shown in green, and the solved-for quantities are in black. The range observables are obtained from the precise timing of transmitted (t_{iT}) and received (t_{iR}) laser pulses, at each spacecraft by free-running clocks. These one-way laser ranges are the measurements used during analysis to refine *a priori* estimates of the trajectory and clock drift of each spacecraft. After Degnan (2002).
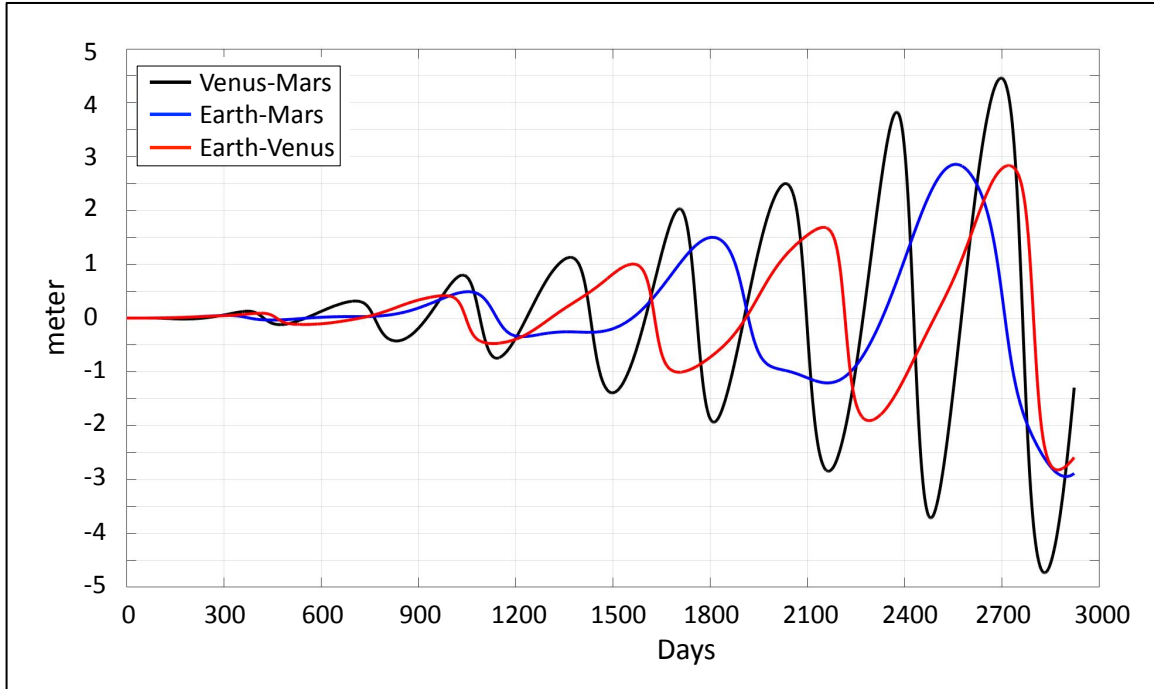
Fig. 5
Changes in the distances between the three Trilogy planets on the same chart.
Note the steady growth of the amplitudes of the oscillation of the synodic periods
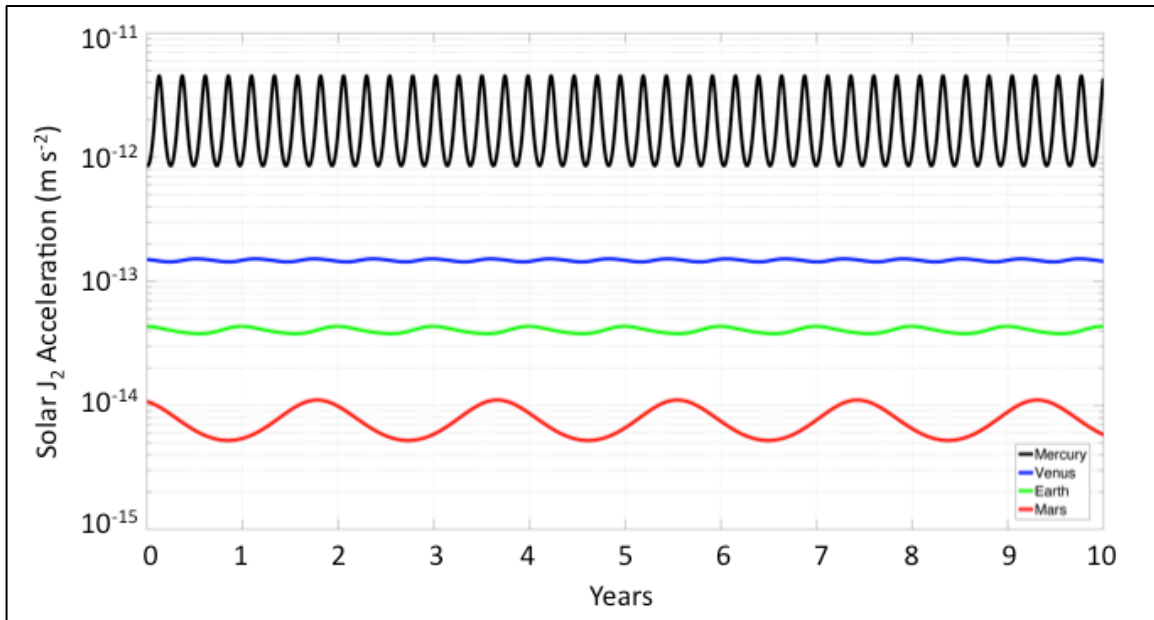and the rates of change reach 20 cm/d after about 8 years.

Fig. 6
Solar J_2 total accelerations on the four terrestrial planets.