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Model of Collaborative Trajectory Options Program Performance

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Motivation

Recently, the Air Traffic Management community has made important progress in collaborative trajectory management through the introduction of an FAA traffic management initiative called a Collaborative Trajectory Options Program (CTOP) (Smith, 2014). CTOP allocates delay and reroutes around multiple FCA (Flow Constrained Area) -based airspace constraints in order to balance demand with available capacity. Similar to what is done with Airspace Flow Programs (AFPs), air traffic managers can create an FCA in a CTOP and control any air traffic that crosses that boundary by setting a flow rate for it. However, CTOP has the ability to manage multiple FCAs within a single program, permitting different parts of the program to be changed as conditions evolve. It also assigns delays or reroutes to flights in order to dynamically manage the capacity-demand imbalance as conditions change. For example, as conditions get better, CTOP can reroute traffic off of lengthy reroutes and back onto shorter routes, thereby decreasing their delays in the system.

A CTOP is also collaborative in that it permits airlines to provide a set of preferred reroute options (called a Trajectory Options Set or TOS) around an FCA. Whereas a traditional flight plan contains a single route, altitude and speed, a TOS contains multiple trajectory options [Figure 1] with each option containing a different route, altitude or speed. Furthermore, each trajectory option may contain the "start" and "end" times in which they are willing to accept for that particular option. These are described in the TVST and TVET columns in Figure 1. Airlines also specify a Relative Trajectory Cost (RTC) for each trajectory option that specifies cost of each route relative to the most preferred option. RTC is in terms of equivalent ground delay minutes. For example, figure 1 lists five different routes and associated RTC costs. Second route option would be preferred over the first route option if ground delay assigned to it is less than 25 minutes as compared to the ground delay assigned to the first route. CTOP assignment algorithm would add RTC

to assigned ground delay to calculate total cost for each route and then assign the route with the lowest cost to an aircraft.

Thus, CTOP permits better management of the overall trajectory of flights by considering both routing and departure delay options simultaneously. To benefit from CTOP, an airline will need to do some advance

RTC	RMNT	TVST	TVET	Route	ALT	SPEED
0				GLD SLN J24 MCI J24 STL	350	435
				J134 FLM J24 HVQ SHNON2		
25				GLD SLN J24 MCI J80 VHP	350	435
				APE AIR J162 MGW VERNI		
				ESL SHNON2		
35				PLAIN4 HCT J128 OBH J10 IOW	310	430
				BDF J64 WHETT J30 APE AIR		
				MGW MGW121 VERNI ESL		
				ROYIL2		
50		1945	2145	YELLO6 HANKI OBH J10 IOW BDF	350	425
				J64 WHETT J30 APE AIR MGW		
				MGW121 VERNI ESL ROYIL2		
65		2030	2200	YELLO6 HANKI ONL J148 MCW	310	430
				J16 BAE J34 AIR MGW		
				MGW121 VERNI ESL ROYIL2		

Figure 1.Trajectory Option Set

planning, on days when constraints are anticipated. Airlines do have the option to not participate in CTOP by just filing only their flight plan. In that case, filed plan will serve as a "single-option" TOS. Airlines will have to accept whatever the ground delay is assigned for this option and thus their chances of being assigned ground delay are higher. To participate with CTOP, airlines need to submit a set of route options their TOS, in advance of the flight.

Adoption of CTOPs in airspace has been hampered by a lack of willingness of a majority of airlines to participate in CTOP as there is a lack of information about benefits of CTOP. At present, only selected airlines are considering participating in CTOP. One open research question is how much benefit an airline gets by making a decision to participate in CTOP. Another question is identifying situations in which CTOP is a better alternative to traditional TMIs. For effective use of CTOP, it would be useful to understand how different factors such as capacity and TOS participation influence CTOP performance. Therefore, it would be helpful to develop a model of CTOP performance in terms of these factors.

This report is organized as follows. Section 1 is the introduction. Section 2 discusses our overall approach. Section 3 discusses how different factors influence CTOP performance. Section 4 describes theoretical analysis. Next section describes a model developed from simulation data. A sixth section describes examples of analysis using CTOP performance models. Finally, the seventh section is a conclusion.

I. Approach

As CTOP has only been used in a few tests, there is limited data from actual CTOP use. However, it is possible to use theoretical analysis and simulations to study CTOP performance. Main benefit of theoretical analysis is that it would be relevant to CTOP use in a wide range of operational conditions whereas generality of conclusions drawn from simulation of specific scenarios is not always clear. On the other hand, we will necessarily be making many simplifying assumptions to reduce the complexity of theoretical analysis. These assumptions would introduce a certain amount of error in the analysis result. We are able to run simulations with more realistic scenarios that include changing demand on airspace as well as changing relative trajectory cost distributions. However, simulations make certain assumptions as well and can vary from real system behavior in some ways. Overall approach in this report is to begin by doing exploratory analysis identifying factors that influence CTOP performance and then use both theoretical

approach and simulation data to create models of CTOP performance. We also use simulation results to assess the degree of error in theoretical analysis.

II. Factors Influencing CTOP Performance

In this section, we used two separate tools to examine influence of a number of factors on CTOP performance.data. nCTOP (NASA CTOP) is the NASA simulation of CTOP assignment algorithm. (Smith, 2016) Another tool, the Multi-Aircraft Control System (MACS) which is a high-fidelity air traffic control simulation environment for prototyping scheduling systems and simulating air traffic, was used to simulate the air traffic. A specific scenario is used based on actual traffic data of aircraft arriving to EWR airport on July 14, 2015. Simulated CTOP has three FCAs - a constrained FCA on the west flow and two unconstrained FCAs on north and south flows. Factors studied are capacity, TOS participation, demand, relative trajectory costs and CTOP duration.





Figure 2. Delays in Different Capacity Deficit Scenarios

For this study, we define capacity deficit to be (demand - capacity)/ demand * 100. Figure 2 shows simulation results under different capacity deficit conditions. With non-zero capacity deficit, ground delays decrease as percent TOS participation increases. However, once percent TOS participation is large enough to make up capacity demand imbalance, further increases in percent TOS participation results in relatively small decrease in average delays. If the percent of TOS participation is smaller than a critical value and

results in unacceptable level of delays, CTOP may not be a desirable traffic management initiative as compared to alternate options traffic managers can choose to use.



Figure 3. Delays vs Capacity Demand Imbalance

When capacity is higher than demand, flights filing TOSs would route out when there are large enough delays. Demand remaining after TOS filing flights have routed out is (demand – number of tos filers). Here, we define demand capacity Imbalance to be ((demand - number of tos filers) - capacity) when (demand – number of tos filers) > capacity. Figure 3 shows a plot of maximum delays observed in simulation data as a function of demand capacity imbalance. As expected, delays increase as imbalance increases. In this data, maximum delay correlates well with imbalance and has a linear fit with r = .96.

Impact of Relative Trajectory Cost

Consider a flight that has filed a TOS with two trajectories. Its most preferred route is going through a constrained FCA and second most preferred route is going through a non-constrained FCA. If rtc cost associated with the route going through a non-constrained FCA is r, this flight would take its second most preferred route whenever ground delay assigned to the most preferred route is higher than r. We define average RTC for a set of flights to be the average of rtc associated with second most preferred route in TOSs for the flights. To study how RTC influences the performance, we compared differences in costs and delays in two simulations. Both simulations are run with air traffic data of aircraft flying to EWR on 7/14/2015. CTOP consists of three FCAs controlling north, west and south flows to EWR. The North and South flows, controlled by the arrival meter fix FCAs at the SHAFF and DYLIN fixes, are set sufficiently high to be effectively unconstrained (7 and 6 aircraft per quarter hour, respectively). The FCA flow rates for the West flow, controlled by the arrival meter fix FCA at the PENNS fix is set at 3 aircraft per hour. In the first simulation scenario, 11 aircraft belonging to UAL, SW and AA with average RTC of 22.5 minutes submit TOSs. In the second simulation scenario, 11 aircraft belonging to regional airlines with average RTC of 9 minutes submit TOSs. The reason for differences in RTC between the two groups is that regional airlines fly shorter routes where flight time difference between two most preferred routes is smaller.

Table 1 shows associated system performance at the end of simulation. When CTOP allocates new routes to flights with excessive ground delays, these re-routed flights have reduced ground delays but longer flight times and higher RTC cost. Therefore, use of CTOP results in reduction in ground delays and increase in RTC costs owing to increased flight times of the flights that get rerouted. Net cost saving for a flight from use of a CTOP allocated route can be calculated by considering both ground delays and RTC for these flights. In the first simulation, only 4 aircraft got routed out of constrained FCA as a result of CTOP whereas 9 aircraft got routed out in the second simulation. Total ground delays assigned to different aircraft in the first simulation is 1165 minutes whereas that in the second simulation is 1061 minutes. The reason we have a smaller amount of ground delays in the second simulation is that there are a higher number of aircraft taking alternative routes in the second simulation. Another factor studied was the additional flight time flown by rerouting flights. This was smaller in the second simulation because TOS filing flights in this case had shorter flights and smaller RTCs. Combined effect of both ground delays and flight times was captured in net cost savings. In the first simulation, net cost savings from the use of CTOP is 62 minutes whereas it is 261 minutes in the case of second simulation. In summary, when we have a smaller relative trajectory cost (RTC) associated with aircraft filing TOSs, number of aircraft taking an alternative to a route through congested FCA is larger and ground delay savings as well as net cost savings are higher.

TOS submitters	Av RTC difference for rerouting flights (min)	Total ground delays (min)	Number of aircraft reroutes	Flight Time difference for rerouting flights (min)	Ground delay savings (min)	Net cost saving for all flights (min)
AA, UAL, SW (11acft)	22.5	1165	4	59	151	62
UPS, ASQ (11 acft)	9	1061	7	41	323	261

Table 1 Impact of RTC Cost

Impact of CTOP Duration

In scenarios where there is capacity demand imbalance even after TOS filing aircraft route out, length of aircraft queue increases over time and associated ground delays increase as well. Thus, in this case, even though CTOP reduces delays compared to the situation when there is no CTOP, we can still have significant amount of ground delays that increase over time. Figure 4 shows relation of average ground delay and CTOP duration in a scenario with capacity demand imbalance. As duration increases, average ground delays seems to increase proportionately.





III. Theoretical Model

In the previous section, we identified a number of factors that influence system performance when CTOP is used. We will now use theoretical analysis to create a model relating these factors and system performance. The analysis done here is based on the following scenario. A CTOP has a constrained FCA C and another unconstrained FCA UC When flights that have their most preferred route going through C have large enough ground delay assigned, these would prefer to take alternate route through UC. Thus, UC allows flights in constrained FCA to route out. Capacity is *c* aircraft per hour for the constrained FCA. Demand is d aircraft per hour through the constrained FCA with evenly spaced flows. Of these, there are *tf* aircraft per hour filing TOSs. Let *max-rtc* be the maximum rtc associated with any of the alternative trajectories. We also define dtf as demand of aircraft per hour when all TOS filing aircraft are assigned high delays to routes through constrained FCA and choose an alternate route. Therefore, dtf = d - tf. We will now analyze two different cases (1) dtf > c Capacity demand imbalance remain even after TOS filing aircraft reroute.

Case 1: Capacity demand imbalance after rerouting

Initial Period: As dtf > c, there will be demand capacity imbalance even if all TOS filers route out. Therefore, ground delays assigned to aircraft would keep increasing over time. During the initial period when assigned ground delays are less than *max-rtc*, not all TOS filers would route out as some do not have an alternative TOS option with rtc < assigned ground delay. Let *qi* be the queue formed in the period when assigned delays are less than *max-rtc*

Post-initial Period: After assigned ground delays are >= max-rtc, all TOS filing flights (*tf* per hour) route out and demand would reduce to dtf = (d - tf) per hour. As this demand is higher than capacity c, aircraft queue length would increase at the rate of (dtf - c) aircraft/hour. Therefore, queue length after n hours in post-initial period would be q = (dtf - c) * n + qi.

As capacity is *c* aircraft per hour, time period allowed between successive aircraft is 1/c hours when scheduling policy is to attempt to space aircraft evenly. Maximum assigned delay would be that assigned to the last aircraft in the queue. Thus, maximum delay would be ((dtf - c) * n + qi)/c hours. If we ignore the initial period as insignificant, maximum delay is approximately ((dtf - c) * n)/c hours. Average delay

for aircraft going through constrained area would be a = ((dtf - c) * n) / 2 c hours. We will refer to this as equation (1).

As capacity is c aircraft per hour, number of aircraft going through the constrained area would be c aircraft per hour. *tf* flights would be re-routed per hour. Thus, throughput of flights that were originally planning to go through west gate is (c + tf) aircraft per hour.

Flights routing out incur cost corresponding to its specified *rtc* value. Average *rtc* cost for these aircraft would correspond to $b = av_{tos filing flights}$ (*rtc*) If assigned *rtc* is k * fltdiff, this cost would be $k^* av_{tos filing flights}$ (*fltdiff*) where *fltdiff* is the flight time difference between two alternative route options for each tos filing flight.

Case 2: Demand capacity imbalance resolved with re-routing flights

Case: $d = c + k^* tf (0 \le k \le 1)$

Initially, assigned ground delays increase and percent of tos filers routing out increase as well.

Let *k*-th percentile of *rtc* values be *rtc-k*. This does not mean that exactly k% of flights for each time period have rtc less than rtc-k. However, to simplify analysis, we assume that *k*% of aircraft during the time period of interest have rtc values less than *rtc-k*.

If all aircraft are assumed to be assigned ground delay of *rtc-k*, we will have k^*tf aircraft routing out. As d - k * tf = c, there will be capacity demand balance and ground delays would remain stable and would equal *rtc-k*.

As a group, *rtc* values for flights routing out would be at the most *rtc-k* but can vary from 0 to *rtc-k* Average *rtc* for these flights would depend on the exact distribution of *rtc* values. For example, if the distribution is uniform, average rtc would be *rtc-k* / 2. In general, individual airlines would have different distribution of *rtc* values. Correspondingly, different airlines would have different percent of flights routing out and different average *rtc* for routing out flights belonging to the airline.

In this section, we created models of system performance using a set of simplifying assumptions. In the next section, we will create models using simulation data.

IV. Model Derived from Simulation Data

While theoretical analysis lends to models that are general, it makes a number of simplifying assumptions. There are several ways in which CTOP simulation is more realistic as compared to our theoretical analysis. In the simulation, traffic is not evenly spaced. That makes capacity demand imbalance change continuously. Also, distribution of flight time differences and RTC values in simulation does not remain constant from hour to hour. Furthermore, percent TOS participation may also vary over time.

Simulation data was created with multiple settings of capacities, duration and percent tos filers. Following model fits the data with adjusted r-squared of .89:

Equation 2: gd * c = 29386 * h - 788 * h * tosfilers -384 * h * c + 8618

where

h = time in hours from start of run.

gd = assigned ground delay (seconds) for aircraft going through FCA at time h

c = west flow capacity/ hour

Tosfilers = number of tosfilers

As equation (2) has the same form as equation (1) and it fits data with a high r-squared value, we can conclude that the form of theoretically expected model described in equation (1) is consistent with simulation data. Substituting demand values in equation (1) would give us following equation.

Equation 3: gd * c = ((d - tf - c) * n) / 2 hr = 36000 * h - 1800* tf * h - 1800* h * c

Differences between equations (2) and (3) show the extent of error introduced by simplifications in the coefficients of our theoretical model.

To illustrate how delay predicted by simulation and theoretical approaches compare, we will examine the impact of partial TOS participation in CTOP.



Figure 5. Simulation vs Theoretical Predictions

Figure 5 shows results from theoretical analysis and nCTOP simulation for a 4 hour scenario. West flow is constrained whereas other flows are not constrained. For the west flow, average demand is 19 aircraft per hour and capacity is 12 aircraft per hour. We run nCTOP several times varying number of TOS filers. As the number of TOS filers have been increase from 0 aircraft/hour to 6 aircraft/hour, delays drop sharply,

but these drop little after TOS filer rate increases above 6 non-exempt aircraft/hr. Route-out RTC cost is mostly constant. We also see a reasonably good match between theoretical prediction and nCTOP simulation. This shows that our theoretical model has acceptable accuracy for predicting trends in the data as well as absolute delays. It shows that errors in predictions are much smaller than errors in coefficients.

V. Examples of Analysis Using CTOP Performance Models

Benefits of filing TOSs

By comparing the blue and orange curves in Figure 5, we can observe that the benefit TOS filers have relative to non-TOS filers in the specific simulation scenario. This benefit is more than 60 minutes in situations where percent of TOS filers is small. This relative benefit decreases as percent of TOS filers increase. After number of TOS filers increase to 6 aircraft per hour, TOS filers as a group would have much smaller benefit over those not filing TOSs. Thus, as percent TOS participation increases, overall system performance improves while relative benefit to TOS filers declines. Models we have described allow us to understand the relative benefit of filing TOSs in a wider set of situations. In the situation where demand capacity is not resolved by aircraft routing out, average benefit to TOS filing is given by a –b where a = ((d - c - tf) * n)/2 c hours and b = av tos filing flights (*rtc*). In the situation where demand capacity is resolved by aircraft routing is given by a –b where a = rtc-k hours and b = av tos filing flights (*rtc*).

Impact of uncertainties in RTC values

Airlines regard creating accurate relative cost of trajectories to be challenging because of uncertainties about factors impacting RTC and about business models relating these factors to RTCs. RTC may depend on other factors in addition to airborne delay and ground delay. RTC associated with a flight TOS is created based on expected flight times and delays. Differences between expected and actual flight times and delays introduce errors in RTC values. The error in relative cost of ground and flight time delay can be as high as 50% given common assumption of ratio of flight time delay cost and ground delay e.g. If missed connections are likely after a certain period, it would increase relative trajectory cost. Thus, requirement that RTC be a fixed number may introduce error. Airlines do not have a model of how some of these factors impact their business. Some airlines may lack software needed to consider complex factors in setting RTC values because of costs associated. Given such factors, it is reasonable to assume that RTC associated with a flight TOS would have errors. One question is the impact of such errors have on operations.

Following theoretical analysis of the scenario where demand capacity imbalance remains after TOS filing flights route out, average ground delays associated with non-TOS filing flights would not depend on filed RTC values whereas RTC cost associated with the flights routing out would be average of actual RTCs associated with alternative trajectories. Therefore, delay cost of flights that route out would be sensitive to changes in RTC values whereas ground delay of non-TOS filing flights would not be sensitive. Also, specifying incorrect RTC would not impact delays.

In the scenario where demand capacity balance is achieved after some TOS filing flights route out, ground delays for flights not routing out correspond to k-th percentile of filed RTC values where k reflects the demand capacity imbalance as defined earlier. Also, only the flights with RTC values below this threshold would route out. In the cases where RTC errors do not affect either this threshold or whether RTC associated with a trajectory is lower or higher relative to the threshold, errors would not have any impact of CTOP performance. On the other hand, if RTC errors do change the threshold, it would affect average ground delays of flights that do not route out and it can also influence which flights route out.

Comparison of CTOP with alternative TMIs

Models we have developed can also be used to compare performance of CTOP with alternative TMIs.

For example, required reroutes is an alternative TMI that could be considered in situations with constrained airspace regions. With required reroute directive from FAA, flights are required to take an alternative route. Even though airline may not specify a RTC with the new trajectory, one could calculate the RTC and use it in the analysis. Flights routing out incur cost corresponding to their *rtc* values. Average *rtc* cost for these aircraft would correspond to r = av tos filing flights (*rtc*) If assigned *rtc* is k * fltdiff, this cost would be $k^* av_{tos filing flights}$ (*fltdiff*) where *fltdiff* is the flight time difference between two alternative route options for each tos filing flight. This cost can then be compared with average ground delay cost with CTOPs. Comparison shows that required reroutes would result in reduced delays in situations where there are not enough TOS filing flights.

VI. Conclusion

Adoption of CTOPs in airspace has been hampered by a lack of willingness of a majority of airlines to participate in CTOP as there is a lack of information about benefits of CTOP. At present, there are only selected airlines that are considering participating in CTOP. One open research question is how much benefit an airline get by making a decision to participate in CTOP. Another question is identifying situations in which CTOP is a better alternative to traditional TMIs. For effective use of CTOP, it would be useful to understand how different factors such as capacity and TOS participation influence CTOP performance. Therefore, it would be helpful to develop a model of CTOP performance in terms of these factors.

In this study, we developed models of CTOP performance using theoretical analysis and simulations.. We found a good match between theoretical models and simulations results. Theoretical model is applicable for a wider set of capacity reduction and demand scenarios than simulations and allows answering queries in the context of these. Models were used to identify minimum TOS participation that would needed for acceptable performance of CTOP. We also examined how factors such as CTOP duration and relative trajectory costs impact CTOP performance.

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