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A Trajectory Algorithm to Support En Route and Terminal Area Self-Spacing Concepts: Fourth Revision

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Nomenclature

2D:	2 dimensional
4D:	4 dimensional
ADS-B:	Automatic Dependence Surveillance Broadcast
BOD:	Bottom-Of-Descent
CAS:	Calibrated Airspeed
DTG:	Distance-To-Go
FAF:	Final Approach Fix
MSL:	Mean Sea Level
RF:	Radius-to-Fix
STAR:	Standard Terminal Arrival Routes
TAS:	True Airspeed
TCP:	Trajectory Change Point
TOD:	Top-Of-Descent
TTG:	Time-To-Go
VTCP:	Vertical Trajectory Change Point

Subscripts

Subscripts associated with waypoints and TCPs, e.g., TCP_2 , denote the location of the waypoint or TCP in the TCP list. Larger numbers denote locations closer to the end of the list, with the end of the list being the runway threshold. Subscripts in variables indicate that the variable is associated with the TCP with that subscript, e.g., $Altitude_2$ is the altitude value associated with TCP_2 .

Units and Dimensions

Unless specifically defined otherwise, units (dimensions) are as follows:

time: seconds

position: degrees, + north and + east

altitude: feet, above MSL

distance: nautical miles

speed: knots

track: degrees, true, beginning at north, positive clockwise

Abstract

This document describes an algorithm for the generation of a four dimensional trajectory. Input data for this algorithm are similar to an augmented Standard Terminal Arrival (STAR) with the augmentation in the form of altitude or speed crossing restrictions at waypoints on the route. This version of the algorithm now accommodates routes that are totally in the cruise regime. The algorithm calculates the altitude, speed, along path distance, and along path time for each waypoint. Wind data at each of these waypoints are also used for the calculation of ground speed and turn radius.

Introduction

Concepts for self-spacing of aircraft operating into airport terminal areas have been under development since the 1970's (refs. 1-19). Interest in these concepts has recently been renewed due to a combination of emerging, enabling technology (Automatic Dependent Surveillance Broadcast data link, ADS-B) and the continued growth in air traffic with the ever increasing demand on airport (and runway) throughput. Terminal area self-spacing has the potential to provide an increase in the accuracy of runway threshold crossing times, which can lead to a decrease of the variability of the runway threshold crossing times. This decrease of the variability of the runway threshold crossing times can then lead to an increase in runway capacity through a reduction of the spacing buffers needed to assure safe separation during landing operations. Current concepts use a trajectory based technique that allows for the extension of self-spacing capabilities beyond the terminal area to a point prior to the top of the en route descent.

The overall NASA Langley concept for a trajectory-based solution for en route and terminal area self-spacing is fairly simple and was documented in references 20-22. By assuming a 4D trajectory for an aircraft and knowing that aircraft's position, it is possible to determine where that aircraft is on its trajectory. Knowing the position on the trajectory, the aircraft's estimated time-to-go (TTG) to a point can then be determined. To apply this to a self-spacing concept, a TTG is calculated for a leading aircraft and for the ownship. Note that the trajectories do not need to be the same. The nominal spacing time and spacing error can then be computed as:

nominal spacing time = planned spacing time interval + traffic TTG.

spacing error = ownship TTG – nominal spacing time.

The foundation of this spacing concept is the ability to generate a 4D trajectory. The algorithm presented in this paper uses as input a simple, augmented 2D path definition (i.e., a traditional Standard Terminal Arrival Route (STAR), with relevant speed and altitude crossing constraints) along with a forecast wind speed profile for each waypoint. The algorithm then computes a full 4D trajectory defined by a series of trajectory change points (TCPs). The input speed (Mach or Calibrated Airspeed (CAS)) or altitude crossing constraint includes the deceleration rate or vertical angle value required to meet the constraint. The TCPs are computed such that speed values, Mach or CAS, and altitudes change linearly between them. TCPs also define the beginning and ending segments of turns, with the midpoint defined as a fly-by waypoint. The algorithm also uses the waypoint forecast wind speed profile in a linear interpolation to calculate the wind speed at the altitude the computed trajectory crosses the waypoint. Wind speed values are then used to calculate the ground speeds along the path.

The major complexity in computing a 4D trajectory involves the interrelationship of ground speed with the path distance around turns. In a turn, the length of the estimated ground path and the associated turn radius will interact with the waypoint winds and with any change in the specified speed during the turn, i.e., a speed crossing-restriction at the waypoint. Either of these conditions will cause a change in the estimated turn radius. The change in the turn radius will affect the length of the ground path which can

then interact with the distance to the deceleration point, which thereby affects the turn radius calculation. To accommodate these interactions, the algorithm uses a multi-pass technique in generating the 4D path, with the ground path estimation from the previous calculation used as the starting condition for the current calculation.

Algorithm Overview

The basic functions for this trajectory algorithm are shown in figure 1. Figure 1 also contains logic and some simple calculations that are not included in the body of this document, e.g., "restore the crossing angles." Also note that waypoints are considered to be TCPs but not all TCPs are waypoints.

For the 2D input, the first and last waypoints must be fully constrained, i.e., have both a speed and altitude constraint defined. With the exception of the first waypoint, which is the waypoint farthest from the runway threshold, constraints must also include a variable that defines the means for meeting that constraint. For altitude constraints, this is the inertial descent angle; for speed constraints, it is the CAS deceleration rate. A separate, single Mach-to-CAS transition speed (CAS) value may also be input for profiles that involve a constant Mach / CAS descent segment. Additionally, an altitude / CAS restriction (e.g., in the U.S., the 10,000 ft / 250 kt restriction) may also be entered.

The algorithm computes the altitude and speed for each waypoint. It also calculates every point along the path where an altitude or speed transition occurs. These points are considered vertical TCPs (VTCPs). TCPs also define the beginning and ending segments of turns, with the midpoint defined as a fly-by waypoint. Turn data are generated by dividing the turn into two parts (from the beginning of the turn to the midpoint and from the midpoint to the end of the turn) to provide better ground speed (and resulting turn radius) data relative to a single segment estimation. A fixed, average bank angle value is used in the turn radius calculation. The algorithm also uses the forecast wind speed profile for a waypoint in a linear interpolation to calculate the wind speed at the altitude the computed trajectory crosses the waypoint (if the crossing altitude is not at a forecast altitude). For non-waypoint TCPs, the generator uses the forecast wind speed profile from the two waypoints on either side of the TCP in a double linear interpolation based on altitude and distance (to each waypoint). Of significant importance for the use of the data generated by this algorithm is that altitude and speeds (Mach or CAS) change linearly between the TCPs, thus allowing later calculations of DTG or TTG for any point on the path to be easily performed.

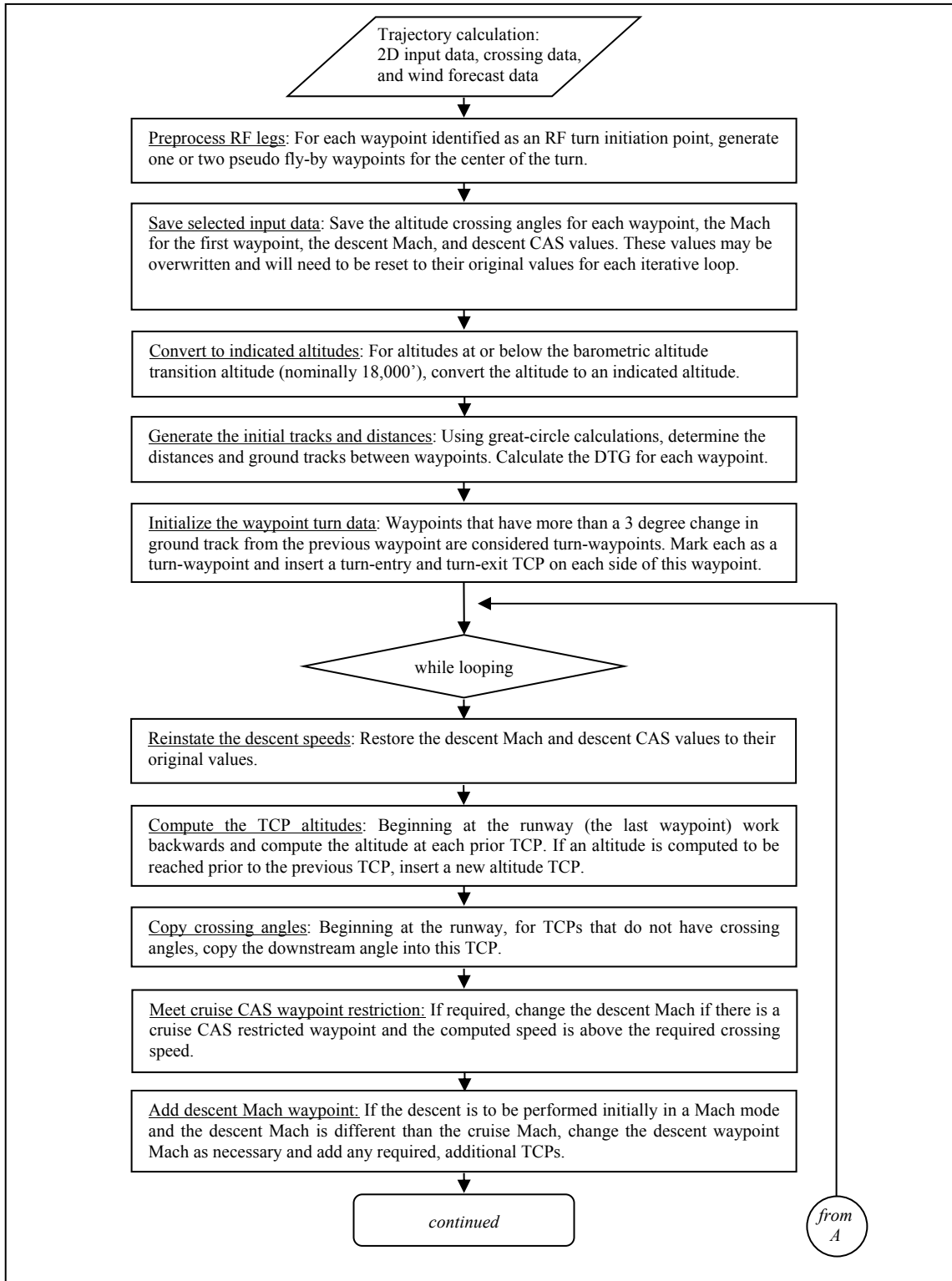


Figure 1. Basic functions.

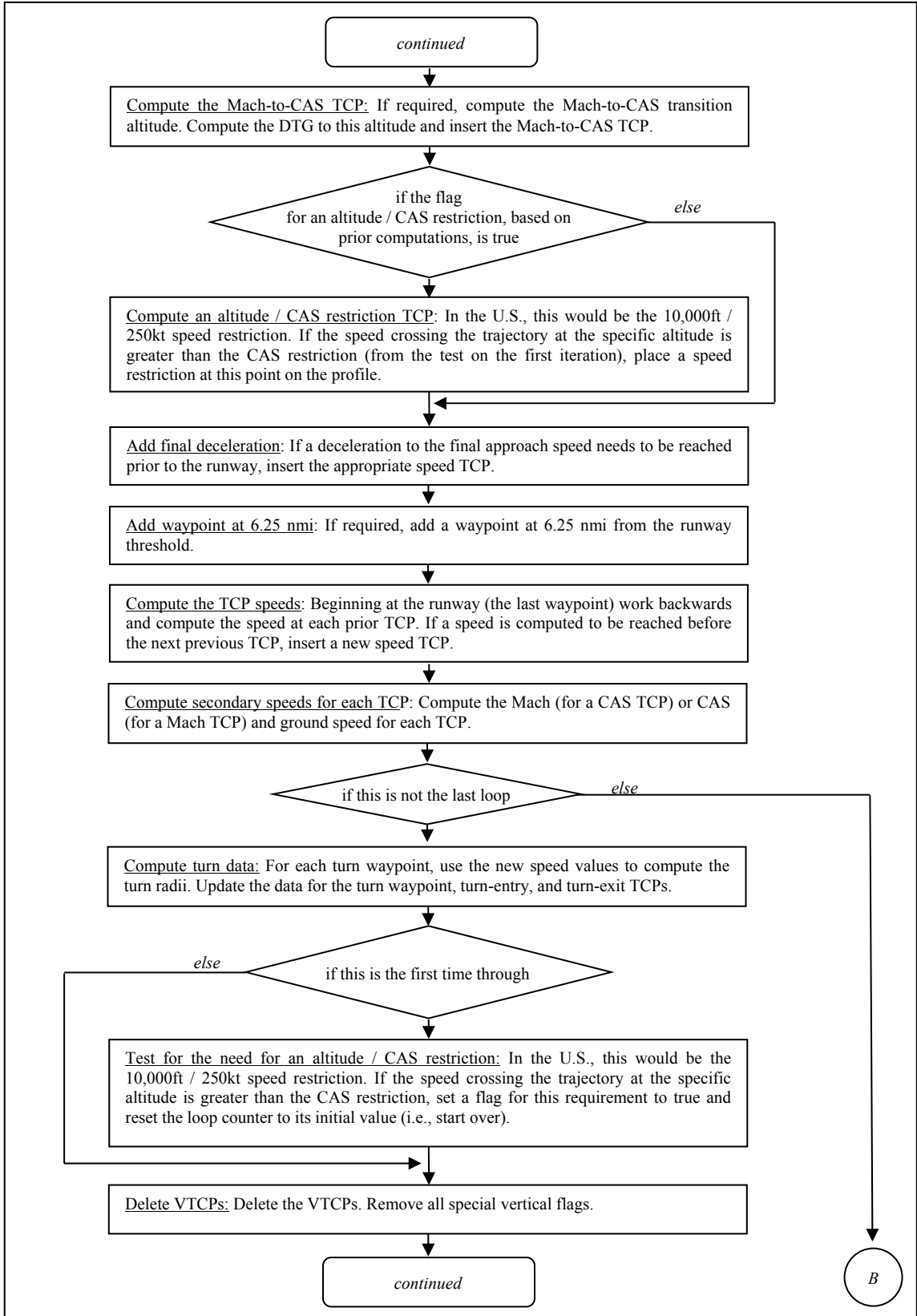


Figure 1 (continued). Basic functions.

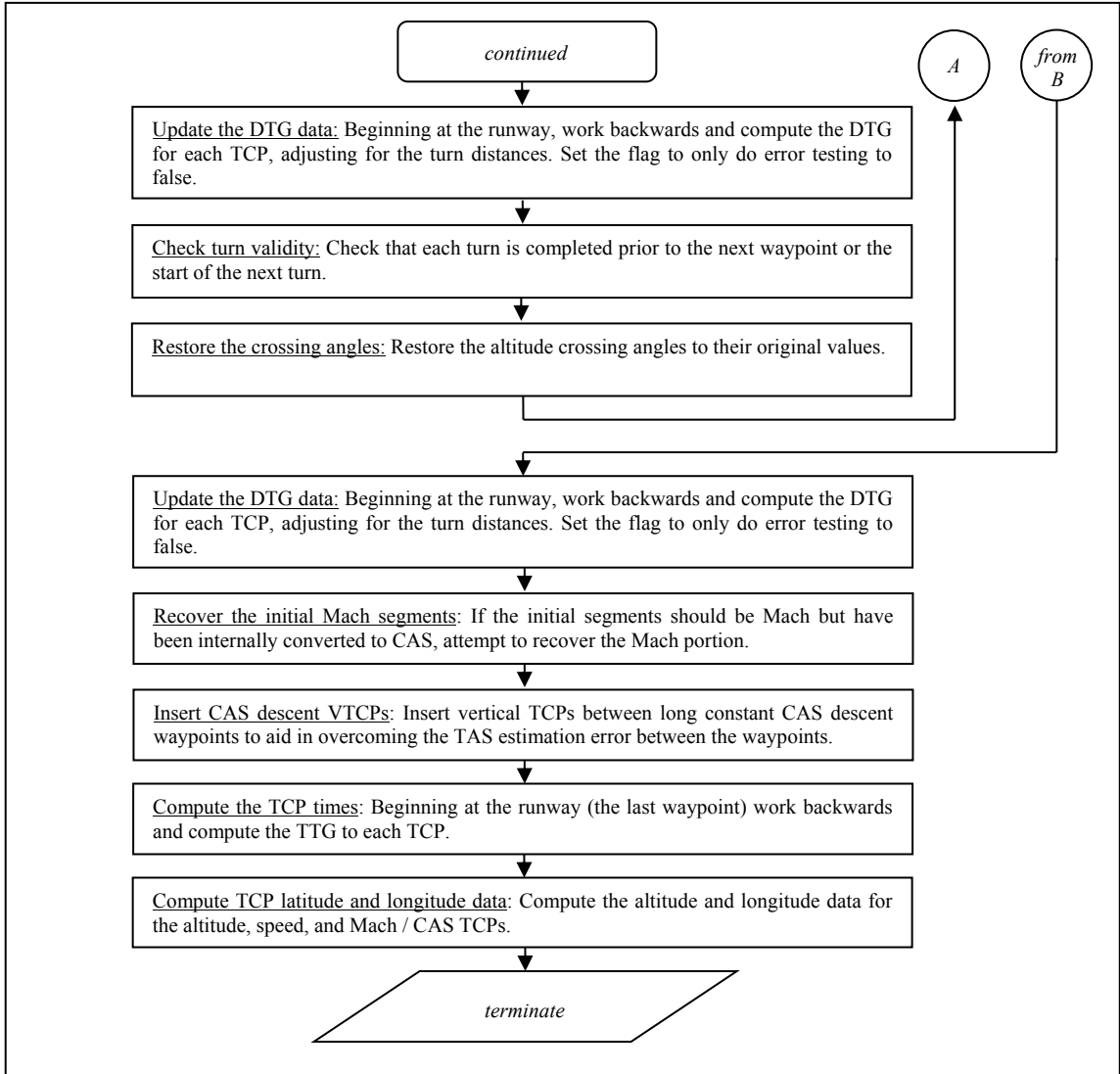


Figure 1 (concluded). Basic functions.

Algorithm Input Data

The algorithm takes as input a list of waypoints, their trajectory-specific data, and associated wind profile data. The list order must begin with the first waypoint on the trajectory and end with the runway threshold waypoint. The trajectory-specific data includes: the waypoint's name and latitude / longitude data, e.g., *Latitude₂* and *Longitude₂*, with the "2" subscript denoting that this is for the second waypoint; an altitude crossing restriction, if one exists, and its associated crossing angle, e.g., *Crossing Altitude₂* and *Crossing Angle₂*; and a speed crossing restriction (Mach or CAS), if one exists, and its associated CAS rate, e.g., *Crossing CAS₂* and *Crossing Rate₂*. A value of 0 as an input for an altitude or speed crossing constraint denotes that there is no constraint at this point. A *Crossing Mach* may not occur after any non-zero *Crossing CAS* input. The units for *Crossing Rate* are knots per second.

In this algorithm, a radius-to-fix (RF) segment is indicated by the addition of a center-of-turn position, e.g., *Center of Turn Latitude₂* and *Center of Turn Longitude₂*, for the input waypoint at the initiation of the turn. Additional requirements for the RF segment are provided in a subsequent section.

To accommodate a descent from the cruise altitude, a Mach value, *Mach Descent Mach*, may be specified that is different from the cruise Mach value. A CAS value may also be specified for the Mach-to-CAS transition speed, *Mach Transition CAS*, during the descent. Additionally, a CAS speed limit at a defined altitude may also be included. In the U.S., this would typically be set to 250 kt at 10,000 ft.

For routes that terminate at the runway threshold, an input variable, *Final Deceleration Type*, is used to accommodate three different means to achieve the speed at the threshold: RUNWAY, where the final approach speed is met at the runway threshold; STABLE XXXX, where the final approach speed is met at a trajectory altitude value defined in the XXXX variable; and AT FAF, where the final deceleration begins at the final approach fix. To support unusual approach geometries where the final approach fix (FAF) is not the waypoint immediately prior to the runway, the FAF name may be input. Also for routes that terminate at the runway threshold, the input variable *AddMopsRWY625* may be used to invoke the generation of a special waypoint at 6.25 nmi before the landing threshold of the runway. This latter capability to support this special waypoint at 6.25 nmi before the threshold, along with associated crossing altitude and speed conditions, is a requirement of the RTCA *Minimum Operational Performance Standards (MOPS) for Flight-deck Interval Management (FIM)* (ref. 23).

For the wind forecast, a minimum of two altitude reports (altitude, wind speed, and wind direction) should be provided at each waypoint. The altitudes should span the estimated altitude crossing at the associated waypoint. The algorithm assumes that the input data are valid.

Internal Algorithm Variables

The significant variables computed by this algorithm are as follows:

Data related to the overall path include:

Mach Transition Altitude *the computed altitude where the transition from Mach to CAS occurs*

NmiToFeet *6076.115486*

Data specific to each TCP include:

Altitude *the computed altitude at the TCP*

CAS *the computed CAS at the TCP*

<i>DTG</i>	<i>the computed, cumulative distance from the last waypoint to the TCP</i>
<i>Ground Speed</i>	<i>the computed ground speed at the TCP</i>
<i>Ground Track</i>	<i>the computed ground track at the TCP</i>
<i>Mach</i>	<i>the computed Mach at the TCP</i>
<i>TTG</i>	<i>the computed, cumulative time from the last waypoint to the TCP</i>

Additionally, the algorithm denotes TCPs in accordance with how they are generated and are marked accordingly in the waypoint variable *WptType*. *WptType* identifiers are:

- Input, from the input waypoint data;
- An internally generated, radius-to fix (RF) center of turn waypoint;
- Turn-entry, identifying a TCP that marks the start of a turn;
- Turn-exit, identifying a TCP that marks the end of a turn; and
- Vertical TCPs (VTCPs), denoting a change in the altitude or speed profile.

TCPs may also be marked with a vertical identifier, *VSegType*, denoting one of the following:

- Altitude, denoting a change in the descent angle;
- Speed, denoting a change in the CAS or Mach;
- Top of descent point, TOD;
- Altitude CAS restriction, denoting a speed change due to a speed restriction at a specific altitude, e.g., 250 kt at 10,000'; and
- Mach-to-CAS, denoting the Mach-to-CAS transition point.

TCPs are also denoted relative to the associated primary speed value, i.e., the crossing speed is Mach or CAS derived.

There are also several input variables that may become overwritten within the algorithm that are required to be restored for subsequent calculation cycles within the algorithm. These variables include the following:

- *Saved Altitude Crossing Angle*, which is the saved input value of *Crossing Angle* for each of the TCP's.
- *Saved Mach Descent Mach*, which is the saved input value of *Mach Descent Mach*.
- *Saved Mach Transition CAS*, which is the saved input value of *Mach Transition CAS*.
- *Saved Mach at First Waypoint*, which is the saved input Mach value for the first waypoint, i.e., *Crossing Mach_{first waypoint}*, assuming that one exists.

Description of Major Functions

The functions shown in figure 1 are described in detail in this section. The functions are presented in the order as shown in figure 1. Secondary functions are described in a subsequent section. In these descriptions, the waypoints, which are from the input data and are fixed geographic points, are considered to be TCPs but not all TCPs are waypoints. Nesting levels in the pseudo-code description are denoted by the level of indentation of the document formatting. Additionally, long sections of logic may end with *end of* statements to enhance the legibility of the text.

Preprocess RF Legs

A radius-to-fix (RF) turn segment is a constant radius turn between two waypoints, with lines tangent to the arc around a center of turn point (fig. 2). This function determines if a valid RF turn exists and if so, calculates a pseudo-waypoint relative to the center-of-turn point and inserts it into the waypoint list. The calculated pseudo-waypoint then allows the remainder of the turn calculations performed by this algorithm to be processed as a standard turn. This function is performed in the following manner:

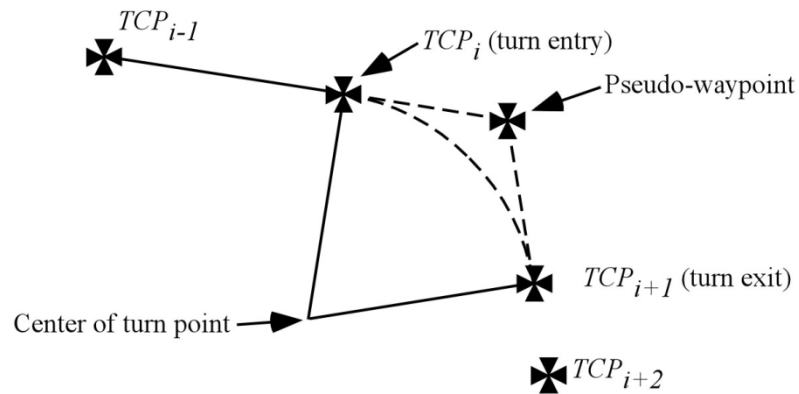


Figure 2. Example of an RF turn.

error = false

Big Turn Error = false

A set of RF turn waypoints is identified by the inclusion of a non-zero value for the latitude and longitude for the center of turn point in the data for the RF turn initiation waypoint. Because three waypoints are needed in an RF turn calculation, two each for the determination of the inbound and outbound track angles, testing is only performed to the next to the last waypoint.

for (i = index number of the first waypoint + 1; i ≤ index number of the last waypoint - 1; i = i + 1)

Determine if this is an RF turn waypoint via the inclusion of the turn center's latitude and longitude data.

if ((Center Of Turn Latitude_i ≠ 0) and (Center Of Turn Longitude_i ≠ 0))

Determine the turn direction.

$$a_1 = \arctangent2(\text{sine}(\text{Longitude}_i - \text{Longitude}_{i-1}) * \text{cosine}(\text{Latitude}_i), \text{cosine}(\text{Latitude}_{i-1}) * \text{sine}(\text{Latitude}_i) - \text{sine}(\text{Latitude}_{i-1}) * \text{cosine}(\text{Latitude}_i) * \text{cosine}(\text{Longitude}_i - \text{Longitude}_{i-1}))$$

$$a_3 = \text{arctangent2}(\text{sine}(\text{Longitude}_{i+1} - \text{Longitude}_i) * \text{cosine}(\text{Latitude}_{i+1}), \text{cosine}(\text{Latitude}_i) * \text{sine}(\text{Latitude}_{i+1}) - \text{sine}(\text{Latitude}_i) * \text{cosine}(\text{Latitude}_{i+1}) * \text{cosine}(\text{Longitude}_{i+1} - \text{Longitude}_i))$$

$$\text{deltax} = \text{DeltaAngle}(a_1, a_3)$$

where the secondary function *DeltaAngle* is described in a subsequent section.

If *deltax* is positive, this is a right-hand turn.

$$\text{if } (\text{deltax} \geq 0) \text{ TurnSign} = 1$$

$$\text{else TurnSign} = -1$$

Calculate the instantaneous angle at the ending waypoint.

$$a_2 = \text{arctangent2}(\text{sine}(\text{Longitude}_{i+1} - \text{Center Of Turn Longitude}_i) * \text{cosine}(\text{Latitude}_{i+1}), \text{cosine}(\text{Center Of Turn Latitude}_i) * \text{sine}(\text{Latitude}_{i+1}) - \text{sine}(\text{Center Of Turn Latitude}_i) * \text{cosine}(\text{Latitude}_{i+1}) * \text{cosine}(\text{Longitude}_{i+1} - \text{Center Of Turn Longitude}_i)) + \text{TurnSign} * 90$$

Adjust a_2 such that $0 \geq a_2 \geq 360$

$$\text{deltaa} = \text{DeltaAngle}(a_1, a_2)$$

Correct the *deltaa* value if it is in the wrong direction.

$$\text{if } ((\text{TurnSign} > 0) \text{ and } (\text{deltaa} < 0))$$

$$\text{deltaa} = \text{deltaa} + 360$$

$$\text{else if } ((\text{TurnSign} < 0) \text{ and } (\text{deltaa} > 0))$$

$$\text{deltaa} = \text{deltaa} - 360$$

If the turn is greater than 170°, break it into two parts so that the standard turn calculations can be performed.

$$\text{if } (|\text{deltaa}| > 170) \text{ BigTurn} = \text{true}$$

If the turn is less than 3° or more than 260°, it is in error.

$$\text{if } ((|\text{deltaa}| < 3) \text{ or } (|\text{deltaa}| > 260)) \text{ error} = \text{true}$$

Perform a center-of-turn test.

$$\text{if } (\text{error} = \text{false})$$

The radius for point 1 must equal the radius for point 2.

$$r_1 = \arccosine(\text{sine}(\text{Center Of Turn Latitude}_i) * \text{sine}(\text{Latitude}_i) + \text{cosine}(\text{Center Of Latitude}_i) * \text{cosine}(\text{Latitude}_i) * \text{cosine}(\text{Center Of Turn Longitude}_i - \text{Longitude}_i))$$

$$r_2 = \arccosine(\text{sine}(\text{Center Of Turn Latitude}_i) * \text{sine}(\text{Latitude}_{i+1}) + \text{cosine}(\text{Center Of Turn Latitude}_i) * \text{cosine}(\text{Latitude}_{i+1}) * \text{cosine}(\text{Center Of Turn Longitude}_i - \text{Longitude}_{i+1}))$$

The radii are considered not equal if the difference is greater than 200 ft. The overall RF leg is considered in error if the turn radius is greater than 10 nmi.

$$\text{if } (|r_1 - r_2| > (200 / \text{NmiToFeet})) \text{ or } (r_1 > 10) \text{ error} = \text{True}$$

if (error = false)

If the turn is greater than 170°, generate two waypoints, otherwise, just generate one waypoint.

$$\text{if } (\text{BigTurn}) \text{ } n = 2$$

$$\text{else } n = 1$$

$$a = \text{TurnSign} * 90$$

for (k = 1; k ≤ n; k = k + 1)

Calculate the pseudo-RF waypoint.

The following is the angle from the turn center toward the pseudo waypoint.

$$a_3 = a_1 - a$$

Adjust a_3 such that $0 \geq a_3 \geq 360$

if (BigTurn)

$$\text{if } (k = 1) \text{ } a_{1b} = a_3 + 0.25 * \text{delta}a$$

$$\text{else } a_{1b} = a_3 + 0.75 * \text{delta}a$$

else

There is just one new waypoint, split the turn in half.

$$a_{1b} = a_3 + 0.5 * \text{delta}a$$

Adjust a_{1b} such that $0 \geq a_{1b} \geq 360$

if (k = 1)

*RadialRadialIntercept(Latitude_i, Longitude_i, a₁,
Center Of Turn Latitude_i, Center Of Turn Longitude_i, a_{1b},
Latitude_{rf}, Longitude_{rf}),*

noting that *Latitude_{rf}* and *Longitude_{rf}* are returned values.

else

*RadialRadialIntercept(Latitude_{i+1}, Longitude_{i+1}, a₂ + 180,
Center Of Turn Latitude_{i-1}, Center Of Turn Longitude_{i-1}, a_{1b},
Latitude_{rf}, Longitude_{rf}),*

The new waypoint is inserted at location $i+1$ in the waypoint list. This inserted waypoint will appear as an input waypoint to the remainder of the algorithm. The waypoint is inserted between waypoint _{i} and waypoint _{$i+1$} from the original list. The function *InsertWaypoint* should be appropriate for the actual data structure implementation of this function.

InsertWaypoint(i + 1)

Note that *Wpt_{i+1}* is the newly created waypoint.

Mark Wpt_{i+1} as though it was an input waypoint and give it a unique name.

Also mark this waypoint as a special, RF turn center waypoint. This special marking is used in subsequent sections to denote that the center-of-turn point has already been calculated.

TurnType_{i+1} = rf-turn-center

Latitude_{i+1} = Latitude_{rf}

Longitude_{i+1} = Longitude_{rf}

Copy the wind data from Wpt_i, the RF initiation waypoint, to Wpt_{i+1}, the pseudo-waypoint.

Save the center of turn data. The Turn Data values are associated with each waypoint or TCP record and contain, if appropriate, data relating to turn conditions for that TCP.

Turn Data Center Latitude_{i+1} = Center Of Turn Latitude_i

Turn Data Center Longitude_{i+1} = Center Of Turn Longitude_i

Increment i because a waypoint was added and the new waypoint at $i + 1$ should not be processed again.

$i = i + 1$

end of for (k = 1; k ≤ n; k = k + 1)

end of if (error = false)

end of if ((Center Of Turn Latitude_i ≠ 0) and (Center Of Turn Longitude_i ≠ 0))

end of for (i = index number of the first waypoint + 1; ...)

Save Selected Input Data

This is an initialization function that saves the original input values for the altitude crossing angle of each waypoint, the Mach for the first waypoint, the descent Mach, and descent CAS. These values are saved because the input values may be overwritten internal to the algorithm and will need to be reset to their original values for each iterative loop. The function is performed in the following manner:

for (i = index number of the first waypoint; i ≤ index number of the last waypoint; i = i + 1)
Saved Altitude Crossing Angle_i = Crossing Angle_i

Saved Mach Descent Mach = Mach Descent Mach

Saved Mach Transition CAS = Mach Transition CAS

Saved Mach at First Waypoint = Crossing Mach_{first waypoint}

Convert to Indicated Altitudes

This is an initialization function that converts altitudes at and below the barometric altitude transition altitude, *barometric transition altitude*, (nominally 18,000'), to indicated altitudes using the waypoint barometric setting from the input data. The function is performed in the following manner:

Initialize the value *Last Altitude* to a very large number.

Last Altitude = 99999

for (i = index number of the first waypoint; i ≤ index number of the last waypoint; i = i + 1)

Calculate the indicated altitude only if the waypoint has an altitude constraint.

*if (((i = index number of the first waypoint) or (Crossing Angle_i > 0) or
(Crossing Angle_i = AUTO DESCENT ANGLE)) and
(Crossing Altitude_i ≤ barometric transition altitude)) then*

*Crossing Altitude_i =
ConvertPressureToIndicatedAltitude(Crossing Altitude_i, barometric setting_i),*

where *ConvertPressureToIndicatedAltitude* is a standard aeronautical function to convert pressure altitude to indicated altitude.

*if (Crossing Altitude_i > barometric transition altitude)
Crossing Altitude_i = barometric transition altitude*

if (Crossing Altitude_i > LastAlt) Crossing Altitude_i = LastAlt

LastAlt = Crossing Altitude_i

Generate Initial Tracks and Distances

This is an initialization function that initializes the *Mach Segment* flag, denoting that the speed in this segment is based on Mach, and calculates the point-to-point distances and ground tracks between input waypoints. Great circle equations are used for these calculations, noting that the various dimensional conversions, e.g., degrees to radians, are not shown in the following text.

Generate the initial distances, the center-to-center distances, and ground tracks between input waypoints

for ($i = \text{index number of the first waypoint}; i \leq \text{index number of the last waypoint}; i = i + 1$)

Start with setting the Mach segments flags to false.

$\text{Mach Segment}_i = \text{false}$

Compute the waypoint-center to waypoint-center distances.

if ($i = \text{index number of the first waypoint}$) $\text{Center to Center Distance}_i = 0$

else

$\text{Center to Center Distance}_i =$
 $\arccosine(\text{sine}(\text{Latitude}_{i-1}) * \text{sine}(\text{Latitude}_i) + \text{cosine}(\text{Latitude}_{i-1}) * \text{cosine}(\text{Latitude}_i) * \text{cosine}(\text{Longitude}_{i-1} - \text{Longitude}_i))$

$\text{Ground Track}_{i-1} =$
 $\arctangent2(\text{sine}(\text{Longitude}_i - \text{Longitude}_{i-1}) * \text{cosine}(\text{Latitude}_i), \text{cosine}(\text{Latitude}_{i-1}) * \text{sine}(\text{Latitude}_i) - \text{sine}(\text{Latitude}_{i-1}) * \text{cosine}(\text{Latitude}_i) * \text{cosine}(\text{Longitude}_i - \text{Longitude}_{i-1}))$

end of for ($i = \text{index number of the first waypoint}; i \leq \text{index number of the last waypoint}; i = i + 1$)

Now set the runway's ground track.

$\text{Ground Track}_{\text{last waypoint}} = \text{Ground Track}_{\text{last waypoint} - 1}$

The cumulative distance, DTG, is computed as follows:

$\text{DTG}_{\text{last waypoint}} = 0$

for ($i = \text{index number of the last waypoint}; i > \text{index number of the first waypoint}; i = i - 1$)

$\text{DTG}_{i-1} = \text{DTG}_i + \text{Center to Center Distance}_i$

Initialize Waypoint Turn Data

The *Initialize Waypoint Turn Data* function is used to determine if a turn exists at a waypoint and if so, inserts turn-entry and turn-exit TCPs. Waypoints that have more than a 3 degree change in ground track between the previous waypoint and the next waypoint are considered turn-waypoints. The function is performed in the following manner:

$i = \text{index number of the first waypoint} + 1$

$\text{Last Track} = \text{Ground Track}_{\text{first waypoint}}$

Note that the first and last waypoints cannot be turns.

while ($i < \text{index number of the last waypoint}$)

$\text{Track Angle After} = \text{Ground Track}_i$

$a = \text{DeltaAngle}(\text{Last Track}, \text{Track Angle After})$

Check for a turn that is greater than 170 degrees.

if ($|a| > 170$)

Set an error and ignore the turn.

Mark this as an error condition.

$a = 0$

If the turn is more than 3-degrees, compute the turn data.

if ($|a| > 3$)

$\text{half turn} = a / 2$

$\text{Track Angle Center} = \text{Last Track} + \text{half turn}$

This is the center of the turn, e.g., the original input waypoint.

$\text{Ground Track}_i = \text{Track Angle Center}$

$\text{Turn Data Track1}_i = \text{Last Track}$

$\text{Turn Data Track2}_i = \text{Track Angle After}$

If this is not an RF turn, then the turn radius needs to be calculated.

if ($\text{TurnType}_i \neq \text{rf-turn-center}$) $\text{Turn Data Turn Radius}_i = 0$

$\text{Turn Data Path Distance}_i = 0$

Insert a new TCP at the end of the turn.

The new TCP is inserted at location $i+1$ in the TCP list. The TCP is inserted between TCP_i and TCP_{i+1} from the original list. The function *InsertWaypoint* should be appropriate for the actual data structure implementation of this function.

InsertWaypoint(i + 1)

Note that TCP_{i+1} is the new TCP.

$TCP_{i+1} = \textit{turn-exit}$

$DTG_{i+1} = DTG_i$

$\textit{Ground Track}_{i+1} = \textit{Track Angle After}$

The start of the turn TCP is as follows,

InsertWaypoint(i)

$TCP_i = \textit{turn-entry}$

Note that the original TCP is now at index $i + 1$.

$DTG_i = DTG_{i+1}$

$\textit{Ground Track}_i = \textit{Last Track}$

$\textit{Last Track} = \textit{Track Angle After}$

$i = i + 2$

end of if ($|a| > 3$)

else $\textit{Last Track} = \textit{Ground Track}_i$

$i = i + 1$

end of while ($i < \textit{index number of the last waypoint}$)

Effectively, this function:

- Marks each turn-waypoint and sets its ground track angle to the computed angle at the midpoint of the turn.
- Inserts a co-distance turn-entry TCP before this turn-waypoint with the ground track angle for this turn-entry TCP set to the value of the inbound ground track angle.
- Inserts a co-distance turn-exit TCP after this turn-waypoint with the ground track angle for this turn-exit TCP set to the value of the outbound ground track angle.

An example illustrating the inserted turn-start and turn-end TCPs is shown in figure 3.

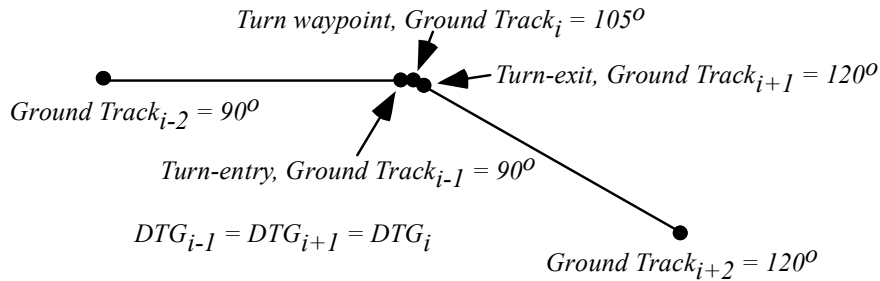


Figure 3. Initialized turn waypoint.

Reinstate the Descent Speeds

The *Restore the Descent Speeds* function simply replaces the current values for Mach Descent Mach, Mach Transition CAS, and Crossing Mach_{first waypoint} with the values that were saved in the function *Save Selected Input Data*.

Compute TCP Altitudes

Beginning with the last waypoint, the *Compute TCP Altitudes* function computes the altitudes at each previous TCP and inserts any additional altitude TCPs that may be required to denote a change in the altitude profile. The function uses the current altitude constraint (TCP_i in fig. 4), searches backward for the previous constraint (TCP_{i-3} in fig. 4), and then computes the distance required to meet this previous constraint. The altitudes for all of the TCPs within this distance are computed and added to the data for the TCPs. If the along-path distance to meet the previous constraint is not at a TCP, a new altitude VTCP is inserted at this distance. An example of this is shown in figure 5. In addition, if the Crossing Angle for a waypoint is set to -99, this denotes that the algorithm is to internally compute the Crossing Angle between this and the next higher, altitude constrained waypoint, noting that this option should only be used in situations where the relevant waypoint pairs are known to procedurally have a fixed angle between them. This function is performed in the following steps:

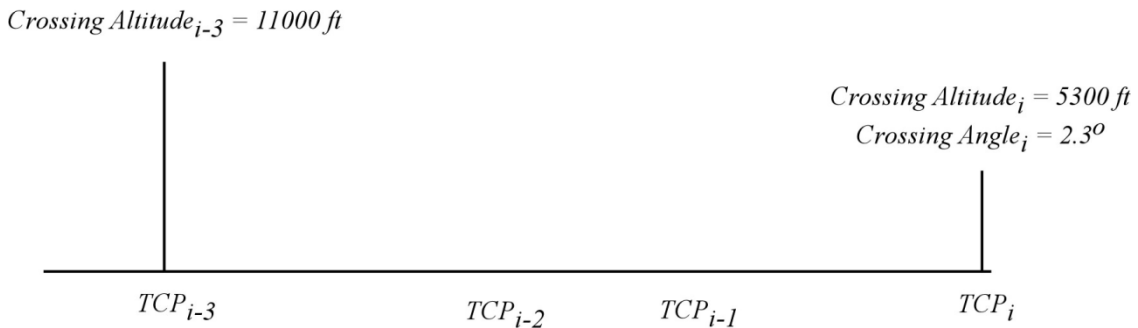


Figure 4. Input altitude crossing constraints.

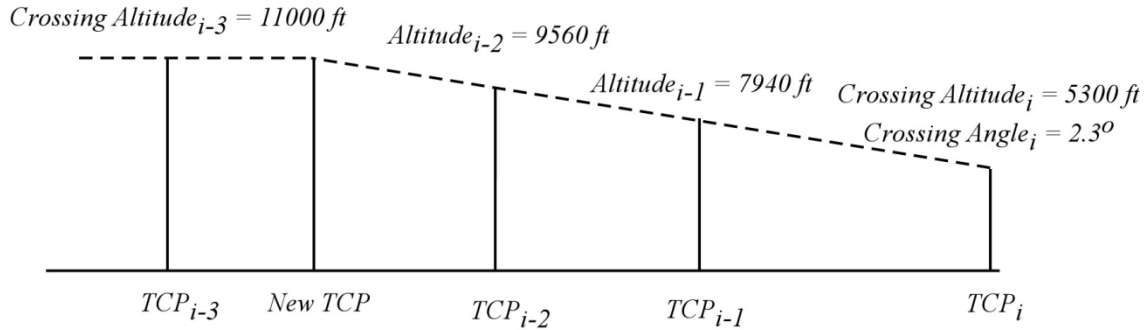


Figure 5. Computed altitude profile with TCP added.

Set the current constraint index number, cc , equal to the index number of the last waypoint,

$cc = \text{index number of the last waypoint}$

Set the altitude of this waypoint to its crossing altitude,

$Altitude_{cc} = \text{Crossing Altitude}_{cc}$

Set a flag denoting that the TOD point has not been identified

$Have\ TOD = \text{false}$

While ($cc > \text{index number of the first waypoint}$)

If this is the TOD, mark this point.

if $Have\ TOD$ is false and $Altitude_{cc}$ is equal or greater than $Altitude_i$

$Have\ TOD = \text{true}$

mark this as the TOD point.

Determine if the previous constraint cannot be met.

If ($Altitude_{cc} > \text{Crossing Altitude}_{cc}$)

The constraint has not been made.

If this is the last pass through the algorithm, mark this as an error condition.

$Altitude_{cc} = \text{Crossing Altitude}_{cc}$

Find the prior waypoint index number pc that has an altitude constraint, e.g., a crossing altitude ($\text{Crossing Altitude}_{pc} \neq 0$). This may not always be the previous (i.e., $cc - 1$) waypoint.

Initial condition is the previous TCP.

$$pc = cc - 1$$

while (($pc >$ index number of the first waypoint) and (($TCP_{pc} \neq$ input waypoint) or ($Crossing\ Altitude_{pc} = 0$))) $pc = pc - 1$

Save the previous crossing altitude,

$$Prior\ Altitude = Crossing\ Altitude_{pc}$$

Save the current crossing altitude (*Test Altitude*) at TCP_{cc} and the descent angle (*Test Angle*) noting that the first and last waypoints always have altitude constraints and except for the first waypoint, all constrained altitude points must have descent angles.

$$Test\ Altitude = Crossing\ Altitude_{cc}$$

$$Test\ Angle = Crossing\ Angle_{cc}$$

If the Test Angle value, i.e., AUTO DESCENT ANGLE, denotes that this angle is to be computed internally as a linear descent between the two altitude constrained waypoints then the following calculations are performed:

if ($Test\ Angle = AUTO\ DESCENT\ ANGLE$)

$$dx = DTG_{pc} - DTG_{cc}$$

$$dy = Prior\ Altitude - Test\ Altitude$$

$$Test\ Angle = arctangent2(dy, NmiToFeet * dx)$$

$$Crossing\ Angle_{cc} = Test\ Angle$$

Test for an extreme angle, e.g., 7.5°.

if ($Test\ Angle >$ maximum allowable descent angle) mark this as an error condition.

Compute all of the TCP altitudes between the current TCP and the previous crossing waypoint.

$$k = cc$$

while $k > pc$

If the previous altitude has already been reached, set the remaining TCP altitudes to the previous altitude.

if ($Prior\ Altitude \leq Test\ Altitude$)

$$for\ (k = k - 1; k > pc; k = k - 1)\ Altitude_k = Test\ Altitude$$

Set the altitude at the last test point.

$$Altitude_{pc} = Test\ Altitude$$

else

Compute the distance from TCP_k to the *Prior Altitude* using the altitude difference between the *Test Altitude* and the *Prior Altitude* with the *Test Angle*. If there is no point at this distance, add a TCP at that distance.

Compute the distance dx to make the altitude.

$$dx = (Prior\ Altitude - Test\ Altitude) / (NmiToFeet * \tan(Test\ Angle))$$

Compute the altitude z at the previous TCP.

$$z = ((DTG_{k-1} - DTG_k) * NmiToFeet) * \tan(Test\ Angle) + Test\ Altitude$$

If there is a TCP prior to this distance or if z is very close to the *Prior Altitude*, compute and insert its altitude.

if $((DTG_{k-1} < (DTG_k + dx))$ or $(|z - Prior\ Altitude| < \text{some small value})$

if $(|z - Prior\ Altitude| < \text{some small value})$ $Altitude_{k-1} = Prior\ Altitude$

else $Altitude_{k-1} = z$

Check to see if the constraint has been reached with a 100 ft tolerance; if not, set an error condition.

if $((k-1) = pc)$

if $(|Altitude_{pc} - Crossing\ Altitude_{pc}| > 100ft)$ mark this as an error condition

Always set the crossing exactly to the crossing value.

$$Altitude_{pc} = Crossing\ Altitude_{pc}$$

Update the Test Altitude.

$$Test\ Altitude = Altitude_{k-1}$$

Decrement the counter to set it to the prior TCP.

$$k = k - 1$$

end of if $((DTG_{k-1} < (DTG_k + dx))$ or $(|z - Prior\ Altitude| < \text{some small value})$

else

The altitude constraint is reached prior to the TCP, a new VTCP will need to be inserted at that point. The distance to the new TCP is,

$$d = DTG_k + dx$$

Compute the ground track at distance d along the trajectory and save it as *Saved Ground Track*.

Saved Ground Track = *GetTrajGndTrk*(d)

Insert a new VTCP at location k in the TCP list. The VTCP is inserted between TCP_{k-1} and TCP_k from the original list. The function *InsertWaypoint* should be appropriate for the actual data structure implementation of this function.

InsertWaypoint(k)

Update the data for the new VTCP which is now TCP_k .

if ($VSegType_k = no\ type$) $VSegType_k = ALTITUDE$

$DTG_k = d$

$Altitude_k = Prior\ Altitude$

Add the ground track data which must be computed if the new VTCP occurs within a turn. The functions *WptInTurn* and *ComputeGndTrk* are described in subsequent sections.

if (*WptInTurn*(k)) $Ground\ Track_k = ComputeGndTrk(k, d)$

else $Ground\ Track_k = Saved\ Ground\ Track$

Compute and add the wind data at distance d along the path to the data of TCP_k .

GenerateWptWindProfile(d, TCP_k)

$Test\ Altitude = Prior\ Altitude$

Since TCP_k , has now been added prior to pc , the current constraint counter cc needs to be incremented by 1 to maintain its correct position in the list.

$cc = cc + 1$

The function loops back to *while* $k > pc$.

Now go to the next altitude change segment on the profile.

$cc = k$

The function loops back to *while* $cc > index\ number\ of\ the\ first\ waypoint$.

Copy Crossing Angles

The *Copy Crossing Angles* is a simple function that starts with the next to last TCP and copies the subsequent crossing angle if the current TCP does not have a crossing angle. E.g.,

for (i = index number of the last waypoint - 1; i ≥ index number of the first waypoint; i = i - 1)

if (Crossing Angle_i = 0) Crossing Angle_i = Crossing Angle_{i+1}

Meet Cruise CAS Waypoint Restriction

The *Meet Cruise CAS Waypoint Restriction* function changes, if required, the descent Mach if there is a high altitude, CAS restricted waypoint and the computed speed is above the required crossing speed for that CAS waypoint.

The calling function provides as input and retains the subsequent outputs for the following variables: *TodId*, *TodMach*, *TodMachRate*, and *MachCasAtTod*. The variable *TodId* is the name of the top-of-descent waypoint (TOD) and is initialized as an empty string by the calling program. This *Meet Cruise CAS Waypoint Restriction* function may modify the Mach and speed change rate that occurs at the TOD, *TodMach* and *TodMachRate*, respectively, and these values are then passed to subsequent functions that require these data. The variable *MachCasAtTod* is a flag that if true, indicates that the Mach-to-CAS transition occurs at the TOD point. This variable is used by the functions *Add Descent Mach Waypoint* and *Compute Mach-to-CAS TCP*.

If the input Mach value for the first waypoint is not valid, i.e., the path does not start with a Mach segment, the function terminates with *MachCasAtTod* set to false. Otherwise, the following is performed.

if (Crossing Mach_{first waypoint} = 0) terminate this function. Otherwise,

Set the initial values.

MachCasAtTod = false

MachCasModified = false

CasIndex = index number of the first waypoint

AltAtMach = 0.

LastMach = 0

z = 0

done = false

If the TOD Mach data have been modified in a previous invocation of *Add Descent Mach Waypoint*, indicated by a non-empty value for *TodId*, reset their values.

if (TodId ≠ empty)

fini = false

i = index number of the first waypoint

Find the waypoint with the name defined in *TodId*.

while ($(i \leq (\text{index number of the last waypoint}))$ and ($\text{fini} = \text{false}$))

if ($\text{Id}_i = \text{TodId}$)

$\text{fini} = \text{true}$

$\text{Crossing Mach}_i = \text{TodMach}$

$\text{Crossing CAS}_i = 0$

$\text{Crossing Rate}_i = \text{TodMachRate}$

$\text{TodId} = \text{empty string}$

$i = i + 1$

end of if ($\text{TodId} \neq \text{empty}$)

Find the first CAS waypoint.

$\text{fini} = \text{false}$

$i = \text{index number of the first waypoint}$

while ($(i \leq \text{index number of the last waypoint})$ and ($\text{fini} = \text{false}$))

if ($\text{Crossing CAS}_i > 0$)

$\text{CasIndex} = i$

$\text{fini} = \text{true}$

$i = i + 1$

Determine if the trajectory is already at the CAS altitude, i.e., the initial altitude is the CAS altitude, and if so, start in a CAS mode, not Mach.

if ($\text{Crossing Altitude}_{\text{first waypoint}} = \text{Altitude}_{\text{CasIndex}}$)

$\text{done} = \text{true}$

for ($k = \text{index number of the first waypoint}; k < \text{CasIndex}; k = k + 1$)

if ($\text{Crossing Mach}_k > 0$)

Change the route data so that the trajectory is starting in a CAS mode.

Invoke the secondary function *MachToCas*. This function is described in a subsequent section.

$Crossing\ CAS_k = MachToCas(Crossing\ Mach_k, Altitude_{CasIndex})$

$Crossing\ Mach_k = 0$

$MachSegment_k = false$

end of if (Crossing Mach_k > 0)

if (done = false)

Find the last Mach value.

$fini = false$

$i = \text{index number of the first waypoint}$

while ((i < index number of the last waypoint) and (fini = false))

if (Crossing CAS_i > 0) fini = true

else if (Crossing Mach_i > 0) LastMach = Crossing Mach_i

$i = i + 1$

Determine the descent Mach value.

if (Mach Descent Mach \neq 0) DescentMach = Mach Descent Mach

else DescentMach = LastMach

Determine the Mach-to-CAS transition CAS value.

if (Mach Transition CAS > 0)

$MachCas = Mach\ Transition\ CAS$

if (Mach Transition CAS < Crossing CAS_{CasIndex}) MachCas = Crossing CAS_{CasIndex}

else MachCas = Crossing CAS_{CasIndex}

Find the last Mach altitude.

$fini = false$

$i = \text{index number of the first waypoint}$

while ((i \leq index number of the last waypoint) and (fini = false))

if (Crossing CAS_i > 0) fini = true

else if (Crossing Altitude_i > 0) AltAtMach = Crossing Altitude_i

$i = i + 1$

Determine if the Mach is slower than the descent CAS.

Invoke the secondary function *FindMachCasTransitionAltitude* which calculates the altitude where the Mach and CAS are equal. This function is described in a subsequent section.

$z = \text{FindMachCasTransitionAltitude}(\text{MachCas}, \text{DescentMach})$

if ($z > \text{Crossing Altitude}_{\text{first waypoint}}$)

The path is already below the transition altitude, change the route data so it starts in a CAS mode.

for ($k = \text{index number of the first waypoint}; k < \text{index number of the last waypoint}; k = k + 1$)

$\text{done} = \text{true}$

if ($\text{Crossing Mach}_k > 0$)

$\text{Crossing CAS}_k = \text{MachCas}$

$\text{Crossing Mach}_k = 0$

$\text{MachSegment}_k = \text{false}$

end of if ($\text{done} = \text{false}$)

if ($\text{done} = \text{false}$)

Find the last Mach value.

$\text{fini} = \text{false}$

$i = \text{index number of the first waypoint}$

while ($(i \leq \text{index number of the last waypoint}) \text{ and } (\text{fini} = \text{false})$)

if ($\text{Crossing CAS}_i > 0$) $\text{fini} = \text{true}$

else if ($\text{Crossing Mach}_i > 0$) $\text{LastMach} = \text{Crossing Mach}_i$

$i = i + 1$

Determine the descent Mach.

if ($\text{Mach Descent Mach} \neq 0$) $\text{DescentMach} = \text{Mach Descent Mach}$

else $\text{DescentMach} = \text{LastMach}$

Find the Mach-to-CAS transition CAS.

if (Mach Transition CAS > 0) MachCas = Mach Transition CAS

Make sure that the crossing restriction can be obtained.

if (Mach Transition CAS < Crossing CAS_{CasIndex}) MachCas = Crossing CAS_{CasIndex}

else MachCas = Crossing CAS_{CasIndex}

Find the last Mach altitude.

fini = false

i = index number of the first waypoint

while ((i ≤ index number of the last waypoint) and (fini = false))

if (Crossing CAS_i > 0) fini = true

else if (Crossing Altitude_i > 0) AltAtMach = Crossing Altitude_i

i = i + 1

Determine if the Mach is slower than the descent CAS.

z = FindMachCasTransitionAltitude(MachCas, DescentMach)

if (z > Crossing Altitude_{first waypoint})

The path is already below the transition altitude, change the route data so it is starting in a CAS mode.

for (k = index number of the first waypoint; k < index number of the last waypoint; k = k + 1)

done = true

if (Crossing Mach_k > 0)

Crossing CAS_k = MachCas

Crossing Mach_k = 0

MachSegment_k = false

end of if (done = false)

If the path still starts with a Mach segment, which may have already been modified in this function, test for other special cases.

if (done = false)

If required, handle the special case of an accelerated descent.

if (DescentMach > LastMach)

Invoke the secondary function *HandleDescentAccelDecel*. This function handles the special case of a Mach acceleration in the descent where the first CAS crossing restriction cannot be met. This function is described in a subsequent section. This function may modify the waypoint data.

HandleDescentAccelDecel(CasIndex, LastMach, MachCasModified, DescentMach, MachCas)

If the descent data are changed, recalculate z .

if (MachCasModified)

$z = \text{FindMachCasTransitionAltitude}(\text{MachCas}, \text{DescentMach})$

Next, update the waypoint data.

$\text{Mach Descent Mach} = \text{DescentMach}$

$\text{Mach Transition CAS} = \text{MachCas}$

end of if (DescentMach > LastMach)

if ($z < \text{Crossing Altitude}_{\text{CasIndex}}$)

At this point, the descent CAS or Mach needs to be changed.

If the descent CAS is faster than the crossing CAS, determine if changing the descent CAS corrects the problem.

fini = false

if (MachCas > Crossing CAS_{CasIndex}) then

$s = \text{MachToCas}(\text{DescentMach}, \text{Crossing Altitude}_{\text{CasIndex}})$

if ($s \geq \text{Crossing CAS}_{\text{CasIndex}}$) then

$\text{MachCas} = s$

$\text{Mach Transition CAS} = s$

fini = true

$m = \text{CasToMach}(\text{MachCas}, \text{Crossing Altitude}_{\text{CasIndex}})$

*if ((*fini = false*) and ($m > \text{DescentMach}$)) then*

$s = \text{MachToCas}(\text{DescentMach}, \text{Crossing Altitude}_{\text{CasIndex}})$

if (s >= Crossing CAS_{CasIndex}) then

Change to descent CAS.

MachCas = s

Mach Transition CAS = s

else

Change the descent Mach.

DescentMach = CasToMach(MachCas, Crossing Altitude_{CasIndex})

else if (fini = false)

DescentMach = CasToMach(MachCas, Crossing Altitude_{CasIndex})

Mach Descent Mach = DescentMach

z = Crossing Altitude_{CasIndex}

Perform an extreme limits test, assuming that a valid Mach value will be between 0.6 and 0.9 Mach.

if ((DescentMach > 0.9) or (DescentMach < 0.6)) mark this as an error condition

end of if (z < Crossing Altitude_{CasIndex})

Make sure that there is sufficient distance to slow from the Mach-to-CAS transition speed to make the crossing CAS.

if ((z ≥ Crossing Altitude_{CasIndex}) and (MachCas > Crossing CAS_{CasIndex}) and (Crossing Rate_{CasIndex} > 0) and (MachCasModified = false))

Find the distance at z. This is an iterative solution.

i = CasIndex - 1

fini = false

Calculate the headwind at the end point. This calculation uses the secondary function *InterpolateWindWptAltitude*, described in a subsequent section.

InterpolateWindWptAltitude(Wind Profile_{CasIndex}, Altitude_{CasIndex}, Ws, Wd, Td)

*HeadWind = Ws * cosine(Wd - GndTrack_{CasIndex})*

CurrentGs = ComputeGndSpeedUsingTrack(Crossing CAS_{CasIndex}, GndTrack_{CasIndex}, Altitude_{CasIndex}, Ws, Wd, Td)

Iterate = false

OnePass = true

MachCasHold = MachCas

LastCut = 0

while (fini = false)

i = CasIndex - 1

while ((i > index number of the first waypoint) and (Altitude_i < z)) i = i - 1

if ((Altitude_i - Altitude_{i+1}) ≤ 0) a = 0

else a = (z - Altitude_{i+1}) / (Altitude_i - Altitude_{i+1})

Calculate the distance, *dx*, required to reach the altitude.

*dx = a * (DTG_i - DTG_{i+1}) + DTG_{i+1} - DTG_{CasIndex}*

InterpolateWindWptAltitude(Wind Profile_{CasIndex}, z, Ws2, Wd2, Td2)

*Hw2 = Ws2 * cosine(Wd2 - GndTrack_i)*

AvgHw = (HeadWind + Hw2) / 2

Invoke the secondary function *EstimateNextCas*. *EstimateNextCas* is an iterative function to estimate the CAS value at the next waypoint.

CasTest = EstimateNextCas(Crossing CAS_{CasIndex}, CurrentGs, true, MachCasHold, AvgHw, z, dx, Crossing Rate_{CasIndex})

If it is required, set up the iteration values, where these values are in CAS.

if (OnePass = true)

if (CasTest < MachCas) Iterate = true

else fini = true

OnePass = false

Calculate the iteration step size.

LastCut = |MachCas - CasTest|

Limit the step size to no smaller than 2 kt.

if (LastCut < 2) LastCut = 2

if (Iterate)

if (MachCas \geq CasTest) s = MachCas - LastCut

else s = MachCas + LastCut

*LastCut = 0.5 * LastCut*

if (s > MachCasHold) s = MachCasHold

Determine if the Mach-to-CAS estimate is valid.

if (((s + 0.25) \geq MachCas) and (|s - MachCas| < 1))

fini = true

Calculate the Mach-to-CAS altitude for the current estimate.

z = FindMachCasTransitionAltitude (MachCas, DescentMach)

Determine if a deceleration is needed prior to the TOD. Add a 50 ft buffer value.

if (z > (AltAtMach + 50))

Find the TOD waypoint.

fini2 = false

j = index number of the first waypoint

while ((j < index number of the last waypoint) and (fini2 = false))

if (Waypoint_j is marked as the TOD point) fini2 = true

else j = j + 1

The altitude index for the test is the TOD altitude point.

if (fini2 and (i = j))

Mach Descent Mach = CasToMach(Mach Transition CAS, AltAtMach)

MachCasAtTod = true

end of if (z > (AltAtMach + 50))

end of if (((s + 0.25) \geq MachCas) and (|s - MachCas| < 1))

else

Mach Transition CAS = s

MachCas = s

$z = \text{FindMachCasTransitionAltitude}(\text{MachCas}, \text{DescentMach})$

$\text{if } (z > \text{Altitude}_i) z = \text{Altitude}_i$

$j = j + 1$

Add a test to limit the number of iterations to 10.

$\text{if } (j \geq 10) \text{fini} = \text{true}$

end of if (Iterate)

end of while (fini = false)

end of if (done = false)

Add Descent Mach Waypoint

The *Add Descent Mach Waypoint* function changes the descent waypoint Mach if the descent Mach, *Mach Descent Mach*, is different than the cruise Mach. This function is only invoked if the variable *MachCasAtTod* is false. The function also will add any required, additional TCPs.

The calling program provides as input and retains the subsequent outputs for the following variables: *TodId*, *TodMach*, and *TodMachRate*. The variable *TodId* is the name of the top-of-descent waypoint and is initialized as a null string by the calling program. Since this function may overwrite the Mach and speed change rate for an input waypoint, these variables allow the function to retain the original values for Mach and speed change rate and to then reset these variables to their original values prior to recalculating new values.

If the Mach value for the first waypoint is not set, i.e., the path does not start with a Mach segment, or there is no defined descent Mach, i.e., *Mach Descent Mach* = 0, the function terminates. Otherwise,

If the previous TOD data for an input waypoint have been changed, these data are restored to their original values.

$\text{fini} = \text{false}$

$i = \text{index number of the first waypoint}$

The last designated Mach waypoint,

$\text{LastMachIndex} = \text{index number of the first waypoint}$

The first designated CAS waypoint,

$\text{FirstCasIndex} = \text{index number of the first waypoint}$

$\text{TodIndex} = 0$

Find the Mach and CAS waypoints.

$\text{fini} = \text{false}$

i = index number of the first waypoint

while ((*i* ≤ index number of the last waypoint) and (*fini* = false))

if (Crossing Mach_{*i*} > 0) LastMachIndex = *i*

else if (Crossing CAS_{*i*} > 0)

 FirstCasIndex = *i*

fini = true

i = *i* + 1

Find the TOD waypoint and Mach.

fini = false

i = index number of the first waypoint

while ((*i* < index number of the last waypoint) and (*fini* = false))

if ((Altitude_{*i*} < Altitude_{first waypoint}) or (Cas Cross_{*i*} > 0))

 if (Altitude_{*i*} ≠ Altitude_{first waypoint}) TodIndex = *i* - 1

 else TodIndex = *i*

fini = true

else if (Crossing Mach_{*i*} > 0) MachAtTod = Crossing Mach_{*i*}

i = *i* + 1

If the vertical segment type has not been defined, mark this as the TOD.

if ((TodIndex > 0) and (VSegType_{TodIdx} = no type)) VSegType_{TodIdx} = TOD ALTITUDE

Check for errors. There cannot be a programmed descent Mach if there is a downstream Mach restriction.

if ((LastMachIndex > TodIndex) or (FirstCasIndex ≤ TodIndex)) mark this as an error condition

else

Save the Mach values for all input waypoints so that they may be reset on subsequent passes back to their original input values.

if (WptType_{TodIndex} = input waypoint)

 Id_{TodIndex} = TodId

$TodMach = Crossing\ Mach_{TodIndex}$

$TodMachRate = Crossing\ Rate_{TodIndex}$

if (($WptType_{TodIndex} = input\ waypoint$) and ($Crossing\ Rate_{TodIndex} > 0$))

$CAS\ Rate = Crossing\ Rate_{TodIndex}$

else $CAS\ Rate = 0.75\ kt / sec$ (a default value)

The following is added to force a subsequent speed calculation.

$Crossing\ Rate_{TodIndex} = CAS\ Rate$

If the aircraft will slow during the descent, do the following:

if ($MachAtTod \geq Mach\ Descent\ Mach$)

Overwrite the TOD Mach value.

$Crossing\ Mach_{TodIndex} = Mach\ Descent\ Mach$

else

This is a special case where the aircraft is accelerating to the descent Mach.

Invoke the secondary function *DoTodAcceleration*. This function is described in a subsequent section.

$DoTodAcceleration(TodIdx, MachAtTod)$

$Crossing\ Mach_{TodIndex} = MachAtTod$

Compute Mach-to-CAS TCP

If a Mach-to-CAS transition is required, this function computes the Mach-to-CAS altitude and inserts a Mach-to-CAS TCP. This function is only performed if the input data starts with a Mach *Crossing Speed* for the first waypoint. The function determines the appropriate Mach and CAS values, calculates the altitude that these values are equal, and then determines the along-path distance where this altitude occurs on the profile. Input into this function includes the variable *MachCasAtTod*. This variable is set in the function *Meet Cruise CAS Waypoint Restriction* and indicates that, if true, the Mach-to-CAS transitions occurs at the TOD point.

The following variables are initialized:

$Mach\ Transition\ Altitude = 0$

where this variable a part of the global path data.

The *Mach Segment* for each TCP is initialized to *false*.

for ($i = index\ number\ of\ the\ first\ waypoint; i \leq index\ number\ of\ the\ last\ waypoint; i = i + 1$)

Mach Segment_i = false

Other local variables are initialized.

f_{ini} = false

First CAS = 0

Last Mach = 0

CAS Constraint Flag = true

Mach Index = 0, where this variable is used to designate the last Mach waypoint.

Cas Index = -1, where this variable is used to designate the first CAS waypoint.

CAS Constraint Flag = true

If this is the special case where the TOD is the Mach to CAS transition point, insert the TCP here. This special case is determined in the function *Meet Cruise CAS Waypoint Restriction*.

if (MachCasAtTod) then

Find the TOD.

i = index number of the first waypoint

while ((i ≤ index number of the last waypoint) and (f_{ini} = false))

if (VSegType_i = TOD ALTITUDE) f_{ini} = true

else i = i + 1

InsertWaypoint(i+1)

Copy all of the data from Wpt_i into Wpt_{i+1}

Now set the data in Wpt_{i+1} to the updated values.

VSegType_{i+1} = MACH_CAS

Crossing Mach_{i+1} = Mach Descent Mach

Crossing CAS_{i+1} = Mach Transition CAS

Mach_{i+1} = Mach Descent Mach

CAS_{i+1} = Mach Transition CAS

Use the default CAS rate if the current rate is 0.

if (Crossing Rate_{i+1} = 0) Crossing Rate_{i+1} = 0.25 kt/sec

Mach Transition Altitude = Altitude_{i+1}

Set the Mach flag to true up to and including this point.

for (j = index number of the first waypoint; j <= i+1; j++) Mach Segment_j = true

end of if (MachCasAtTod)

else if (Crossing Mach_{first waypoint} > 0) then

Perform the standard test for the Mach / CAS transition point.

CAS Constraint Flag = false

i = index number of the first waypoint

while ((i <= index number of the last waypoint) and (fini = false))

if (Crossing Mach_i > 0) then

Last Mach = Crossing Mach_i

Mach Index = i

else if (Crossing CAS_i > 0) then

First CAS = Crossing CAS_i

CAS Rate = Crossing Rate_i

CAS Index = i

CAS Constraint Flag = true

fini = true

i = i + 1

end of while

if (Mach Transition CAS > 0) First CAS = Mach Transition CAS

if (CAS Constraint Flag) then

z = FindMachCasTransitionAltitude(First CAS, Last Mach)

Determine if the very first waypoint is already below the Mach-to-CAS transition altitude and z is greater or equal to 28,000 ft.

if ((Mach Index = 0) and (z > Altitude_{first waypoint}) && (z >= 28000 ft)) then

Change the first waypoint to CAS, using the descent CAS value if it is valid.

if (Mach Transition CAS > 0.) Crossing CAS_{first waypoint} = Mach Transition CAS

else Crossing CAS_{first waypoint} = First CAS

Set the entire speed profile to CAS.

fini = false

i = index number of the first waypoint

while ((fini = false) and (i < (index number of the last waypoint - 1)))

if (Crossing Mach_i > 0) Crossing Mach_i = 0

if (Crossing CAS_i ≠ 0) fini = true

Mach Transition Altitude = z

Mach Transition CAS = 0

end of if ((Mach Index = 0)...

Otherwise, determine if there is a Mach / CAS transition error.

else if ((z > Altitude_{Mach Index}) or (z < 18000 ft)) then

skip = false

Determine if the trajectory is already at a level altitude.

j = Mach Index

while ((j > index number of the first waypoint) and (WptType_j ≠ Input)) j = j - 1

if (Altitude_j = Altitude_{CAS Index}) then

spd = MachToCas(Crossing Mach_{Mach Index}, Altitude_j)

if (spd >= Crossing CAS_{CAS Index}) then

Convert the Mach to a CAS crossing.

Crossing Mach_j = Crossing Mach_{Mach Index}

Crossing CAS_j = spd

Crossing Rate_j = Crossing Rate_{CAS Index}

Crossing Altitude_j = Altitude_{CAS Index}

```

if (Crossing Anglej = 0) then
    if (Crossing AngleCAS Index ≠ 0) Crossing Anglej = Crossing AngleCAS Index
    else if (Crossing AngleMach Index ≠ 0) Crossing Anglej = Crossing AngleMach Index
    else Crossing Anglej = 2.4 degrees
end if (Crossing Anglej = 0)
VSegTypej = MACH_CAS
Machj = Last Mach
CASj = spd
Mach Transition Altitude = Altitudej
Mach Transition CAS = spd
for (k = index number of the last waypoint; k < j; k++) Mach Segmentk = true
skip = true
end of if (spd ≥ Crossing CASCAS Index)
end of if (Altitudej = AltitudeCAS Index)
if (skip = false) Set an error indicating a bad Mach-to-CAS transition.
end of else if ((z > AltitudeMach Index)...
else
    i = index of the first waypoint + 1
    fini = false
    while ((i < index of the last waypoint) and (fini = false))
        if (Altitudei > z) i = i + 1
        else fini = true
    Calculate the distance to Altitudei.
    z2 = Altitudei-1 - Altitudei
    if (z2 ≤ 0) rz = 0
    else rz = (z - Altitudei) / z2

```

$d = rz * (DTG_{i-1} - DTG_i) + DTG_i$

$GndTrk = GetTrajGndTrk(d)$

$InsertWaypoint(i)$

$WptType_i = VTCP$

$VSegType_i = MACH_CAS$

$TurnType_i = no\ turn$

$Crossing\ Mach_i = Last\ Mach$

$Crossing\ CAS_i = First\ CAS$

$Crossing\ Rate_i = CAS\ Rate$

$DTG_i = d$

$Altitude_i = z$

$Crossing\ Angle_i = Altitude\ Crossing\ Angle_{i+1}$

$Ground\ Track_i = GndTrk$

$Mach_i = Last\ Mach$

$CAS_i = First\ CAS$

$Mach\ Transition\ Altitude = z$

$Mach\ Transition\ CAS = First\ CAS$

Compute and add the wind data at distance d along the path to the data of TCP_i .

$GenerateWptWindProfile(DTG_i, TCP_i)$

Set the Mach flag for these TCPs.

for (j = index number of the first waypoint; j < i; j++) Mach Segment_j = true

end of else

end of if (CAS Constraint Flag)

else

There are only Mach segments, set the Mach flags to true.

for (j = index number of the first waypoint; j < index number of the last waypoint; j++)

Mach Segment_j = true
end of else if (Crossing Mach_{first waypoint} > 0)

Compute Altitude / CAS Restriction TCP

If an altitude / CAS restriction is required, the *Compute Altitude / CAS Restriction TCP* function computes the altitude / CAS restriction point and inserts an altitude / CAS TCP. This is the (U.S.) point where the trajectory transitions through 10,000 ft and a 250 kt restriction is required. This function is only performed if the previously computed flag *Need10KRestriction* is true. The function determines the along-path distance where this altitude / CAS occurs on the profile. A TCP is then inserted into the TCP list at this point. The restriction values are *Descent Crossing Altitude* and *Descent Crossing CAS*.

Find the first TCP that is below the *Descent Crossing Altitude* in the list.

i = index number of the first waypoint

k = i

fini = false

while ((i < index number of the last waypoint) and (fini = false))

if (Altitude_i < ConvertPressureToIndicatedAltitude(Descent Crossing Altitude,
barometric setting_i))

k = i

fini = true

i = i + 1

Find the last CAS restriction prior to the first waypoint below *Descent Crossing Altitude*.

i = k - 1

fini = false

Last CAS = 0

while ((i > 0) and (fini = false))

if (Crossing CAS_i > 0)

Last CAS = Crossing CAS_i

fini = true

i = i - 1

Determine if an altitude or CAS TCP is required. If it is, add it.

if ((TCP_k is a Mach segment) and (Last CAS > Descent Crossing CAS))

$$i = k$$

$DescentCrossingAltitude = ConvertPressureToIndicatedAltitude(Descent\ Crossing\ Altitude,$
 $barometric\ setting_i)$

Find the distance to this altitude.

$$x = Altitude_{i-1} - Altitude_i$$

$$if\ (x \leq 0)\ ratio = 0$$

$$else\ ratio = (Descent\ Crossing\ Altitude - Altitude_i) / x$$

$$d = ratio * (DTG_{i-1} - DTG_i) + DTG_i$$

Compute the ground track at distance d along the trajectory and save it as *Saved Ground Track*.

$$Saved\ Ground\ Track = GetTrajGndTrk(d)$$

Insert a new TCP at location i in the TCP list. The TCP is inserted between TCP_{i-1} and TCP_i from the original list. The function *InsertWaypoint* should be appropriate for the actual data structure implementation of this function.

$$InsertWaypoint(i)$$

Mark this TCP as the altitude / CAS restriction TCP.

$$VSegType_i = altitude\ CAS\ restriction$$

$$TurnType_i = no\ turn$$

Add the data for this new TCP.

$$Crossing\ Mach_i = 0$$

$$Crossing\ CAS_i = Descent\ Crossing\ CAS$$

Use a high value, arbitrary CAS rate.

$$CAS\ Rate_i = 0.75\ kt / sec$$

$$DTG_i = d$$

$$Altitude_i = Descent\ Crossing\ Altitude$$

$$Crossing\ Angle_i = Crossing\ Angle_{i+1}$$

Set the Mach flag for TCP_i to false

$$Ground\ Track_i = Saved\ Ground\ Track$$

$$Mach_i = 0$$

$CAS_i = \text{Descent Crossing CAS}$

Compute and add the wind data at distance d along the path to the data of TCP_i .

$GenerateWptWindProfile(DTG_i, TCP_i)$

Add Final Deceleration

The Add Final deceleration function generates the appropriate speed TCP's for the case where either the deceleration to the final approach speed is to begin at the Final Approach Fix or the deceleration is to end at a specific altitude, *Stable Altitude*. This latter option is to support the case, which is typical for air transport operations, where a stable approach is required at and below a specific altitude. This function may only be invoked if the last waypoint is the runway threshold and the input crossing speed is a valid CAS value.

if (((Final Deceleration Option = AT FAF) or (Final Deceleration Option = STABLE)) and (Crossing $CAS_{last\ waypoint} > 0$)) then

The speed specified at the last waypoint, which must be the runway, is the target speed for these options. This speed should be the corrected final approach speed, *CFAS*.

$CFAS = \text{Crossing } CAS_{last\ waypoint}$

Find the waypoint index number for the waypoint used as the FAF. The default value is the input waypoint just before the last waypoint. If there is a FAF waypoint named in the input data, *NamedFaf*, then use that waypoint.

$FafWpt = \text{index number of the last waypoint} - 1$

if (NamedFaf) then

Find this waypoint by name.

$found = false$

$k = FafWpt$

while ((found = false) and ($k > \text{index number of the first waypoint}$))

if (NamedFaf = Id_k) found = true

else $k = k - 1$

if (found) $FafWpt = k$

else

This is the default waypoint. Find it in the input data.

while (($FafWpt > \text{index number of the first waypoint}$) and ($WptType_{FafWpt} \neq \text{input waypoint}$))

$FafWpt = FafWpt - 1$

The following is for the deceleration at the FAF.

if (Final Deceleration Option = AT FAF) then

$$\text{delta} = \text{Crossing CAS}_{\text{FafWpt}} - \text{CFAS}$$

Find the time required to reach the final speed.

$$t = \text{delta} / \text{Crossing Rate}_{\text{last waypoint}} / 3600$$

Find the FAF altitude.

if (Crossing Altitude_{FafWpt} > 0)

$$\text{AltitudeFaf} = \text{Crossing Altitude}_{\text{FafWpt}}$$

else if (Crossing Angle_{last waypoint} ≤ 0)

$$\text{AltitudeFaf} = \text{Crossing Altitude}_{\text{last waypoint}}$$

else

$$\text{AltitudeFaf} = \text{Crossing Altitude}_{\text{last waypoint}} + (\text{DTG}_{\text{FafWpt}} * \text{NmiToFeet}) * \text{tangent}(\text{Crossing Angle}_{\text{last waypoint}})$$

Calculate the ground speed at the runway.

$$\text{InterpolateWindWptAltitude}(\text{Wind Profile}_{\text{last waypoint}}, \text{Altitude}_{\text{last waypoint}}, \text{Ws}, \text{Wd}, \text{Td})$$

$$\text{GsRny} = \text{ComputeGndSpeedUsingTrack}(\text{Crossing CAS}_{\text{last waypoint}}, \text{GndTrack}_{\text{last waypoint}}, \text{Altitude}_{\text{last waypoint}}, \text{Ws}, \text{Wd}, \text{Td})$$

Calculate the ground speed at the FAF.

$$\text{InterpolateWindWptAltitude}(\text{Wind Profile}_{\text{FafWpt}}, \text{Altitude}_{\text{FafWpt}}, \text{Ws}, \text{Wd}, \text{Td})$$

$$\text{GsFaf} = \text{ComputeGndSpeedUsingTrack}(\text{Crossing CAS}_{\text{FafWpt}}, \text{GndTrack}_{\text{FafWpt}}, \text{Altitude}_{\text{FafWpt}}, \text{Ws}, \text{Wd}, \text{Td})$$

Calculate the distance from the FAF toward the runway where the final speed will be reached.

$$x2 = (\text{GsFaf} + \text{GsRny}) / 2 * t$$

Calculate the distance from the runway.

$$\text{dtg} = \text{DTG}_{\text{FafWpt}} - x2$$

Now find this distance in the TCP's.

$$\text{TmpWpt} = \text{index number of the last waypoint}$$

while ((DTG_{TmpWpt} < dtg) and (TmpWpt > index number of the first waypoint))

$$TmpWpt = TmpWpt - 1$$

Now find the next downstream input waypoint.

*while ((WptType_{TmpWpt} ≠ input waypoint) and
(TmpWpt < index number of the last waypoint))*

$$TmpWpt = TmpWpt + 1$$

$$GndTrk2 = GndTrack_{TmpWpt}$$

Using the just computed estimates, recalculate the DTG.

if (Crossing Angle_{last waypoint} ≤ 0) Delta Z = 0

*else Delta Z = (x2 * NmiToFeet) * tangent(Crossing Angle_{last waypoint})*

$$Altitude2 = AltitudeFaf - Delta Z$$

Find the wind value between the two points.

InterpolateWindWptAltitude(Wind Profile_{FafWpt}, Altitude2, Spd0, Dir0, TDev0)

InterpolateWindWptAltitude(Wind Profile_{TmpWpt}, Altitude2, Spd1, Dir1, TDev1)

*if (dtg > 0) InterpolateWindAtRange(dtg, DTG_{FafWpt}, Spd0, Dir0, TDev0,
0, Spd1, Dir1, TDev1, WindSpd, WindDir, TempDev)*

else

$$WindSpd = Spd1$$

$$WindDir = Dir1$$

$$TempDev = TDev1$$

Calculate the ground speed at the deceleration point.

*DecelGs = ComputeGndSpeedUsingTrack(CFAS, GndTrk2, Altitude2, WindSpd,
WindDir, TempDev)*

Calculate the average ground speed.

$$AvgGs = (GsFaf + DecelGs) / 2$$

Calculate the distance for the speed change.

$$x2 = AvgGs * t$$

Calculate the distance from the runway for this speed point.

$$dtg = DTG_{FAFWpt} - x2$$

end of if (Final Deceleration Option = AT FAF)

else

Calculate the data for the stabilized altitude option.

$$StableAlt = Crossing Altitude_{last\ waypoint} + Stable Altitude$$

$$dtg = (Stable Altitude / NmiToFeet) / tangent(Crossing Altitude_{last\ waypoint})$$

Find the waypoint prior to the stable altitude.

$$TmpWpt = index\ number\ of\ the\ last\ waypoint$$

while ((DTG_{TmpWpt} < dtg) and (TmpWpt > index number of the first waypoint))

$$TmpWpt = TmpWpt - 1$$

Save the ground track at this point.

$$GndTrk2 = Ground\ Track_{TmpWpt}$$

Calculate the wind data at the two positions.

$$InterpolateWindWptAltitude(Wind\ Profile_{FAFWpt}, StableAlt, Spd0, Dir0, TDev0)$$

$$InterpolateWindWptAltitude(Wind\ Profile_{TmpWpt}, StableAlt, Spd1, Dir1, TDev1)$$

Interpolate the winds between the two waypoints.

$$if\ (dtg > 0)\ InterpolateWindAtRange(dtg, DTG_{FAFWpt}, Spd0, Dir0, TDev0, \\ 0, Spd1, Dir1, TDev1, WindSpd, WindDir, TempDev)$$

else

$$WindSpd = Spd1$$

$$WindDir = Dir1$$

$$TempDev = TDev1$$

Calculate the ground speed at the deceleration point.

$$DecelGs = ComputeGndSpeedUsingTrack(CFAS, GndTrk2, StableAlt, WindSpd, \\ WindDir, TempDev)$$

end of else { Calculate the data for the stabilized altitude option }

Add the appropriate speed TCP if its position is between the FAF and the runway and the CFAS is slower than the speed at the FAF.

if ((dtg > 0) and (dtg ≤ DTG_{FAfWpt}) and (Crossing CAS_{FAfWpt} > CFAS)) then

Save the original ground track value at this distance.

GndTrk = GetTrajGndTrk(dtg)

Find the position in the TCP list to insert this waypoint.

i = index number of the last waypoint

while ((DTG_i < dtg) and (i > index number of the first waypoint)) i = i - 1

Define the correct insertion point.

i = i + 1

InsertWaypoint(i)

WptType_i = VTCP

if (VSegType_i = no type) VSegType_i = FINAL SPEED

TurnType_i = no turn

Crossing Mach_i = 0.

Crossing CAS_i = Crossing CAS_{last waypoint}

Crossing Rate_i = Crossing Rate_{last waypoint}

DTG_i = dtg

Calculate the altitude at this point.

if ((DTG_{i-1} - DTG_{i+1}) ≤ 0) x2 = 0

else x2 = (DTG_i - DTG_{i+1}) / (DTG_{i-1} - DTG_{i+1})

*Altitude_i = x2 * Altitude_{i-1} + (1 - x2) * Altitude_{i+1}*

Mach Segment_i = false

Crossing Angle_i = Crossing Angle_{i+1}

Ground Track_i = GndTrk

Ground Speed_i = DecelGs

$Mach_i = 0$

$CAS_i = \text{Crossing } CAS_i$

Add the wind data at this distance.

Compute and add the wind data at the new TCP's DTG.

$GenerateWptWindProfile(DTG_i, TCP_i)$

end of adding the TCP

else mark this as an error condition

end of if ((Final Deceleration Option = AT FAF) or (Final Deceleration Option = STABLE))

Add Waypoint at 6.25 nmi

The Add Waypoint at 6.25 nmi function generates a special waypoint at 6.25 nmi before the landing threshold of the runway. This function is invoked if the input variable *AddMopsRWY625* is true. This capability to support this special waypoint at 6.25 nmi before the threshold, along with associated crossing altitude and speed conditions, is a requirement of the RTCA *Minimum Operational Performance Standards (MOPS) for Flight-deck Interval Management (FIM)* (ref. 23). This function may only be invoked if the last waypoint is the runway threshold and the input crossing speed is a valid CAS value.

if (*AddMopsRWY625* and ($CAS_{last\ waypoint} > 0$)) then

$error = false$

$LastNum = \text{index number of the last waypoint}$

Determine where the 6.25 nmi needs to be placed in the TCP list.

$found = false$

$i1 = LastNum$

while (($found = false$) and ($i1 > \text{index number of the first waypoint}$))

if (($WptType_{i-1} = \text{input waypoint}$) and ($DTG_{i-1} > 6.25 \text{ nmi}$)) $found = true$

$i1 = i1 - 1$

if ($found = false$) $error = true$

Find the upstream waypoint with a speed constraint.

$j = i1$

$found2 = false$

while (($found2 = false$) and ($j \geq \text{index number of the first waypoint}$))

if ((WptType_j = input waypoint) and (Crossing CAS_j > 0)) found2 = true
else j = j - 1
if (found2 = false) error = true
spd = Crossing CAS_j
 The MOPS requires that the crossing speed cannot be faster than 170 kt.
if (spd > 170 kt) spd = 170 kt
 Find the downstream CAS rate.
j = i1 + 1
found2 = false
while ((found2 = false) and (j ≤ index number of the last waypoint))
 if ((WptType_j = input waypoint) and (Crossing CAS_j > 0.0)) found2 = true
 else j = j + 1
if (found2 = false) error = true
spdrate = Crossing Rate_j
 Set the rate to a minimum of 0.75 kt / sec.
if (spdrate < 0.75 kt /sec) spdrate = 0.75 kt / sec
 Find the downstream descent data.
j = i1 + 1
found2 = false
while ((found2 = false) and (j < index number of the last waypoint))
 if ((WptType_j = input waypoint) and (Crossing Altitude_j > 0)) found2 = true
 else j = j + 1
if (found2 = false) error = true
 This point needs to be crossed at an altitude of at least 2000 ft above the runway altitude.
alt = Crossing Altitude_{last waypoint} + 2000 ft
if (alt ≤ Crossing Altitude_j) then

$alt = Crossing\ Altitude_j$
 $angle = Crossing\ Angle_j$
else
 $angle = Crossing\ Angle_j$
if ($angle < Crossing\ Angle_{last\ waypoint}$) $angle = Crossing\ Angle_{last\ waypoint}$

Check the actual calculated altitude.

$z = alt - Crossing\ Altitude_j$

if ($z > 0$) then

$d = 6.25\ nmi - DTG_j$

if ($d > 0$) then

$a = arctangent(z, NmiToFeet * d)$

if ($a > angle$) $angle = a$

Find the waypoint after this in the input waypoint data.

$found2 = false$

$j1 = index\ number\ of\ the\ last\ waypoint$

while (($found = false$) and ($j1 \geq index\ number\ of\ the\ first\ waypoint$))

if ($Id_{j1} = Id_{i1}$) $found2 = true$

else $j1 = j1 - 1$

if ($found = false$) $error = true$

$j0 = j1$

Find the waypoint after this point.

$found2 = false$

$i0 = index\ number\ of\ the\ last\ waypoint$

while (($found2 = false$) and ($i0 \geq index\ number\ of\ the\ first\ waypoint$))

if (($WptType_{i0} = input\ waypoint$) and ($Id_{j0} = Id_{i0}$)) $found2 = true$

else $i0 = i0 - 1$

if (found2 = false) error = true

If there are not errors, insert the 6.25 nmi point.

if (error= false)

GndTrk = GetTrajGndTrk(6.25 nmi)

Find the position to insert this waypoint.

i = index number of the last waypoint

while ((DTG_i < 6.25 nmi) and (i > index number of the first waypoint)) i = i - 1

The correct insertion point is the next downstream point.

i = i + 1

InsertWaypoint(i)

WptType_i = VTCP

VSegType_i = RUNWAY625

TurnType_i = no turn

Crossing Mach_i = 0

Crossing CAS_i = spd

Crossing Rate_i = spdrate

DTG_i = 6.25 nmi

Altitude_i = alt

Crossing Altitude_i = alt

Mach Segment_i = false

Crossing Angle_i = angle

Ground Track_i = GndTrk

Mach_i = 0

CAS_i = Crossing CAS_i

Add the wind data at this distance.

GenerateWptWindProfile(DTG_i, TCP_i)

InterpolateWindWptAltitude(Wind Profile_i, Crossing Altitude_i, WindSpd, WindDir, TempDev)

*Ground Speed_i = ComputeGndSpeedUsingTrack(Crossing CAS_i, Ground Track_i,
Crossing Altitude_i, WindSpd, WindDir, TempDev)*

If there is a programmed deceleration at the original FAF and the FAF is farther from the runway than 6.25 nmi, remove the previously computed final deceleration point.

if ((Final Deceleration Option = AT FAF) or (Final Deceleration Option = STABLE)) then

Find the index number for the FAF. Initialize the index to an invalid number, -1.

FafWptNum = -1

Is this the special case with a named FAF, *NamedFaf*, in the input?

if (NamedFaf) then

Find this waypoint by name.

found = false

k = index number of the last waypoint

while ((found = false) and (k > index number of the first waypoint))

if (NamedFaf = Id_k) found = true

else k = k - 1

if (found) FafWptNum = k

else

FafWptNum = index number of the last waypoint

*while ((FafWptNum > index number of the first waypoint) and
(WptType_{FafWptNum} ≠ input waypoint))*

found2 = false

i = index number of the last waypoint

*while ((found2 = false) and (FafWptNum > index number of the first waypoint) and
(i > index number of the first waypoint))*

VSegType_i = FINAL SPEED) found2 = true

else i = i - 1

if (found and (DTG_{FafWptNum} > 6.25 nmi)) RemoveWaypoint(i)

where the RemoveWaypoint function simply deletes the TCP at the index i .

end of if (error= false)

else mark this as an error condition

Compute TCP Speeds

The *Compute TCP Speeds* function is similar to *Compute TCP Altitudes* in its design. Beginning with the last waypoint, this function computes the Mach or CAS at each previous TCP and inserts any additional speed TCPs that may be required to denote a change in the speed profile. The function uses the current speed constraint, searches backward for the previous constraint, and then computes the distance required to meet this previous constraint. The speeds for all of the TCPs within this distance are computed and added to the data for the TCPs. If the along-path distance to meet the previous constraint is not at a TCP, a new speed VTCP is inserted at this distance. This function invokes two secondary functions, described in the subsequent text, with the invocation dependent on the constraint speed, whether it is a Mach or a CAS value. This function is performed in the following steps:

Set the current constraint index number, cc , equal to the index number of the last waypoint,

$cc = \text{index number of the last waypoint}$

The speed of the first waypoint is set to its crossing speed.

if (Crossing Mach_{first waypoint} > 0)

$Mach_{\text{first waypoint}} = \text{Crossing Mach}_{\text{first waypoint}}$

$CAS_{\text{first waypoint}} = \text{MachToCas}(Mach_{\text{first waypoint}}, \text{Altitude}_{\text{first waypoint}})$

else

$CAS_{\text{first waypoint}} = \text{Crossing CAS}_{\text{first waypoint}}$

$Mach_{\text{first waypoint}} = \text{CasToMach}(CAS_{\text{first waypoint}}, \text{Altitude}_{\text{first waypoint}})$

A flag signifying that Mach segment computation has begun is initially set to false,

$\text{Doing Mach} = \text{false}$

Check for special case where there are no CAS segments.

if ((Crossing CAS_{cc} = 0) and (Crossing Mach_{cc} > 0.0)) then

$CAS_{cc} = \text{MachToCas}(\text{Crossing Mach}_{cc}, \text{Crossing Altitude}_{cc})$

$Mach_{cc} = \text{Crossing Mach}_{cc}$

$\text{DoingMach} = \text{true}$

else CAS_{cc} = Crossing CAS_{cc}

while (cc > index number of the first waypoint)

Set the Mach flag if the current TCP is the Mach-to-CAS transition point.

if ($TCP_{cc} = \text{Mach Transition CAS}$) *Doing Mach* = true

if (*Doing Mach*) *ComputeTcpMach*(cc)

else *ComputeTcpCas*(cc)

end of while $cc > \text{index number of the first waypoint}$

Compute Secondary Speeds

The *Compute Secondary Speeds* function adds the Mach values to CAS TCPs, the CAS values to Mach TCPs, and the ground speed values to all TCPs. This function is performed in the following steps:

Doing Mach = false

Working backwards from the runway, compute the relevant speeds.

for ($i = \text{index number of the last waypoint}; i \geq \text{index number of the first waypoint}; i = i - 1$)

Set the flag if the current TCP is the Mach-to-CAS transition point.

if ($TCP_i = \text{Mach Transition CAS}$) *Doing Mach* = true

if (*Doing Mach*) $Cas_i = \text{MachToCas}(\text{Mach}_i, \text{Altitude}_i)$

else $\text{Mach}_i = \text{CasToMach}(Cas_i, \text{Altitude}_i)$

Compute the ground track.

if ($i = \text{index number of the first waypoint}$) $track = \text{Ground Track}_i$

else if ($\text{WptInTurn}(i)$ or ($TCP_i = \text{turn-exit}$)) $track = \text{Ground Track}_i$

else $track = \text{Ground Track}_{i-1}$

Compute the ground speed. This also requires the computation of the wind at this point.

InterpolateWindWptAltitude(*Wind Profile*, *Altitude*, *Wind Speed*, *Wind Direction*,
Temperature Deviation)

$\text{Ground Speed}_i = \text{ComputeGndSpeedUsingTrack}(Cas_i, track, \text{Altitude}_i, \text{Wind Speed},$
 $\text{Wind Direction}, \text{Temperature Deviation})$

end of for ($i = \text{index number of the last waypoint}; i \geq \text{index number of the first waypoint}; i = i - 1$)

Compute Turn Data

The *Compute Turn Data* function computes the turn data for each turn waypoint and modifies the associated waypoint's turn data sub-record. This function performs as follows:

$KtsToFps = 1.69$

Nominal Bank Angle = 22

index = index number of the first waypoint + 1

while (index < index number of the last waypoint)

Find the next input waypoint with a turn.

while ((index < index number of the last waypoint) and ((TCP_{index} ≠ input waypoint) or (not WptInTurn(index)))) index = index + 1

If there are no errors and there is a turn of more than 3-degrees, compute the turn data.

if (index < index number of the last waypoint)

Find the start of the turn.

i = index - 1

while (TCP_i ≠ turn-entry) i = i - 1

start = i

The following are all approximations and are based on a general, constant radius turn.

The start of turn to the midpoint data is as follows, noting that the ground speeds for all points must be valid at this point.

The overall distance d for this part of the turn is,

$$d = DTG_{start} - DTG_{index}$$

The special case with 0 distance between the points is,

$$if (d \leq 0) AvgGsFirstHalf = (Ground Speed_{start} + Ground Speed_{index}) / 2$$

else

The overall average ground speed is computed as follows, noting that it is the sum of segment distance / overall distance * average segment ground speed.

$$AvgGsFirstHalf = 0$$

for (j = start; j ≤ (index - 1); j = j + 1)

$$dx = DTG_j - DTG_{j+1}$$

$$AvgGsFirstHalf = AvgGsFirstHalf + (dx / d) * (Ground Speed_j + Ground Speed_{j+1}) / 2$$

Now, find the end of the turn.

$i = index + 1$

while ($TCP_i \neq turn\text{-}exit$) $i = i + 1$

end = i

Now, find the midpoint to the end of the turn.

The overall distance for this part of the turn is,

$$d = DTG_{index} - DTG_{end}$$

Test for the special case, 0 distance between the points.

if ($d \leq 0$)

$$AvgGsLastHalf = (Ground\ Speed_{index} + Ground\ Speed_{end}) / 2$$

else

Compute the overall average ground speed noting that it is the sum of the segment distances / overall distance * average segment ground speed.

$$AvgGsLastHalf = 0$$

for ($j = index; j \leq (end - 1); j = j + 1$)

$$dx = DTG_j - DTG_{j+1}$$

$$AvgGsLastHalf = AvgGsLastHalf + (dx / d) * (Ground\ Speed_j + Ground\ Speed_{j+1}) / 2$$

end of for ($j = index; j \leq (end - 1); j = j + 1$)

end of else if ($d \leq 0$)

$$full\ turn = DeltaAngle(Ground\ Track_{start}, Ground\ Track_{end})$$

$$half\ turn = full\ turn / 2$$

Compute the outputs from the average ground speed values.

$$Average\ Ground\ Speed = (AvgGsFirstHalf + AvgGsLastHalf) / 2$$

Save the ground speed data in the turn data for this waypoint.

$$Turn\ Data\ Average\ Ground\ Speed_{index} = Average\ Ground\ Speed$$

Compute the turn radius and associated data. This set of calculations is not performed if the waypoint is a special, RF center-of-turn turn waypoint.

if ($TurnType_i \neq rf\text{-}turn\text{-}center$)

The general equation is $\text{turn rate} = c \tan(\text{bank angle}) / v$. If the bank angle is a constant, $\text{turn rate} = c0 / v$. The *Nominal Bank Angle* = 22 degrees.

$$c0 = 57.3 * 32.2 / \text{KtsToFps} * \text{tangent}(\text{Nominal Bank Angle})$$

Test for a negative ground speed.

if (*Average Ground Speed* ≤ 0)

$$\text{Turn Data Turn Time}_{index} = 0$$

$$\text{Turn Data Turn Radius}_{index} = 0$$

else

$$w = c0 / \text{Average Ground Speed}$$

The time to make the turn is,

$$\text{Turn Data Turn Time}_{index} = |\text{full turn}| / w$$

The turn radius is,

$$\text{Turn Data Turn Radius}_{index} = (57.3 * \text{KtsToFps} * \text{Average Ground Speed}) / (\text{NmiToFeet} * w)$$

The along-path distance for the turn is,

$$\text{Turn Data Path Distance}_{index} = |\text{full turn}| * \text{Turn Data Turn Radius}_{index} / 57.3$$

else

These are the data for an RF turn. The along-path distance for the turn is,

$$\text{Turn Data Path Distance}_{index} = |\text{full turn}| * \text{Turn Data Turn Radius}_{index} / 57.3$$

Calculate the time to make the turn.

Test for a negative ground speed.

if (*Average Ground Speed* ≤ 0) *Turn Data Turn Time*_{index} = = 0

else

$$\text{Turn Data Turn Time}_{index} =$$

$$\text{Turn Data Path Distance}_{index} / \text{Average Ground Speed} * 3600$$

Save the turn data for the first half of the turn, denoted by the "1" in the variable name.

$$\text{Turn Data CasI}_{index} = \text{CAS}_{start}$$

$$\text{Turn Data Average Ground Speed } I_{index} = \text{AvgGsFirstHalf}$$

$$\text{Turn Data Track } I_{index} = \text{Ground Track}_{start}$$

The *Straight Distance* values are the distances from the turn-entry TCP to the waypoint and from the waypoint to the turn-exit TCP. See the example in figure 6.

$$\text{Turn Data Straight Distance } I_{index} = \text{Turn Data Turn Radius }_{index} * \text{tangent}(|\text{half turn}|)$$

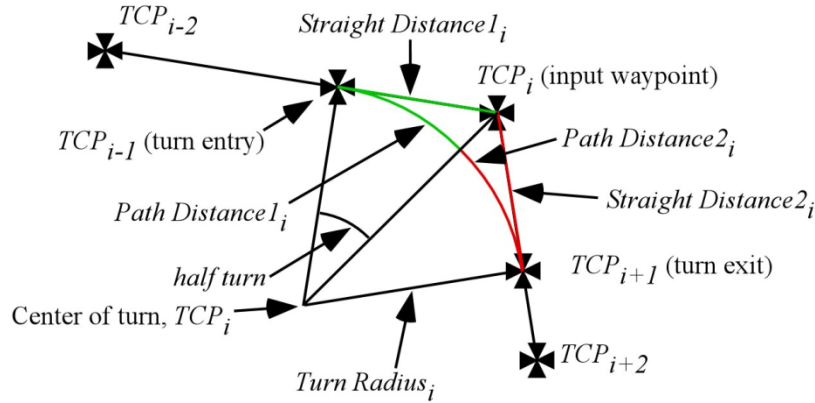


Figure 6. Turn distances for waypoint_i.

The *Path Distance* values are the along-the-path distances from the turn-entry TCP to a point one-half way along the turn and from this point to the turn-exit TCP. See the example in figure 6.

$$\text{Turn Data Path Distance } I_{index} = |\text{half turn}| * \text{Turn Data Turn Radius}_{index} / 57.3$$

Compute the midpoint waypoint data. This set of calculations is not performed if the waypoint is a special, RF center-of-turn waypoint.

if (TurnType_i ≠ rf-turn-center)

Test for a negative ground speed.

$$\text{if (AvgGsFirstHalf} \leq 0) \text{ Turn Data Turn Time } I_{index} = 0$$

else

$$w = c0 / \text{AvgGsFirstHalf}$$

$$\text{Turn Data Turn Time } I_{index} = |\text{half turn}| / w$$

else

These are the data for an RF turn.

$$\text{Turn Data Turn Time } I_{index} = \text{Turn Data Path Distance } I_{index} / \text{AvgGsFirstHalf} * 3600$$

The data for the midpoint to the end of the turn, denoted by the "2" in the variable name, are as follows:

$$\text{Turn Data Cas2}_{index} = \text{CAS}_{end}$$

$$\text{Turn Data Average Ground Speed2}_{index} = \text{AvgGsLastHalf}$$

$$\text{Turn Data Track2}_{index} = \text{Ground Track}_{end}$$

The distances for the second half of the turn are the same as for the first, but their calculations are recomputed here for clarity.

$$\text{Turn Data Straight Distance2}_{index} = \text{Turn Data Turn Radius}_{index} * \text{tangent}(|\text{half turn}|)$$

$$\text{Turn Data Path Distance2}_{index} = |\text{half turn}| * \text{Turn Data Turn Radius}_{index} / 57.3$$

Compute the data for the last half of the turn. Again, this set of calculations is not performed if the waypoint is a special, RF center-of-turn waypoint.

if (TurnType_i ≠ rf-turn-center)

 Test for a negative ground speed.

if (AvgGsFirstHalf ≤ 0) Turn Data Turn Time2_{index} = 0

else

$$w = c0 / \text{AvgGsLastHalf}$$

$$\text{Turn Data Turn Time2}_{index} = |\text{half turn}| / w$$

else

 These are the data for an RF turn.

$$\text{Turn Data Turn Time2}_{index} = \text{Turn Data Path Distance2}_{index} / \text{AvgGsLastHalf} * 3600$$

The DTG values are as follows:

$$\text{DTG}_{start} = \text{DTG}_{index} + \text{Turn Data Path Distance1}_{index}$$

$$\text{DTG}_{end} = \text{DTG}_{index} - \text{Turn Data Path Distance2}_{index}$$

Since the turn waypoints have been moved, the wind data need to be updated for the new locations.

if (TCP_{start} ≠ input waypoint) GenerateWptWindProfile(DTG_{start}, TCP_{start})

if (TCP_{end} ≠ input waypoint) GenerateWptWindProfile(DTG_{end}, TCP_{end})

end of if (index < index number of the last waypoint)

index = index + 1

end of while (index < index number of the last waypoint)

Test for Altitude / CAS Restriction Requirement

The *Test for Altitude / CAS Restriction Requirement* function determines if the addition of an altitude / CAS restriction point is required. This is the (U.S.) point where the trajectory transitions through 10,000 ft and a 250 kt restriction is required. This function determines the value of the *Need10KRestriction* flag. The function can only be called after an initial, preliminary trajectory has been generated. The restriction values are *Descent Crossing Altitude* and *Descent Crossing CAS*.

Need10KRestriction = false

if ((Descent Crossing Altitude > 0) and (Descent Crossing CAS > 0)) ok = true

else ok = false

If we don't start above 10,000ft, skip this whole routine.

if (ok and (Altitude_{first waypoint} > ConvertPressureToIndicatedAltitude(Descent Crossing Altitude, barometric setting_{first waypoint})) then

Find the first point below Descent Crossing Altitude

f_{ini} = false

i = 0

while ((i < index number of the last waypoint) and (f_{ini} = false))

Crossing Altitude = ConvertPressureToIndicatedAltitude(Descent Crossing Altitude, barometric setting_i)

if (Altitude_i < Crossing Altitude) then

Find the distance to this altitude.

x = Altitude_{i-1} - Altitude_i

if (x ≤ 0) ratio = 0

else ratio = (Crossing Altitude - Altitude_i) / x

*s = ratio * (CAS_{i-1} - CAS_i) + CAS_i*

if (s > (Descent Crossing Cas + 2)) Need10KRestriction = true

f_{ini} = true

i = i + 1

Delete VTCPs

The *Delete VTCPs* function deletes the altitude, speed, and Mach-to-CAS TCPs. The remaining TCPs will only consist of input waypoints, turn-entry, and turn-exit TCPs. This function also removes any flags

that associate any remaining TCPs with a speed or altitude change, e.g., a waypoint marked as the 10,000 ft, 250 kt restriction.

Update DTG Data

The *Update DTG Data* function is performed after the turn data have been updated and the VTCPs have been deleted. Only input, turn-entry, and turn-exit TCPs should be in the list at this time. If the input test flag, *TestOnly*, is true, then only the testing portions of this function are used.

if (TestOnly = false) DTG_{first waypoint} = 0

i = index number of the last waypoint

while (i > index number of the first waypoint)

Determine if there is a turn at either end and adjust accordingly.

if (WptInTurn(i))

if (TestOnly = false) DTG_{i-1} = DTG_i + Turn Data Path Distance_i

The following is the difference between going directly from the waypoint to going along the curved path.

PriorDistanceOffset = Turn Data Straight Distance_i - Turn Data Path Distance_i

else PriorDistanceOffset = 0

Find the next input waypoint.

n = i - 1

while (TCP_n ≠ input waypoint) n = n - 1

if (WptInTurn(n))

The following is the difference between going directly from the waypoint to going along the curved path.

DistanceOffset = Turn Data Straight Distance_{2n} - TurnData.PathDistance_{2n}

The DTG to the input waypoint is then:

if (TestOnly = false) DTG_n = (Center to Center Distance_i - PriorDistanceOffset - DistanceOffset) + DTG_i

If the *DistanceOffset* is greater than *Center to Center Distance_i*, then the turn is too big.

if (DistanceOffset > Center to Center Distance_i) mark this as an error condition

The turn-exit DTG is then,

if (TestOnly = false) DTG_{n+1} = DTG_n - Turn Data Path Distance_{2n}
else if (TestOnly = false)

The next waypoint is not in a turn.

DTG_n = Center to Center Distance_i - PriorDistanceOffset + DTG_i

i = n

end of while (i > 0)

Check Turn Validity

The *Check Turn Validity* function is performed after the turn data have been updated and the VTCPs have been deleted. Only input, turn-entry, and turn-exit TCPs should be in the list at this time. The function simple checks that there are no turns within turns by examining the DTG values.

for (i = index number of the first waypoint; i < index number of the last waypoint; i = i + 1)

if (DTG_i < DTG_{i+1}) mark this as an error condition

Restore the Crossing Angles

The *Restore the Crossing Angles* function simply replaces the current value for each waypoint's crossing angle with the value that was saved in the function *Save Selected Input Data*.

Recover the Initial Mach Segments

This function, *Recover the Initial Mach Segments*, attempts to recover the Mach portion of the trajectory if the initial segments should be Mach but have been internally converted to CAS in the function *Meet Cruise CAS Waypoint Restriction*. This function uses the Mach value that was saved at the start of this program from the first waypoint of the original route. This saved Mach value, *First Waypoint Mach*, is compared to the Mach equivalent value of the CAS at the initial waypoints and if these Mach values are the same, these waypoints are marked as Mach segments instead of CAS segments.

Only perform this function if the calculated trajectory does not start with a Mach segment but the original route does start with a Mach value.

if ((Mach Segment_{first waypoint} = false) and (First Waypoint Mach ≠ 0))

Mach = CasToMach(Crossing CAS_{first waypoint}, Altitude_{first waypoint})

Determine if this value is close to the original Mach or if there is a different but valid cruise Mach.

DoTest = false

if (Mach ≈ First Waypoint Mach) DoTest = true

else if ((Mach ≥ 0.80 Mach) and (Altitude_{first waypoint} ≥ 29000 ft)) then

Find the TOD, the speed needs to be the same as the starting speed.

fini = false

i = index number of the first waypoint + 1

while ((i < (index number of the last waypoint - 1)) and (fini = false))

DoTest = true

if (Altitude_i ≠ Altitude_{first waypoint}) fini = true

else if (CAS_i ≠ CAS_{first waypoint}) then

fini = true

DoTest = false

i = i + 1

end of else if ((Mach >= 0.80 Mach)...

if (DoTest)

fini = false

i = index number of the last waypoint

First Cas = Crossing CAS_{first waypoint}

If there is no Mach transition altitude set, set the transition values.

if (Mach Transition Altitude = 0)

Mach Descent Mach = First Waypoint Mach

Mach Transition Cas = First Cas

Mach Transition Altitude = Altitude_{first waypoint}

while ((i < (index number of the last waypoint - 1)) and (fini = false))

Test that the CAS computed for the waypoint is the same as the *First Cas*, that except for the first waypoint that there is not speed crossing condition at the waypoint, and that the altitude computed for the waypoint is the same as the altitude for the first waypoint.

*if ((Cas_i = First Cas) and ((i = index number of the last waypoint) or
((Crossing Mach_i = 0) and (Crossing CAS_i = 0))) and
(Altitude_i = Crossing Altitude_{first waypoint}))*

If the previous conditions are turn, set this waypoint as a Mach segment.

Mach Segment_i = true

Change the speed crossing values for the first waypoint.

if (Crossing CAS_i > 0)

Crossing CAS_i = 0

Crossing Mach_i = First Waypoint Mach

end of if ((Cas_i = First Cas)...))

else fini = true

i = i + 1

Insert CAS Descent VTCPs

This function inserts vertical TCPs between constant CAS descent waypoints to improve the TAS estimation when using the data provided by this algorithm. This updating occurs at 3,000 ft intervals.

Update Altitude = 3000

Find the first CAS point.

j = 0

while ((Mach Segment_i = true) and (j < index number of the last waypoint)) j = j + 1

for (i = j; i < (index number of the last waypoint - 1); i = i + 1)

DeltaZ = Altitude_i - Altitude_{i+1}

Update at 3000 ft intervals but skip the update if the waypoint is within 500 ft of the test altitude.

if ((DeltaZ ≥ (Update Altitude + 500)) and (Cas_i ≈ Cas_{i+1}))

z = Altitude_i - Update Altitude

dx = DTG_i - DTG_{i+1}

*a = arctangent2 (DeltaZ, NmiToFeet * dx)*

d = DTG_i - Update Altitude / tan(a) / NmiToFeet

Compute the ground track at distance *d* along the trajectory and save it as *Saved Ground Track*.

Saved Ground Track = GetTrajGndTrk(d)

k = i + 1

Insert a new VTCP at location *k* in the TCP list. The VTCP is inserted between TCP_{*k-1*} and TCP_{*k*} from the original list. The function *InsertWaypoint* should be appropriate for the actual data structure implementation of this function.

InsertWaypoint(k)

Update the waypoint-type data in the new waypoint.

$WptType_k = VTCP$

$VSegType_k = TAS\ adjustment$

$TurnType_k = no\ turn$

Update the crossing data in the new waypoint.

$Crossing\ Mach_k = 0$

$Crossing\ CAS_k = 0$

$Crossing\ Rate_k = 0$

$CAS_k = CAS_{k+1}$

$DTG_k = d$

$Altitude_k = z$

$Mach_k = CasToMach(CAS_k, Altitude_k)$

$Mach\ Segment_k = false$

$Crossing\ Angle_k = Crossing\ Angle_{k+1}$

$Ground\ Track_k = Saved\ Ground\ Track$

Compute and add the wind data at this waypoint.

$GenerateWptWindProfile(DTG_k, TCP_k)$

Compute the wind at the waypoint altitude and then waypoint's ground speed.

$InterpolateWindWptAltitude(Wind\ Profile_k, Altitude_k, Ws, Wd, Td)$

$Ground\ Speed_k = ComputeGndSpeedUsingTrack(CAS_k, Ground\ Track_{k-1}, Altitude_k, Ws, Wd, Td)$

Compute TCP Times

The function *Compute TCP Times* calculates the time to each TCP. The calculations begin at the runway (the last waypoint), working backwards, and compute the TTG to each TCP.

$TTG_{last\ waypoint} = 0$

for ($i = index\ number\ of\ the\ last\ waypoint; i > index\ number\ of\ the\ first\ waypoint; i = i - 1$)

$$\text{Average Ground Speed} = (\text{Ground Speed}_{i-1} + \text{Ground Speed}_i) / 2$$

$$x = \text{DTG}_{i-1} - \text{DTG}_i$$

Test for an error condition where the distance is less than 0. This error only occurs if the segment ends overlap.

if ($x < 0$)

Find the previous input waypoint in case it is needed in a later test. Also determine if this previous waypoint is an RF turn point.

PreviousIsRf = *false*

fini = *false*

$j = i - 1$

while (*fini* = *false*)

if ($j < \text{index number of the first waypoint}$) *fini* = *true*

else if ($(\text{WptType}_j = \text{input waypoint}) \text{ and } (\text{TurnType}_j = \text{rf-turn-center})$) *then*

PreviousIsRf = *true*

fini = *true*

else if ($\text{WptType}_j = \text{input waypoint}$) *fini* = *true*

$j = j - 1$

end of while (*fini* = *false*)

If the distance is close to 0, e.g., within 500 ft for a normal segment pair, set the distance to the previous distance value and ignore the error.

if ($x \geq (-500 \text{ ft} / \text{NmiToFeet})$)

$\text{DTG}_i = \text{DTG}_{i-1}$

$x = 0$

Allow a larger margin of error of 1500 ft for the beginning of an RF turn.

else if ($(x \geq -1500 \text{ ft} / \text{NmiToFeet}) \text{ and } (\text{TurnType}_i = \text{turn-entry}) \text{ and } (\text{Center Of Turn Latitude}_i \neq 0)$)

$\text{DTG}_i = \text{DTG}_{i-1}$

$x = 0$

Allow a larger margin of error of 1500 ft if the end of the previous segment is the end of an RF turn and it overlaps the start of another turn.

*else if (($x \geq -1500 \text{ ft} / \text{NmiToFeet}$) and ($\text{TurnType}_i = \text{turn-entry}$) and
($i > \text{index number of the first waypoint}$) and ($\text{TurnType}_{i-1} = \text{turn-exit}$) and
 PreviousIsRf) then*

Overwrite the previous end of turn data with the subsequent start of turn data.

$$DTG_{i-1} = DTG_i$$

$$\text{Altitude}_{i-1} = \text{Altitude}_i$$

$$CAS_{i-1} = CAS_i$$

$$\text{Ground Speed}_{i-1} = \text{Ground Speed}_i$$

$$\text{Ground Track}_{i-1} = \text{Ground Track}_i$$

$$\text{Mach}_{i-1} = \text{Mach}_i$$

$$\text{Mach Segment}_{i-1} = \text{Mach Segment}_i$$

$$x = 0$$

else mark this as an error condition

$$\text{Delta Time} = 3600 * x / \text{Average Ground Speed}$$

$$TTG_{i-1} = TTG_i + \text{Delta Time}$$

Compute TCP Latitude and Longitude Data

With the exception of the input waypoints, the *Compute TCP Latitude and Longitude Data* function computes the latitude and longitude data for all of the TCPs.

In Turn = false

Last Base = index number of the first waypoint

Next Input = index number of the first waypoint

Turn Index = index number of the first waypoint

Turn is Clockwise = true

Turn Adjustment = 0

Base Latitude = Latitude_{Last Base}

Base Longitude = Longitude_{Last Base}

for ($i = \text{index number of the first waypoint}; i \leq \text{index number of the last waypoint}; i = i + 1$)

if ($TCP_i = \text{turn-entry}$)

Turn Adjustment = 0

InTurn = True

Find the major waypoint for this turn.

Next Input = $i + 1$

while ($(TCP_{\text{Next Input}} \neq \text{input waypoint})$ and ($\text{Next Input} \leq \text{index number of the last waypoint}$))

Next Input = Next Input + 1

Turn Index = Next Input

$a = \text{DeltaAngle}(\text{Ground Track}_i, \text{Ground Track}_{\text{Next Input}})$

$x = \text{Turn Data Turn Radius}_{\text{Turn Index}} / \text{cosine}(a)$

if ($a > 0$) Turn Clockwise = true

else Turn Clockwise = false

if (Turn Clockwise) $a1 = \text{Ground Track}_{\text{Turn Index}} + 90$

else $a1 = \text{Ground Track}_{\text{Turn Index}} - 90$

Now compute the relative latitude and longitude values. The function *RelativeLatLon* is described in a subsequent section.

RelativeLatLong(Latitude_{Turn Index}, Longitude_{Turn Index}, $a1$, x), returning Center Latitude and Center Longitude

end of if ($TCP_i = \text{turn-entry}$)

if (In Turn)

Turn Adjustment = 0

if (Turn Clockwise) $a1 = \text{Ground Track}_i - 90$

else $a1 = \text{Ground Track}_i + 90$

if ($WptType_i = \text{input waypoint}$)

Turn Data Center Latitude _{i} = Center Latitude

Turn Data Center Longitude _{i} = Center Longitude

RelativeLatLong(Center Latitude, Center Longitude, $a1$, Turn Data Turn Radius_{Turn Index}),

returning Turn Data Latitude_i and Turn Data Longitude_i

end of if (WptType_i = input waypoint)

*else RelativeLatLon(Center Latitude, Center Longitude, a1, Turn Data Turn Radius_{Next Input}),
returning Latitude_i and Longitude_i*

if (TCP_i = turn-exit)

*Turn Adjustment = Turn Data Straight Distance_{2Turn Index} -
Turn Data Path Distance_{2Turn Index}*

In Turn = false

Last Base = Next Input

Base Latitude = Latitude_{Last Base}

Base Longitude = Longitude_{Last Base}

end of if (In Turn)

else

if (WptType_i = input waypoint)

Turn Adjustment = 0

Last Base = i

Base Latitude = Latitude_{Last Base}

Base Longitude = Longitude_{Last Base}

else

*RelativeLatLong(Base Latitude, Base Longitude, Ground Track_{i-1}, DTG_{Last Base} - DTG_i +
Turn Adjustment), returning Latitude_i and Longitude_i*

end of for (i = index number of the first waypoint; i ≤ index number of the last waypoint; i = i + 1)

Description of Secondary Functions

The secondary functions are listed in alphabetical order. Note that standard aeronautical functions, such as CAS to Mach conversions, *CasToMach*, are not expanded in this document but may be found numerous references, e.g., reference 24. It may also be of interest to include atmospheric temperature or temperature deviation in the wind data input and calculate the temperature at the TCP crossing altitudes to improve the calculation of the various speed terms.

BodDecelerationDistance

The function `BodDecelerationDistance` estimates the distance required for the special case of a deceleration to a CAS restricted waypoint from the Mach-to-CAS transition. This function is invoked from `HandleDescentAccelDecel`, which passes in the index number for the bottom-of-descent (TOD) waypoint, `BodIndex`, the Mach transition to CAS altitude, `Mach Transition Altitude`, and the CAS at the Mach transition to CAS, `TransitionCas`. The function returns the distance from the index point of the deceleration, `Distance`.

Estimate the distance to the new Mach value. Begin by finding the time to do the deceleration.

$$t = (TransitionCas - Crossing CAS_{BodIdx}) / Crossing Rate_{BodIdx}$$

Compute the wind speed and direction at the current altitude.

$$InterpolateWindWptAltitude(Wind Profile_{BodIdx}, Altitude_{BodIdx}, Ws, Wd, Td)$$

Calculate the ground track at the current point.

$$if (WptInTurn(BodIdx)) track = Ground Track_{BodIdx-1}$$

$$else track = Ground Track_{BodIdx}$$

Calculate the ground speed over this segment.

$$BodGs = ComputeGndSpeedUsingTrack(Crossing CAS_{BodIdx}, track, Altitude_{BodIdx}, Ws, Wd, Td)$$

$$DescentGs = ComputeGndSpeedUsingTrack(TransitionCas, track, Mach Transition Altitude, Ws, Wd, Td)$$

Calculate the average groundspeed, `AvgGS`.

$$AvgGs = (BodGs + DescentGs) / 2$$

The distance estimate is $AvgGs * t$.

$$Distance = AvgGs * t / 3600$$

ComputeGndSpeedUsingMachAndTrack

The `ComputeGndSpeedUsingMachAndTrack` function computes a ground speed from track angle (versus heading), `track`, Mach, `Mach`, altitude, `Altitude`, and wind data, `Wind Speed`, `Wind Direction`, and `Temperature Deviation`.

$$CAS = MachToCas(Mach, Altitude)$$

$$Ground Speed = ComputeGndSpeedUsingTrack(CAS, track, Altitude, Wind Speed, Wind Direction, Temperature Deviation)$$

ComputeGndSpeedUsingTrack

The `ComputeGndSpeedUsingTrack` function computes a ground speed from track angle (versus heading), `track`, CAS, `CAS`, altitude, `Altitude`, and wind data, `Wind Speed`, `Wind Direction`, and `Temperature Deviation`.

$b = \text{DeltaAngle}(\text{track}, \text{Wind Direction})$

$\text{if } (\text{CAS} \leq 0) r = 0$

$\text{else } r = (\text{Wind Speed} / \text{CasToTas Conversion}(\text{CAS}, \text{Altitude})) * \text{sine}(b)$

Limit the correction to something reasonable.

$\text{if } (|r| > 0.8) r = 0.8 * r / |r|$

$\text{heading} = \text{track} + \text{arcsine}(r)$

$a = \text{DeltaAngle}(\text{heading}, \text{Wind Direction})$

$\text{TAS} = \text{CasToTas Conversion}(\text{CAS}, \text{Altitude}, \text{Temperature Deviation})$

$\text{Ground Speed} = (\text{Wind Speed}^2 + \text{TAS}^2 - 2 * \text{Wind Speed} * \text{TAS} * \text{cosine}(a))^{0.5}$

ComputeGndTrk

The *ComputeGndTrk* function computes the ground track at the along-path distance equal to *distance*., where *distance* must lie between TCP_{i-1} and TCP_{i+1} . It is assumed that the value for *Ground Track_i* is invalid. The function uses a linear interpolation based on DTG_{i-1} and DTG_{i+1} , with the index value *i* input into the function and where the distance, *distance*, must lie between these points.

$d = DTG_{i-1} - DTG_{i+1}$

$\text{if } (d \leq 0) \text{Ground Track} = \text{Ground Track}_{i-1}$

else

$a = (1 - (\text{distance} - DT_{i+1}) / d) * \text{DeltaAngle}(\text{Ground Track}_{i-1}, \text{Ground Track}_{i+1})$

$\text{Ground Track} = \text{Ground Track}_{i-1} + a$

ComputeTcpCas

The index variable *cc* is passed into and out of the *ComputeTcpCas* function. Beginning with the last waypoint, this function computes the CAS at each previous TCP and inserts any additional speed TCPs that may be required to denote a change in the speed profile. The function uses the current speed constraint, searches backward for the previous constraint, and then computes the distance required to meet this previous constraint. The speeds for all of the TCPs within this distance are computed and added to the data for the TCPs. If the along-path distance to meet the previous constraint is not at a TCP, a new speed VTCP is inserted at this distance. Because there is no general closed form solution to compute distances to meet the deceleration constraints, an iterative technique is used in this function. This function is performed in the following steps:

While ($cc > \text{index number of the first waypoint}$) and ($TCP_{cc} \neq \text{Mach Transition CAS}$)

Determine if the previous constraint cannot be met.

If ($CAS_{cc} > \text{Crossing CAS}_{cc}$)

If this is the last pass through the algorithm, mark this as an error condition

$$CAS_{cc} = \text{Crossing } CAS_{cc}$$

Find the prior waypoint index number pc that has a CAS constraint, e.g., a crossing CAS ($\text{Crossing } CAS_{pc} \neq 0$). This may not always be the previous (i.e., $cc - 1$) waypoint.

The initial condition is the previous TCP.

$$pc = cc - 1$$

while ($(pc > \text{index number of the first waypoint})$ and $(TCP_{pc} \neq \text{Mach Transition CAS})$
and $(\text{Crossing } CAS_{pc} = 0)$) $pc = pc - 1$

Save the previous crossing speed,

$$\text{Prior Speed} = \text{Crossing } CAS_{pc}$$

Save the current crossing speed (Test Speed) at TCP_{cc} and the deceleration rate (Test Rate) noting that the first and last waypoints always have speed constraints and except for the first waypoint, all constrained speed points must have deceleration rates.

$$\text{Test Speed} = \text{Crossing } CAS_{cc}$$

$$\text{Test Rate} = \text{Crossing Rate}_{cc}$$

Compute all of the TCP speeds between the current TCP and the previous crossing waypoint.

$$k = cc$$

while $k > pc$

If the previous speed has already been reached, set the remaining TCP speeds to the previous speed.

if ($\text{Prior Speed} \leq \text{Test Speed}$)

for ($k = k - 1$; $k > pc$; $k = k - 1$)

$$CAS_k = \text{Test Speed}$$

$$\text{Mach}_k = \text{CasToMach}(CAS_k, \text{Altitude}_k)$$

Set the speeds at the last test point.

$$CAS_{pc} = \text{Test Speed}$$

$$\text{if } (\text{Mach}_{pc} = 0) \text{ Mach}_{pc} = \text{CasToMach}(CAS_{pc}, \text{Altitude}_{pc})$$

else

Estimate the distance required to meet the crossing restriction using the winds at the current altitude. This is a first-estimation.

Compute the time to do the deceleration.

$$t = (\text{Prior Speed} - \text{Test Speed}) / \text{Test Rate}$$

Compute the wind speed and direction at the current altitude.

$$\text{InterpolateWindWptAltitude}(\text{Wind Profile}_k, \text{Altitude}_k, \text{Wind Speed}_1, \text{Wind Direction}_1, \text{Temperature Deviation}_1)$$

The ground track at the current point is,

$$\text{if } (\text{WptInTurn}(k)) \text{ Track} = \text{Ground Track}_k$$

$$\text{else Track} = \text{Ground Track}_{k-1}$$

$$\text{Current Ground Speed} = \text{ComputeGndSpeedUsingTrack}(\text{Test Speed}, \text{Track}, \text{Altitude}_k, \text{Wind Speed}_1, \text{Wind Direction}_1, \text{Temperature Deviation}_1)$$

Compute the wind speed and direction at the prior altitude.

$$\text{InterpolateWindWptAltitude}(\text{Wind Profile}_{k-1}, \text{Altitude}_k, \text{Wind Speed}_1, \text{Wind Direction}_1, \text{Temperature Deviation}_1)$$

The ground speed at the prior point.

$$\text{Prior Ground Speed} = \text{ComputeGndSpeedUsingTrack}(\text{Prior Speed}, \text{GndTrack}_{k-1}, \text{Altitude}_{k-1}, \text{Wind Speed}_1, \text{Wind Direction}_1, \text{Temperature Deviation}_1)$$

$$\text{Average Ground Speed} = (\text{Prior Ground Speed} + \text{Current Ground Speed}) / 2$$

The distance estimate, dx , is $\text{Average Ground Speed} * t$.

$$dx = \text{Average Ground Speed} * t / 3600$$

Recalculate the distance required to meet the speed using the previous estimate distance dx .

Begin by computing the altitude, $AltD$, at distance dx .

$$\text{if } (\text{Altitude}_k \geq \text{Altitude}_{k-1}) \text{ AltD} = \text{Altitude}_k$$

else

$$\text{AltD} = (\text{NmiToFeet} * dx) * \text{tangent}(\text{Crossing Angle}_k) + \text{Altitude}_k$$

$$\text{if } (\text{AltD} \geq \text{Altitude}_{k-1}) \text{ AltD} = \text{Altitude}_k$$

The new distance x is $DTG_k + dx$.

Compute the winds at $AltD$ and distance x .

*InterpolateWindAtDistance(AltD, x, Wind Speed2, Wind Direction2,
Temperature Deviation2)*

The track angle at this point, with *GetTrajGndTrk* defined in this section:

Track2 = GetTrajGndTrk(x)

The ground speed at altitude *AltD* is then,

*Prior Ground Speed = ComputeGndSpeedUsingTrack(Prior Speed, Track2, AltD,
Wind Speed2, Wind Direction2, Temperature Deviation2)*

Average Ground Speed = (Prior Ground Speed + Current Ground Speed) / 2

*dx = Average Ground Speed * t / 3600*

If there is a TCP prior to *dx*, compute and insert its speed.

If the distance is very close to the waypoint, just set the speed.

if ((DTG_{k-1} < (DTG_k + dx + some small value))

if (|DTG_{k-1} - DTG_k - dx| < some small value) CAS_{k-1} = Prior Speed

else

Compute the speed at the waypoint using $v^2 = v_0^2 + 2ax$ to get *v*.

The headwind at the end point is,

*HeadWind2 = Wind Speed2 * cosine(Wind Direction2 - Ground Track_{k-1})*

dx = DTG_{k-1} - DTG_k

The value of *CAS_{k-1}* is computed using function *EstimateNextCas*, described in this section.

*CAS_{k-1} = EstimateNextCas(Test Speed, Current Ground Speed, false,
Prior Speed, Head Wind2, Altitude_k, dx, Crossing Rate_{cc})*

Determine if the constraint is met.

if ((k-1) = pc)

Determine the allowable crossing window, accounting for special conditions.

*if (((pc + 1) < index number of the last waypoint) and
(VSegType_{pc} = MACH_CAS)) CrossingWindow = 5*

else CrossingWindow = 1

Was the crossing window speed met? If not, set this as an error.

if ($|CAS_{pc} - Crossing\ CAS_{pc}| > CrossingWindow$)
Mark this as an error condition

Always set the crossing exactly to the crossing speed.

$$CAS_{pc} = Crossing\ CAS_{pc}$$

Set the test speed to the computed speed.

$$Test\ Speed = CAS_{k-1}$$

Back up the index counter to the next intermediate TCP.

$$k = k - 1$$

end of if ($(DTG_{k-1} < (DTG_k + dx + some\ small\ value))$)

else

The constraint occurs between this TCP and the previous TCP. A new VTCP needs to be added at this point.

The along path distance d where the VTCP is to be inserted is:

$$d = DTG_k + dx$$

Save the ground track value at this distance.

$$Saved\ Ground\ Track = GetTrajGndTrk(d)$$

Insert a new VTCP at location k in the TCP list. The VTCP is inserted between TCP_{k-1} and TCP_k from the original list. The function *InsertWaypoint* should be appropriate for the actual data structure implementation of this function.

$$InsertWaypoint(k)$$

Update the data for the new VTCP which is now TCP_k .

$$WptType_k = VTCP$$

$$if\ (VSegType_k = no\ type)\ VSegType_k = SPEED$$

$$TurnType_k = no\ turn$$

$$DTG_k = d$$

The altitude at this point is computed as follows, recalling that the new waypoint is TCP_k :

$$if\ (Altitude_{k+1} \geq Altitude_{k-1})\ Altitude_k = Altitude_{k-1}$$

*else Altitude_k = (NmiToFeet * dx) * tangent(Crossing Angle_{k+1}) + Altitude_{k+1}*

CAS_k = Prior Speed

Add the ground track data which must be computed if the new VTCP occurs within a turn. The functions *WptInTurn* and *ComputeGndTrk* are described in subsequent sections.

if (WptInTurn(k)) Ground Track_k = ComputeGndTrk(k, d)

else Ground Track_k = Saved Ground Track

Compute and add the wind data at distance *d* along the path to the data of *TCP_k*.

GenerateWptWindProfile(d, TCP_k)

Test Speed = Prior Speed

Since *TCP_k*, has now been added prior to *pc*, the current constraint counter *cc* needs to be incremented by 1 to maintain its correct position in the list.

cc = cc + 1

end of while k > pc.

Now go to the next altitude change segment on the profile.

cc = k

end of while cc > index number of the first waypoint

ComputeTcpMach

The index variable *cc* is passed into and out of the *ComputeTcpMach* function. This function is similar to *ComputeTcpCas* with the exception that the computed Mach rate will need to be recomputed with any change of altitude. Beginning with the last Mach waypoint (the Mach waypoint that is closest to the runway in terms of DTG), this function computes the Mach at each previous TCP and inserts any additional speed TCPs that may be required to denote a change in the speed profile. The function uses the current speed constraint, searches backward for the previous constraint, and then computes the distance required to meet this previous constraint. The speeds for all of the TCPs within this distance are computed and added to the data for the TCPs. If the along-path distance to meet the previous constraint is not at a TCP, a new speed VTCP is inserted at this distance. Because there is no general closed form solution to compute distances to meet the deceleration constraints, an iterative technique is used in this function. This function is performed in the following steps:

While (cc > index number of the first waypoint)

Determine if the previous constraint cannot be met.

If (Mach_{cc} > Crossing Mach_{cc})

If this is the last pass through the algorithm, mark this as an error condition

$Mach_{cc} = \text{Crossing Mach}_{cc}$

Find the prior waypoint index number pc that has a Mach constraint, e.g., a crossing Mach ($\text{Crossing Mach}_{pc} \neq 0$). This may not always be the previous (i.e., $cc - 1$) waypoint.

Initial condition is the previous TCP.

$pc = cc - 1$

$finished = false$

while ($(pc > \text{index number of the first waypoint})$ and $(TCP_{pc} \neq \text{Mach Transition CAS})$
and $(\text{Crossing CAS}_{pc} = 0)$) $pc = pc - 1$

Save the previous crossing speed,

$Prior\ Speed = \text{Crossing Mach}_{pc}$

Save the current crossing speed ($Test\ Speed$) at TCP_{cc} and the deceleration rate ($Test\ Rate$) noting that the first and last waypoints always have speed constraints and except for the first waypoint, all constrained speed points must have deceleration rates.

$Test\ Speed = \text{Crossing Mach}_{cc}$

$Test\ Rate = \text{CasToMach}(\text{Altitude}_{cc}, \text{Crossing Rate}_{cc})$

Compute all of the TCP speeds between the current TCP and the previous crossing waypoint.

$k = cc$

while $k > pc$

If the previous speed has already been reached, set the remaining TCP speeds to the previous speed.

if ($Prior\ Speed \leq Test\ Speed$)

for ($k = k - 1$; $k > pc$; $k = k - 1$)

$Mach_k = Test\ Speed$

$CAS_k = \text{MachToCas}(Mach_k, \text{Altitude}_k)$

Mark TCP_k as a Mach segment.

Set the speeds at the last test point.

$Mach_{pc} = Test\ Speed$

$CAS_{pc} = \text{MachToCas}(Mach_{pc}, \text{Altitude}_{pc})$

else

Estimate the distance required to meet the crossing restriction using the winds at the current altitude. This is a first-estimation.

Compute the time to do the deceleration.

$$t = (\text{Prior Speed} - \text{Test Speed}) / \text{Test Rate}$$

Compute the wind speed and direction at the current altitude.

InterpolateWindWptAltitude(Wind Profile_k, Altitude_k, Wind Speed_l, Wind Direction_l, Temperature Deviation_l)

The ground track at the current point is,

if (WptInTurn(k)) Track = Ground Track_k

else Track = Ground Track_{k-1}

Current Ground Speed = ComputeGndSpeedUsingMachAndTrack(Test Speed, Track, Altitude_k, Wind Speed_l, Wind Direction_l, Temperature Deviation_l)

Compute the wind speed and direction at the prior altitude.

InterpolateWindWptAltitude(Wind Profile_{k-1}, Altitude_k, Wind Speed_l, Wind Direction_l, Temperature Deviation_l)

The ground speed at the prior altitude and speed is,

Prior Ground Speed = ComputeGndSpeedUsingMachAndTrack(Prior Speed, GndTrack_{k-1}, Altitude_{k-1}, Wind Speed_l, Wind Direction_l, Temperature Deviation_l)

Average Ground Speed = (Prior Ground Speed + Current Ground Speed) / 2

The distance estimate, *dx*, is *Average Ground Speed * t*.

$$dx = \text{Average Ground Speed} * t / 3600$$

Compute the distance required to meet the speed using the previous estimate distance *dx*.

Begin by computing the altitude, *AltD*, at distance *dx*.

if (Altitude_k ≥ Altitude_{k-1}) AltD = Altitude_k

else

$$\text{AltD} = (\text{NmiToFeet} * dx) * \text{tangent}(\text{Crossing Angle}_k) + \text{Altitude}_k$$

if (AltD ≥ Altitude_{k-1}) AltD = Altitude_k

Compute the average Mach rate.

$$MRate1 = CasToMach(Crossing Rate_{cc}, Altitude_k)$$

$$MRate2 = CasToMach(Crossing Rate_{cc}, AltD)$$

$$Test Rate = (MRate1 + MRate2) / 2$$

$$t = (Prior Speed - Test Speed) / Test Rate$$

The new distance x is $DTG_k + dx$.

Compute the winds at $AltD$ and distance x .

$$InterpolateWindAtDistance(AltD, x, Wind Speed2, Wind Direction2, \\ Temperature Deviation2)$$

The track angle at this point, with $GetTrajGndTrk$ defined in this section, is:

$$Track2 = GetTrajGndTrk(x)$$

The ground speed at altitude $AltD$ is then,

$$Prior Ground Speed = ComputeGndSpeedUsingMachAndTrack(Prior Speed, Track2, \\ AltD, Wind Speed2, Wind Direction2, Temperature Deviation2)$$

$$Average Ground Speed = (Prior Ground Speed + Current Ground Speed) / 2$$

$$dx = Average Ground Speed * t / 3600$$

If there is a TCP prior to dx , compute and insert its speed.

If the distance is very close to the waypoint, just set the speed.

$$if ((DTG_{k-1} < (DTG_k + dx + some small value))$$

$$if (|DTG_{k-1} - DTG_k - dx| < some small value)$$

$$Mach_{k-1} = Prior Speed$$

Mark TCP_k as a Mach segment.

else

Compute the speed at the waypoint using $v^2 = v_0^2 + 2ax$ to get v .

The headwind at the end point is,

$$HeadWind2 = Wind Speed2 * cosine(Wind Direction2 - Ground Track_{k-1})$$

$$dx = DTG_{k-1} - DTG_k$$

Compute the average Mach rate.

$$MRate1 = CasToMach(Crossing Rate_{cc}, Altitude_k)$$

$$MRate2 = CasToMach(Crossing Rate_{cc}, Altitude_{k-1})$$

$$Test Rate = (MRate1 + MRate2) / 2$$

The value of $Mach_{k-1}$ is computed using function *EstimateNextMach*, described in this section.

$$Mach_{k-1} = EstimateNextMach(Test Speed, Current Ground Speed, Prior Speed, Head Wind2, Altitude_k, dx, Test Rate)$$

Determine if the constraint is met.

$$if ((k-1) = pc)$$

Was the crossing speed met within 0.002 Mach? If not, set this as an error.

$$if (|Mach_{pc} - Crossing Mach_{pc}| > 0.002) Mark this as an error condition$$

Always set the crossing exactly to the crossing speed.

$$Mach_{pc} = Crossing Mach_{pc}$$

Set the test speed to the computed speed.

$$Test Speed = Mach_{k-1}$$

Back up the index counter to the next intermediate TCP.

$$k = k - 1$$

end of if ((DTG_{k-1} < (DTG_k + dx + some small value))

else

The constraint occurs between this TCP and the previous TCP. A new VTCP needs to be added at this point.

The along path distance d where the VTCP is to be inserted is:

$$d = DTG_k + dx$$

Save the ground track value at this distance.

$$Saved Ground Track = GetTrajGndTrk(d)$$

Insert a new VTCP at location k in the TCP list. The VTCP is inserted between TCP_{k-1} and TCP_k from the original list. The function *InsertWaypoint* should be appropriate for the actual data structure implementation of this function.

InsertWaypoint(k)

Update the data for the new VTCP which is now TCP_k .

$WptType_k = VTCP$

if ($VSegType_k = no\ type$) $VSegType_k = SPEED$

$TurnType_k = no\ turn$

$DTG_k = d$

The altitude at this point is computed as follows, recalling that the new waypoint is TCP_k :

if ($Altitude_{k+1} \geq Altitude_{k-1}$) $Altitude_k = Altitude_{k-1}$

else $Altitude_k = (NmiToFeet * dx) * tangent(Crossing\ Angle_{k+1}) + Altitude_{k+1}$

$Mach_k = Prior\ Speed$

Mark TCP_k *as a Mach segment.*

Add the ground track data which must be computed if the new VTCP occurs within a turn. The functions *WptInTurn* and *ComputeGndTrk* are described in subsequent sections.

if (*WptInTurn*(k)) $Ground\ Track_k = ComputeGndTrk(k, d)$

else $Ground\ Track_k = Saved\ Ground\ Track$

Compute and add the wind data at distance d along the path to the data of TCP_k .

GenerateWptWindProfile(d, TCP_k)

$Test\ Speed = Prior\ Speed$

Since TCP_k , has now been added prior to pc , the current constraint counter cc needs to be incremented by 1 to maintain its correct position in the list.

$cc = cc + 1$

end of while $k > pc$.

Now go to the next altitude change segment on the profile.

$cc = k$

end of while $cc > index\ number\ of\ the\ first\ waypoint$.

Make sure that the waypoints get marked correctly if there are no CAS waypoints.

if ((begin > index number of the first waypoint) and (cc = index number of the first waypoint)) then

for (k = index number of the first waypoint; k < begin; k++)

$Mach\ Segment_k = true$

DeltaAngle

The *DeltaAngle* function returns angle *a*, the difference between *Angle1* and *Angle2*. The returned value may be negative, i.e., $-180\ degrees \geq DeltaAngle \geq 180\ degrees$.

$a = Angle2 - Angle1$

Adjust "a" such that $0 \geq a \geq 360$

if ($a > 180$) $a = a - 360$

DoTodAcceleration

The *DoTodAcceleration* function handles the special case when there is an acceleration to the descent Mach at the top-of-descent. This function is invoked from *Add Descent Mach Waypoint*, which passes in the index number for the TOD waypoint, *TodIndex*, and the Mach value at the TOD, *MachAtTod*. The function will insert the Mach acceleration point into the waypoint list if a valid acceleration point can be found.

Make an initial estimate of the distance to the new Mach value. The function *TodAccelerationDistance* returns the values *Valid*, *k*, and *dx*.

TodAccelerationDistance(*TodIdx*, *MachAtTod*, *Mach Descent Mach*, *Valid*, *k*, *dx*)

if (*Valid*)

Add the VTCP for the end of the TOD acceleration.

$d = DTG_{TodIdx} - dx$

The original ground track will be needed for the new TCP, so save it.

$OldGroundTrack = GetTrajGndTrk(d)$

Save the wind data at this distance as a temporary TCP.

GenerateWptWindProfile(*d*, *TemporaryTcp*)

The new waypoint is downstream of the current value of *k*.

$k = k + 1$

InsertWaypoint(*k*)

Note that Wpt_k is the newly created waypoint.

$WptType_k = VTCP$

TurnType_k = no turn

If the new waypoint is not already marked as a special vertical type, mark it as a top-of-descent acceleration point.

if (VSegType_k = NONE) VSegType_k = TOD acceleration

DTG_k = d

Calculate the altitude for the new TCP.

*Altitude_k = Altitude_{TodIdx} - (NmiToFeet * dx) * tangent(Crossing Angle_{k+1})*

Mach_k = Mach Descent Mach

Mach Cross_k = Mach Descent Mach

MachSegment_k = true

Set the *Crossing Rate* to the default value of 0.75.

Crossing Rate_k = 0.75

Add the appropriate ground track value.

if (WptInTurn(k)) Ground Track_k = ComputeGndTrk(k, d)

else Ground Track_k = OldGroundTrack

Copy the wind data from *TemporaryTcp* into *Wpt_k*.

end of if (Valid)

else mark this as an error for being unable to accelerate to the descent Mach value.

EstimateNextCas

EstimateNextCas is an iterative function to estimate the CAS value, *CAS*, at the next TCP. Note that there is no closed-form solution for this calculation of CAS. The input variable names described in this function are from the calling routine and are, in order, the target CAS value, *Test CAS*; the ground speed at the estimation starting point, *Current Ground Speed*; an estimation limiting flag, *No Limit Flag*; the CAS at the estimation starting point, *Prior CAS*; the head wind at the estimation starting point, *Head Wind*; the altitude at the estimation starting point, *Altitude*; the distance from the estimation starting point to the point where the CAS is to be estimated, *Distance*; and the deceleration rate to be used in this estimation, *CAS Rate*. Also, the input deceleration value must be greater than 0, *CAS Rate* > 0. The function returns the estimated CAS value.

Guess CAS = Test CAS

Set up a condition to get at least one pass.

*d = -10 * Distance*

$size = 1.01 * (Prior\ CAS - Guess\ CAS)$

$count = 0$

if $((Distance > 0) \text{ and } (CAS\ Rate > 0))$

Iterate a solution. The counter count is used to terminate the iteration if the distance estimation does reach a solution within 0.001 nmi.

while $((|Distance - d| > 0.001) \text{ and } (count < 10))$

if $(Distance > d)$ $Guess\ CAS = Guess\ CAS - size$

else $Guess\ CAS = Guess\ CAS + size$

$size = size / 2$

The estimated time t to reach this speed,

$t = (Guess\ CAS - Test\ CAS) / CAS\ Rate$

The new ground speed,

$Gs2 = CasToTas\ Conversion(guess, Altitude) - Head\ Wind$

$d = ((Current\ Ground\ Speed + Gs2) / 2) * (t / 3600)$

$count = count + 1$

end of the while loop

Limit the computed CAS, if necessary.

if $((NoLimit = false) \text{ and } (Guess\ CAS > Prior\ CAS))$ $Guess\ CAS = Prior\ CAS$

return $Guess\ CAS$

EstimateNextMach

EstimateNextMach is an iterative function to estimate the Mach value, *Mach*, at the next TCP. Note that there is no closed-form solution for this calculation. The input variable names described in this function are from the calling routine. Also, the input deceleration value must be greater than 0, *Mach Rate* > 0.

$Mach = Test\ Speed$

Set up a condition to get at least one pass.

$d = -10 * dx$

$size = 1.01 * (Prior\ Speed - Test\ Speed)$

$count = 0$

if ($(dx > 0)$ and $(Test\ Rate > 0)$)

Iterate a solution. The counter count is used to terminate the iteration if the distance estimation does reach a solution within 0.001 nmi.

while ($(|d - dx| > 0.001)$ and $(count < 10)$)

if ($d > dx$) $Mach = Mach - size$

else $Mach = Mach + size$

$size = size / 2$

The estimated time t to reach this speed,

$t = (Mach - Test\ Speed) / Test\ Rate$

The new ground speed,

$CAS = MachToCas(Mach, Altitude)$

$Gs2 = CasToTas\ Conversion(CAS, Altitude) - Head\ Wind2$

$d = ((Current\ Ground\ Speed + Gs2) / 2) * (t / 3600)$

$count = count + 1$

end of the while loop

Limit the computed $Mach$, if necessary.

if ($Mach > Prior\ Speed$) $Mach = Prior\ Speed$

GenerateWptWindProfile

The function *GenerateWptWindProfile* is used to compute new wind profile data. This function is a double-linear interpolation using the wind data from the two bounding input waypoints to compute the wind profile for a new VTCP, TCP_k . The interpolations are between the wind altitudes from the input data and the ratio of the distance d at a point between TCP_{i-1} and TCP_i and the distance between TCP_{i-1} and TCP_i . E.g.,

- Find the two bounding input waypoints, TCP_{i-1} and TCP_i , between which d lies, e.g., $TCP_{i-1} \geq d \geq TCP_i$.
- Using the altitudes from the wind profile of TCP_i , compute and temporarily save the wind data at these altitudes using the wind data from TCP_{i-1} (e.g., $Wind\ Speed_{Temporary, Altitude1}$).
- Compute the wind speed, wind direction, and temperature deviation for each altitude using the ratio r of the distances. Assuming that the difference between DTG_{i-1} and $DTG_i \neq 0$, and that $DTG_{i-1} > DTG_i$.

$$r = (DTG_{i-1} - d) / (DTG_{i-1} - DTG_i)$$

Iterate the following for each altitude in the profile.

$$\text{Wind Speed}_{k, \text{Altitude}l} = (1 - r) * \text{Wind Speed}_{\text{Temporary}, \text{Altitude}l} + r * \text{Wind Speed}_{i, \text{Altitude}l}$$

$$a = \text{DeltaAngle}(\text{Wind Direction}_{\text{Temporary}, \text{Altitude}l}, \text{Wind Direction}_{i, \text{Altitude}l})$$

$$\text{Wind Direction}_{k, \text{Altitude}l} = \text{Wind Direction}_{k, \text{Altitude}l} + (r * a)$$

$$\begin{aligned} \text{Temperature Deviation}_{k, \text{Altitude}l} = \\ (1 - r) * \text{Temperature Deviation}_{\text{Temporary}, \text{Altitude}l} + r * \text{Temperature Deviation}_{i, \text{Altitude}l} \end{aligned}$$

Figure 7 is an example of the computation data for the wind computation at a 9,000 ft altitude. In this example, TCP_{i-1} has wind data at 10,000 and 8,000 ft and TCP_i has wind data at 9,000 ft.

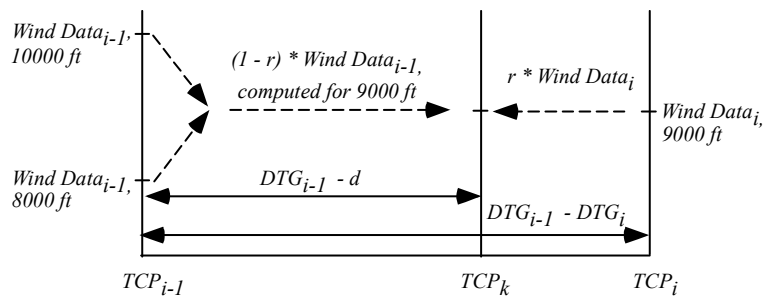


Figure 7. Example of computing a single wind data altitude.

GetTrajectoryData

The *GetTrajectoryData* function computes the trajectory data at the along-path distance equal to d and saves these data in a temporary TCP record. The function uses a linear interpolation based on the DTG values of the two TCPs bounding this distance and the distance d to compute the trajectory data at this point.

GetTrajGndTrk

The *GetTrajGndTrk* function computes the ground track at the along-path distance, $distance$.

if ($(distance < 0)$ or $(distance > DTG_{\text{first waypoint}})$) $Ground Track = Ground Track_{\text{first waypoint}}$

else

Find where $distance$ is on the path.

$i = \text{index number of the last waypoint}$

while ($distance > DTG_i$) $i = i - 1$

if ($distance = DTG_i$) $Ground Track = Ground Track_i$

else

$$x = DTG_i - DTG_{i+1}$$

$$\text{if } (x \leq 0) \text{ } r = 0$$

else r = (distance - DTG_{i+1}) / x

if (r > 1) r = 1

*dx = (1 - r) * DeltaAngle(Ground Track_i, Ground Track_{i+1})*

Ground Track = Ground Track_i + dx

HandleDescentAccelDecel

The function *HandleDescentAccelDecel* is designed to handle the special case of a Mach acceleration in the descent where the first CAS crossing restriction cannot be met. The calling program provides as input and retains the subsequent outputs for the following variables: *CasIndex*, *CruiseMach*, *MachCasModified*, *DescentMach*, and *MachCas*. The variable *CasIndex* is the index value in the TCP list for the first CAS constrained waypoint. The variable *CruiseMach* is the last Mach crossing restriction value prior to the first CAS segment. The variable *MachCasModified* is a flag returned by this function if the *DescentMach* or *MachCas* values are changed. The variables *DescentMach* and *MachCas* are the planned descent Mach and planned Mach-to-CAS transition CAS, respectively, and these values may be modified by this function.

Initialize variables.

i = 0

z = 0

fini = false

MachCasModified = false

Perform up to two iterations to calculate any required Mach or CAS change in the descent.

while ((fini = false) and (i < 2))

Calculate *z* at the descent Mach and the Mach-to-CAS CAS.

z = FindMachCasTransitionAltitude(MachCas, DescentMach)

Determine if *z* is below the CAS crossing restriction.

if (z < Altitude_{CasIndex})

Set the CAS to the value at this altitude, knowing the crossing restriction can't be met.

MachCas = MachToCas(DescentMach, Altitude_{CasIndex})

else if (z > Altitude_{Cross_{first} waypoint})

Set the Mach to the descent CAS at the cruise altitude.

m = CasToMach(MachCas, Altitude_{first waypoint})

$if (m > CruiseMach) DescentMach = m$
 $if (MachCas < Crossing CAS_{CasIndex})$
 $MachCas = Crossing CAS_{CasIndex}$
 $i = i + 1$
 $else fini = true$
 $end\ of\ while\ ((fini = false)\ and\ (i < 2))$
 Find the TOD TCP.
 $fini = false$
 $TodIndex = 0$
 $i = index\ number\ of\ the\ first\ waypoint$
 $while\ ((i < index\ number\ of\ the\ last\ waypoint)\ and\ (fini = false))$
 $if\ ((Altitude_i < Altitude_{first\ waypoint})\ or\ (Crossing\ CAS_i > 0))$
 $if\ ((Altitude_i \neq Altitude_{first\ waypoint}))\ TodIndex = i - 1$
 $else\ TodIndex = i$
 $fini = true$
 $i = i + 1$
 $end\ of\ while\ ((i < index\ number\ of\ the\ last\ waypoint)\ and\ (fini = false))$

Calculate the entire decent distance.

$$d = DTG_{TodIndex} - DTG_{CasIndex}$$

Estimate the distance, D_{accel} , to the new Mach value.

$$TodAccelerationDistance(TodIndex, CruiseMach, MachDescentMach, Valid, AccelIndex, D_{accel})$$

Estimate the distance, D_{decel} , to the CAS crossing speed.

$$BodDecelerationDistance(CasIndex, z, Mach\ Transition\ CAS, D_{decel})$$

$$fini = false$$

$$m = DescentMach$$

The nominal speed values won't work, there is insufficient distance to obtain the acceleration and then slow to the crossing speed. Iterate until a solution is found.

while ((fini = false) and (d < (Daccel + Ddecel)))

Iterate the solution.

Slightly change the Mach and then find the CAS.

m = m - 0.002

if (m < Cruise Mach)

m = Cruise Mach

fini = true

Estimate the distance to the new Mach value.

TodAccelerationDistance(TodIndex, Cruise Mach, m, Valid, AccelIndex, Daccel)

Find the altitude where the acceleration ends.

*z = Crossing Altitude_{first waypoint} - (Daccel / d) * (Crossing Altitude_{first waypoint} - Crossing Altitude_{CasIndex})*

CAS = MachToCas(m, z)

Estimate the distance to the CAS crossing speed.

BodDecelerationDistance(CasIndex, z, CAS, Ddecel)

if (d ≥ (Daccel + Ddecel))

fini = true

Modify the descent Mach and CAS values.

modified = true

DescentMach = m

Add a buffer to the CAS so that subsequent Mach-to-CAS calculation won't cause an error.

MachCas = CAS + 0.1

end of if (d ≥ (Daccel + Ddecel))

InterpolateWindAtDistance

The function *InterpolateWindAtDistance* is used to compute the wind speed, wind direction, and temperature deviation at an altitude, *Altitude*, for a specific distance, *Distance*, along the path. This function is a linear interpolation using the wind data from the input waypoints that bound the along-path distance.

Find the bounding input waypoints.

$i0 = \text{index number of the first waypoint}$

$j = \text{index number of the first waypoint}$

$\text{fini} = \text{false}$

if ($\text{Distance} < 0$) $\text{Distance} = 0$

while ($(\text{fini} = \text{false})$ and $(j < (\text{index number of the last waypoint} - 1))$)

if ($(\text{WptType}_j = \text{input waypoint})$ and $(\text{DTG}_j \geq \text{Distance})$) $i0 = j$

if ($\text{DTG}_j < \text{Distance}$) $\text{fini} = \text{true}$

end of the while loop

$i1 = i0 + 1$

$j = i1$

$\text{fini} = \text{false}$

while ($(\text{fini} = \text{false})$ and $(j < \text{index number of the last waypoint})$)

if ($(\text{WptType}_j = \text{input waypoint})$ and $(\text{DTG}_j \leq \text{Distance})$)

$i1 = j$

$\text{fini} = \text{true}$

end of if

$j = j + 1$

end of the while loop

if ($i1 > \text{index number of the last waypoint}$) $i1 = \text{index number of the last waypoint}$

if ($i0 = i1$) $\text{InterpolateWindWptAltitude}(\text{TCP}_{i0}, \text{Altitude})$

else

Interpolate the winds at each waypoint.

$\text{InterpolateWindWptAltitude}(\text{TCP}_{i0}, \text{Altitude})$, returning Spd0 , Dir0 , and Td0

$\text{InterpolateWindWptAltitude}(\text{TCP}_{i1}, \text{Altitude})$, returning Spd1 , Dir1 , and Td1

Interpolate the winds between the two waypoints.

$$r = (DTG_{i0} - Distance) / (DTG_{i0} - DTG_{i1})$$

$$Wind\ Speed = ((1 - r) * Spd0) + (r * Spd1)$$

$$a = DeltaAngle(Dir0, Dir1)$$

$$Wind\ Direction = Dir0 + (r * a)$$

$$Temperature\ Deviation = ((1 - r) * Td0) + (r * Td1)$$

InterpolateWindAtRange

The function *InterpolateWindAtRange* is used to compute the wind speed, *WindSpd*, wind direction, *WindDir*, and temperature deviation, *TempDev*, at a distance along path, *Distance*, between two sets of wind data sets, denoted by the subscripts 1 and 2, where $DTG_1 \geq Distance \geq DTG_2$. This function is a linear interpolation using the wind data from the input.

if (($DTG_1 = DTG_2$) or ($Distance = DTG_1$))*then*

$$WindSpd = WindSpd_1$$

$$WindDir = WindDir_1$$

$$TempDev = TempDev_1$$

else if ($Distance = DTG_2$) *then*

$$WindSpd = WindSpd_2$$

$$WindDir = WindDir_2$$

$$TempDev = TempDev_2$$

else

Interpolate the values.

$$r = (DTG_1 - Distance) / (DTG_1 - DTG_2)$$

$$WindSpd = (1 - r) * WindSpd_1 + (r * WindSpd_2)$$

$$a = DeltaAngle(WindDir_1, WindDir_2)$$

$$WindDir = WindDir_1 + (r * a)$$

$$TempDev = ((1 - r) * TempDev_1) + (r * TempDev_2)$$

InterpolateWindWptAltitude

The function *InterpolateWindWptAltitude* is used to compute the wind speed, wind direction, and temperature deviation at an altitude, *Altitude*, for a specific TCP. This function is a linear interpolation using the wind data from the current TPC.

Find the index numbers, $p0$ and $p1$, for the bounding altitudes.

$$p0 = 0$$

$$p1 = 0$$

for ($k = 1$; $k \leq \text{Number of Wind Altitudes}_i$; $k = k + 1$)

$$\text{if } (\text{Wind Altitude}_{i,k} \leq \text{Altitude}) \text{ } p0 = k$$

$$\text{if } ((\text{Wind Altitude}_{i,k} \geq \text{Altitude}) \text{ and } (p1 = 0)) \text{ } p1 = k$$

if ($p1 = 0$) $p1 = \text{Number of Wind Altitudes}_i$

If $\text{Altitude} = \text{Wind Altitude}_{p0}$ or if $\text{Altitude} = \text{Wind Altitude}_{p1}$ then the wind data from that point is used. Otherwise, Altitude is not at an altitude on the wind profile of TCP_i , i.e., $z = \text{Wind Altitude}_{i,k}$, then:

$$\text{if } (\text{Wind Altitude}_{p1} \leq \text{Wind Altitude}_{p0}) \text{ } r = 0$$

$$\text{else } r = (\text{Altitude} - \text{Wind Altitude}_{p0}) / (\text{Wind Altitude}_{p1} - \text{Wind Altitude}_{p0})$$

$$\text{Wind Speed} = ((1 - r) * \text{Wind Speed}_{p0}) + (r * \text{Wind Speed}_{p1})$$

$$a = \text{DeltaAngle}(\text{Wind Direction}_{p0}, \text{Wind Direction}_{p1})$$

$$\text{Wind Direction} = \text{Wind Direction}_{p0} + (r * a)$$

$$\text{Temperature Deviation} = ((1 - r) * \text{Temperature Deviation}_{p0}) + (r * \text{Temperature Deviation}_{p1})$$

FindMachCasTransitionAltitude

The function *FindMachCasTransitionAltitude* is used to compute the altitude where the input Mach, Mach, and CAS, Cas, values would be equivalent

$$z = (1 - (((0.2 * ((\text{Cas}/661.48)^2) + 1)^{3.5}) - 1) / (((0.2 * (\text{Mach}^2) + 1)^{3.5}) - 1))^{0.19026}) / 0.00000687535$$

return the value of z.

RadialRadialIntercept

The function *RadialRadialIntercept* determines if two place-and-radial sets, each defined by latitude, longitude, and a track angle, will intersect and if so, calculates the latitude and longitude of the intercept point. Inputs are values of latitude, Latitude, longitude, Longitude, and angle, Angle; one set of each for the two place-and-radial sets. If a valid intercept can be calculated, then the intercept point's latitude and longitude are output, NewLatitude and NewLongitude, and the function returns a valid indication. Otherwise, the function returns an invalid indication.

Calculate the distance and the track angle between the two input positions.

$$\text{distance}_{1,2} = \arccosine(\text{sine}(\text{Latitude}_1) * \text{sine}(\text{Latitude}_2) + \text{cosine}(\text{Latitude}_1) * \text{cosine}(\text{Latitude}_2) * \text{cosine}(\text{Longitude}_1 - \text{Longitude}_2))$$

$$\text{track}_{1,2} = \arctangent2(\text{sine}(\text{Longitude}_2 - \text{Longitude}_1) * \text{cosine}(\text{Latitude}_2), \text{cosine}(\text{Latitude}_1) * \text{cosine}(\text{Longitude}_2 - \text{Longitude}_1) + \text{sine}(\text{Latitude}_1) * \text{sine}(\text{Latitude}_2))$$

$$\text{sine}(\text{Latitude}_2) - \text{sine}(\text{Latitude}_1) * \text{cosine}(\text{Latitude}_2) * \text{cosine}(\text{Longitude}_2 - \text{Longitude}_1))$$

Check for error in the intercept calculation.

error = false

$$\text{track}_1 = \text{Angle}_1 - \text{track}_{1,2} + 90$$

Adjust track₁ such that $0 \geq \text{track}_1 \geq 360$

$$\text{track}_2 = \text{Angle}_2 - \text{track}_{1,2} + 90$$

Adjust track₂ such that $0 \geq \text{track}_2 \geq 360$

Determine the quadrant.

$$\text{ang}_1 = \text{track}_2 + 180$$

Adjust ang₁ such that $0 \geq \text{ang}_1 \geq 360$

if ($(|\text{DeltaAngle}(\text{track}_1, \text{track}_2)| < 2)$ or $(|\text{DeltaAngle}(\text{track}_1, \text{ang}_1)| < 2)$)

Determine if the angles are really 180 degrees apart.

$$\text{ang}_2 = \text{Angle}_2 + 180$$

Adjust ang₂ such that $0 \geq \text{ang}_2 \geq 360$

$$\text{ang}_3 = \text{DeltaAngle}(\text{Angle}_1, \text{ang}_2)$$

$$\text{ang}_4 = \text{DeltaAngle}(\text{Angle}_1, \text{track}_{1,2})$$

if ($(|\text{ang}_3| > 2)$ or $(|\text{ang}_4| > 2)$) error = true

if (error = false)

RelativeLatLong(Latitude₁, Longitude₁, track_{1,2}, distance_{1,2} / 2, NewLatitude, NewLongitude)

else

Determine the quadrant.

if (track₁ ≤ 90) quadrant1 = 1

else if (track₁ ≤ 180) quadrant1 = 2

else if (track₁ ≤ 270) quadrant1 = 3

else quadrant1 = 4

if (track₂ ≤ 90) quadrant2 = 1

```

else if (track2 ≤ 180) quadrant2 = 2
else if (track2 ≤ 270) quadrant2 = 3
else quadrant2 = 4
if (quadrant1 = 1)
    if ((quadrant2 = 2) or (quadrant2 = 3)) error = true
    if ((quadrant2 = 1) and (chktk1 < chktk2)) error = true
else if (quadrant1 = 2)
    if ((quadrant2 = 1) or (quadrant2 = 4)) error = true
    if ((quadrant2 = 2) and (chktk1 > chktk2)) error = true
else if (quadrant1 = 3)
    if ((quadrant2 = 1) or (quadrant2 = 2) or (quadrant2 = 4)) error = true
    if (track1 > track2) error = true
else
    if ((quadrant2 = 1) or (quadrant2 = 2) or (quadrant2 = 3)) error = true
    if (track1 < track2) error = true
if (error = false)
    trx1 = |Angle1 - track1,2|
    Adjust trx1 such that 0 ≥ trx1 ≥ 360
    trx2 = |Angle2 - (track1,2 + 180)|
    Adjust trx2 such that 0 ≥ trx2 ≥ 360
    if (trx1 > 180) trx1 = 360 - trx1
    if (trx2 > 180) trx2 = 360 - trx2
    ang5 = 180 - trx1 - trx2
    if ((ang5 = 0) or ((ang5-180) = 0) or (distance1,2 = 0)) error = true
if (error = false)
    distance2 = distance1,2 * sine(trx2) / sine(ang5)

```

```

if (distance2 ≤ 0) distance2 = - distance2

if (distance2 > max_intercept_range) error = true

else RelativeLatLon(Latitude1, Longitude1, Angle1, distance2, NewLatitude,
NewLongitude)

if (error) return false

else return true

```

RelativeLatLon

The function *RelativeLatLon* computes the latitude and longitude from input values of latitude, *BaseLat*, longitude, *BaseLon*, angle, *Angle*, and range, *Range*.

```

if (Angle = 180) Latitude = -Range / 60 + BaseLat

else Latitude = ((Range * cos(Angle)) / 60) + BaseLat

if ((BaseLat = 0) or (BaseLat = 180)) Longitude = BaseLon

else if (Angle = 90) Longitude = BaseLon + Range / (60 * cos(BaseLat))

else if (Angle = 270) Longitude = BaseLon - Range / (60 * cos(BaseLat))

else

    r1 = tangent(45 + 0.5 * Latitude)

    r2 = tangent(45 + 0.5 * BaseLat)

    if ((r1 = 0) or (r2 = 0)) Longitude = 20, just some number, mark this as an error condition.

    else Longitude = BaseLon + (180 / pi * (tangent(Angle) * (log(r1) - log(r2))))

```

TodAccelerationDistance

The *TodAccelerationDistance* function estimates the distance required for the special case of an acceleration from the top-of-descent Mach to the descent Mach at the top-of-descent. This function is invoked from *HandleDescentAccelDecel* and *DoTodAcceleration*, which passes in the index number for the TOD waypoint, *TodIndex*, and the Mach value at the TOD, *MachAtTod*. The function returns a validity flag to indicate if a TOD acceleration is valid, *Valid*, and if valid, the indices in the TCP list where the acceleration occurs, *AccelIndex*, and the distance from the index point of the acceleration, *Distance*.

Perform an initialization of flags and counters.

```
fni = false
```

```
skip = true
```

$k = TodIndex$

Make an initial guess of the distance to the new Mach value.

$Descent\ Speed = Mach\ Descent\ Mach$

$Mach\ Rate_1 = CasToMach(0.75\ kt / sec, Altitude_{TodIndex})$

Compute the time required to do the deceleration.

$t = (Mach\ Descent\ Mach - MachAtTod) / Mach\ Rate_1$

Compute the wind speed and direction at the current altitude.

$InterpolateWindWptAltitude(Wind\ Profile_{TodIndex}, Altitude_{TodIndex}, Wind\ Speed, Wind\ Direction, Temperature\ Deviation)$

Get the ground track at the current point.

$if\ (WptInTurn(Waypoint_{TodIndex}))\ track = Ground\ Track_{TodIndex + 1}$

$else\ track = Ground\ Track_{TodIndex}$

$TOD\ Ground\ Speed = ComputeGndSpeedUsingMachAndTrack(MachAtTod, track, Altitude_{TodIndex}, Wind\ Speed, Wind\ Direction, Temperature\ Deviation)$

$Descent\ Ground\ Speed = ComputeGndSpeedUsingMachAndTrack(Mach\ Descent\ Mach, track, Altitude_{TodIndex}, Wind\ Speed, Wind\ Direction, Temperature\ Deviation)$

The average ground speed is as follows:

$Average\ Ground\ Speed = (TOD\ Ground\ Speed + Descent\ Ground\ Speed) / 2$

The distance estimate, dx , is $Average\ Ground\ Speed * t$ with a conversion to nmi.

$dx = Average\ Ground\ Speed * t / 3600$

Now compute better estimates, doing this twice to refine the estimation.

$for\ (i = 1; i \leq 2; i = i + 1)$

$skip = false$

Determine if this distance is beyond the next downstream waypoint.

$k = TodIndex$

$d = DTG_{TodIndex} - dx$

$while\ ((k < (index\ number\ of\ the\ last\ waypoint - 1))\ and\ (DTG_{k+1} > d))$

$if\ ((k \neq TodIndex)\ and\ (Crossing\ Rate_k > 0))\ skip = True$

$$k = k + 1$$

Compute the wind speed and direction at the new altitude.

InterpolateWindWptAltitude(Waypoint_k, Altitude_k, Wind Speed, Wind Direction, Temperature Deviation)

The ground speed at this point is:

Descent Ground Speed = ComputeGndSpeedUsingMachAndTrack(Mach Descent Mach, Ground Track_k, Altitude_k, Wind Speed, Wind Direction, Temperature Deviation)

The average ground speed is:

Average Ground Speed = (TOD Ground Speed + Descent Ground Speed) / 2

The distance, *dx*, is:

$$dx = \textit{Average Ground Speed} * t / 3600$$

If there is a valid deceleration point, add it.

Valid = not skip

AccelIndex = k

Distance = dx

WptInTurn

The *WptInTurn* function simply determines if the waypoint is between a turn-entry TCP and a turn-exit TCP. If this is true, then the function returns a value of true, otherwise it returns a value of false.

fini = false

within = false

j = i + 1

while ((fini = false) and (j < (index number of the last waypoint)))

if (TurnType_j = turn-entry) fini = true

else if (TurnType_j = turn-exit)

fini = true

within = true

j = j + 1

return within

Summary

The algorithm described in this document takes as input a list of waypoints, their trajectory-specific data, and associated wind profile data. This algorithm calculates the altitude, speed, along path distance, and along path time for each waypoint and every point along the path where the speed, altitude, or ground track changes. A full 4D trajectory can then be generated by the techniques described. A software prototype has been developed from this documentation.

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