A Telescope at the Solar Gravitational Lens:
Problems and Solutions

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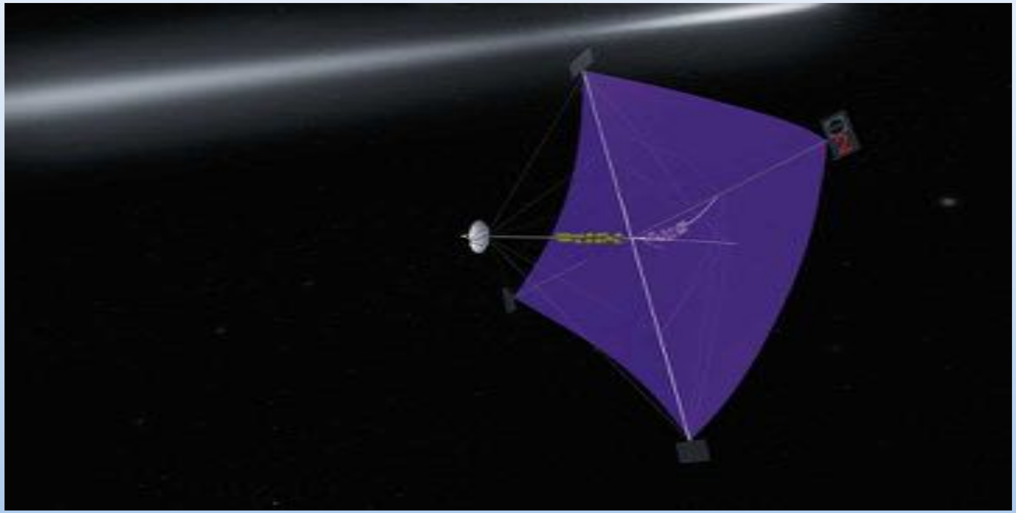

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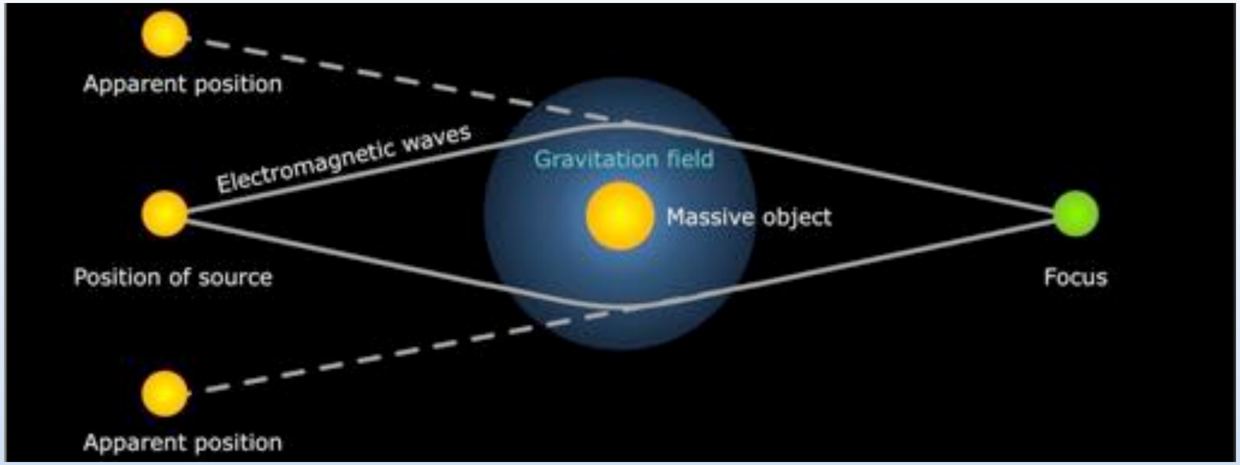
Background

- The fact that the gravity of a massive body deflects light is one of the consequences of Einstein's general theory of relativity. This effect means that the sun (or any massive body) can effectively act as a lens.



- For the sun, the focal distance of the lens starts about 550 astronomical units (AU) from the sun (the "minimum gravitational focal distance").

Background

- Starting with Eshleman in 1979, there have been several suggestions that a mission to the gravitational focus could use the sun as a powerful telescope.
- The proposed benefit is that the gravitational lens of the sun is, in some sense, a telescope with an area comparable to the size of the sun, far larger than any telescope currently conceived.
- Could be used to image the surfaces of extrasolar planets?

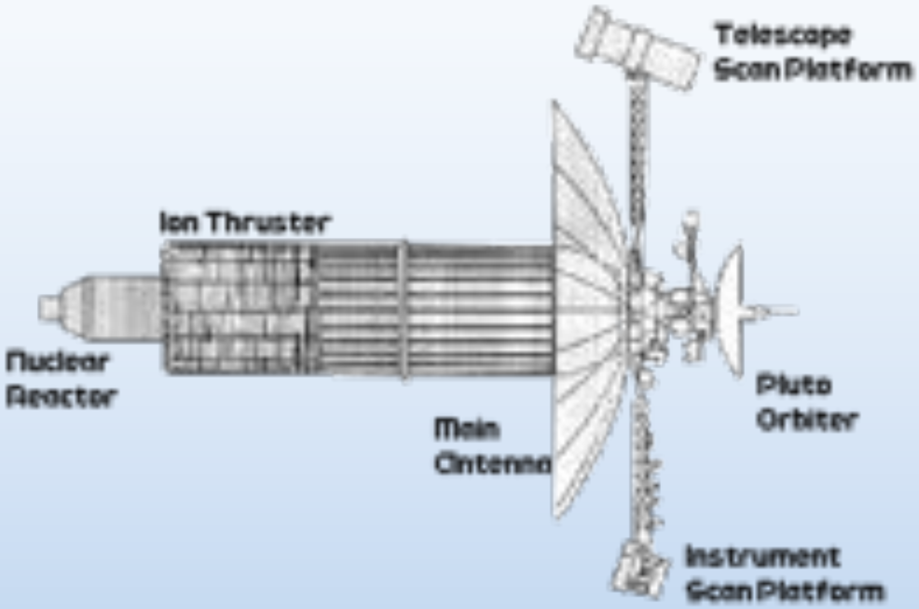
Yes: the *theoretical* magnification would be high enough to not just image extrasolar planets, but to map their surface

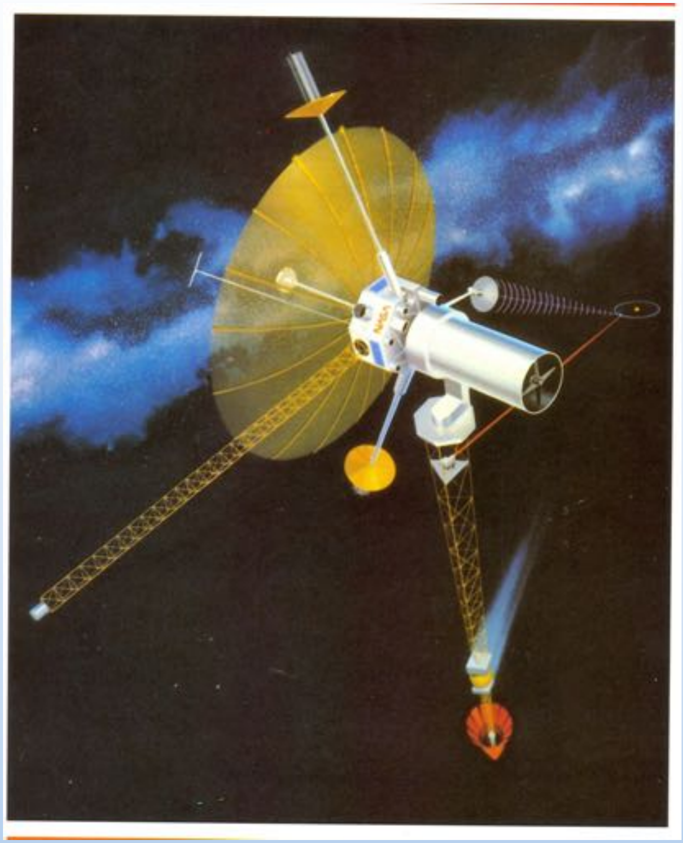
...but there are practical difficulties.

Challenge of the Mission

- A mission to 550 AU– more than ten times farther than Pluto-- is difficult. The mission is interesting specifically **because** it is difficult.
  - The lure of the mission is that the gravitational focus of the sun is one of very few possible places to send a mission that is beyond the Kuiper belt, but at less-than-interstellar distances:
  - An interstellar precursor mission: "merely" tens of billions of kilometers, not trillions of kilometers
- The challenge of the mission to the necessary distance is difficult, but possible: proposed methods of reaching the focal distance include high specific-impulse electric propulsion or use of a solar sail.

Proposed TAU (Thousand Astronomical Unit) mission





- JPL conceptual mission design 1987

Solar- or laser-pushed lightsail

Solar sail making a close perihelion pass to the sun

Physics

The gravitational deflection angle of light passing a massive body is

- $\theta = (4GM/c^2)(1/r)$ (1)

where r is the distance by which the rays being focused miss the center of the sun. The distance F to the gravitational focus is:

- $F = (4GM/c^2)^{-1} r^2$ (2)

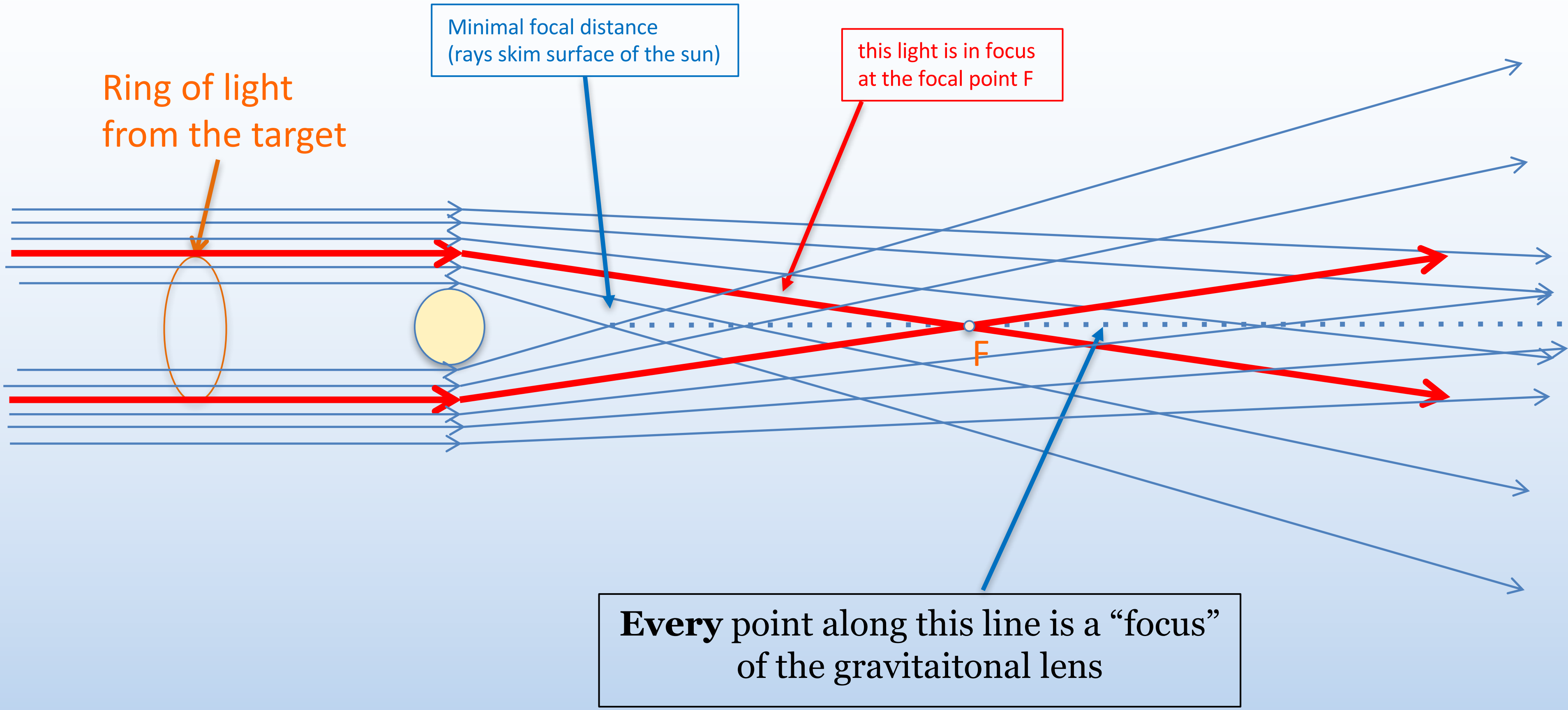
The condition required to use of the gravitational lens as a telescope is that the light from the distant target reaching the focus must not intersect the surface of the sun, and hence r must be greater than the radius of the sun (about 700,000 km).

Inserting $r = R_{sun}$ defines the minimum gravitational focus.

For the sun, this minimum focal distance (at which the focussed rays just skim the surface of the sun) is ~550 AU

--any distance beyond this will **also** be on the focal line

Gravitational lens:
not a single focus, but a **line of focus points**



As the focus point moves more distant from the sun, the rays that are in focus are the ones from rings passing progressively farther from the sun

(not to scale)

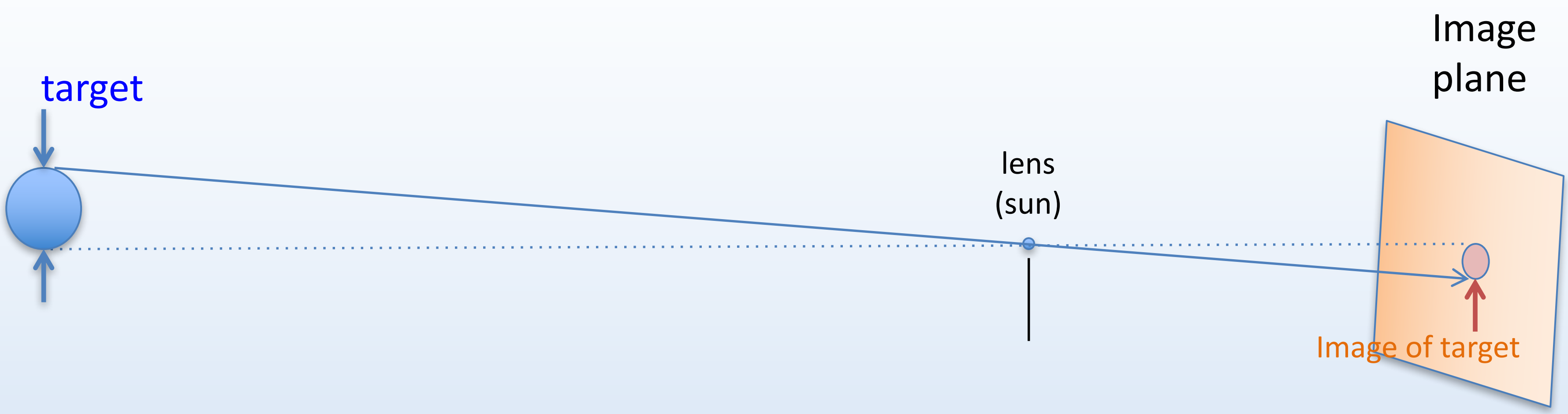
Pointing

- The lens is pointed at the target exactly behind the sun.
- to re-aim to a new target 1° away, the spacecraft at the minimum focal distance would have to move laterally 10 AU
  - equivalent to the distance from Earth to Saturn.
  - In practice, such a telescope is not repointable.
- Thus: **a telescope at the gravitational focus is necessarily going to be a single-purpose telescope**, with the target of observation selected before the mission is launched.

To aim at an Earth-sized planet 10 light years away, the telescope will have to be positioned in space to within an accuracy of about 10 km

Difficulties associated with the pointing and corresponding image size were previously pointed out by Giancarlo Genta and Giovanni Vulpetti, "Some Considerations on Sun Gravitational Lens Missions," *Journal of the British Interplanetary Society, Vol. 55* (2002), pp 131-136.

Gravitational lens –geometrical optics
(not to scale)



- The lens is a telescope with a focal length ≥ 550 AU
- The gravitational lens produces an image at the focal plane.
- The size of the image can be calculated from geometrical optics.

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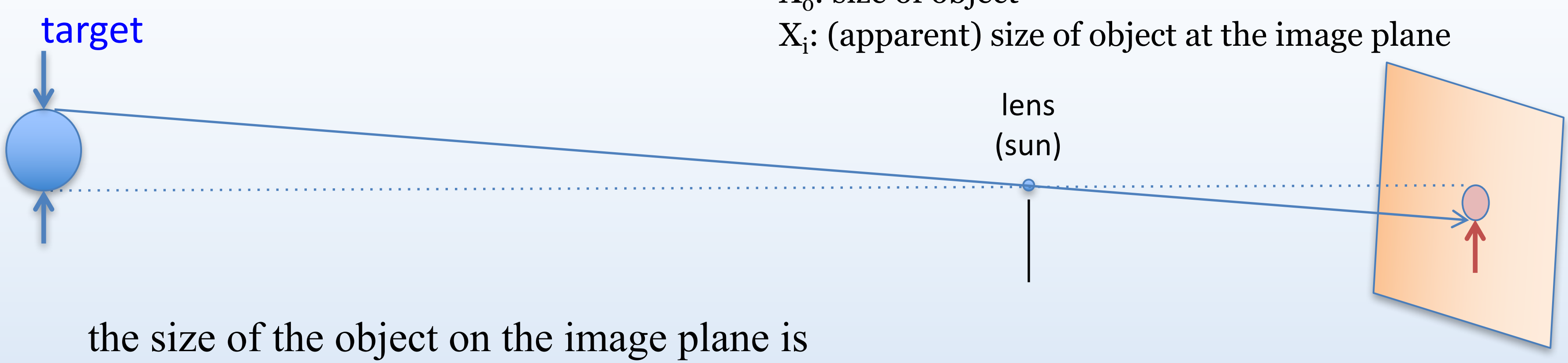
Gravitational lens – focal geometry

(not to scale)

F: focal distance (distance from lens to image plane)
d: distance from lens to object being observed
X_o: size of object
X_i: (apparent) size of object at the image plane



the size of the object on the image plane is

$$X_i = X_o\ (F/d)$$

Example case:

Focal plane F= 630 AU (0.01 light year)

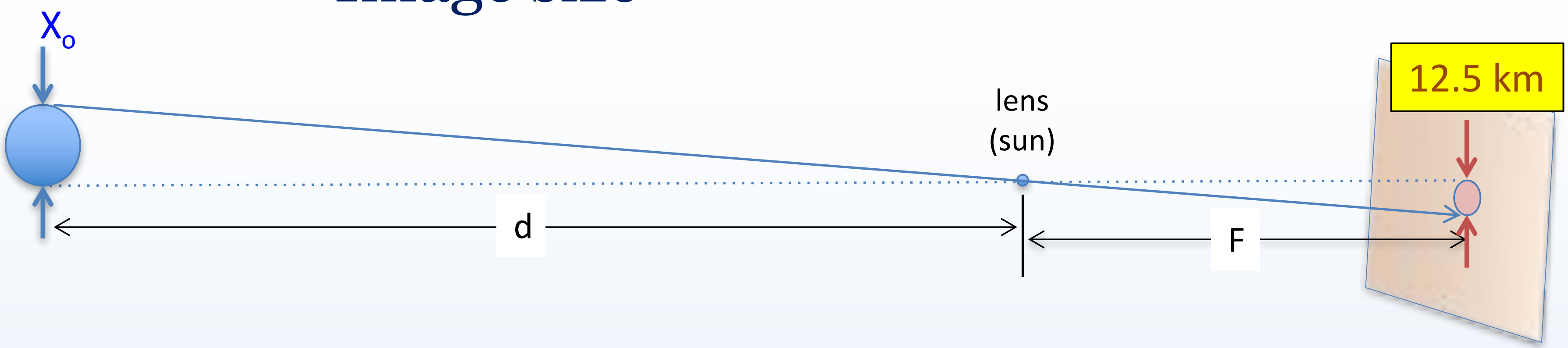
the target: planet around a star at d= 10 LY (~ Epsilon Eridani)

Image produced at the focal plane is smaller than the planet by factor of 1000.

If the exoplanet imaged is the diameter of the Earth:

- **the image at the focal plane will be 12.5 km in diameter.**

Image size

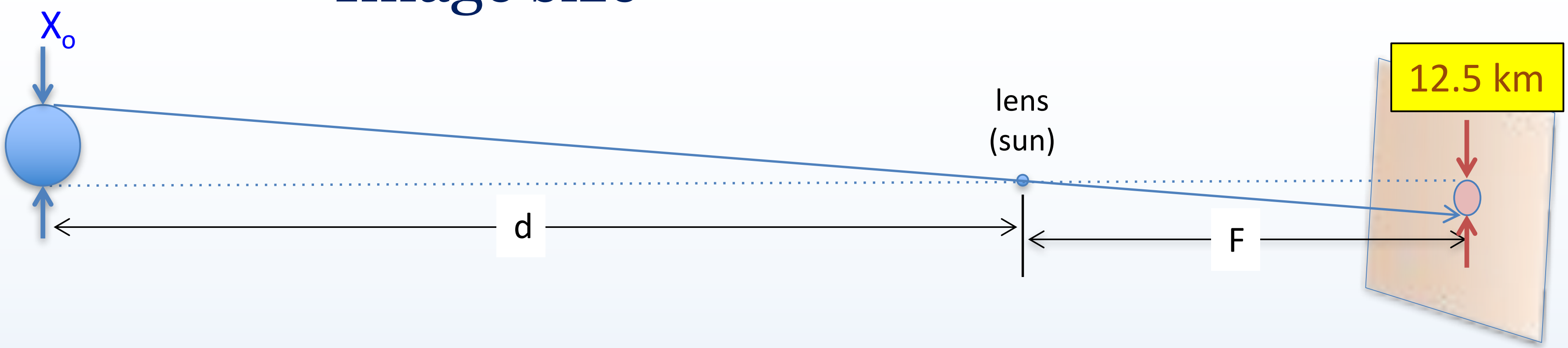


The large size of the image produces practical difficulties in a real mission geometry. Unlike a "real world" telescope, it is unlikely that we can make a focal plane sensor 12 kilometers in diameter.

The gravitational lens telescope does not work like an ordinary telescope, with an imaging array at the focal plane

The image at the focal plane is larger than the spacecraft

Image size



The large size of the image produces practical difficulties in a real mission geometry. Unlike a "real world" telescope, it is unlikely that we can make a focal plane sensor 12 kilometers in diameter.

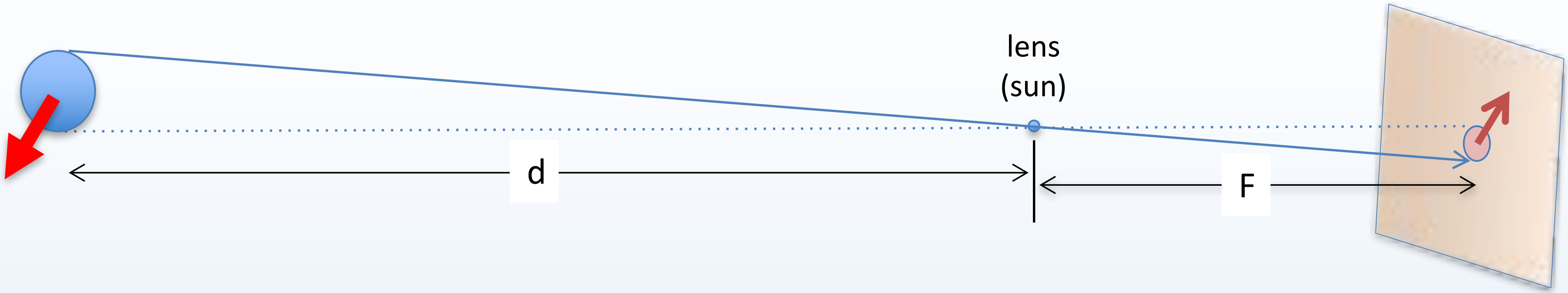
The result is that the telescope does not image the planet, but instead a small fraction of the planet.

- For the example case, a focal plane detector one meter in dimension would image a 1-km area on the surface of the planet, *etc**.
- Imaging the full planet would require a focal plane that was a 12.5 km by 12.5 km array of individual telescopes

One way of mapping the planet might be to scan ("raster") an imager across the planetary focal plane.
Another way would be to send a large number of small spacecraft (a "swarm") to slightly different places on the focal line, each imaging a single pixel of the planet

*(As noted in later, focal blur will make the actual area imaged much larger).

Motion of image



The planet will not stay in the field of view for very long.

- If the planet is orbiting at the same orbital velocity as the Earth, 30 km/sec, the 1-km section of planet being imaged will traverse a 1-meter focal plane in 33 milliseconds.
- Rather than imaging the location on the planet, the spot will be motion blurred.

The entire diameter of the planet will pass across a given spot on the focal plane in 42 seconds

Can we use this feature? This means that at the image plane, the image of the planet will scan across a detector by its own motion. To produce a full 2D image would thus only require a linear array of detectors

On-axis gain

- The gravitational lens is interesting because it has very high gain
- The geometrical magnification of a point exactly aligned with the center of the sun is theoretically infinite.
- But the magnification of any non-zero area is finite.

Definitions:
Magnification, amplification, and gain

- The signal *amplification* is defined as the flux received at the focus of the optical system, divided by the flux that would be received without the optical system.
- The *gain* is the amplification expressed in logarithmic units.
- *Magnification* is the angular size of the target object as viewed at the focus, divided by the angular size without the optical system.
  - (In this case, we are interested in the *area* magnification, expressed in terms of solid angle, not the linear magnification.)
- Because of the brightness theorem, amplification and magnification are equal.

The focused light from behind the sun forms an *Einstein Ring*

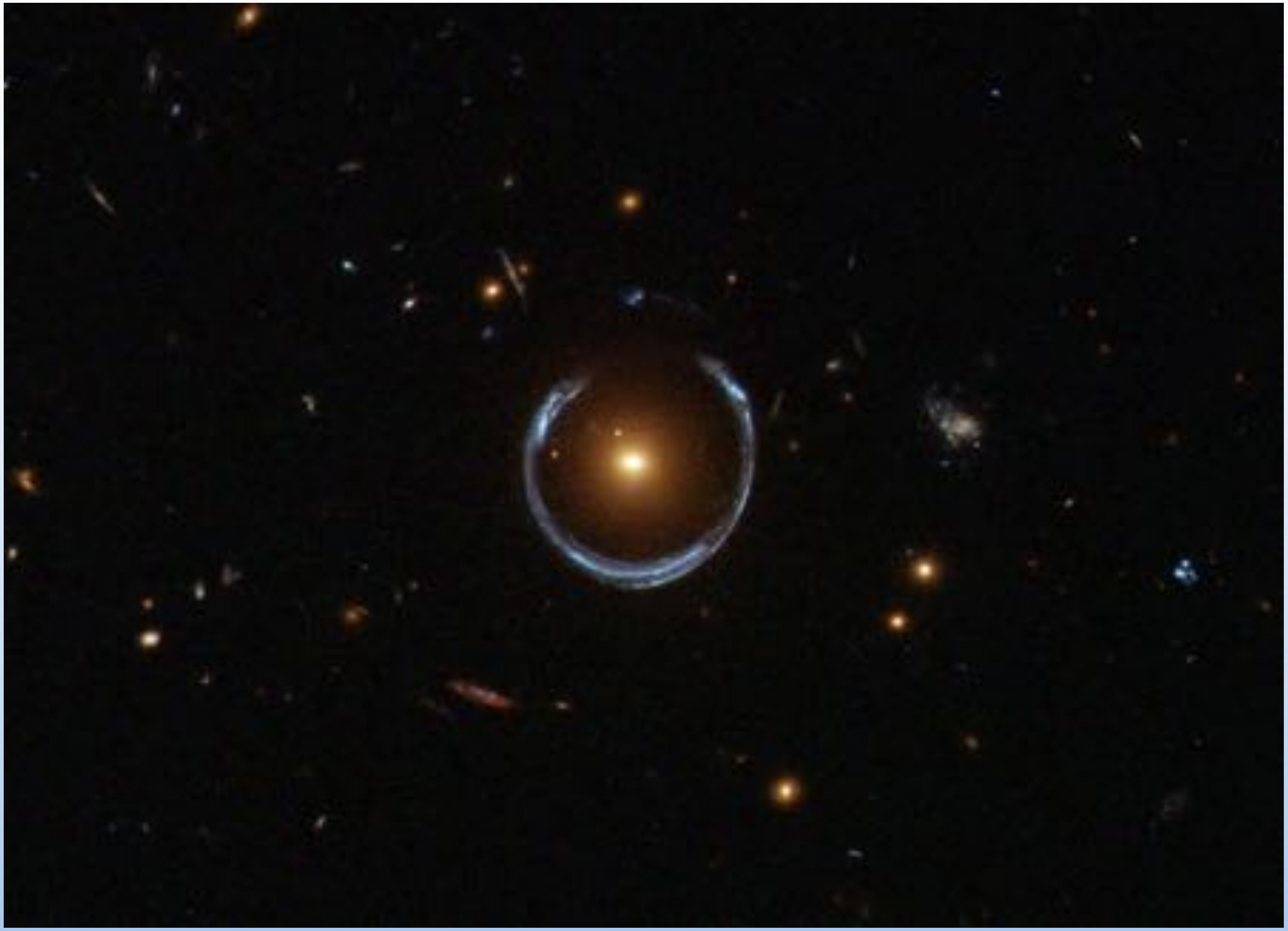
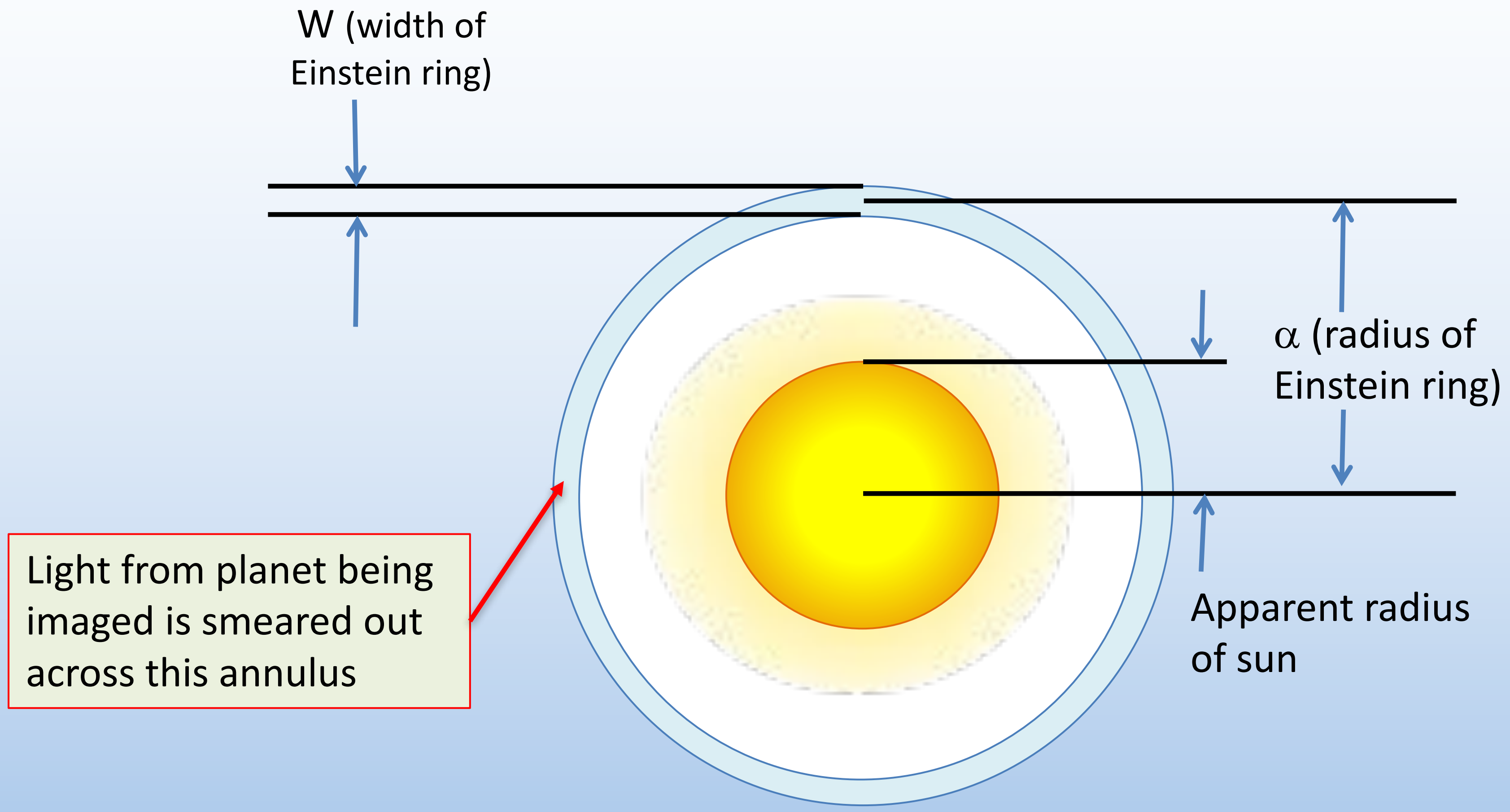


Image Credit: [ESA/Hubble & NASA]

An Einstein ring, viewed from the Hubble Space Telescope
https://apod.nasa.gov/apod/ap111221.html

Gravity lens: view of target from focus

(not to scale)

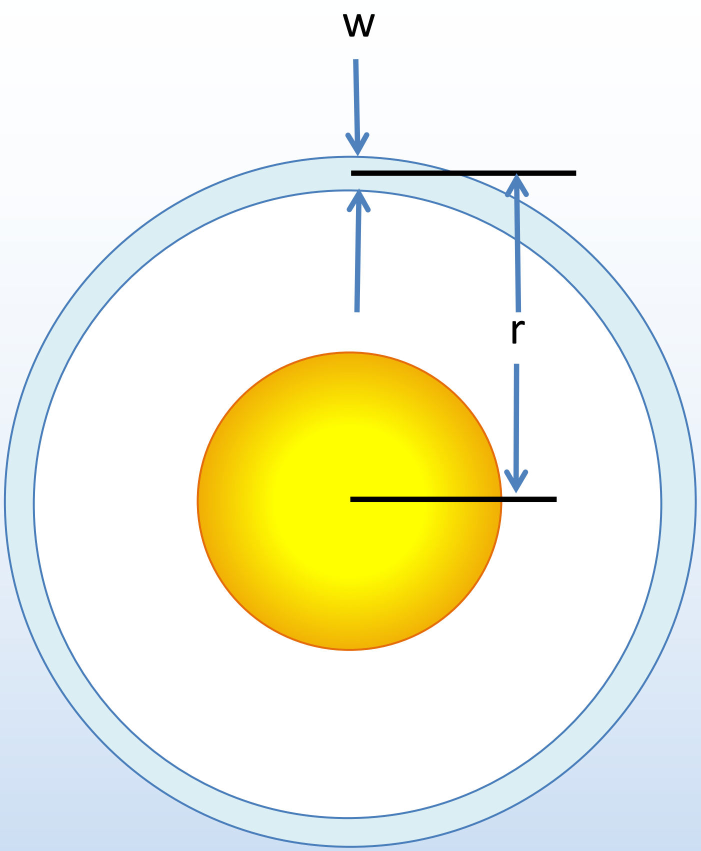


At the minimum gravitational focus distance, this Einstein ring just touches the visible surface of the sun

Magnification

- Example case: telescope is imaging an extrasolar planet at 10 Light year distance
- Magnification is the (angular) area of the Einstein Ring (the smeared image of the planet) divided by the (angular) area of the planet with no lens, Ao/Ai.

Area of annulus



Gravity lens: view from focus of Einstein ring produced by target (not to scale)

r = angular radius of Einstein ring
w= angular width of Einstein ring
Ao = (solid angle) area of Einstein ring

Area of annulus Ao = 2πrw

but for w = a/2
(for a = angular diameter of source)
thus Ao = πar

- Total area magnification equals Ao/Ai
  - Ai= (angular) area of source
  - for source of diameter d at distance x, angular diameter is a=d/x
  - angular area of source Ai = $\pi a^2/4$
- **Magnification factor Ao/Ai = $(\pi ar)/(\pi a^2/4) = 4r/a$**

- For the case where the focal plane is exactly at the minimum focal distance, the angle α is simply r/F, and the amplification is $2(r_{sun}/r_{planet})(d/F)$.
- For an earth-diameter planet at 10 LY distance, amplification = 6400.

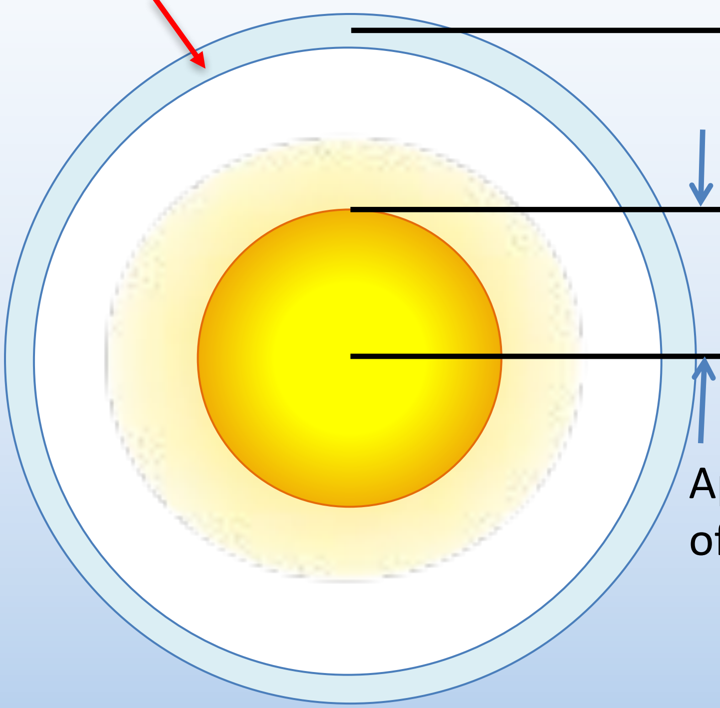
Solar Corona

Image by Espenak, Eclipse 2001

Gravity lens: view of target from focus

(not to scale)

Light from planet being imaged is in this annulus



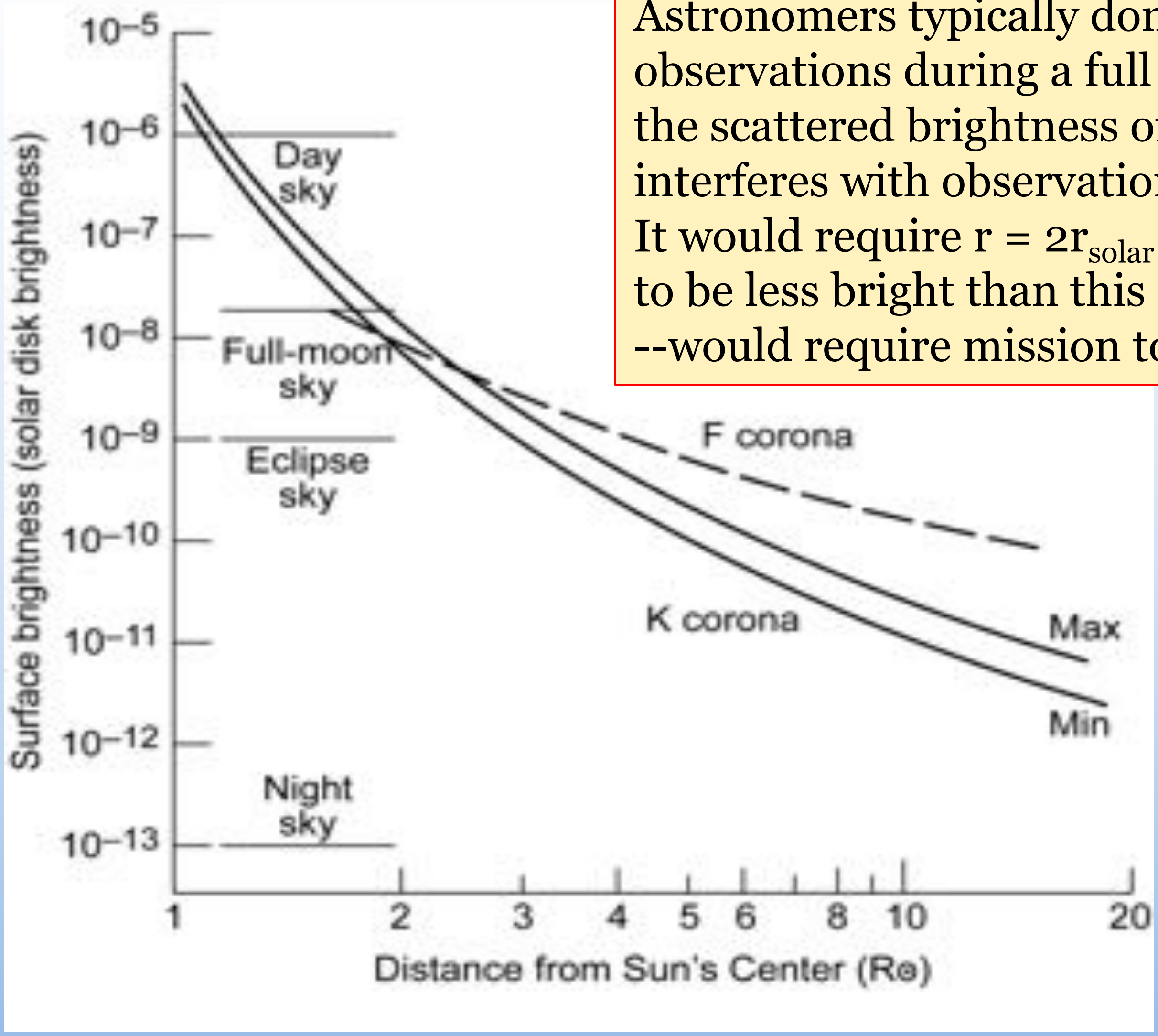
At the minimum gravitational focus, the Einstein ring just touches the visible surface of the sun

View of the solar corona (seen during eclipse)

1. The disk of the sun itself will have to be blocked by a coronagraph (an “artificial eclipse”). This will complicate the design of the mission

2. If the Einstein ring is near the disk of the sun, the corona will be brighter than the planet being imaged. The actual mission will have to be more distant than 550 AU!

Signal to noise ratio: solar corona



Astronomers typically don't do observations during a full moon, because the scattered brightness of the sky interferes with observation.
It would require $r = 2r_{solar}$ for the corona to be less bright than this
--would require mission to **2200 AU**

Image brightness

The amplification of image brightness can also be calculated from the geometry
The amount of light collected from an example planet at 10 light years is increased by the gravitational lens by a factor of 64,000

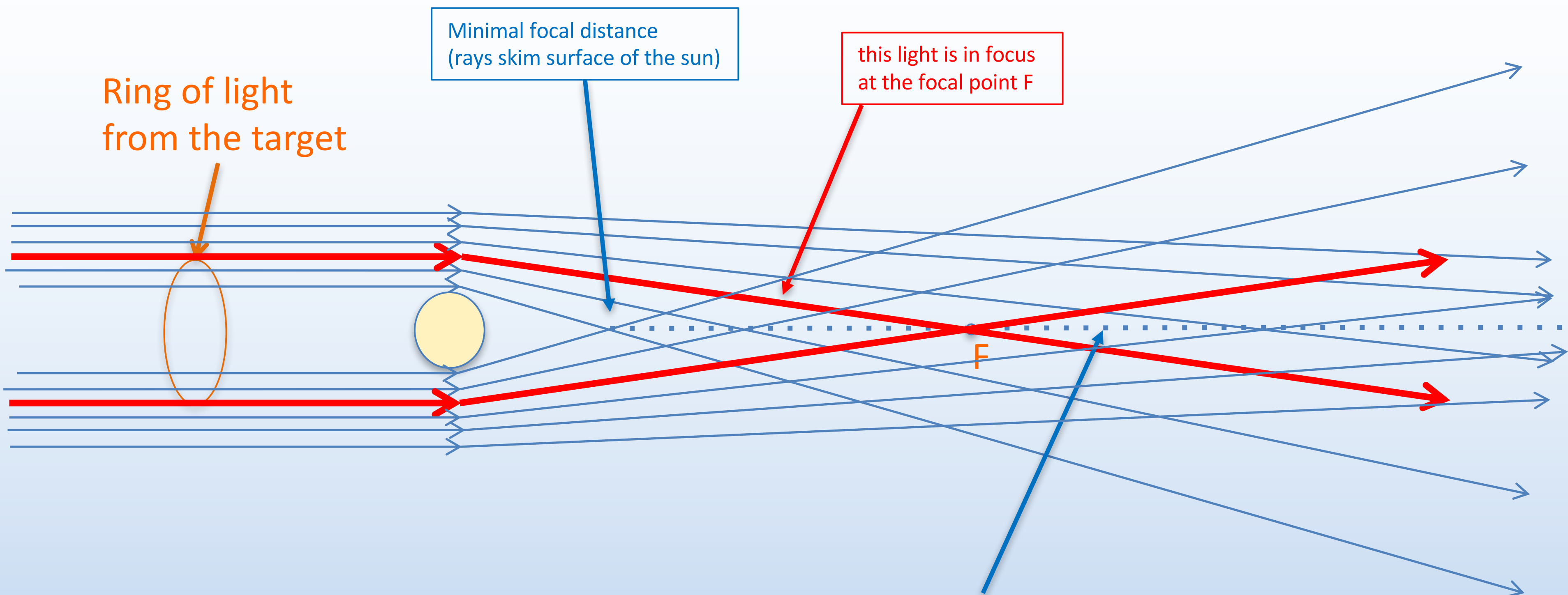
- The gravitational lens means a telescope of 1 meter diameter light collects the same amount of light as a telescope of 80 meter diameter without the lens

But: the speed with which the planet moves past the focal plane reduces the total amount of photons collected

- Astronomical telescopes focus on one spot for long periods– sometimes many days
- Planet will move past focal plane in 40 seconds
- Long integrations are needed for reducing signal to noise ratio

Image by Steve Bryson of August 2017 solar eclipse

Gravitational lens:
line of focus points

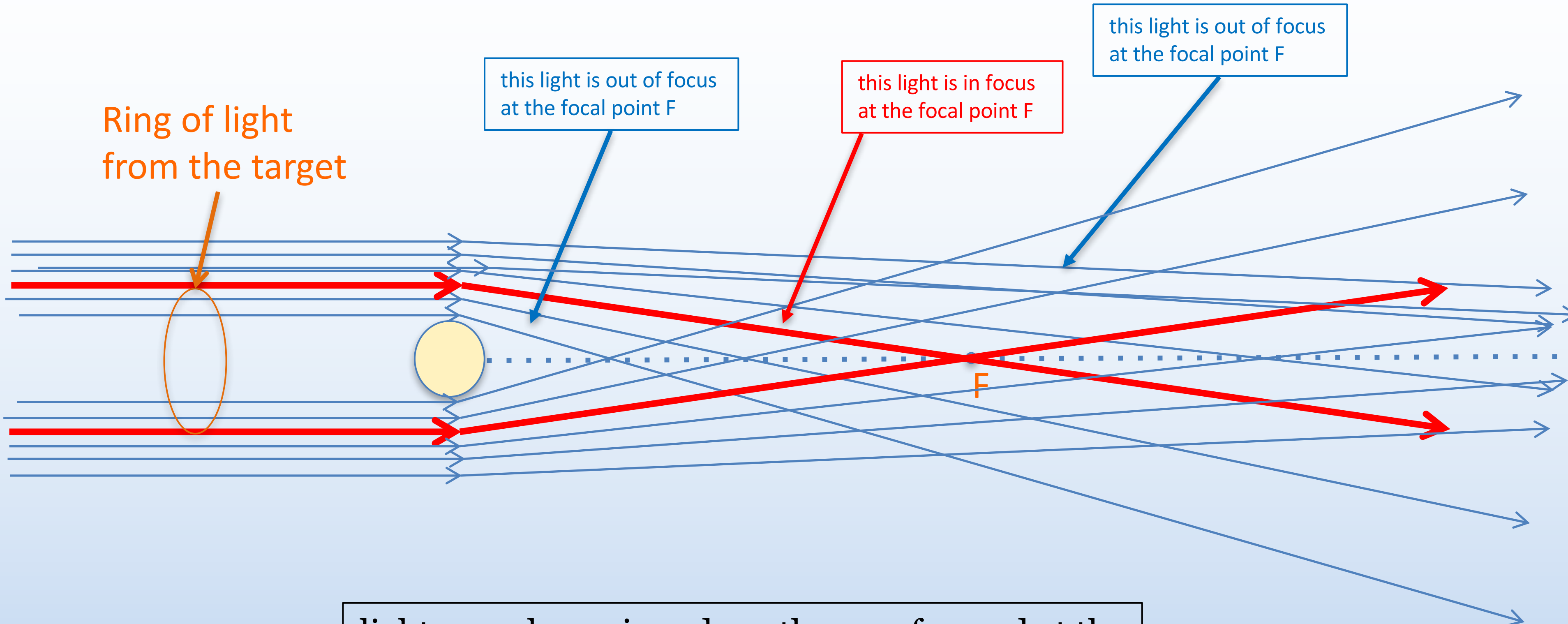


Every point along this line is a "focus" of the gravitaitonal lens

As the focus point moves more distant from the sun, the rays that are in focus are the ones from rings passing progressively farther from the sun

(not to scale)

Gravitational lens:
spherical aberration



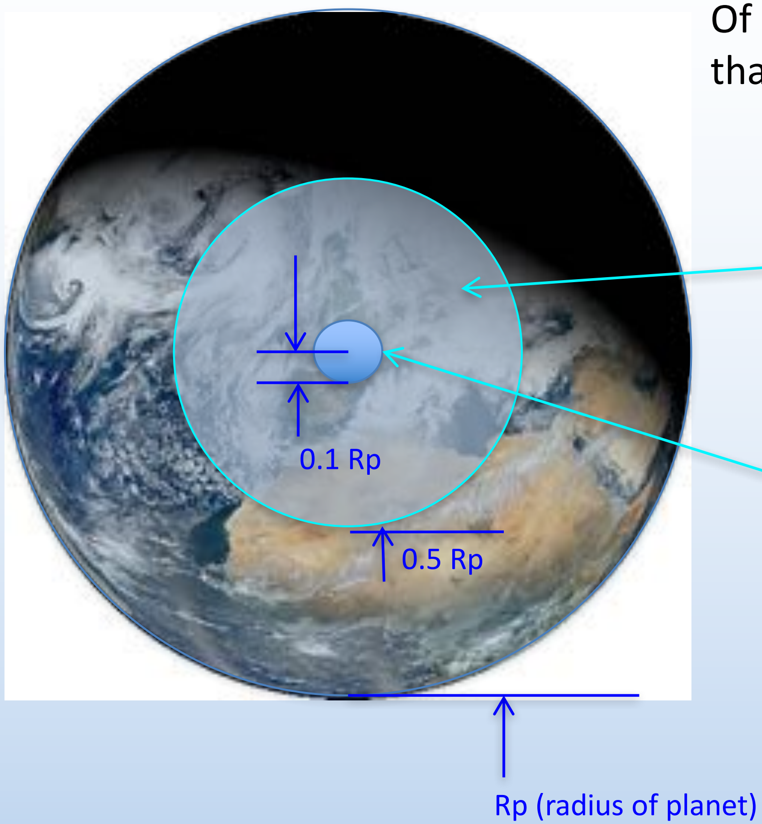
light rays shown in red are the rays focused at the chosen focal distance F

Other rays are out of focus
-this is (negative) spherical aberration

(not to scale)

Focal blur

Disk of planet imaged



Of the light from the disk of the target planet that reaches the focal point:

50% of light comes from area outside this ring (75% of planet's disk area)

50% of light comes from area inside the this ring (25% of disk area)

- **Focal blur (FWHM) is half the radius of the planet imaged**

10% of light comes from the area within the circle at radius 0.1 Rp (1% of planet disk area)

Although the central spot on the target planet is intensified relative to the rest of the planet (geometrical magnification of central spot is infinite... but only for an infinitesimal area); **most of the light received at the focal plane is not from the central spot**

Focal blur is inherent in the gravitational lens (does not change with position or magnification):

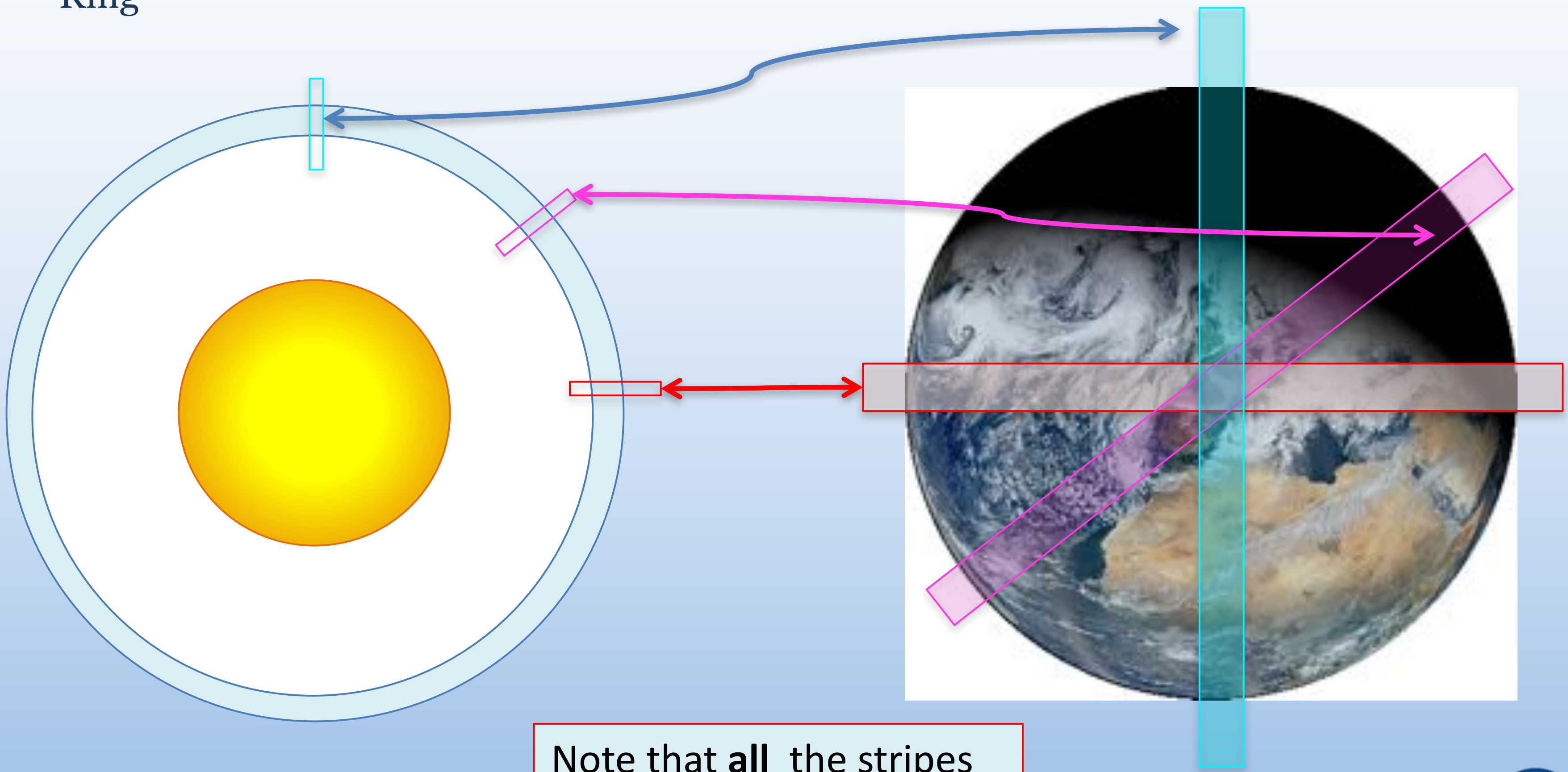
- Correcting the focal blur would require a telescope at the focus that could resolve the width of the Einstein ring
- But because of the radial demagnification of the gravitational lens, the width of the Einstein ring is half the angular width of the planet
- a telescope that could resolve the width of the Einstein ring could image the planet directly, without need for the gravitational lens

Structure of Einstein Ring

in this example case, the planet is centered on the optical axis

- Images this stripe on the planet

- This stripe of the Einstein Ring



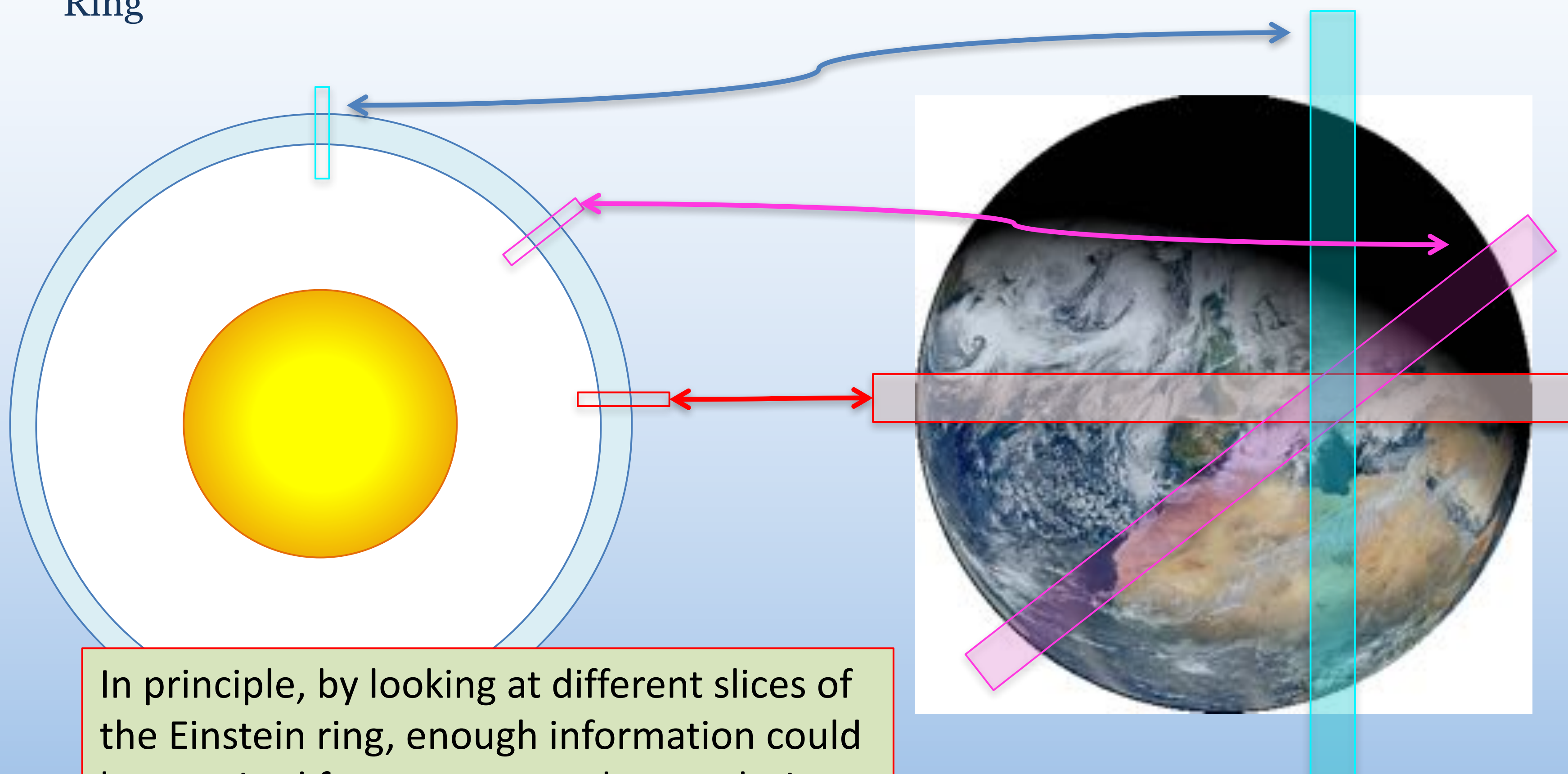
Note that **all** the stripes include the center point

Structure of Einstein Ring

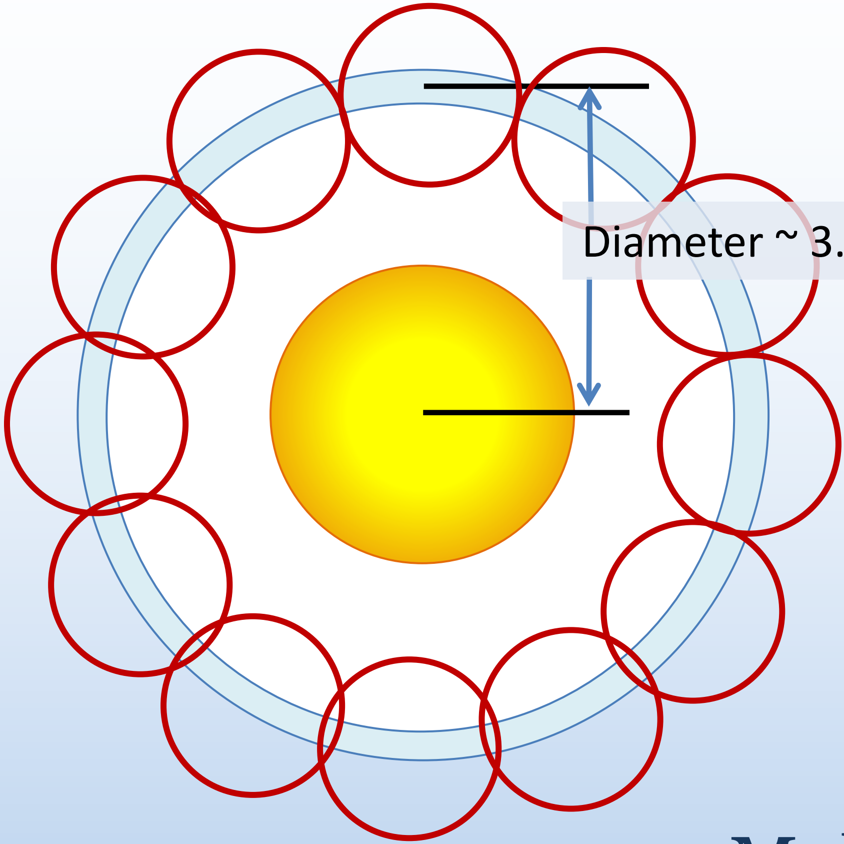
in this example case, the planet is not centered on the optical axis, but is displaced slightly to the right

- Images this stripe on the planet

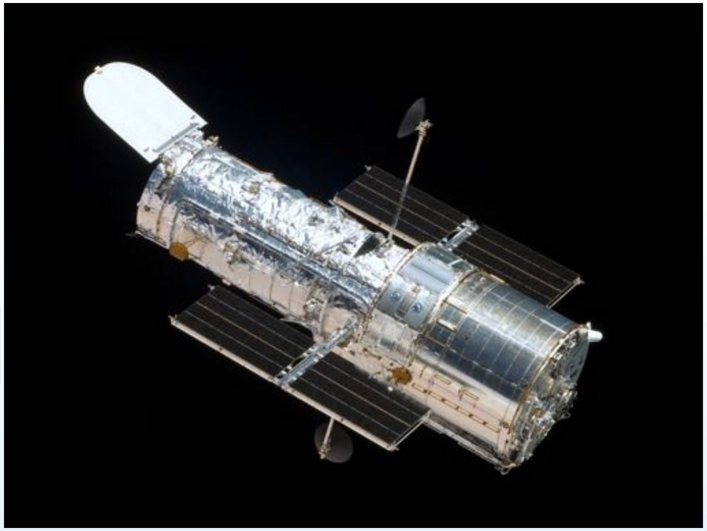
- This stripe of the Einstein Ring



In principle, by looking at different slices of the Einstein ring, enough information could be acquired for a computer deconvolution to reconstruct the planet



Gravity lens: view from focus
of Einstein ring produced by target
(not to scale)



Hubble Space telescope
2.4 meter diameter mirror
diffraction-limited resolution 0.05 arc seconds
*actual resolution 0.1 arc second)

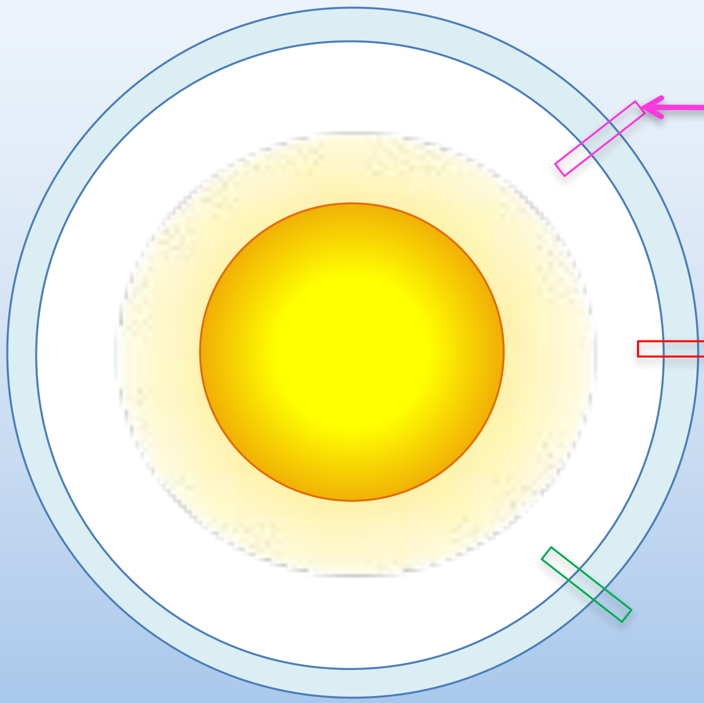
- Multiple spots around the Einstein Ring can be resolved

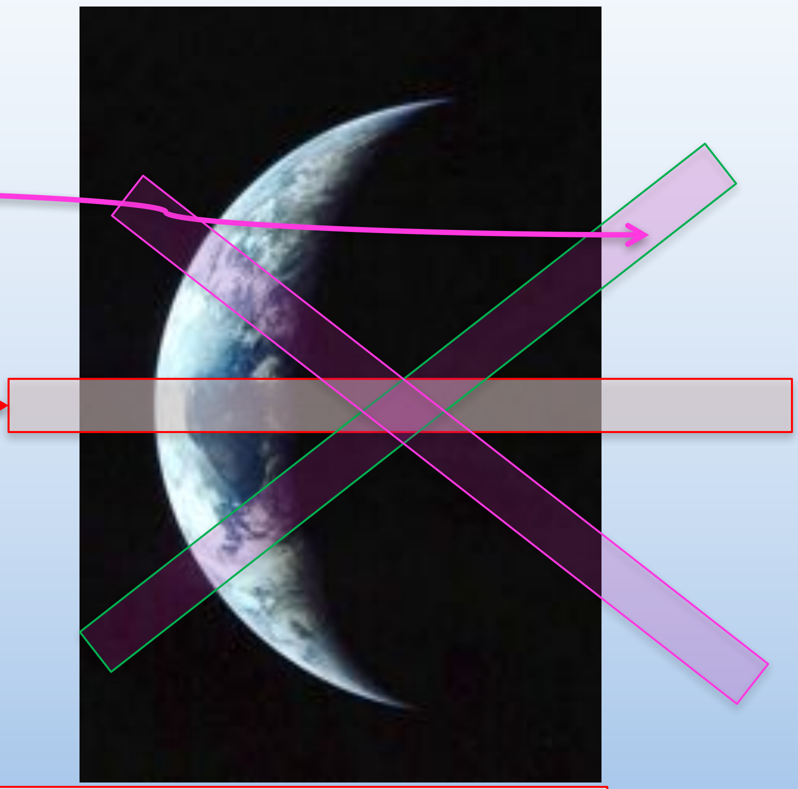
Einstein Ring on a crescent planet

in this example case, the planet is centered on the optical axis

- This stripe of the Einstein Ring

- Images this stripe on the planet





Resolving the Einstein ring on a planet in crescent phase eliminates the mirror ambiguity (because the mirror side is in darkness)

Imaging a crescent planet

- Focal blur is less of a problem on a crescent planet
- Blur is still technically half the planet diameter: but most of the planet is dark
- Dark portion does not contribute

Conclusions

According to general relativity, the gravity of sun forms a lens, which could be used as a telescope.

Here, I have pointed out several difficulties with a mission to use the gravitational lens of the sun to image an extrasolar planet around a nearby star.

The difficulties include:

- the required pointing,
- the size of the image on the focal plane
- the speed of motion of the image across the focal plane,
- the requirement for an occulter to remove the brightness of the sun itself from the image,
- the signal to noise ratio produced by the brightness of the solar corona,

Einstein Ring viewed by Hubble

Conclusions (continued)

- and the fact that the inherent aberration of the lens means that the focal blur of the image will be equal to half the diameter of the planet imaged.

The difficulties are not necessarily fatal flaws: clever approaches may make it possible to use this large telescope and avoid some or all of the problems. In particular, an approach is suggested where imaging different slices of the Einstein Ring would allow the planet's disk to be resolved

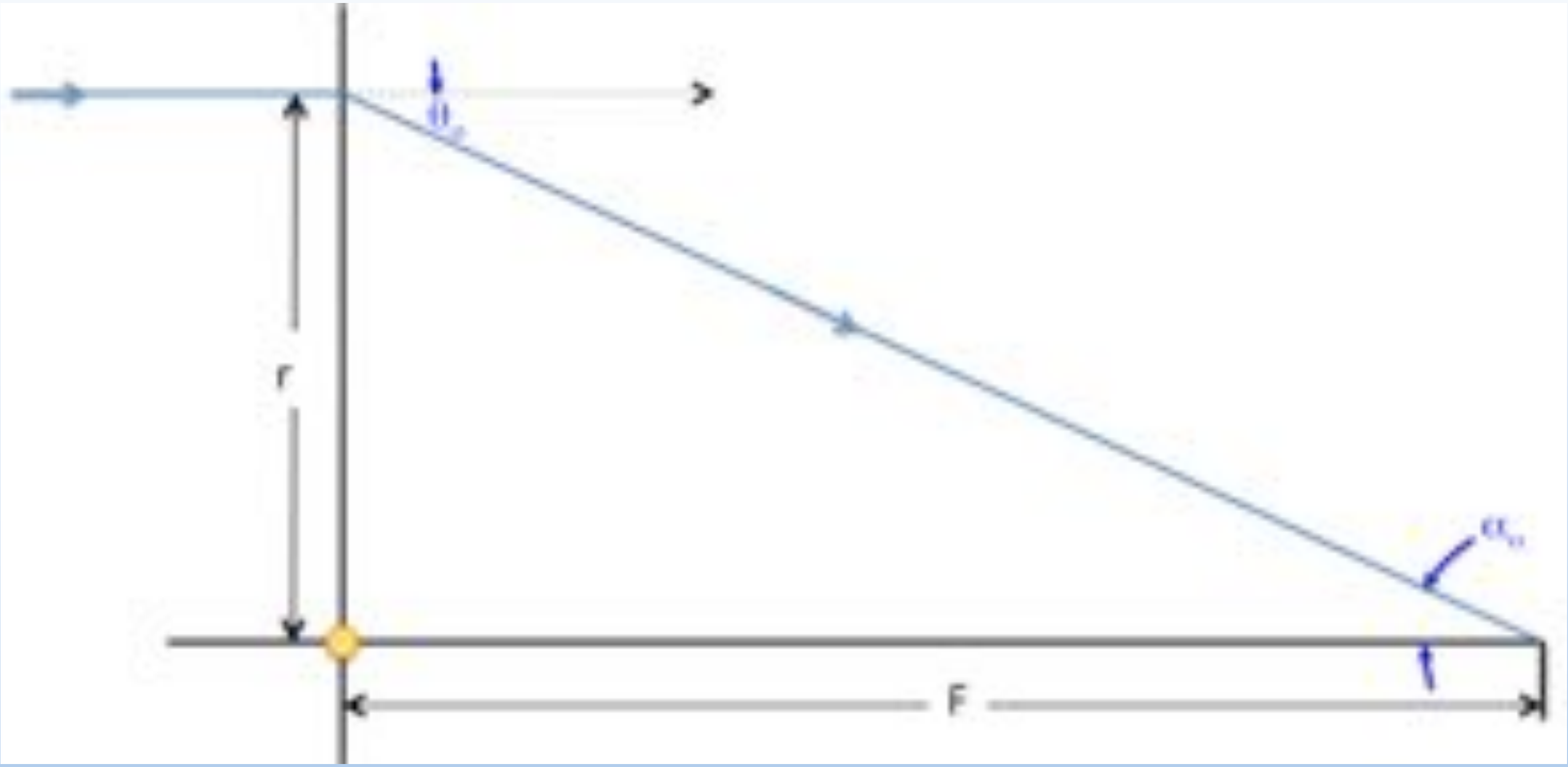
However, it is clear that this mission is more complex than simply sending a spacecraft equipped with an imaging plane to a distance of 550 AU from the sun.

Reference with detailed calculations: Landis, Geoffrey A, "Mission to the Gravitational Focus of the Sun: A Critical Analysis," paper AIAA-2017-1679, AIAA Science and Technology Forum and Exposition 2017, Grapevine TX, Jan. 9-13, 2017.

Backup slides

- Angle definitions
- geometry

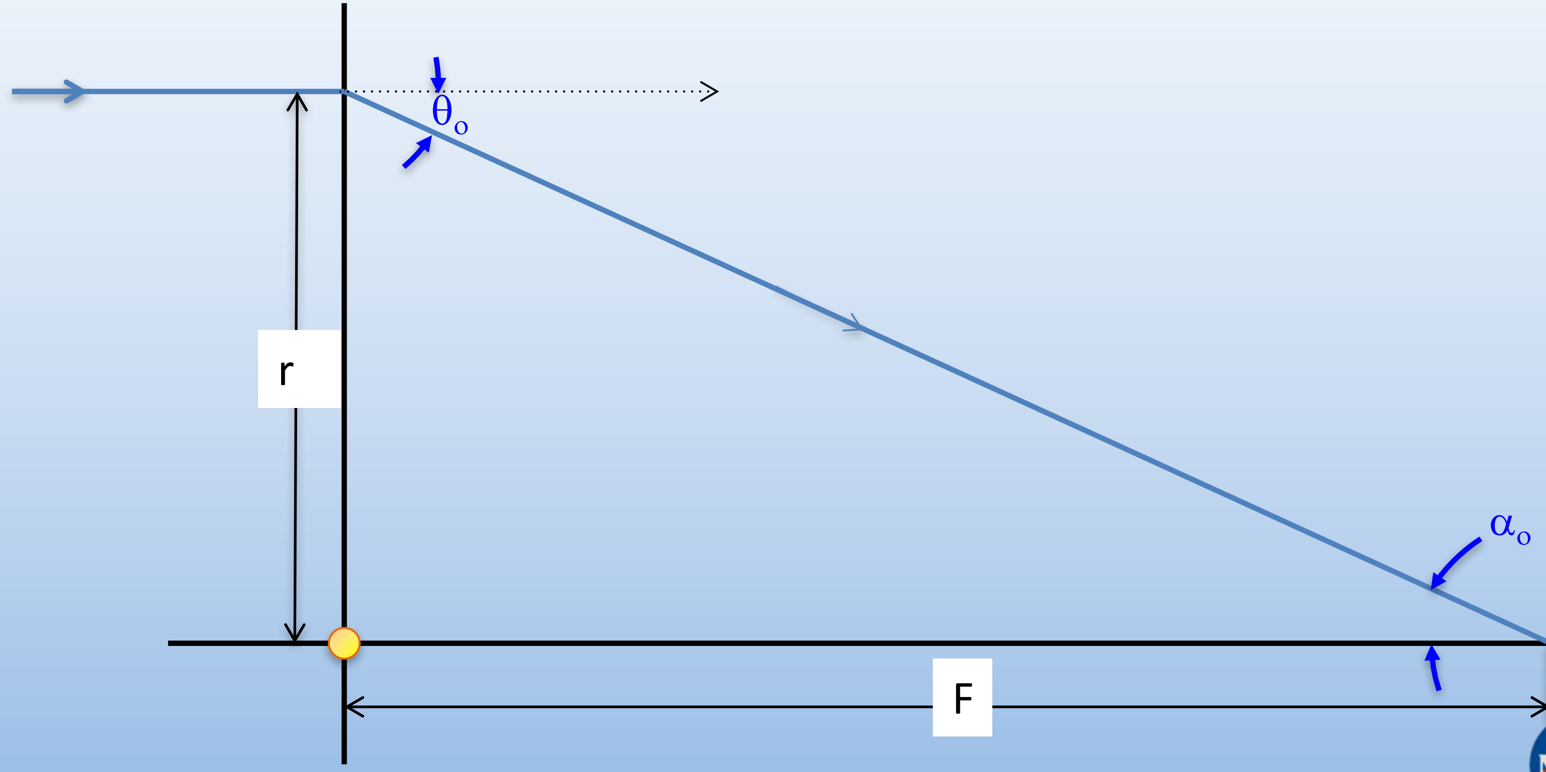
Definition of angles



Definition of deflection angles

Θ = light deflection angle
α = apparent direction of light viewed at focus

- At Einstein ring, $\alpha_o = \Theta_o$

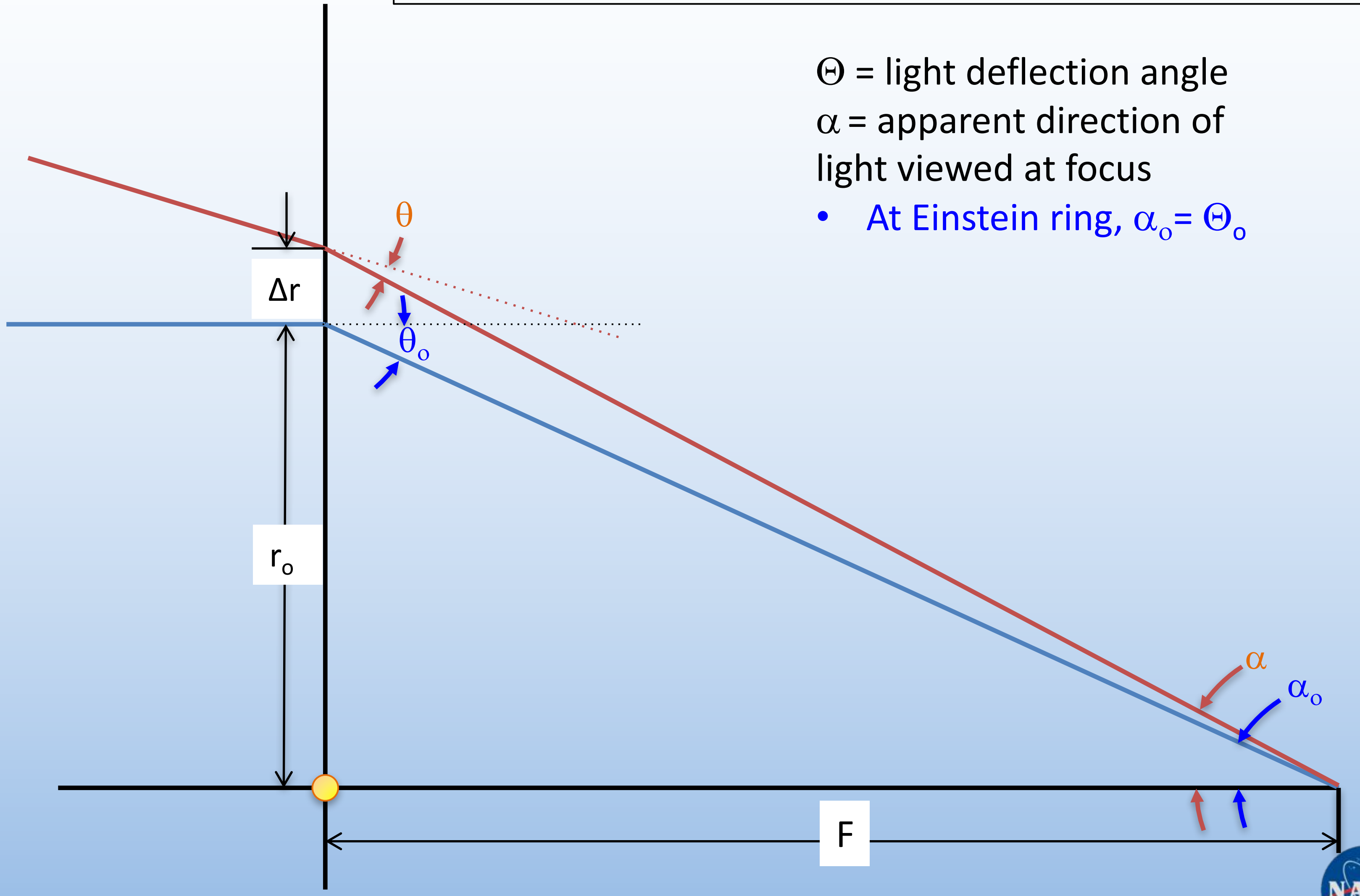


Deflection geometry

(not to scale)

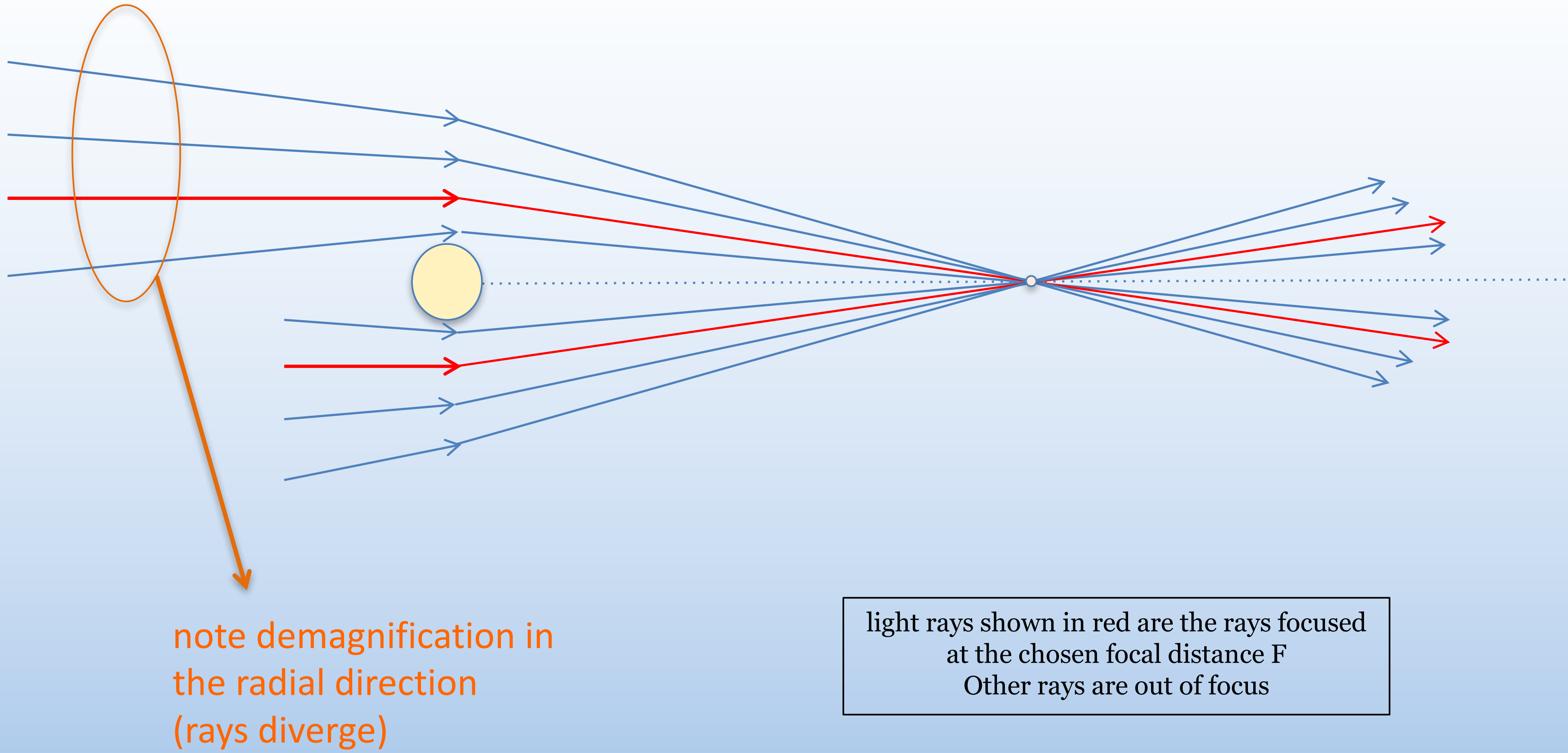
Θ = light deflection angle
α = apparent direction of light viewed at focus

- At Einstein ring, $\alpha_o = \Theta_o$



Gravitational lens – focus

(not to scale)

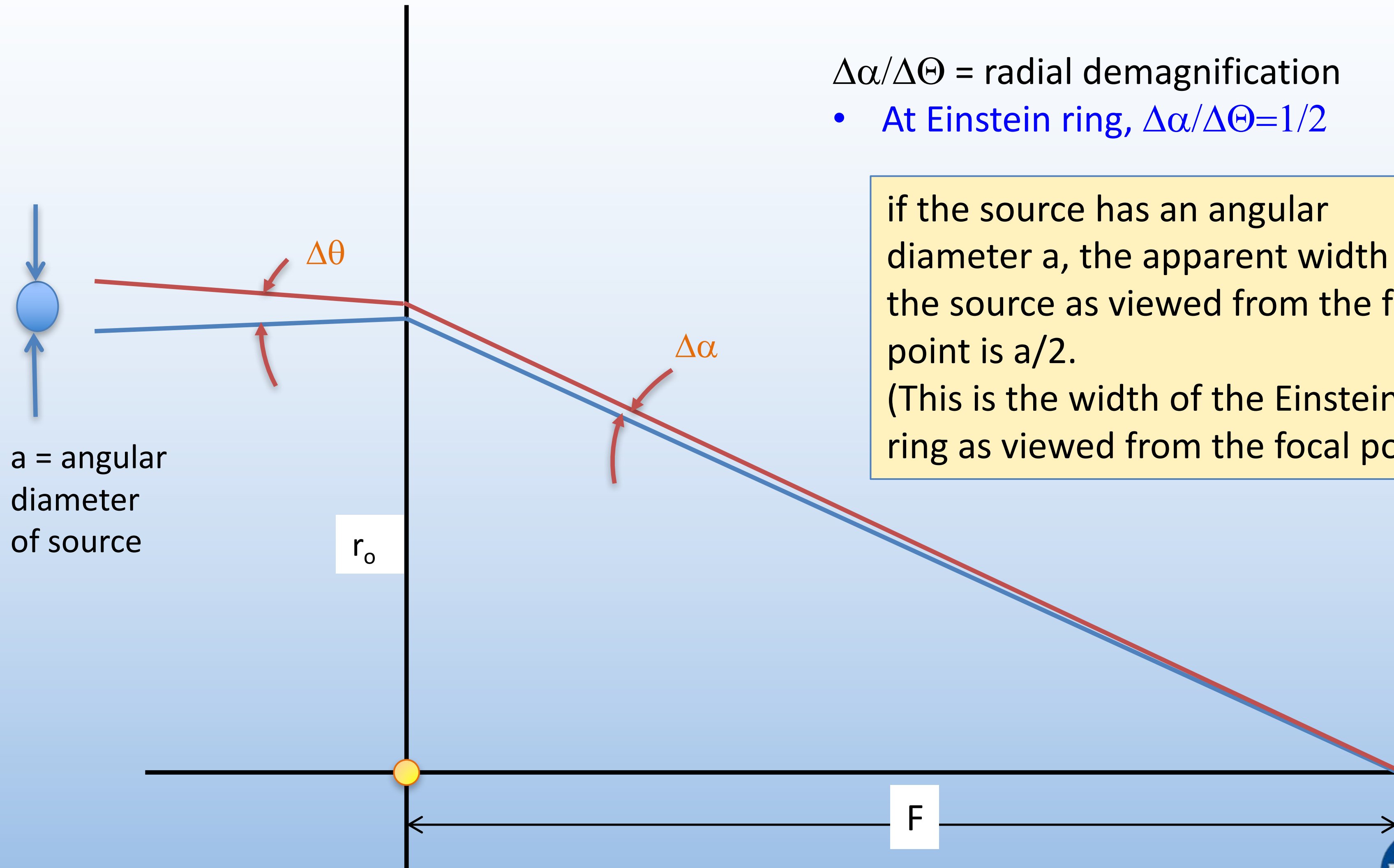


Radial Demagnification

$\Delta\alpha/\Delta\Theta$ = radial demagnification

- At Einstein ring, $\Delta\alpha/\Delta\Theta=1/2$

if the source has an angular diameter a, the apparent width of the source as viewed from the focal point is a/2.
(This is the width of the Einstein ring as viewed from the focal point)



Solar Corona

Image by Steve Bryson of August 2017 solar eclipse