A Telescope at the Solar Gravitational Lens: Problems and Solutions

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Abstract

Due to the bending of light by gravity, the gravity of sun forms a lens. In principle, a spacecraft sent to the distance of the soalr gravitational focus could be used as a gravitational lens telescope. One example of such a mission would be to use the gravitational lens to image an extrasolar planet around a nearby star. The practical difficulties with this concept are discussed, and some approaches to mitigating these difficulties suggested.

1. Background

According to Einstein's general theory of relativity, the gravity of a massive body deflects light. Due to this effect, the gravitational field of the sun can be used as a lens [1]. The minimal distance to the focus of the sun's gravitational lens is about 550 AU, about 82 billion kilometers. The sun continues to act as a lens beyond this minimum; at longer distances, the focused light passes increasingly far from the solar limb. Although this distance is large, it has been proposed by Eshleman [2] and others [3-6] that a mission to the gravitational focus of the sun could use this effect as a lens (*e.g.,* to form a long focal-length telescope) that is much larger than the lens of any physical telescope.

A mission to the gravitational focus of the sun, beyond the edge of the solar system, but far closer than the nearest stars, is attractive because it could be a target for an interstellar precursor. For example, it was listed as a precursor goal in the 1998 JPL workshop "Robotic Interstellar Exploration in the Next Century" [7], because it is one of the few possible targets of any interest that lie beyond Kuiper belt, but still closer than the nearest stars.

Reaching such a large distance in a reasonable time requires advance propulsion technologies, similar to the technologies proposed for interstellar flight. Proposed methods of reaching the focal distance include electric propulsion or laser- or solar sails. For example, by bringing a solar sail to a very close pass to the sun (figure 1), a high solar-escape velocity can be achieved [8,9], allowing a mission to the solar gravitational focus in a time scale of years, rather than decades.

Several possible objectives for such a mission have been proposed. In this paper, I will use as the example case a mission to use the gravitational lens as a telescope to image and map an extrasolar planet, with a baseline assumption that a target planet for observation has been discovered in an orbit about a nearby star before the launch of the mission, but other possible applications, ranging from astronomy to SETI to communications, have been proposed.

Why is the gravitational lens of the sun attractive to use as a telescope? The primary reason is that the lens is extremely large, and hence the aperture for gathering light (or, in the more general case, for gathering radio or other electromagnetic radiation) is large, and the resolution is, in principle, very high.

It is notable that, although the gravitational lens is described as a lens, and the design is in a sense a "telescope," it is considerably different in nature from a conventional lens, or a conventional telescope. A spacecraft does not simply reach the focal point and then look back "through" the gravitational lens to see a magnified image, in the way that looking through a conventional telescope will yield an image.

Previous discussions of the use of the gravitational lens as a telescope have been notably lacking in key design details. It is one purpose of this discussion to address that lack and begin discussion of the mission which takes notice of problems, as well as the potential, of the mission.

In a previous paper [6], the engineering difficulties of a gravitational lens telescope were pointed out, and the amplification, magnification, and geometrical focal blur were calculated.

Figure 1: Artist's conception of a solar sail making a close pass to the sun [10].

2. Resolution, Focus, and Intensity

An important aspect of the gravitational lens telescope is the resolution, which in principle can be very high. The amount by which the focus deviates from an ideal point, and hence the loss of resolution (or "blur") in the image, is characterized by the point spread function (PSF). The PSF of the optical system can be found from analysis of the individual effects which degrade the focus; the total point spread is the convolution of the individual effects. A good approximation is that the errors in focus are independent, and thus the width of the PSF squared can be approximated as the sum of the squares of the individual components.

The diffraction-limited resolution of the gravitational lens has been the subject of several previous analyses [11-15]. It is useful to calculate the diffraction-limited performance, since diffraction sets a lower limit to the possible resolution of any optical system, however, the main result of such analysis is to show that for optical wavelengths, the resolution of the gravitational lens telescope is limited by factors other than diffraction.

Spherical aberration is inherent in the lens action of a gravitational field, and cannot be easily corrected; the loss of focus due to spherical aberration of the lens is larger than the diffraction limitations for optical imaging applications. The focal blur due to the spherical aberration can be simply calculated from the geometry of the Einstein ring, and turns out to be a blur with a characteristic width of exactly half the diameter of the planet imaged, regardless of the distance or the size of the focal plane [6].

Another cause of defocus is due to the variations in the index of refraction of the solar corona. The focused light passes through the plasma off the corona of the sun, and although the index of refraction of the coronal plasma is close to unity, it is not exactly unity. Figure 1 shows the corona; as can be seen, the variations in density of the corona are large. Hence, these variations in the index of the medium that the focused light passes through (over a pathlength on the order of millions of kilometers) produce a randomness in the phase of the focused light that is considerably larger than the wavelength of the light, on the order of 500 nm. Koechlin *et al.* calculate the uniform deflection due to the index of refraction of the corona in the visible as a 10 meters of deviation of the image from the center of the ideal focal point (figure 3 of reference [14]); although not explicitly calculated in their paper, the variation in this index cause a roughly equal amount of focal blur. Other smaller effects, such as the perturbation due to the gravity of other bodies in the solar system, are also discussed by Koechlin *et al.* . The gravity of Jupiter dominates these effects, and adds an astigmatism to the focal point.

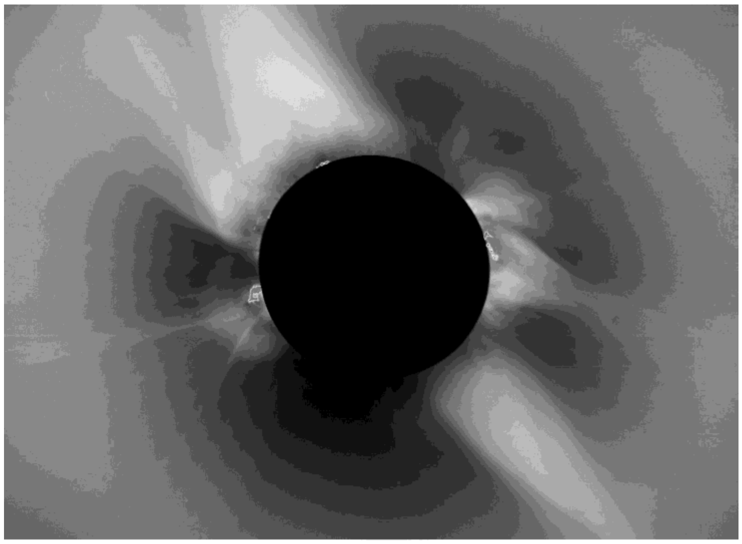

Figure 1: the solar corona and solar prominences, viewed by the UCAR/NCAR High Altitude Observatory during the 1991 solar eclipse.

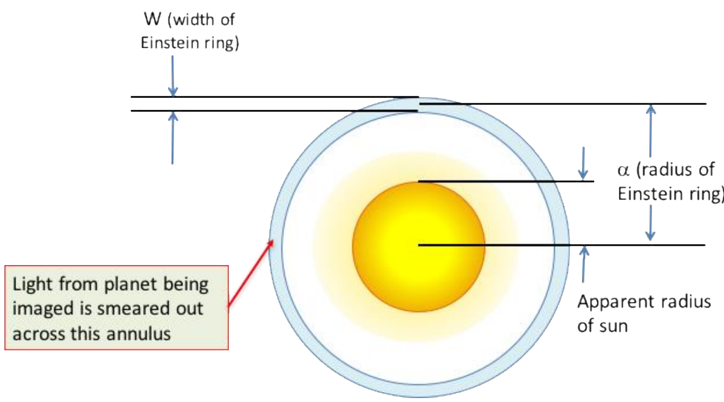

Figure 2: The view from a point along the gravitational focus. The light from the planet being imaged is smeared into an annulus surrounding the solar disk, the "Einstein Ring"; it is this ring which is the light input to the gravitational lens telescope. The radius of the Einstein ring equals the apparent radius of the sun at the minimum focus distance, and increases as the distance increases beyond this. This image is not to scale (if the image were to scale, the width W of the Einstein ring would be to narrow to distinguish).

One expressed advantage of the gravitational lens is the high gain, that is, the amplification of the power received at the telescope, or the number of photons received from the target per unit time*. From the focal point, the target is seen as the Einstein ring surrounding the sun. The amplification occurs because the projected solid angle of the Einstein ring is large than the solid angle subtended by the unmagnified planet.

For a focal plane exactly at the minimum focal distance, this amplification is $2(r_{sun}/r_{planet})(d/F)$. The amplification of the signal from an Earth-diameter planet at 10 LY distance is by a factor of 6400: the gravitational lens multiplies the the number of photons received by a telescope at the focal point by a factor of 6400. Essentially, the telescope receives a number of photons from the whole planet equivalent to that of a telescope of 80 times larger diameter.

3. Difficulties in Implementation

As shown in the previous paper [6], the difficulties in use of the solar gravitational lens as a telescope can be summarized:

1. The size of the image on the focal plane and speed of transit of the image
2. The requirement for an occulter, and the noise due to the corona of the sun
3. The inherent focal blur

4. Mission Approaches to Using the Gravitational Lens

It is clear that using the gravitational lens of the sun as a telescope is considerably more difficult than simply placing a spacecraft at the focal line and looking at the image. The light from the planet being imaged will be distorted into an "Einstein ring" surrounding the sun (Figure 2), and the actual signal being received will consist of the light from that right.

4.1 Image Size

As was pointed out by Genta and Vulpetti [16], the size of the image produced by the gravitational lens is a problem. The size of the image of the planet on the focal plane is proportional to the focal length of the telescope. From geometrical optics, the size of the object being viewed on the image plane, X_i, is related to the size of the object being imaged, X_o, as:

$$X_i = X_o\,(F/d) \qquad (1)$$

where d is the distance to the object and F is the focal length of the telescope, in the case of the gravitational lens telescope, greater than 82 billion kilometers, the minimal focal distance. For the example case of a mission at 0.01 light years (630 astronomical units, slightly farther than the minimal focal distance of 550 AU) targeting a planet at a distance of 10 LY, approximately the distance of Epsilon Eridani, the image of an Earth-sized planet at the focal plane will be 12.5 km in diameter.

The gravitational lens telescope is quite different, then, from a classic telescope: with the image larger than the telescope itself, the imaging plane images only a single pixel of the image.

A telescope that would image the planet in the usual telescope method by placing an imaging array at the focal plane, imager at the focal plane would require a focal plane consisting of an array of elements spanning a plane 12.5 km by 12.5 km. This may not be an insurmountable difficulty, however. The "Breakthrough Starshot" project, for example, envisions high-velocity laser-pushed sails, each of which is about 4 meters in diameter, and each at extremely low in cost. It would be possible to send hundreds, and possibly thousands, of such small probes out,

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* Gain is conventionally expressed in logarithmic units (for astronomical purposes, in magnitude units) while amplification is in linear units; for this paper, I am ignoring the distinction between gain and amplification. If desired, gain in magnitudes can be calculated as 2.5 times the $\log_{10}$ of the amplification factor.

and use each one as a lightbucket to collect light from a single spot on the focal plane.

The planet's orbital motion would carry it across the size of this focal plane in a period of about 40 seconds. This means that an image would have to be acquired very quickly. This motion, however, brings up another possible solution to the problem of the image size: an image could be put together by rastering a detector across the focal plane. Since the planet naturally moves across the focal plane, a line of detectors could image the planet by having the planet's image move across the imaging line.

If the probe carrying the focal plane had sufficient lateral motion capability, a single detector could raster scan the entire image of the planet. However, the pointing and the occultation (discussed below) requirements would make this difficult.

Image brightness is also a problem. The amplification of image brightness can also be calculated from the geometry. The amount of light collected from an example planet at 10 light years is increased by the gravitational lens by a factor of 64,000 times. The gravitational lens means a telescope of 1 meter diameter light collects the same amount of light as a telescope of 80 meter diameter without the lens

But: the speed with which the planet moves past the focal plane reduces the total amount of photons collected. Astronomical telescopes focus on one spot for long periods– sometimes many days. Since the planet will move past focal plane in 40 seconds, the imager at the focal plane cannot perform such long integrations, which are needed for reducing signal to noise ratio.

4.2 Occulter

The minimal gravitational focus distance is defined by the distance from the sun at which the Einstein ring apparently touches the disk of the sun; at distances farther along the line of focus, the apparent size of the disk will be smaller, and the ring will be more visibly separated from the sun. Using the lens as a telescope thus involves collecting the light from ring. This requires pointing the imaging plane of the telescope nearly directly at the sun. The light from the sun will, of course, greatly overwhelm any signal from a planet. The first caution given to anybody on receiving a telescope is always "don't point it directly at the sun" -- but that is, of course, exactly what has to be done in this case. To image the planet, the direct light of the sun must be blocked using either a coronagraph [17] or an occulter. A conventional coronagraph would be difficult, because of the high magnification of the image, although to my knowledge design studies of a coronagraph for this purpose have yet to be done, and it is likely that approaches to solving the problem may be possible. A more straightforward solution would be to use an occulting disk; a second spacecraft in the shape of a circle that is placed between the imaging spacecraft and the sun, exactly blocking the sun: essentially, an artificial eclipse. Use of such occulting spacecraft have been proposed for imaging extrasolar planets, and the main engineering difficulties (such as suppressing the diffraction around the edge of the disk) have been addressed [18,19].

However, the fact that a second occulting spacecraft is needed will considerably complicate the design of any spacecraft that does not sit in a specific place. The approach of sending hundreds of individual sails, each one to collect the light at a single spot on the focal plane, would require an equal number of occulting spacecraft, each one positioned extremely precisely to exactly block the sun.

An occulter does not completely solve the problem, since it will block the light from the disk of the sun, but not the corona. As visible in figure 1, the corona will be a considerable background brightness, and the noise due to the corona of the sun implies limits to the effective usefulness of the lens. Discussions of gravity lens telescopes tend to emphasize the high magnification of the telescope, but it's important to point out that the signal source is still only 3.5 arc seconds across, so even with the high magnification, the absolute intensity of the signal is still small. The coronal light, combined with the required short exposures due to the planet's orbital motion, will make achieving a high signal to noise ratio problematical.

The simplest solution to this problem is to move to a position further along the focal line, in which the Einstein ring being imaged is further from the sun, and hence is moved to a position further out from the brightest part of the inner corona. However, to move to a position where the Einstein ring is, say, one solar radius from the sun makes the mission much more difficult: this would require a mission four times as far, about 2200 AU instead of the 550 AU typically discussed for the mission.

4.3 Focal Blur

From geometrical optics, discussed in the previous paper [6], it is possible to calculate the geometrical component of the focal blur. Technically, the focal blur is due to the large (and negative) spherical aberration of the lens: the part of the lens focusing at a given distance is only a narrow annulus; regions of the lens further from the axis (that is, rays travelling farther to the sun) focus to a distance farther away, while regions closer to the axis (rays closer to the sun) focus to a closer distance.

Although the central spot on the target planet is intensified relative to the rest of the planet; most of the light received at the focal plane is not from the central spot; it is the progressively further out of focus light from different parts of the planet being imaged. Focal blur is inherent in the gravitational lens; it does not change with position or magnification.

Figure 3 shows a candidate planet being imaged, with the amount of light coming from each portion of the image noted. The amount of light coming from an area inside a radius r is directly proportional to r. Thus, 50% of the light reaching the focal point comes from the area inside the 50% diameter; and 50% comes from outside the 50% diameter (regardless of the fact that the inner 50% of the diameter comes to only 25% of the area). Likewise, 10% of the light reaching the detector comes from the central 10% of the diameter, accounting for 1% of the planet's area. Thus, although the central regions are weighed more heavily in the amplification, the outer parts have proportionally larger area. Thus, if we define the focal blur as the circle inside which half the light originates, *the focal blur is exactly half the diameter of the planet, regardless of the size of the planet.*

Correcting the focal blur could be done if the telescope at the focus was able to resolve the width of the Einstein ring. But because of the radial demagnification of the gravitational lens, the width of the Einstein ring is half the angular width of the planet, and hence any telescope that could resolve the width of the Einstein ring could image the planet directly, without need for the gravitational lens.

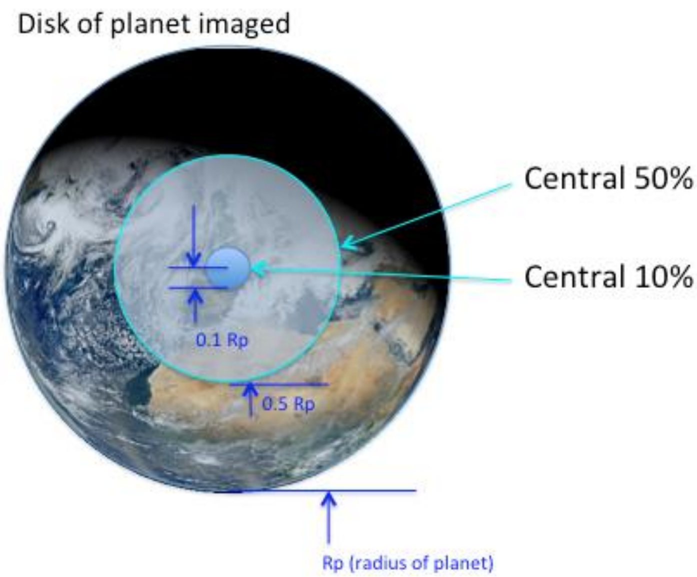

Figure 3. Focal blur of the image of a candidate planet

With sufficiently sophisticated deconvolution technique, it may be possible to sharpen the image by scanning across the planet, using the fact that the portion of the image closer to the axis contributes proportionally more to the total image, and also possibly taking the rotation of the planet into account. However, it is clear that the magnification of the planetary image at kilometer scales cannot be achieved.

4.4. Imaging the Einstein Ring

It was earlier stated that the surface of the planet was smeared out into the Einstein ring. More precisely, the gravitational lens maps the surface of the planet onto the Einstein ring. It is interesting to look in more detail at this mapping.

The point exactly on the optical axis is mapped to the central circle of the Einstein ring. Every other point on the planet is mapped onto the Einstein ring twice, in mirror imaged once inside and once outside of the central circle. This is shown in schematic in figure 4.

The width of the Einstein ring, of course, is far too narrow to be resolved by a telescope at the focal plane. However, it takes only a relatively modest telescope to resolve the circumference of the ring, 2.5 arc seconds at the closest focal distance. Each point on the circumference of the Einstein ring averages the light received from a stripe across the planet's disk (with each stripe repeated twice at positions 180° around the disk). It is possible to think that the planet's area can be reconstructed by these stripes.

At any given position of the telescope at the focus of the gravitational lens, there is a mirror ambiguity in that there is no way to distinguish light from the left end of the stripe from the right end. This ambiguity may be resolved by views from more than one location of telescope at the gravitational focus, or by allowing the moving image of the planet to pass across the telescope, thus changing the center spot.

More to the point, however, in general the image of the planet will not be fully illuminated. For the planet in half phase, the mirror ambiguity is irrelevant, since the dark side of the planet does not contribute. A planet in crescent phase would reduce the ambiguity even further. As the planet rotates, different areas of the surface will rotate into and then out of view. Multiple images of the planet may be required to distinguish the changes due to planetary rotation from the changing cloud patterns on the planet.

Since the Einstein ring being imaged has a radius of about 3.5 arc seconds, the resolution of the planet will depend on the resolution of the telescope at the focal plane. The 2.4-meter mirror of the Hubble Space Telescope, for example, has a diffraction-limited resolution of 0.05 arc seconds and an actual resolution of 0.1 arc seconds. Thus, if the Hubble telescope were used as the imager at the focal point, about 110 segments around the arc of the Einstein ring could be resolved, and hence the resolution at the planet would, if the image could be deconvoluted, be on the order of about $1/220^{th}$ of the circumference (assuming that the planet is imaged in half- or crescent phase, so only half the disk is imaged). In principle, as the planet rotates, this might map the planet to a resolution of about 200 km.

This thus produces a completely different concept for imaging the planet: rather than making an image with multiple detectors at the focal plane, as in a conventional telescope, a single telescope is used, but the light from different parts of the Einstein ring is imaged separately. The signals from the different parts of the ring are then deconvoluted using an algorithm to reconstruct the planet.

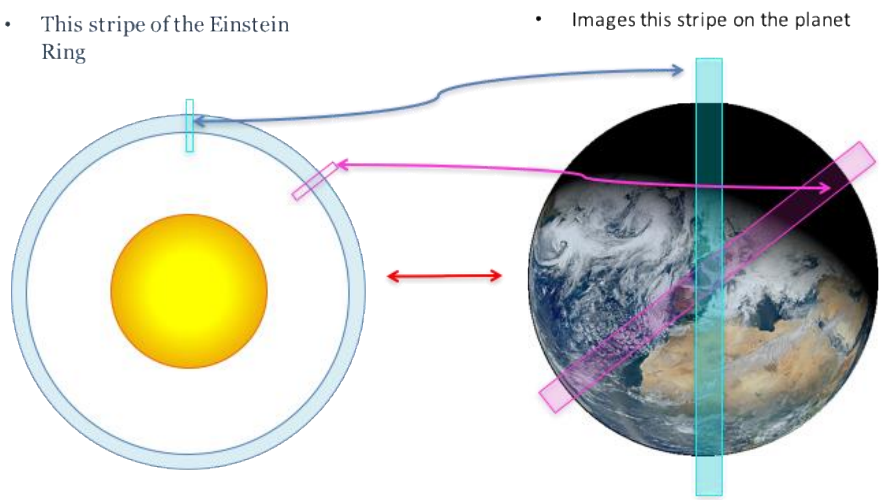

Figure 4. Mapping of the planet to the Einstein ring. Each section of the Einstein ring maps to a stripe across the image of the planet. (Not to scale).

5. Conclusions

Due to the bending of light by gravity, the gravity of sun forms a lens, which, in principle, can be used as a gravitational lens telescope. A mission to the gravitational focus could use the gravitational lens telescope to image an extrasolar planet around a nearby star.

The difficulties include the required pointing, the size of the image on the focal plane, the speed of motion of the image across the focal plane, the requirement for an occulter to remove the brightness of the sun itself from the image, the interference of the brightness of the primary star of the target planet, the signal to noise ratio produced by the brightness of the solar corona, and the fact that the inherent aberration of the lens means that the focal blur of the image will be equal to half the diameter of the planet imaged.

The difficulties are not necessarily fatal flaws: clever approaches may make it possible to use this large telescope and avoid some or all of the problems. In particular, an approach is pointed out where by examining slices of the Einstein Ring, the surface of the planet might be reconstructed. However, it is clear that a mission to use the gravitational lens as a high magnification telescope would be much more complicated than simply simply sending a spacecraft equipped with an imaging plane to the minimal focal distance of 550 AU from the sun.

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