CARA "Brute Force Monte Carlo" Study Results

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Pc Calculation by Monte Carlo: Purpose

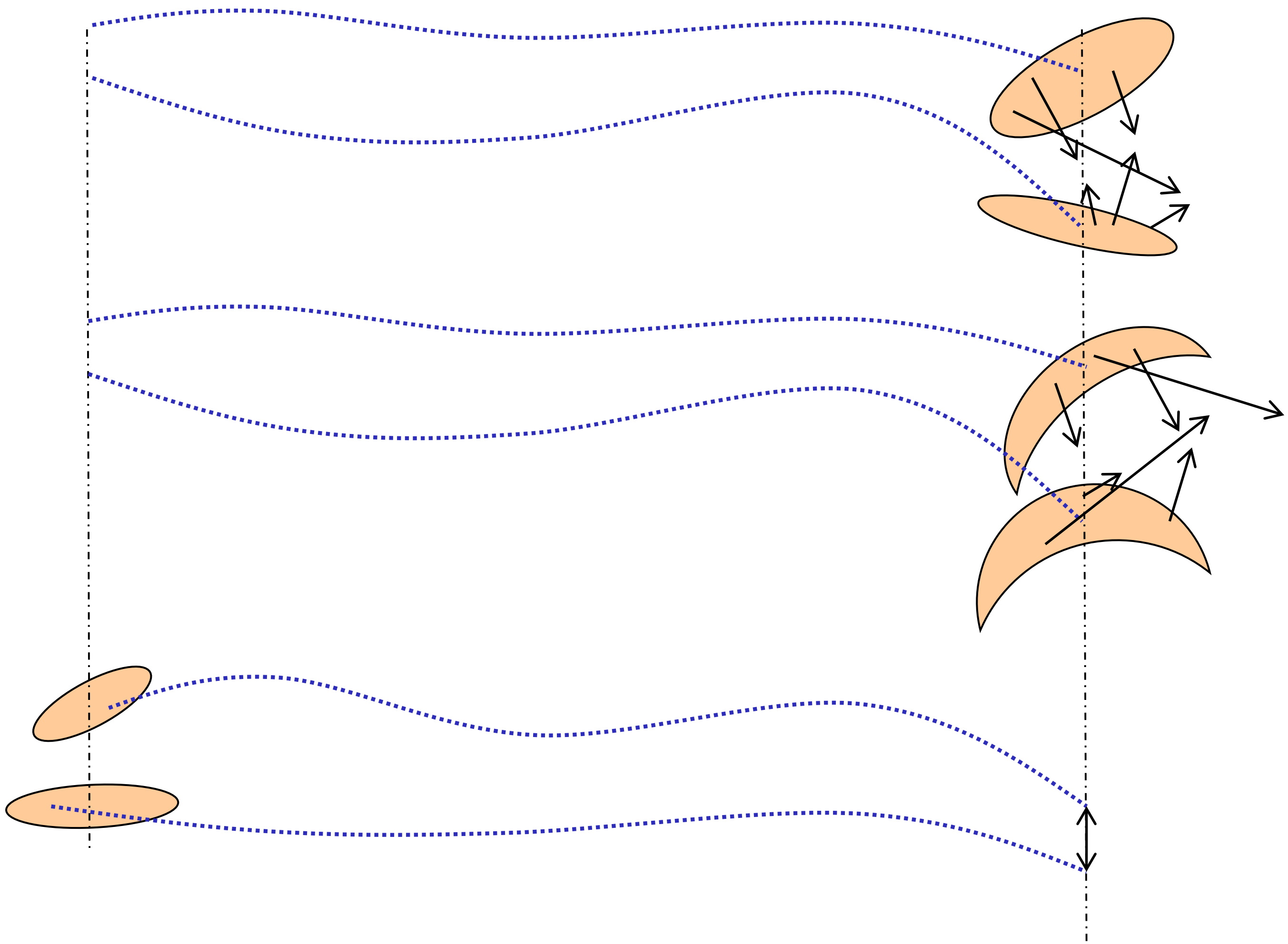
- **Situations in which 2-D Pc assumptions do not hold**
  - Rectilinear motion in encounter region
  - Invariant covariance throughout encounter
  - Some fresh insights generated by current study
- **Situations in which it is difficult to separate one conjunction event from another and a composite Pc is desired**
  - Certain deep space / GEO encounters
- **Situations in which one wishes to compensate for primary and secondary covariance correlation**
  - Subject of separate study, but some brief discussion here

Different Monte Carlo Types

Level II: propagate covariances to TCA; generate MC samples in Cartesian space and find TCA between pairs

Level III: propagate covariances to TCA; generate MC samples in element space and find TCA between pairs

Level IV: Generate samples at epoch; propagate every pair of samples forward to its proper TCA



CARA “Brute Force Monte Carlo” (BFMC)

- **Level IV Monte Carlo capability**
- **Uses complete ASW algorithm set**
  - ASW propagator
  - JBH09 atmospheric density model, including HASDM corrections
  - Global density forecast error and frontal area error consider parameters
- **No reuse of primary and secondary propagated states**
  - Sample reuse invalidates calculated confidence intervals
- **Full 8-parameter perturbations of primary and secondary at epoch**
  - Six equinoctial state parameters
  - Ballistic coefficient and solar radiation pressure coefficient variances
- **Epoch covariances properly inflated with consider parameters**
  - Perturbations due to global density forecast error component shared between primary and secondary covariances
  - This properly accounts for correlation between primary and secondary covariances

BFMC VCM vs CDM Modes

- **BFMC VCM mode**
  - Performs sampling at epoch (approach described on previous slide)
- **BFMC CDM mode**
  - Performs sampling from covariances propagated to TCA (from CDM)
  - Converts to equinoctial element representation before sampling performed
  - Two-body propagation used for sample states
  - Analytic method to estimate and remove correlated portion of covariances
- **CDM mode obviously computationally favorable**
  - Details on next slide
- **Must determine conditions and frequency for which VCM and CDM modes differ**

BFMC Processing Speeds

- **Measured computation rate for the BFMC CDM mode**
  - $R_{CDM} \approx 10^5$ samples/CPUsec (for a single CPU on Linux workstation COOKE)
- **Measured computation rate for the BFMC VCM mode**
  - $R_{VCM} \approx 10$ samples/CPUsec for epoch-to-TCA $\approx$ 0.5 days
  - $R_{VCM} \approx 1$ samples/CPUsec for epoch-to-TCA $\approx$ 5.0 days
- **BFMC's CDM mode is faster than BFMC's VCM mode by a factor of about 10^4 to 10^5**
- **A 20-processor workstation estimating Pc values to an accuracy of 10% (at 95% confidence) requires the following predicted run times:**

	$Pc = 10^{-4}$	$Pc = 10^{-5}$	$Pc = 10^{-6}$	$Pc = 10^{-7}$
CDM Mode	0.5 sec	5 sec	50 sec	8.3 min
VCM (0.5day)	1.4 hours	14 hours	5.8 days	58 days
VCM (5.0day)	14 hours	5.8 days	58 days	580 days

Monte Carlo Analyses

Analysis 1: BFMC CDM mode vs VCM mode comparisons

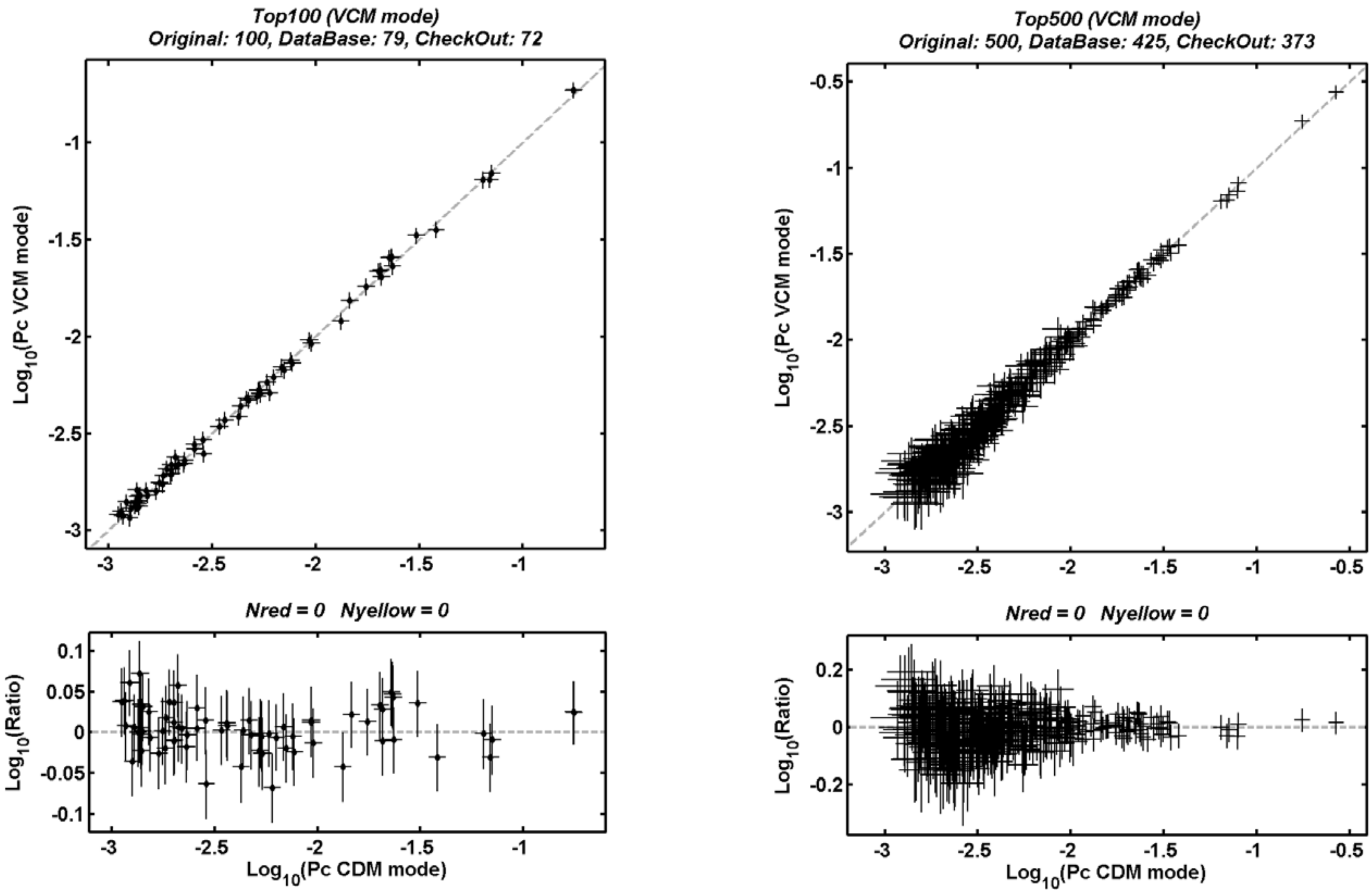
- **Compare CDM mode (from TCA) vs VCM mode (from epoch) Pc estimates**
  - Top 100 original BFMC trade-space conjunctions (from 2017 Aug-Sep)
  - Top 500 conjunctions (from 2017 May – 2018 Mar)
- **Preliminary Conclusion: After passing check-out processing, no significant (*i.e.,* very improbable) deviations have been found between BFMC CDM mode and VCM mode Pc values, in the data sets analyzed so far**

Analysis 2: BFMC CDM mode vs 2D Pc comparisons

- **Compare BFMC CDM mode vs 2D Pc estimates**
  - Top 500 conjunctions (from 2017 May – 2018 Mar)
  - Top 28665 conjunctions (from 2017 May – 2018 Mar)
- **Preliminary Conclusion: A small fraction of BFMC CDM mode Pc estimates deviate significantly (in both directions) from 2D Pc estimates**

BFMC CDM Mode Pc vs VCM Mode Pc Comparisons

Top 500 Conjunctions from 2017 May–2018 Mar (2D Pc > 10^{-3}; 100@10%; 200@20%; 200@30%)

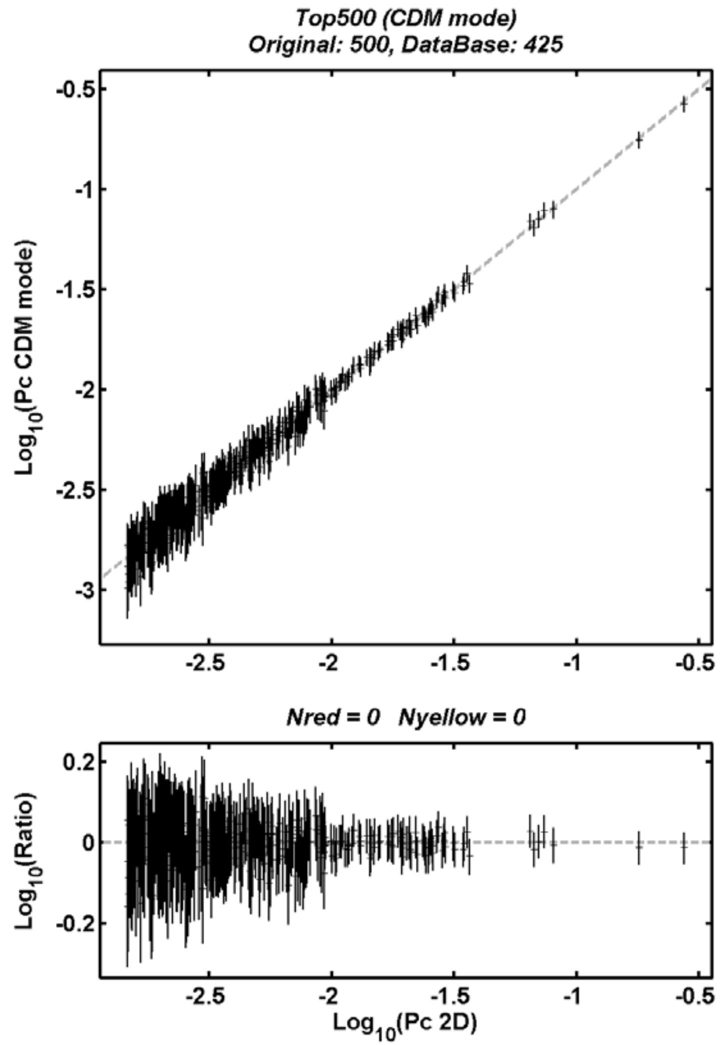


BFMC CDM Mode Pc vs 2D Pc Comparisons

Top 500 Conjunctions from 2017 May–2018 Mar (2D Pc > 10^{-3}; 100@10%; 200@20%; 200@30%)

- **Horizontal: log of CDM mode Pc**
- **Vertical: log of VCM mode Pc**
- **Error bars: 95% confidence**

- **Red: very improbable deviations**
  - Defined as $P_{value} < 10^{-6}$
- **Yellow: improbable deviations**
  - Defined as $10^{-6} < P_{value} < 10^{-3}$
- **Black: all others**
  - Defined as $P_{value} \geq 10^{-3}$



Top 28665 Conjunctions from 2017 May – 2018 Mar (with 2D Pc ≥ 10^{-7}; 3046@10%; 17158@30%; 8451@45%)

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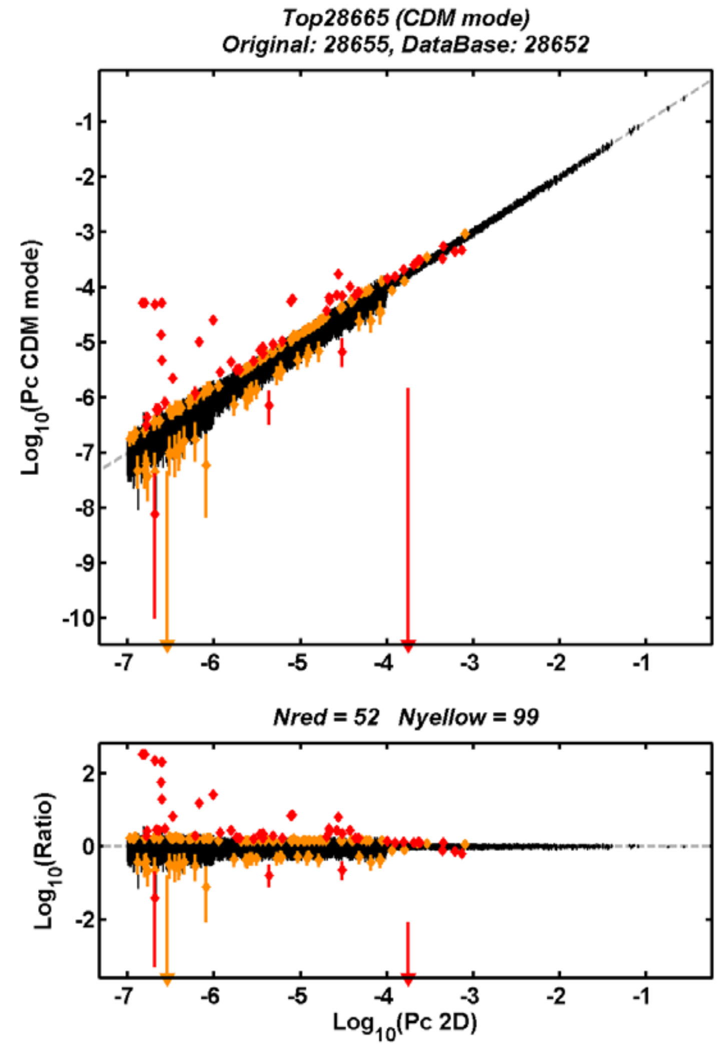


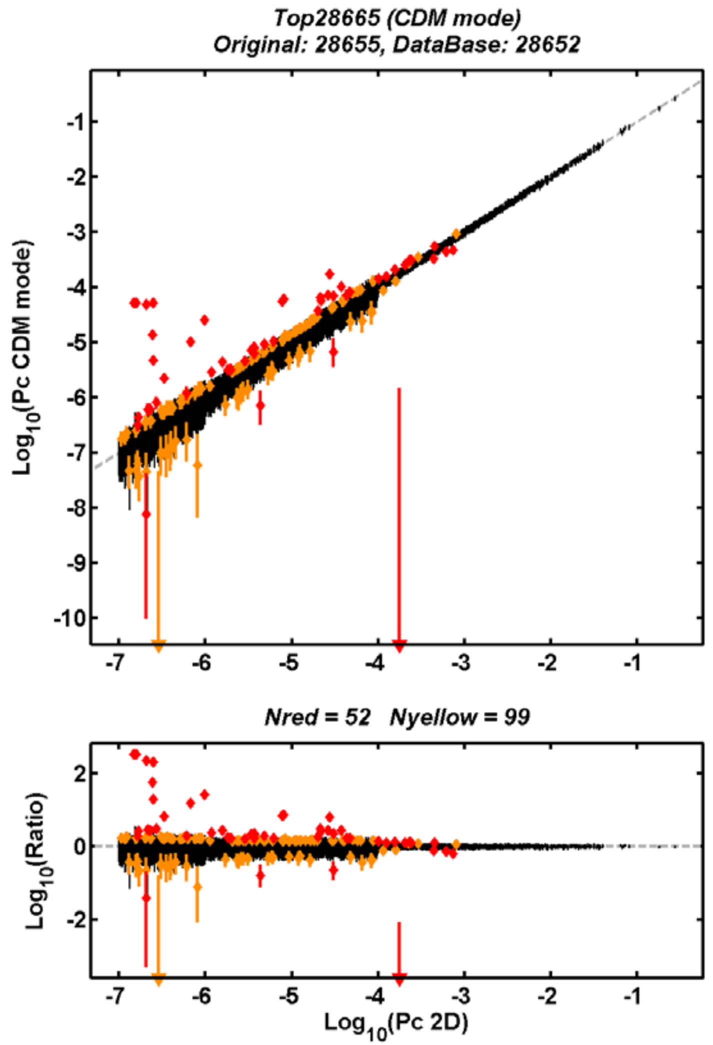
Top 28665 Conjunctions from 2017 May – 2018 Mar (with 2D Pc ≥ 10^{-7}; 3046@10%; 17158@30%; 8451@45%)

- **Phenomenology of CDMPc vs 2DPc:**

- **There are many more significant deviations (i.e., red and yellow points) than expected by chance**

- **All significant deviations seen so far occur for 2D Pc < 10^{-3}**

- **The deviations occur in both directions**
  - CDMPc > 2DPc & CDMPc < 2DPc
  - Some CDMPc values correspond to zero MC counts, so only the 95% upper limit is plotted

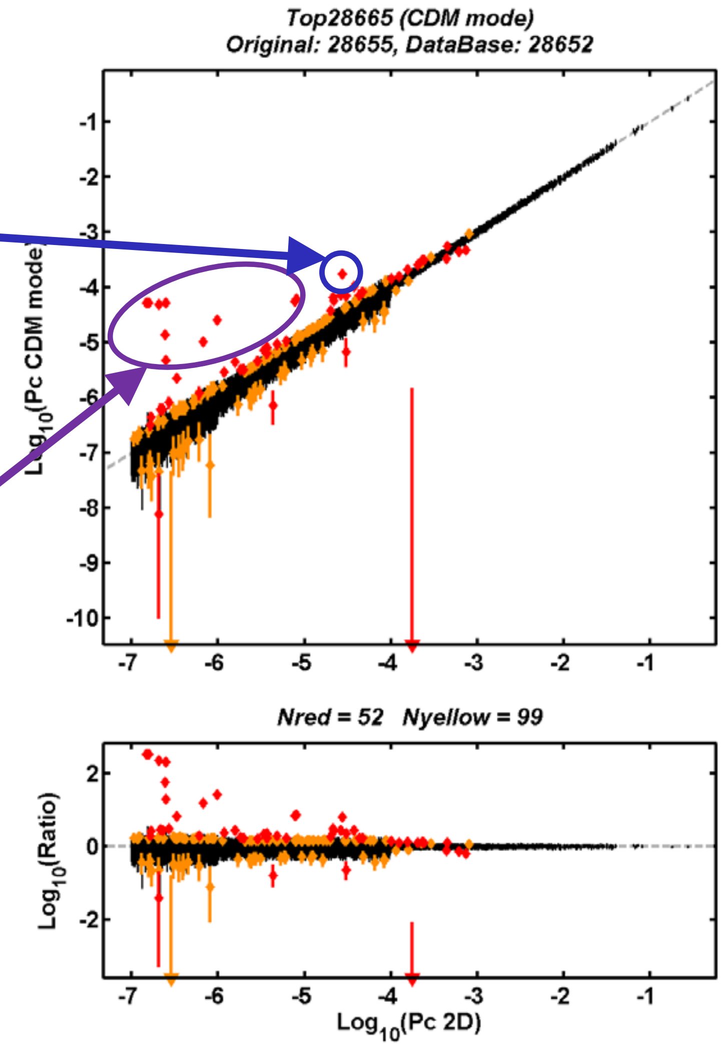


Top 28665 Conjunctions from 2017 May – 2018 Mar (with 2D Pc ≥ 10^{-7}; 3046@10%; 17158@30%; 8451@45%)

- **Phenomenology of CDMPc vs 2DPc:**

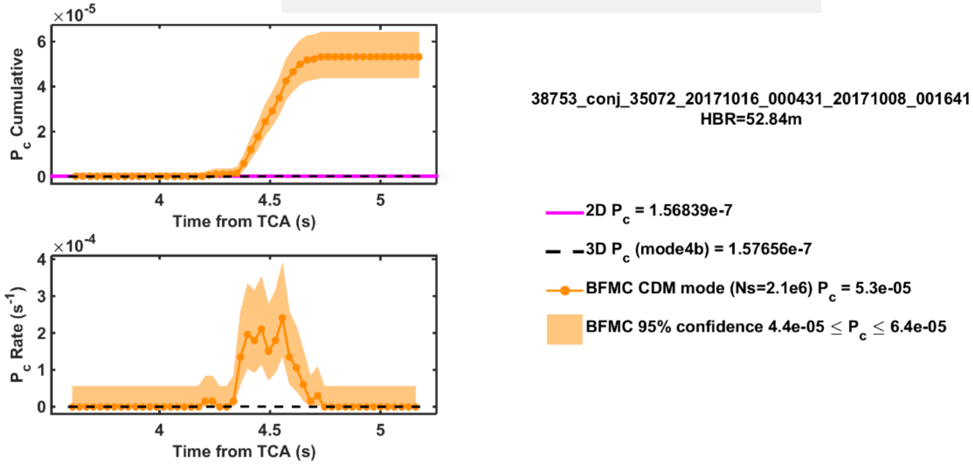
- **The highest deviant CDMPc value corresponds to an ISS conjunction**
  - Object 25544 encountering object 87200
  - See the point circled in blue

- **Many of the deviants involve two specific satellites: Van Allen A and B, which occupy highly eccentric orbits**
  - Objects 38752 & 38753 encountering a variety of other objects
  - See the points circled in purple

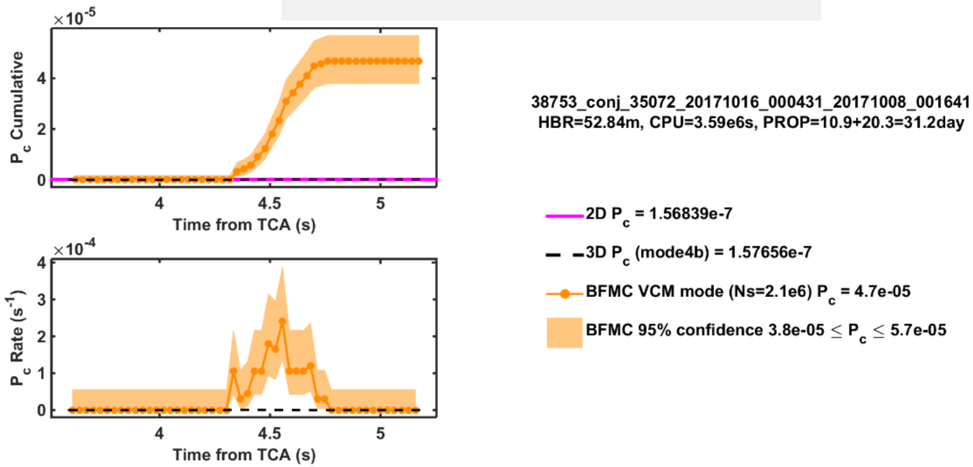


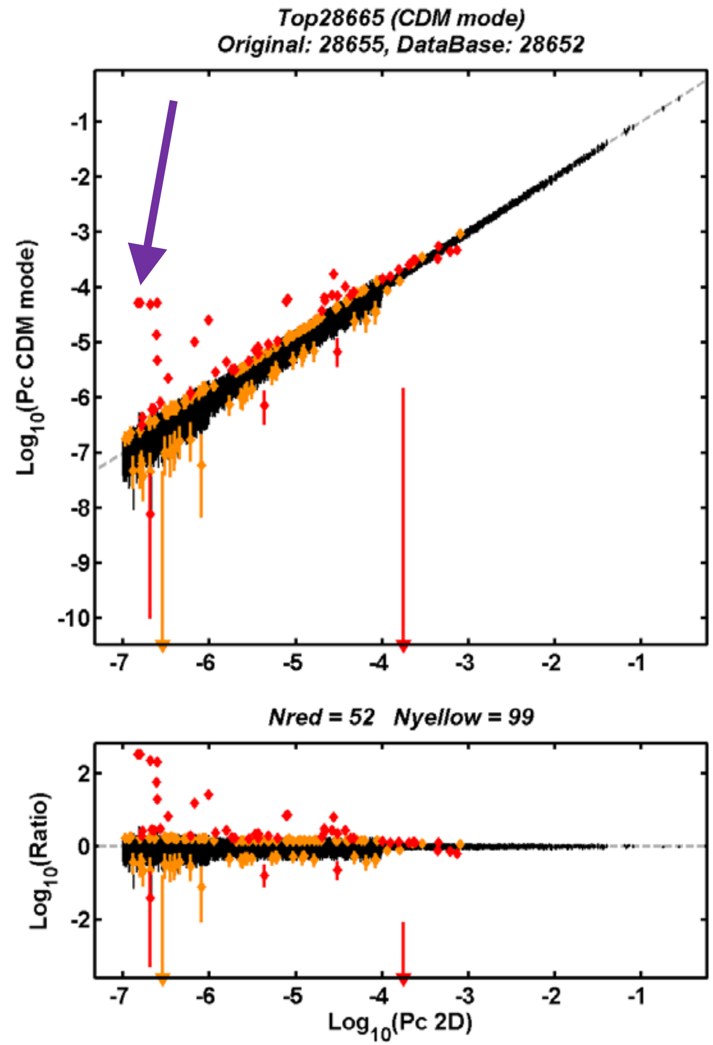
Van Allen B Conjunction with BFMCPc >> 2DPc
(CDMPc ≈ VCMPc; BFMCPc ≈ 320×2DPc)

BFMC CDM Mode



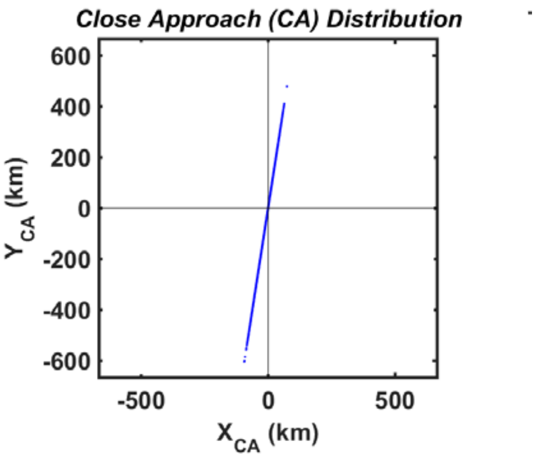
BFMC VCM Mode

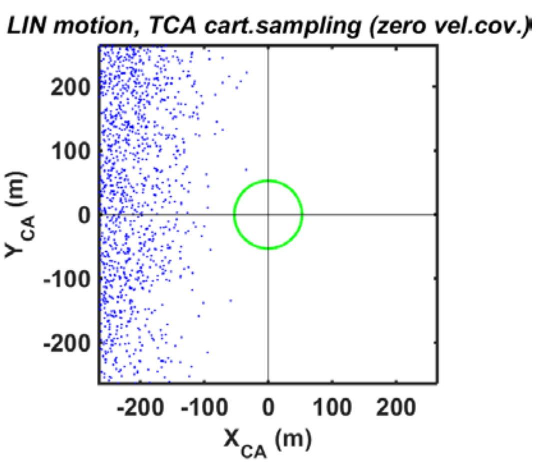




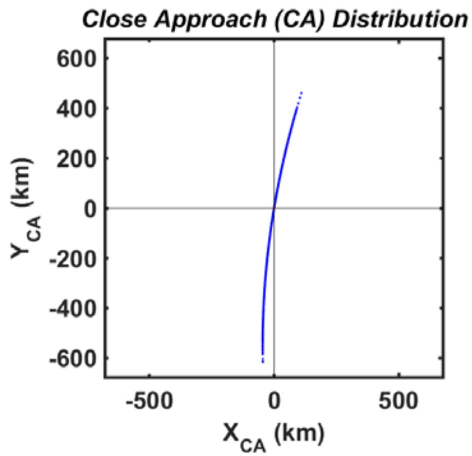
Van Allen B Conjunction with BFMCPc >> 2DPc (CDMPc ≈ VCMPc; BFMCPc ≈ 320×2DPc)

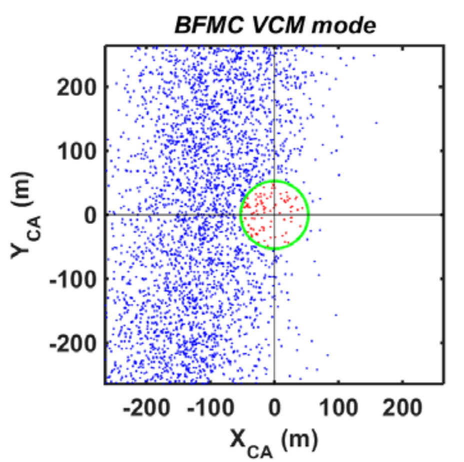
MCWB Linear Mode

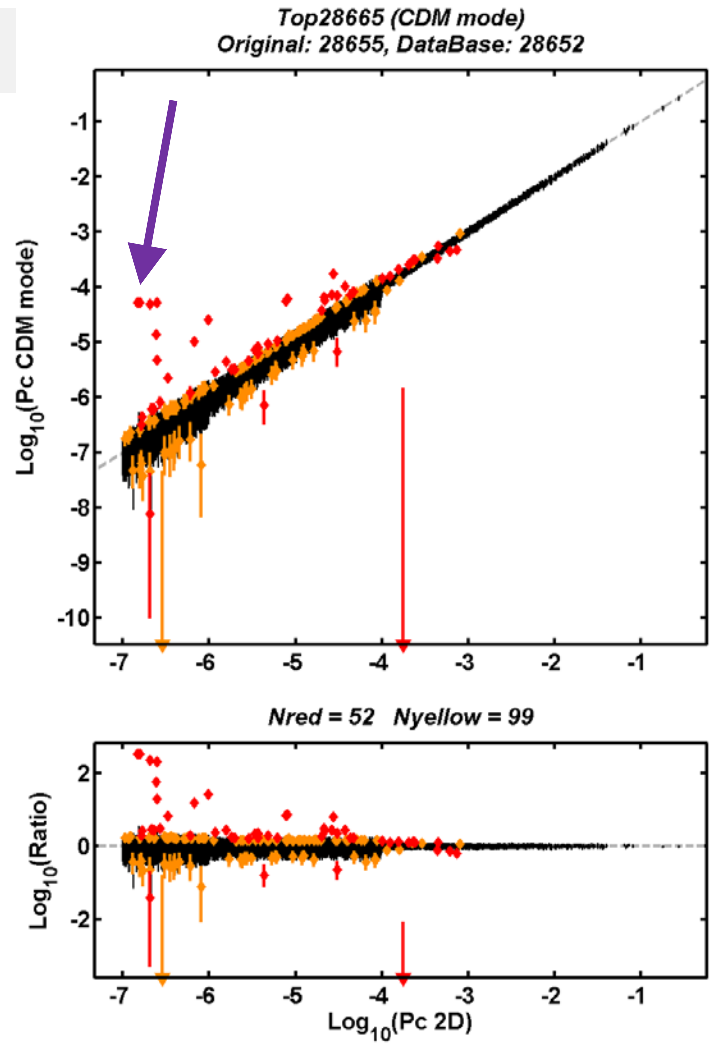




BFMC VCM Mode

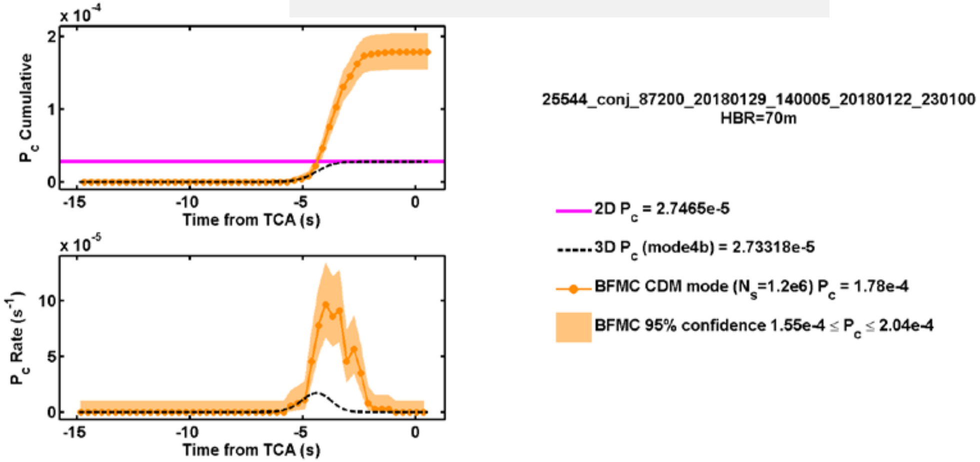




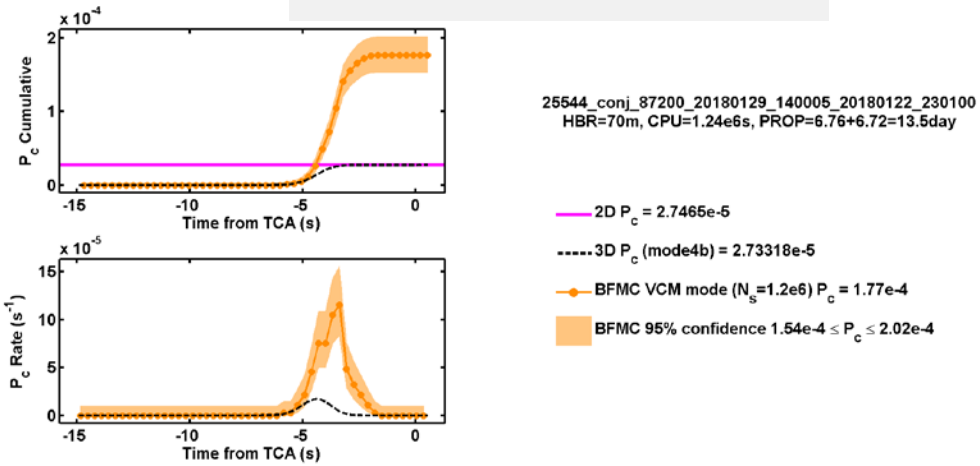


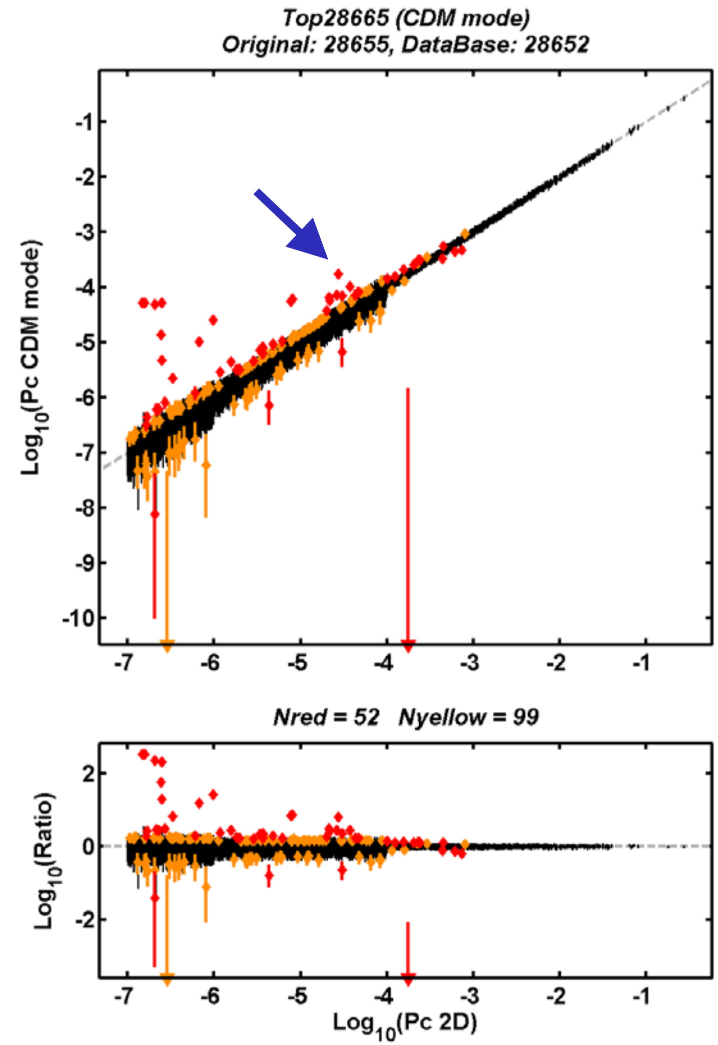
ISS Conjunction with BFMCPc >> 2DPc
(CDMPc ≈ VCMPc; BFMCPc ≈ 6.5×2DPc)

BFMC CDM Mode



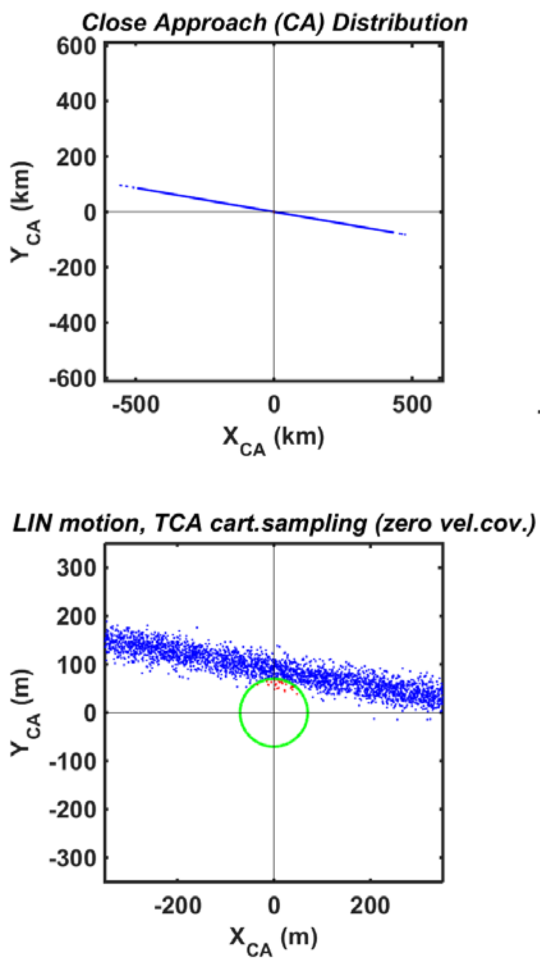
BFMC VCM Mode



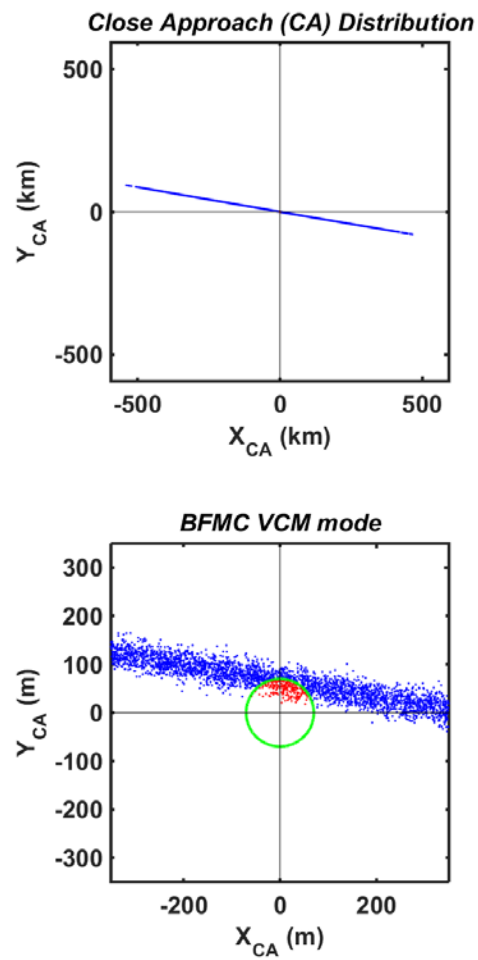


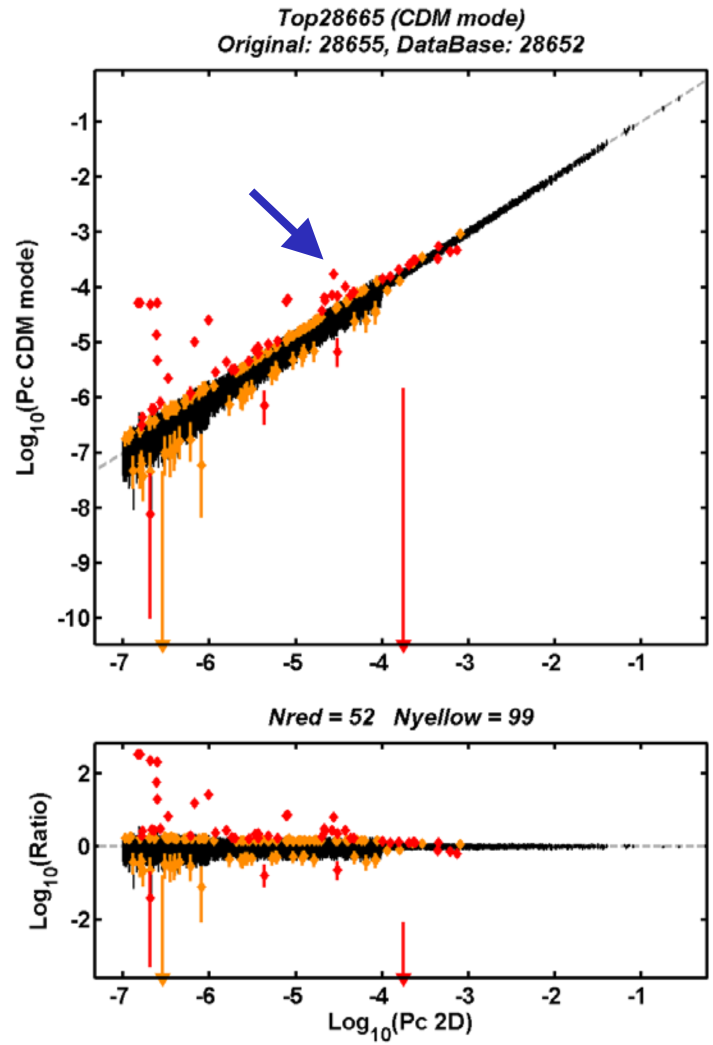
ISS Conjunction with BFMCPc >> 2DPc
(CDMPc ≈ VCMPc; BFMCPc ≈ 6.5×2DPc)

MCWB Linear Mode



BFMC VCM Mode





Results Summary: VCM vs CDM Mode of Monte Carlo

- **No cases discovered so far in which two modes differ appreciably**
- **More investigation required, and it is computationally demanding**
  - Processing requirements function of Pc value
  - All high-Pc conjunctions (> 1E-03) from last year's dataset already evaluated (only period for which all needed supporting files are available)
  - Broadening of set to Pc values of 1E-04 to 1E-05 requires several hours to days to process each conjunction (on 24-processor machine)
- **Five-week run presently executing**
- **Not clear when enough cases examined to allow conclusion of VCM/CDM mode equivalence**
- **However, if this conclusion can be drawn, finding significant**
  - Much more computationally efficient MC approach then available
  - Should discuss whether this calculation should replace 2-D Pc at least for HIEs, and perhaps entirely

Results Summary: CDM Mode of Monte Carlo vs 2-D Pc

- **Relatively small number of cases with significant deviations (0.5%)**
  - And only a handful of these generate operationally significant differences in Pc
- **However, cases exist with orders-of-magnitude Pc differences**
- **Can these cases be predicted by parameter investigation?**
  - Abiding characteristic: large temporal difference between Pc-dot (first derivative of Pc) maximum and TCA
  - Pc-dot maximum appears to be correlated with Mahalanobis distance minimum; circumstance should therefore be predictable
  - Ongoing investigation to determine robustness of this technique
- **If predictable, then MC need be run on only a small subset of events**
  - But so computationally inexpensive that might be advisable for all yellow events, for example

Next Steps

- **Complete both VCM-CDM and CDM-2DPc investigations**
  - To be presented in conference papers this summer/fall
- **Write/distribute recommendations memorandum to missions**
  - Analytic methods to identify situations in which 2-D Pc inadequate
  - Type of Monte Carlo necessary for such situations
- **If helpful, investigate making BFMC available to missions in some form**

BACKUP SLIDES

Cartesian vs Element Representations of States

- **Three-dimensional states require six parameters to specify fully**
  - Seven if one wishes to include the time for which that state is representative
- **Cartesian representations specify 3D position and 3D velocity**
  - Use an orthogonal "Cartesian coordinate system"
  - Straightforward to visualize, and needed for certain force model applications
- **Element representations specify states in terms of geometry**
  - Keplerian elements, which can experience singularities
  - Equinoctial elements, which are transformed versions of traditional elements in order to eliminate most singularities
- **Because orbits are curvilinear and not rectilinear, element representations perform better under linearization**
  - Element-formulated covariances give better covariance realism
- **Working in element representations therefore desirable**
- **Different Monte Carlo results obtained depending on whether sampling performed in Cartesian or equinoctial frame**

Different Monte Carlo Types: Tabular Comparison

Level	Description	Commentary
I	• Linear propagation of position covariances to nominal TCA • Position error draws taken from propagated covariance • Pairs of perturbed states "propagated" to TCA with rectilinear motion (constant state velocity)	Essentially reproduces 2D Pc Can be used to examine distributions of component errors (but analytical solutions for these distributions exist)
II	• Linear propagation of covariances to nominal TCA • Position and velocity error draws taken from propagated covariance • Pairs of perturbed states propagated to TCA with two-body propagator	Considers full state error in executing perturbations and performs actual propagation to find point-pair TCAs
IIIa	• Linear propagation of position covariances to nominal TCA, but natively in element (equinoctial) space • Six-element state perturbations taken from equinoctial covariances; propagated natively in element space to find TCA for each pair	Works natively in curvilinear (element) space; covariances should be more accurately representative of real state error distributions; does not consider correlation between covariances
IIIb	• Linear propagation of covariances to nominal TCA • Covariance converted from Cartesian to equinoctial elements • Large number of error samples taken from equinoctial covariance and back-converted (non-linearly) to Cartesian framework • Back-converted error samples used to create state perturbations; MC at this point follows Level II approach	Published study (Sabol 2010) showed that this resampling approach renders very similar results to an equinocitally-native method (*i.e.,* Level IIIa)
IV	• Primary and secondary states perturbed at epoch • Each pair of perturbations propagated forward, will full non-linear dynamics, to its proper TCA • JSpOC ASW propagator used, along with JBH09 atmospheric density model • Execution times can be extremely long	Sometimes called "brute force MC," it is the gold standard for evaluating actual conjunction Pc; no known limitations Produces reliable comparison results for actual conjunctions only if ASW propagator and atmospheric model used

Monte Carlo Pc Calculation: Confidence Intervals and Sample Reuse (1 of 2)

- **Monte Carlo methods should eventually converge on correct Pc**
  - Question is whether enough trials were run to presume convergence
- **Methods exist for calculating confidence interval for solution**
  - Usually give confidence range and Pc value boundaries
    - *e.g.,* Pc estimate is 1E-04, with 95% confidence that true Pc between 5E-05 and 2E-04
    - To narrow (or improve) the confidence interval, increase the number of trials
  - Because MC for Pc is a binary exercise (each trial either is or is not a hit), results conform to a binomial distribution
  - Multiple approaches for estimating binomial confidence intervals
    - CARA uses Clopper-Pearson approach (embedded in MATLAB “binofit” function)
- **However, these methods work only if MC samples are not reused!**

Monte Carlo Pc Calculation: Confidence Intervals and Sample Reuse (2 of 2)

- **Computationally expensive to propagate large numbers of trajectories forward to TCA**
- **Practitioners tempted to reuse each propagated sample for primary by comparing to each propagated sample for secondary**
  - Single-comparison method: p(1) → s(1), p(2) → s(2), … p(n) → s(n)
  - Multiple-comparison method: p(1) → s(1), p(1) → s(2), … p(1) → s(n),
p(2) → s(1), p(2) → s(2), … p(2) → s(n), &c.
  - If N samples produced for each, this produces N^2 trials
  - Seems like easy and efficient way to obtain more trails with same propagation
- **However, confidence interval calculations invalid with this approach**
  - Demonstrated by R. Carpenter in 2016 paper*
  - MC with this approach may converge on the correct answer, but not possible to know whether enough samples run to trust any given result
- **MC methods for operational use need to have reliable accompanying confidence intervals**

*Schilling *et al.*, "Operational Experience with the Wald Sequential Probability Ratio Test for Conjunction Assessment from the Magnetospheric MultiScale Mission," 2016 AIAA/AAS Astrodynamics Specialist Conference, Long Beach CA, September 2016