# Stability and Control Derivative Estimation for the Bell-Shaped Lift Distribution

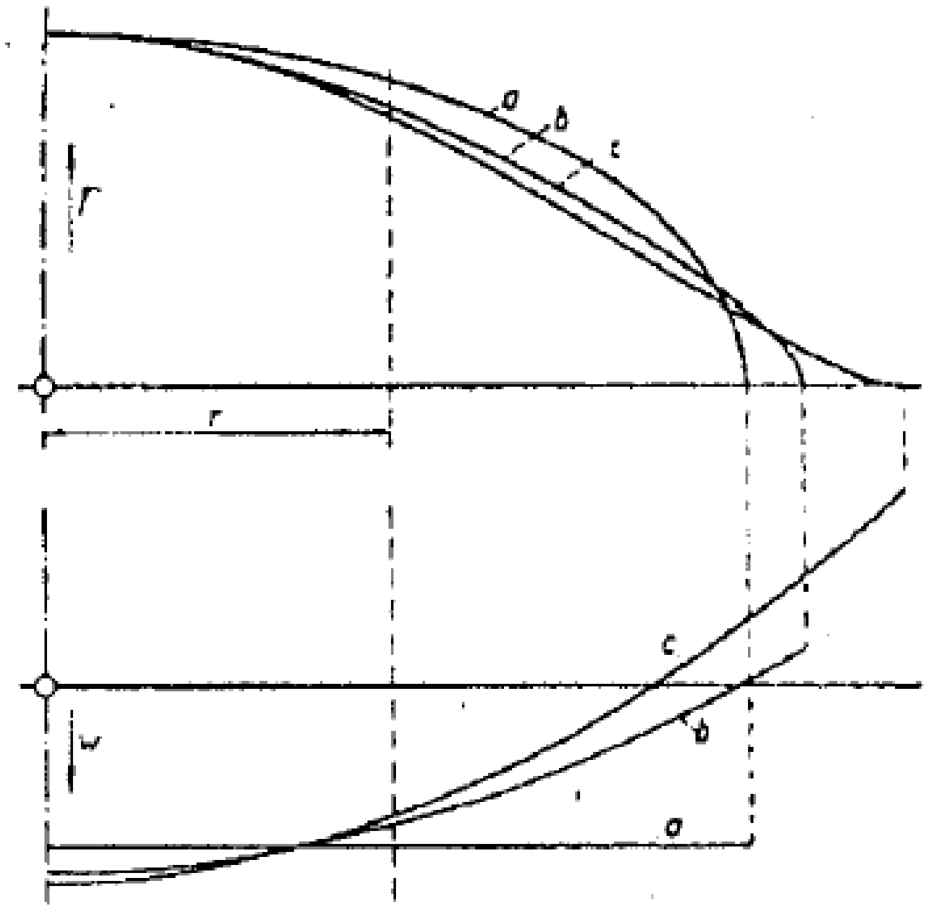

Loren J. Newton

The University of California, Berkeley

NASA Armstrong Flight Research Center

# Introduction: The Bell-Shaped Lift Distribution

- Ludwig Prandtl, 1933
- Minimum induced drag solution for a wing of constrained mass
- Results:
  - **11% less induced drag**, 22% greater span than the elliptical spanload (solution for a wing of defined span)
  - Upwash at the wingtips
  - Proverse yaw & tailless flight



# Introduction: PRANDTL-D

- Preliminary Research AerodyNamic Design To Lower Drag
- Uninhabited, unpowered flying wings with the Bell-Shaped Lift Distribution
  - Prandtl-1: Lightly instrumented proof of concept (12.3’ span)
  - **Prandtl-2: Flight computer-equipped data acquisition (12.3’ span)**
  - Prandtl-3: Pressure/strain data for spanload measurement (25’ span)

# Flight Test Procedures

- Edwards AFB lakebeds
- Average flight ~ 90 sec.
- Elastic high-start launch
- Doublet maneuvers: square wave input to control surfaces
  - Pitch
  - Roll
- 2-3 doublets per flight

# Flight Dynamics

$$\dot{V} = -\frac{\bar{q}s}{m}C_D + g\left(\cos\phi\ \cos\theta\ \sin\alpha\ \cos\beta + \sin\phi\ \cos\theta\ \sin\beta - \sin\theta\ \cos\alpha\ \cos\beta\right)$$

$$\dot{\alpha} = q - \tan\beta\left(p\cos\alpha + r\sin\alpha\right) - \frac{\bar{q}sR}{mV\cos\beta}C_L + \frac{gR}{V\cos\beta}\left(\cos\theta\ \cos\phi\ \cos\alpha + \sin\theta\ \sin\alpha\right)$$

$$\dot{\beta} = p\sin\alpha - r\cos\alpha + \frac{\bar{q}sR}{mV}C_Y + \frac{gR}{V}\left[\cos\beta\ \cos\theta\ \sin\phi - \sin\beta\left(\cos\theta\ \cos\phi\ sin\alpha - \sin\theta\ \cos\alpha\right)\right]$$

$$I_x x\dot{p} - I_{xy}\dot{q} - I_{xz}\dot{r} = \bar{q}sbC_l R + \left[qr\left(I_y y - I_z z\right) + \left(q^2 - r^2\right)I_{yz} + pqI_{xz} - prI_{xy}\right]/R$$

$$I_y y\dot{q} - I_{yz}\dot{r} - I_{xy}\dot{p} = \bar{q}sbC_m R + \left[pr\left(I_z z - I_x x\right) + \left(r^2 - p^2\right)I_{xz} + qrI_{xy} - pqI_{yz}\right]/R$$

$$I_z z\dot{r} - I_{xz}\dot{p} - I_{yz}\dot{q} = \bar{q}sbC_n R + \left[pq\left(I_x x - I_y y\right) + \left(p^2 - q^2\right)I_{xy} + prI_{yz} - qrI_{xz}\right]/R$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

$$\dot{\phi} = p + \tan\theta\left(r\cos\phi + q\sin\phi\right)$$

# Flight Dynamics

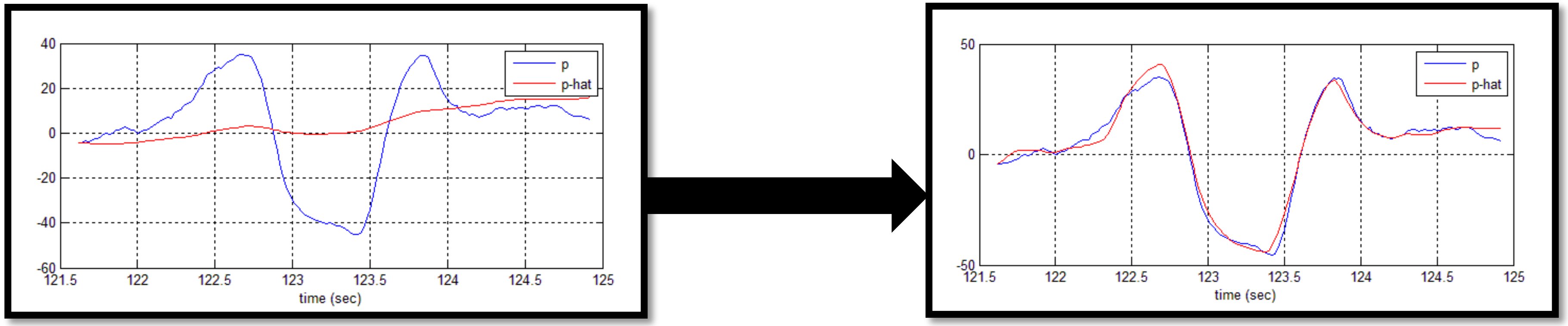
$$C_A = C_{A_0} + C_{A_\alpha}\alpha + \frac{c}{2VR}C_{A_q}q + C_{A_{\delta e}}\delta e$$
$$C_N = C_{N_0} + C_{N_\alpha}\alpha + \frac{c}{2VR}C_{N_q}q + C_{N_{\delta e}}\delta e$$
$$C_m = C_{m_0} + C_{m_\alpha}\alpha + \frac{c}{2VR}C_{m_q}q + C_{m_{\delta e}}\delta e$$

$$C_L = C_N \cos\alpha - C_A \sin\alpha$$
$$C_D = C_A \cos\alpha + C_N \sin\alpha$$

$$C_Y = C_{Y_0} + C_{Y_\beta}\beta + \frac{b}{2VR}\left(C_{Y_p}p + C_{Y_r}r\right) + C_{Y_{\delta a}}\delta a$$
$$C_l = C_{l_0} + C_{l_\beta}\beta + \frac{b}{2VR}\left(C_{l_p}p + C_{l_r}r\right) + C_{l_{\delta a}}\delta a$$
$$C_n = C_{n_0} + C_{n_\beta}\beta + \frac{b}{2VR}\left(C_{n_p}p + C_{n_r}r\right) + C_{n_{\delta a}}\delta a$$

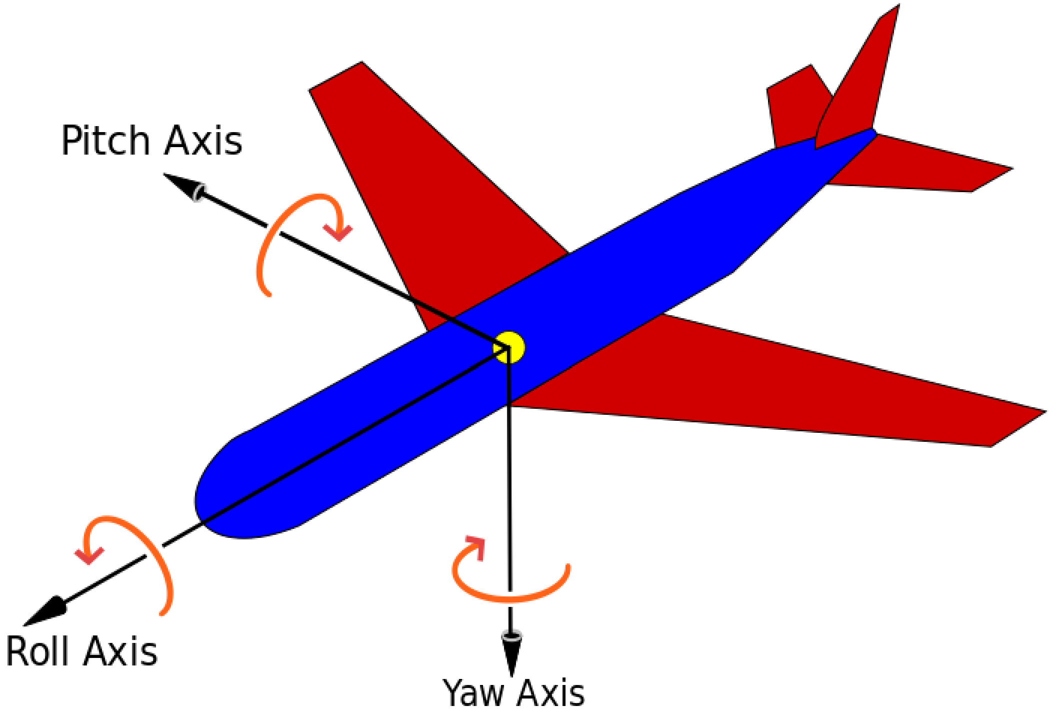
# Parameter Estimation

- Method for determining stability and control derivatives from flight data
- Derivatives are varied in the aircraft equations of state until the mathematical model matches recorded flight data
- NASA Dryden code: MATLAB pEst MX.96



# Flight Data Conversion

- Isolate doublets in data time histories
- Adjust units to pEst convention
- Correct axes and signs to flight control/pEst convention
- Define constants (geometry, mass properties) to pEst



# Stability & Control Derivative Estimation

- Different pEst scripts for lateral and longitudinal maneuvers
  - Lateral: estimated $\beta$, $p$, $r$, $a_y$ signals
  - Longitudinal: estimated $\alpha$, $q$, $a_n$ signals
- User input: selecting estimating weights $W_{ii}$ for each signal $i$
- Algorithm minimizes cost function: summed squared difference between flight data and model estimate, scaled by $W$

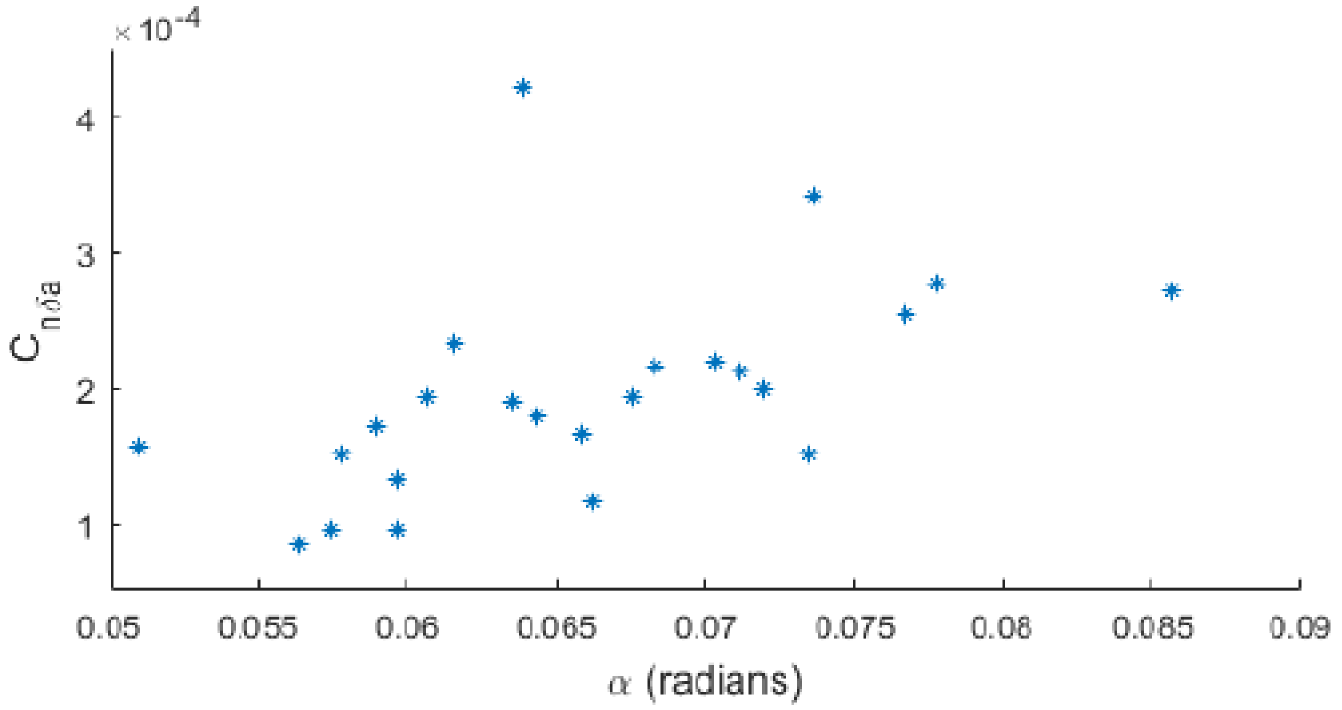
$$J = \frac{1}{2n_z n_t} \sum_{i=1}^{n_t} [z(t_i) - \tilde{z}(t_i)]^T W [z(t_i) - \tilde{z}(t_i)]$$

# Stability & Control Derivative Maps

- Prandtl-2 flew entirely in the linear regime
- Linear regressions were created for each S&C derivative with respect to $\alpha$
- Data points were weighted by the inverse of the Cramer-Rao bound error estimated by pEst
- Applicable to lookup tables in simulation

# Results: Unique Flight Dynamics

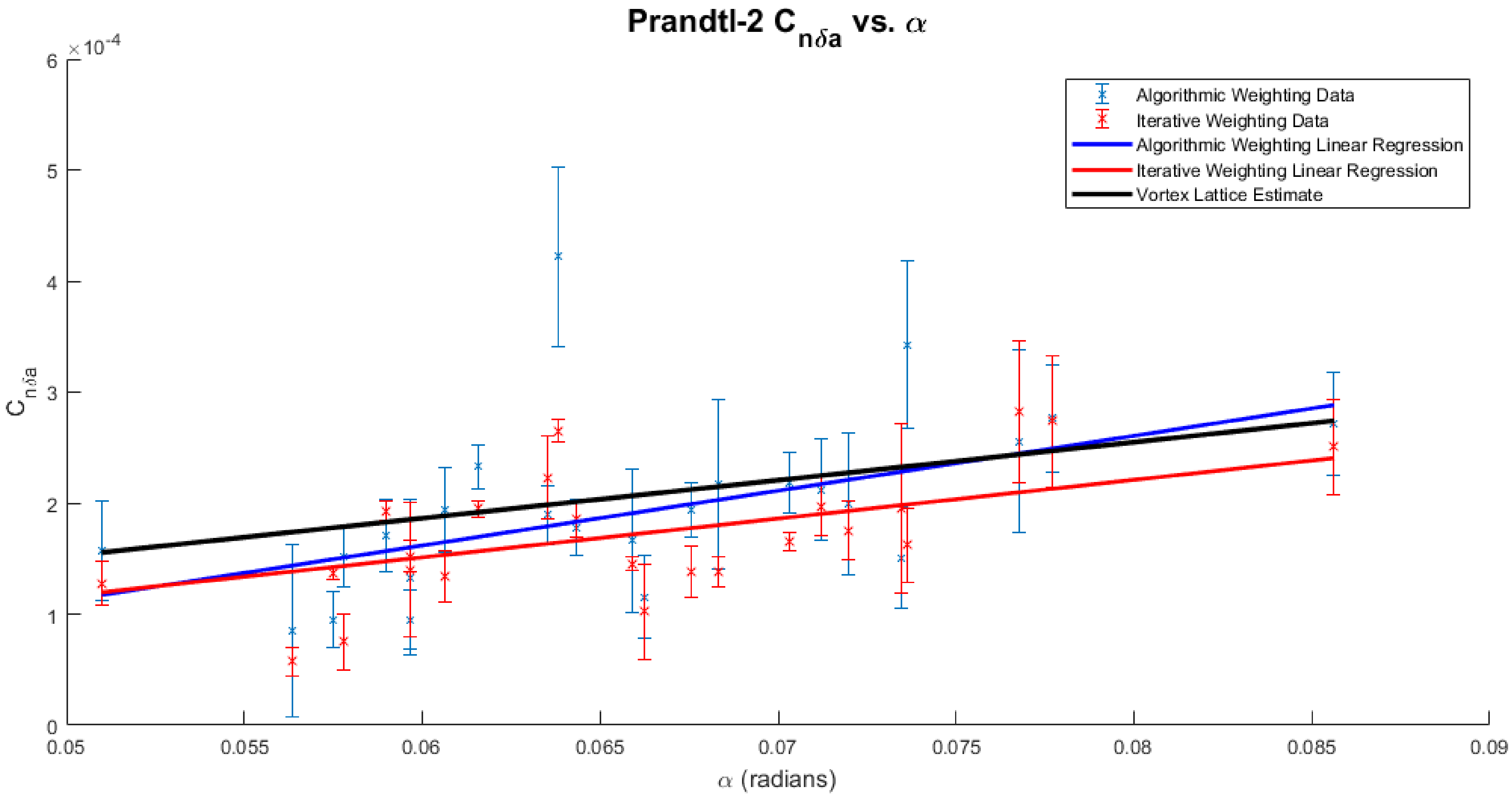
- $C_{n\delta a}$: nondimensional yawing moment due to aileron deflection
  - Quantifies how the aircraft responds in yaw due to a roll command
  - Sign specifies nature of yaw/roll coupling



# Results: Algorithmic User Input

- Demonstrated algorithmic weight selection to accelerate analysis
- Normalize square of signals by $W_{ii} = [range(i)]^{-2}$
  - Cost function distributes error evenly as a percent error of each signal
- 2 data analysis teams: 1 algorithmic weight selection, 1 iterative "trial and error" weight selection

# Results: Algorithmic User Input



# Conclusions & Future Steps

- Prandtl-2 flight testing returned sufficient flight data to quantify the flight dynamics of the Bell-Shaped Lift Distribution equipped vehicle.
- Parameter Estimation was used to determine, from flight data, the characteristic stability and control derivatives. Two different teams with different weight selection schemes produced agreeing results.
- A positive $C_{n\delta a}$ provided quantifiable evidence of proverse yaw.
- Potential future steps: Prandtl-3 spanload measurements, PRANDTL-D flight dynamics simulator, autopilot development

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# Questions?

# References


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# Appendix: Nomenclature

- *A* = axial force
- *b* = reference span
- $C_i$ = nondimensional coefficient of force or moment *i*
- $C_{mn}$= nondimensional stability/control derivative: coefficient of *m* due to *n*
- *c* = reference chord
- *D* = drag force
- *g* = gravitational acceleration
- $I_{jk}$ = moment of inertia
- *L* = lift force
- *l* = rolling moment
- *M* = vehicle mass
- *m* = pitching moment
- *N* = normal force
- *n* = yawing moment
- $n_t$= number of time steps
- $n_z$ = number of signals
- *o* = coefficient bias
- *p* = roll rate
- *q* = pitch rate
- $\bar{q}$ = dynamic pressure
- *R* = conversion parameter: 180/π
- *r* = yaw rate
- *s* = reference area
- *V* = equivalent airspeed
- *W* = weighting matrix
- *Y* = side force
- *z* = measured signal
- $\tilde{z}$ = estimated signal
- *α* = angle of attack
- *β* = angle of sideslip
- *ξ* = set of signal/estimate pairs
- *φ* = roll angle
- *θ* = pitch angle
- *ψ* = yaw angle
- *δe* = elevator deflection
- *δa* = aileron deflection