Computational Fluid Dynamics Analysis of the Stall Characteristics of a Wing Designed Based on Prandtl’s Minimum Induced Drag

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Outline

• Introduction/Background
• Method
• Results
• Conclusion
• Questions
Introduction

• Prandtl’s work on minimum induced drag
  • 1929 publication – elliptical spanwise lift distribution, constrain wing span
  • 1933 publication – bell shaped spanwise lift distribution, constrain bending moment
  • 11% less drag, 22% longer span compared to elliptical distribution for wings of identical weight

• Summary of Prandtl’s result

Lift (L): \( L = (1 - x^2)^{1.5} \)

Downwash angle (DW): \( DW = 1.5 \times (x^2 - 0.5) \)

Lift tapers to zero at wing tip: \( \lim_{x:0 \to b/2} L(x) = 0, \lim_{x:0 \to b/2} \frac{dL(x)}{dx} = 0 \)

Continuous down wash angle at wing tip: \( \lim_{x:0 \to b/2} \frac{dDW(x)}{dx} = \lim_{x:\infty \to b/2} \frac{dDW(x)}{dx} = 0 \)
Introduction

- Lift zero at wing tip
- Slope of lift zero at wing tip
- Downwash becomes upwash at 70.7% span
  - Inboard vortex, no wing tip vortex
  - Proverse yaw due to induced thrust at wing tip caused by upwash
Introduction

- Achieve bell shape loading via nonlinear spanwise twist distribution
- Wing tip is at approximately -10° twist relative to root chord
Introduction

- P-3C from the Preliminary Aerodynamic Design To Lower Drag (PRANDTL-D) program
- Span of 24.6 ft
- MAC of 1.969 ft
- Planform area of 40.5 ft²
- ~30 mph
Method

• OVERFLOW version 2.2l
  – 2nd order central differencing scheme
  – Beam-Warming block tridiagonal scheme
  – Low Mach preconditioner
  – Steady state
  – Spalart-Allmaras turbulence model with rotation/curvature correction

• Best practices
  – High lift workshop grid guideline
  – Best practices for overset meshing

• Warm start procedure
  – Sequential restart at stall w/ smaller $\Delta \alpha$, achieve angle of attack resolution of 0.25°
## Result – Grid Study

<table>
<thead>
<tr>
<th></th>
<th>Parameter</th>
<th>Coarse</th>
<th>Medium</th>
<th>Fine</th>
<th>Finer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Surface</strong></td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td>Stretching ratio</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.05</td>
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<td></td>
<td>Maximum spacing, in</td>
<td>20</td>
<td>10</td>
<td>2.5</td>
<td>2.5</td>
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<tr>
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<td>Minimum spacing, in</td>
<td>0.0157</td>
<td>0.00787</td>
<td>0.00197</td>
<td>0.00197</td>
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<tr>
<td><strong>Volume</strong></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Stretching ratio</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>Marching distance, in</td>
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<td></td>
<td>10.0</td>
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<td>Initial spacing off of the wall, in</td>
<td>6.50E-04</td>
<td>1.90E-04</td>
<td>6.45E-05</td>
<td>3.23E-05</td>
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<tr>
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<td>Final spacing off in the near field grid, in</td>
<td>1.0</td>
<td>0.5</td>
<td>0.33</td>
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<tr>
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<td>y+</td>
<td>1.0</td>
<td>0.3</td>
<td>0.1</td>
<td>0.05</td>
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<tr>
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<td>Level-1 spacing, in</td>
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<tr>
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<td>MINBUF</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>8</td>
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<tr>
<td><strong>Total number of grid points (millions)</strong></td>
<td>4.48</td>
<td>21.9</td>
<td>79.6</td>
<td>190.3</td>
<td></td>
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</table>
Result – Grid Study
### Result – Grid Study

<table>
<thead>
<tr>
<th>Grid</th>
<th>Stall Angle (deg)</th>
<th>$C_{L\text{, stall}}$</th>
<th>$C_{L\text{, stall Error}}$ (%)</th>
<th>$C_{L\text{, max}}$</th>
<th>$C_{L\text{, max Error}}$ (%)</th>
<th>$C_{D\text{, stall}}$</th>
<th>$C_{D\text{, stall Error}}$ (%)</th>
<th>$C_{m\text{, stall}}$</th>
<th>$C_{m\text{, stall Error}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coarse</td>
<td>17.25</td>
<td>1.0106</td>
<td>-3.05</td>
<td>1.0265</td>
<td>2.24</td>
<td>0.12020</td>
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<tr>
<td>medium</td>
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<td>finer</td>
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<td>0.12011</td>
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<td>-0.2223</td>
<td>--</td>
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</tbody>
</table>
Result – Pressure Contour, Upper Surface

- Flow separations start at 6.0°
- Most separation at ~40% span
- Flow at tip stays attached for all AoA shown
Result – Surface Pressure at tip

- Tip produces no lift at 8.0° AoA
- Lift varies almost linearly between 0.0° and 16.0°
Conclusion

• Wing designed based on Prandtl’s minimum induced drag configuration simulated at high angle of attack
• Adequate grid resolution achieved
• $C_L$ break at 17.25°
• Large flow separation ~40% semi-span
• Flow at wing tip remains attached through the lift break
Acknowledgement

• Albion Bowers
  – NASA-TP-2016-219072 - On Wings of Minimum Induced Drag - Spanload Implications for Aircraft and Birds