

# Computational Fluid Dynamics Analysis of the Stall Characteristics of a Wing Designed Based on Prandtl's Minimum Induced Drag

Seung Y. Yoo



# Outline

- Introduction/Background
- Method
- Results
- Conclusion
- Questions

# NASA

#### Introduction

- Prandtl's work on minimum induced drag
  - 1929 publication elliptical spanwise lift distribution, constrain wing span
  - 1933 publication bell shaped spanwise lift distribution, constrain bending moment
    - 11% less drag, 22% longer span compared to elliptical distribution for wings of identical weight

#### Summary of Prandtl's result

Lift (L): 
$$L = (1 - x^2)^{1.5}$$

Downwash angle (DW): DW =  $1.5 * (x^2 - 0.5)$ 

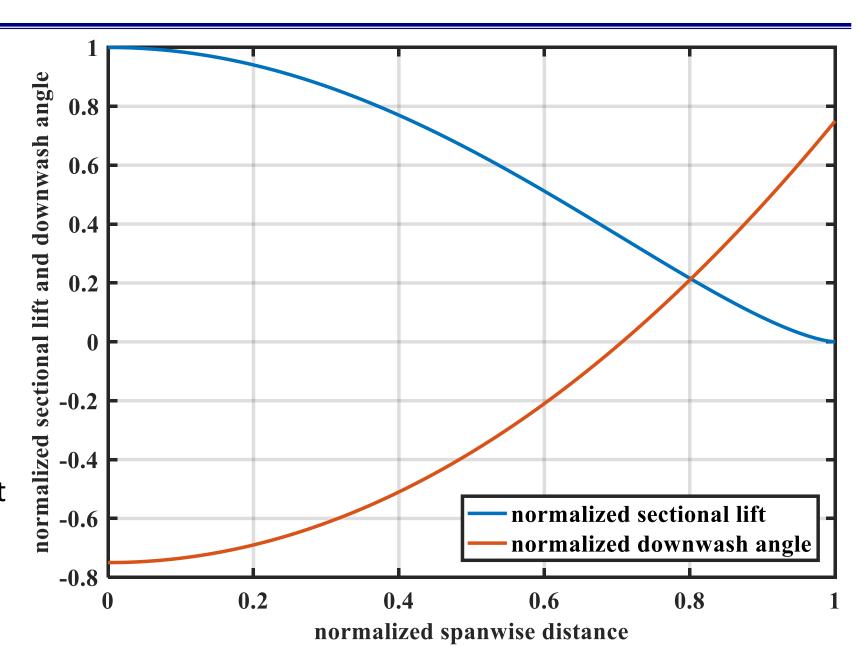
Lift tapers to zero at wing tip: 
$$\lim_{x:0\to b/2} L(x) = 0$$
,  $\lim_{x:0\to b/2} \frac{dL(x)}{dx} = 0$ 

Continuous down wash angle at wing tip:  $\lim_{x:0\to b/2} \frac{dDW(x)}{dx} = \lim_{x:\infty\to b/2} \frac{dDW(x)}{dx} = 0$ 



#### Introduction

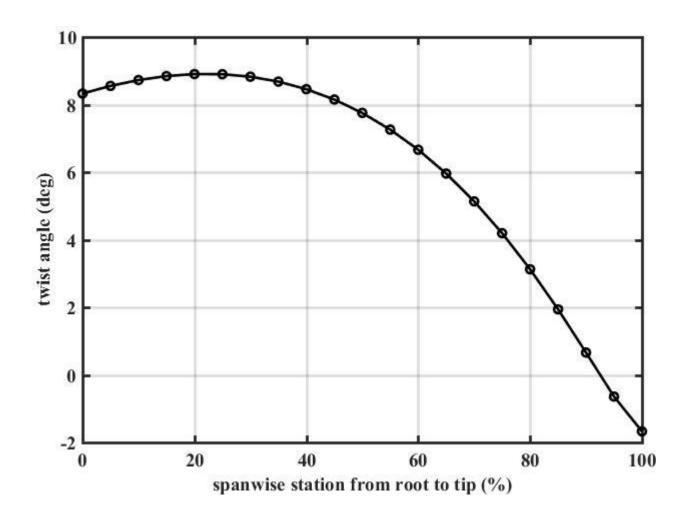
- Lift zero at wing tip
- Slope of lift zero at wing tip
- Downwash becomes upwash at 70.7% span
  - Inboard vortex, no wing tip vortex
  - Proverse yaw due to induced thrust at wing tip caused by upwash





### Introduction

- Achieve bell shape loading via nonlinear spanwise twist distribution
- Wing tip is at approximately -10° twist relative to root chord





# Introduction

- P-3C from the Preliminary Aerodynamic Design To Lower Drag (PRANDTL-D) program
- Span of 24.6 ft
- MAC of 1.969 ft
- Planform area of 40.5 ft<sup>2</sup>
- ~30 mph



# NASA

#### Method

#### OVERFLOW version 2.21

- 2<sup>nd</sup> order central differencing scheme
- Beam-Warming block tridiagonal scheme
- Low Mach preconditioner
- Steady state
- Spalart-Allmaras turbulence model with rotation/curvature correction

#### Best practices

- High lift workshop grid guideline
- Best practices for overset meshing

#### Warm start procedure

– Sequential restart at stall w/ smaller  $\Delta\alpha$ , achieve angle of attack resolution of  $0.25^{\circ}$ 

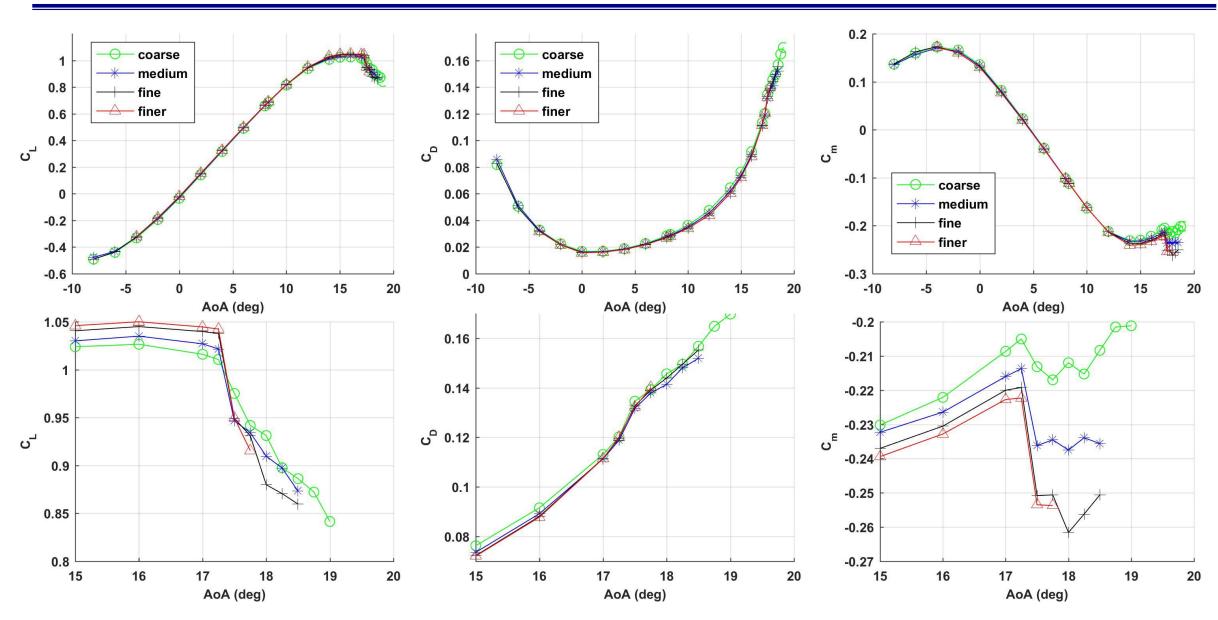


# Result – Grid Study

|         | Parameter                                    | Coarse   | Medium   | Fine     | Finer    |  |  |
|---------|--|----------|----------|----------|----------|--|--|
| Surface | Stretching ratio                             | 1.3      | 1.2      | 1.1      | 1.05     |  |  |
|         | Maximum spacing, in                          | 20       | 10       | 2.5      | 2.5      |  |  |
|         | Minimum spacing, in                          | 0.0157   | 0.00787  | 0.00197  | 0.00197  |  |  |
| Volume  | Stretching ratio                             | 1.3      | 1.2      | 1.1      | 1.05     |  |  |
|         | Marching distance, in                        | 10.0     |          |          |          |  |  |
|         | Initial spacing off of the wall, in          | 6.50E-04 | 1.90E-04 | 6.45E-05 | 3.23E-05 |  |  |
|         | Final spacing off in the near field grid, in | 1.0      | 0.5      | 0.33     | 0.25     |  |  |
|         | y+   | 1.0      | 0.3      | 0.1      | 0.05     |  |  |
|         | Level-1 spacing, in                          | 0.8      | 0.4      | 0.264    | 0.2      |  |  |
|         | MINBUF                                       | 4        | 4        | 6        | 8        |  |  |
| Tot     | tal number of grid points (millions)         | 4.48     | 21.9     | 79.6     | 190.3    |  |  |

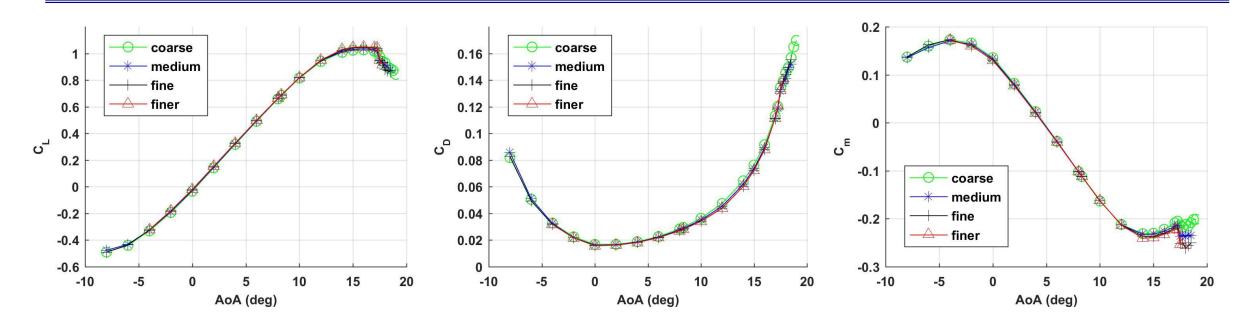


# Result – Grid Study





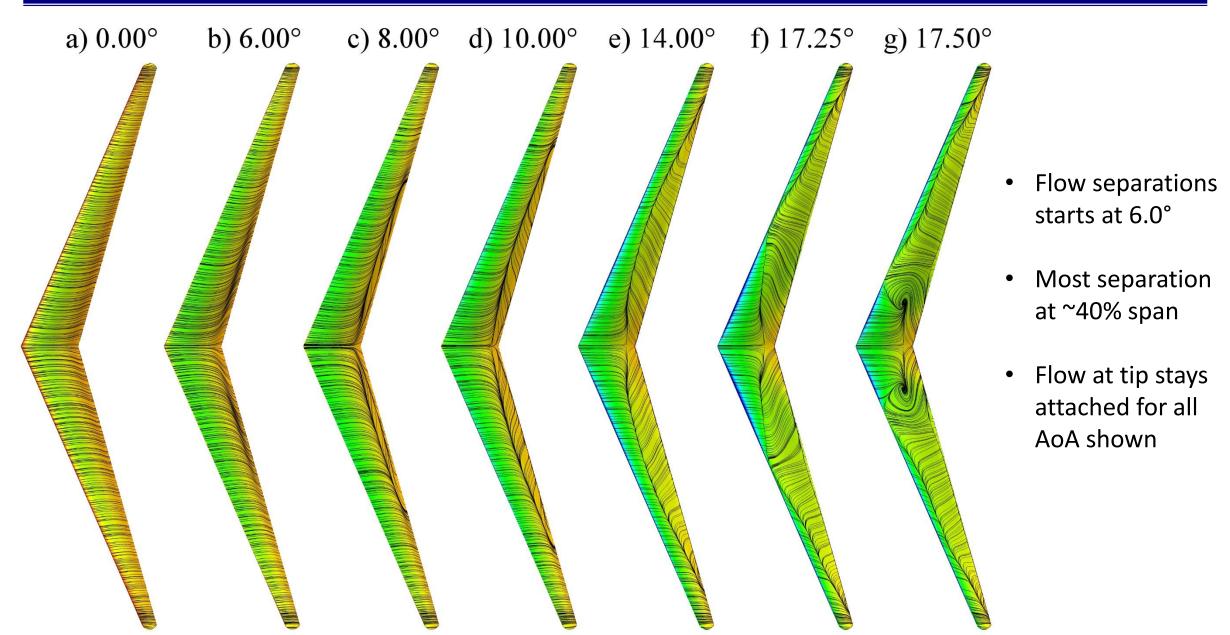
# Result – Grid Study



| Grid   | Stall Angle (deg) | C <sub>L_stall</sub> | C <sub>L_stall</sub><br>Error (%) | C <sub>L_max</sub> | C <sub>L_max</sub><br>Error (%) | C <sub>D_stall</sub> | C <sub>D_stall</sub><br>Error (%) | C <sub>m_stall</sub> | C <sub>m_stall</sub><br>Error (%) |
|--------|-------------------|----------------------|-----------------------------------|--------------------|---------------------------------|----------------------|-----------------------------------|----------------------|-----------------------------------|
| coarse | 17.25             | 1.0106               | -3.05                             | 1.0265             | 2.24                            | 0.12020              | 0.08                              | -0.2050              | -7.78                             |
| medium | 17.25             | 1.0216               | -1.99                             | 1.0350             | 1.43                            | 0.11885              | -1.05                             | -0.2137              | -3.87                             |
| fine   | 17.25             | 1.0378               | -0.44                             | 1.0450             | 0.48                            | 0.11968              | -0.36                             | -0.2192              | -1.39                             |
| finer  | 17.25             | 1.0424               |                                   | 1.0500             |                                 | 0.12011              |                                   | -0.2223              |                                   |

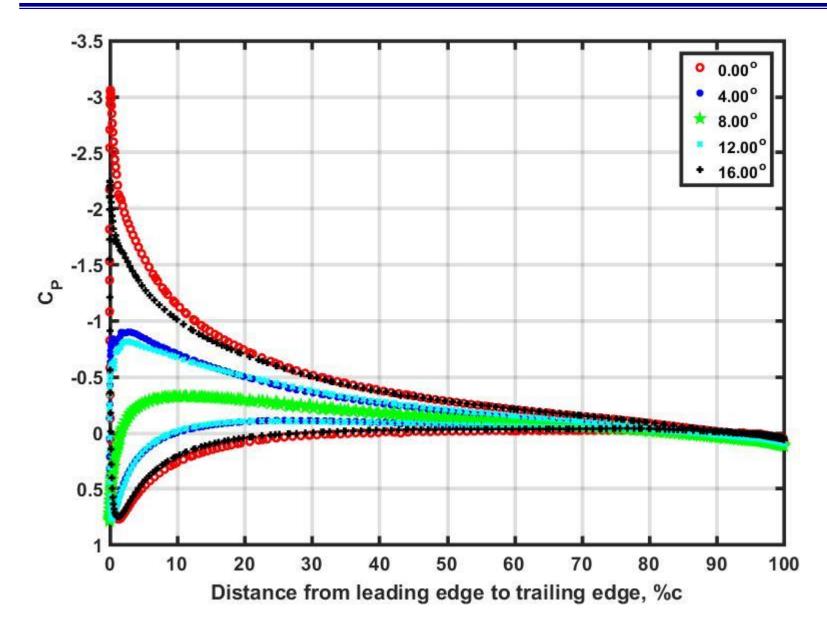


# Result – Pressure Contour, Upper Surface





# Result – Surface Pressure at tip



- Tip produces no lift at 8.0° AoA
- Lift varies almost linearly between 0.0° and 16.0°



#### Conclusion

- Wing designed based on Prandtl's minimum induced drag configuration simulated at high angle of attack
- Adequate grid resolution achieved
- C<sub>1</sub> break at 17.25°
- Large flow separation ~40% semi-span
- Flow at wing tip remains attached through the lift break



# Acknowledgement

- Albion Bowers
  - NASA-TP-2016-219072 On Wings of Minimum Induced Drag Spanload Implications for Aircraft and Birds



### QUESTION?