Green's Function Applicable to Turbofan Exhaust Noise in Jets with an External Center-Body

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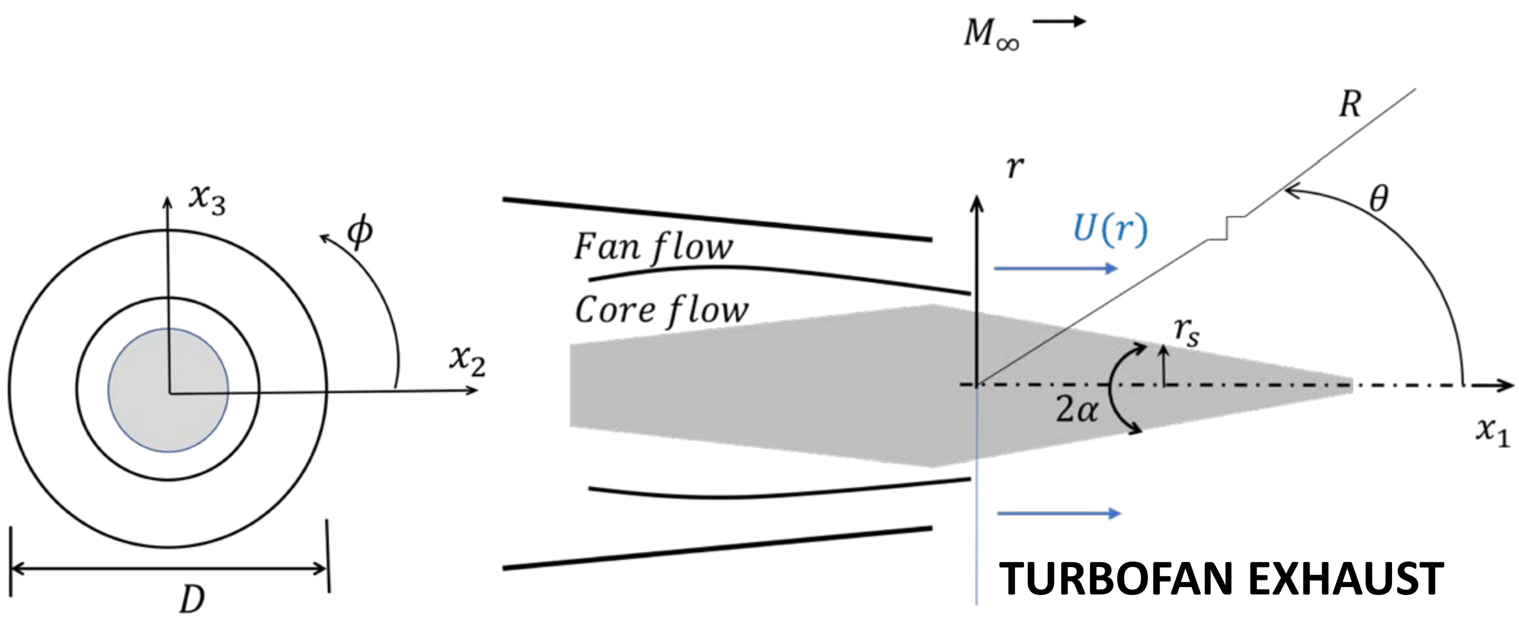
OBJECTIVE

A Numerically Evaluated Propagator Applicable to Engine Exhaust Noise (Fan & Jet):

- External Center-Body (CB)
- Surface Treatment on the CB
- Flight Effect
- Slip vs. No-Slip Condition on the CB Surface



TURBOFAN EXHAUST

OUTLINE

- Locally Parallel Exhaust Flow (Compressible Rayleigh Eq.)
- Far-Field GF $(R \gg \lambda)$
- Two Forms of the Exact Solution
- Point Source GF
- Ring Source GF

GOVERNING EQS.

- **Compressible Rayleigh Eq.**

$$L\pi' = \Gamma, \qquad \pi' \cong \frac{p'(\vec{x},\ t)}{\gamma\bar{p}}$$

$$L \equiv D\left(D^2 - \frac{\partial}{\partial x_j}\left(c^2\frac{\partial}{\partial x_j}\right)\right) + 2c^2\frac{\partial U}{\partial x_j}\frac{\partial^2}{\partial x_1 \partial x_j}\,, \qquad D \equiv \frac{\partial}{\partial t} + U\frac{\partial}{\partial x_1}\,.$$

$$LG(\vec{x},t|\vec{y},\tau) = \delta(\vec{x}-\vec{y})\delta(t-\tau)$$

$$\pi'(\vec{x},t) = \int_{\vec{y}}\int_{\tau} G(\vec{x},t|\vec{y},\tau)\,\Gamma(\vec{y},\tau)d\tau d\vec{y}$$

- **Locally parallel mean flow** $\qquad \vec{x}_T = (x_2, x_3), \qquad U = U(\vec{x}_T), c = c(\vec{x}_T)$

- **Assumes local inner condition to be applicable throughout x_1 axis.**

$$\hat{G}(k_1,\vec{x}_T|\,\vec{y}_T,\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x},t|\vec{y},\tau)e^{-ik_1(x_1-y_1)+i\omega(t-\tau)}\,d(x_1-y_1)d(t-\tau)$$

GOVERNING EQS. (cont'd)

- **Define new GF** $\breve{G}$

$$\hat{G}(k_1, \vec{x}_T | \vec{y}_T, \omega) = \frac{i}{(2\pi)^2} \frac{1}{c(\vec{x}_T)c(\vec{y}_T)} \frac{-\omega + k_1 U(\vec{x}_T)}{\left(-\omega + k_1 U(\vec{y}_T)\right)^2} \breve{G}(k_1, \vec{x}_T | \vec{y}_T, \omega)$$

$$\left(\nabla_T^2 + f(k_1, \vec{x}_T, \omega)\right) \breve{G}(k_1, \vec{x}_T | \vec{y}_T, \omega) = \delta(\vec{x}_T - \vec{y}_T)$$

$$\breve{G}(k_1, \vec{x}_T | \vec{y}_T, \omega) = \breve{G}(k_1, \vec{y}_T | \vec{x}_T, \omega)$$ (Self-Adjoint)

- **Function $f(k_1, \vec{x}_T, \omega)$ depends on the span-wise gradients of the mean velocity & temperature**

- **Axisymmetric Jets**

$$\breve{G}(k_1, \vec{x}_T | \vec{y}_T, \omega) = \sum_{n=-\infty}^{+\infty} \breve{G}_n(k_1, r | r_o, \omega)\, e^{in(\phi - \phi_o)}$$

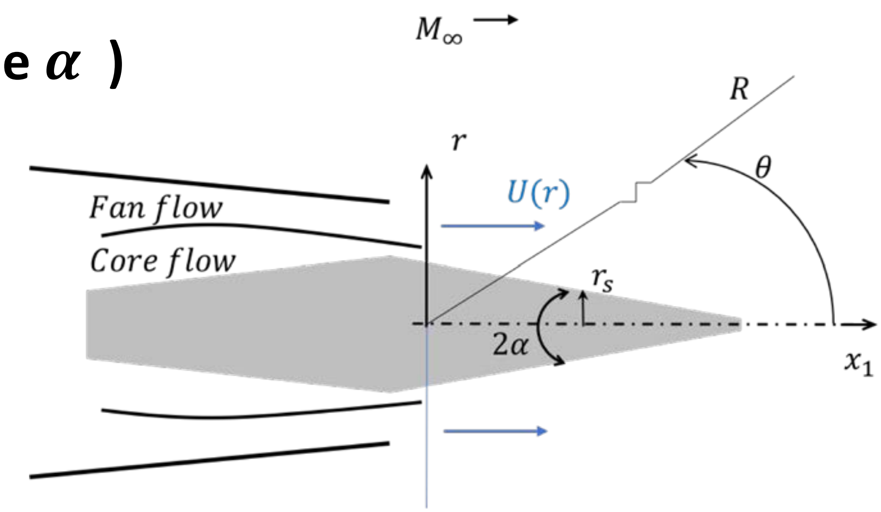
GOVERNING EQS. (AXISYMMETRIC EXHAUST)

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + f(k_1, r, \omega) - \frac{n^2}{r^2}\right)\breve{G}_n(k_1, r|r_o, \omega) = \frac{1}{2\pi r}\delta(r - r_o), \quad n = -\infty, \cdots, +\infty$$

$$\breve{G}_n(k_1, r|r_o, \omega) = \frac{1}{2\pi r_o}\frac{V_n^{(2)}(r)V_n^{(1)}(r_o)}{W\left(V_n^{(1)}, V_n^{(2)}\right)_{r=r_o}}, \qquad r > r_o$$

- **Boundary conditions (conical CB with angle α)**

$U = 0,\ r \to r_s$ And $\hat{p}' = -Z(\omega)\hat{v}'_\perp$



$$\begin{cases} V_n^{(1)}(k_1, r, \omega) = V_n^{(2)} = 1, & r = r_s \\ \frac{\partial}{\partial r}V_n^{(1)} - \psi V_n^{(1)} = 0, & r = r_s \\ V_n^{(2)} = b_n(k_1, \omega)H_n^{(1)}(r\chi_\infty), & r \geq r_2 \end{cases}$$

$$\psi(k_1, r, \omega, \bar{Z}) = \frac{-i}{\cos\alpha}\left(k_1 \sin\alpha + \frac{\kappa_o}{\bar{Z}}\frac{c_\infty^2}{c^2}\right) + \frac{c^{2'}}{2c^2} + \frac{k_1}{\kappa_o}M'(r),$$

$$M(r) = \frac{U(r)}{c_\infty}, \qquad \kappa_o = \frac{\omega}{c_\infty}$$

- **Centerline conditions (no external CB)**

$V_n^{(1)}(k_1, r, \omega) \to r^n$ as $r \to 0$

GOVERNING EQs. (cont'd)

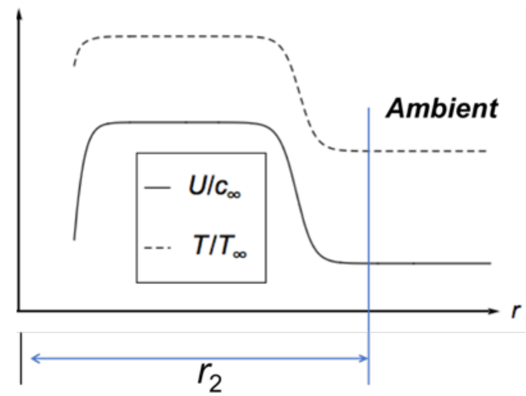
- **Green's Function - Stationary Phase solution** $(R \gg \lambda)$

$$G(\vec{x},\vec{y};\omega)\sim\frac{-1}{4\pi^3}\frac{1}{c_\infty^2\, c(r_o)}\frac{e^{i\kappa_o R\Psi}}{\kappa_o R}\frac{(1-M_\infty\cos\theta^s)}{(1-M(r_o)\cos\theta^s)^2}\frac{1}{\sqrt{1-M_\infty^2\sin^2\theta}}$$
$$\times\sum_{n=0}^{\infty}\varepsilon_n e^{-i\frac{n\pi}{2}}\cos(n\phi)\,V_n^{(1)}(k_1^*,r_o,\omega)\Lambda(V_n^{(1)},H_n^{(1)})\Big|_{k_1=k_1^*}$$

- **Factor Λ is evaluated at the outer jet boundary** $(r \geq r_2)$

$$\Lambda\left(V_n^{(1)},H_n^{(1)}\right)=\frac{1/r}{V_n^{(1)}(k_1,r,\omega)\dfrac{\partial H_n^{(1)}(r\chi_\infty)}{\partial r}-\dfrac{\partial V_n^{(1)}(k_1,r,\omega)}{\partial r}H_n^{(1)}(r\chi_\infty)}$$

$$\chi_\infty^2=(-\kappa_o+k_1M_\infty)^2-k_1^2>0$$



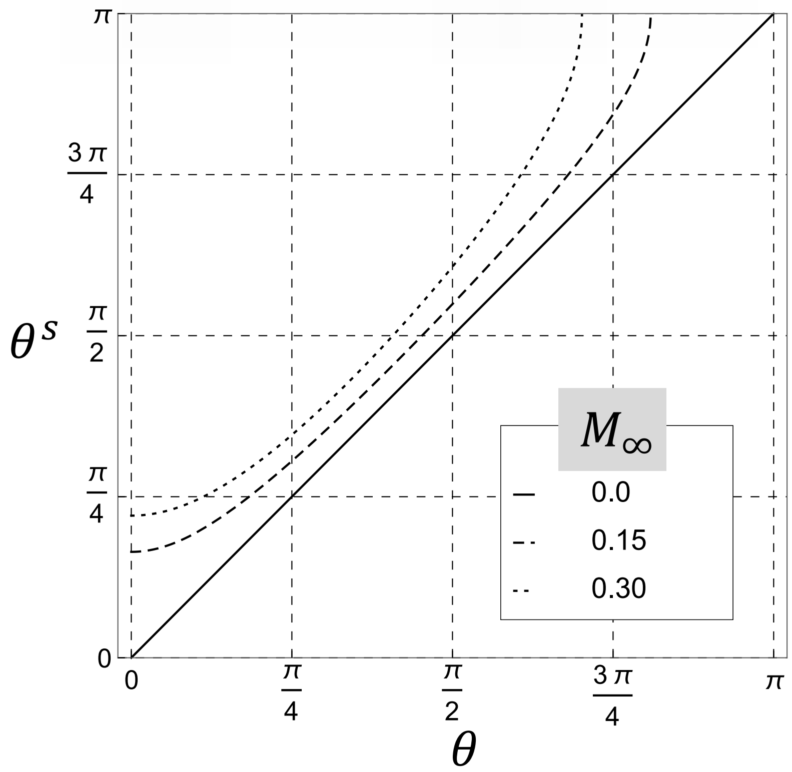
- **Phase factor Ψ in flight**

$$\Psi=\frac{1}{(1-M_\infty^2)}\left(-M_\infty\cos\theta+\sqrt{1-M_\infty^2\sin^2\theta}\right)$$

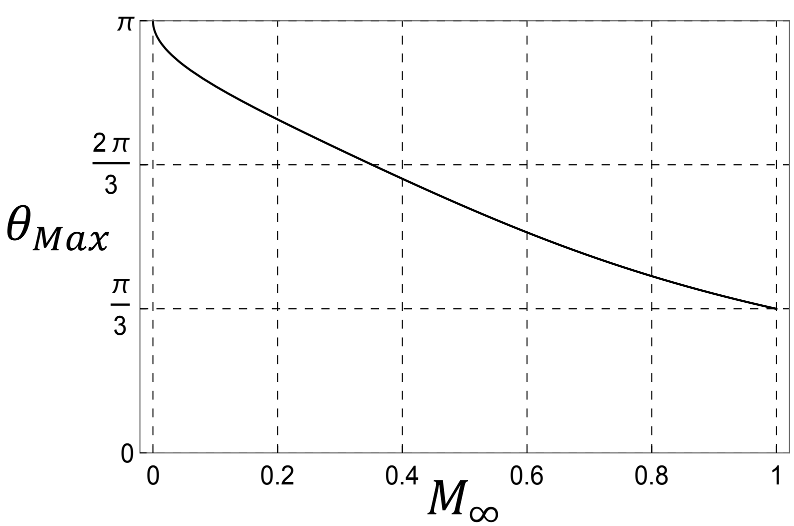
- **Radiation angle θ^s vs. observer angle θ**

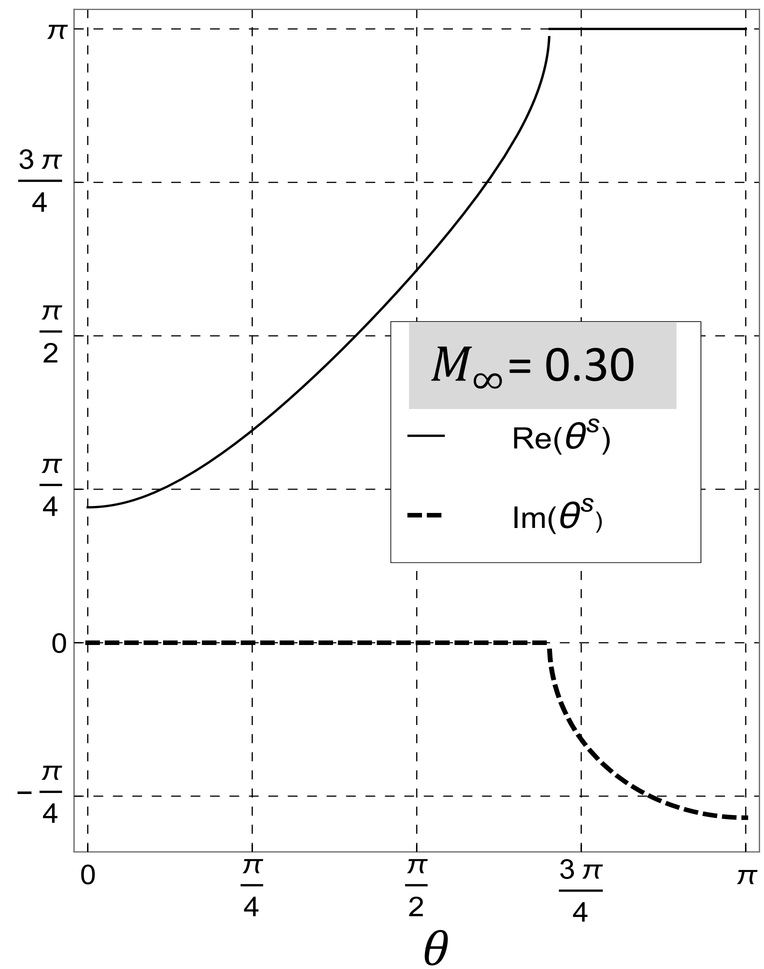
$$\frac{k_1^*}{\kappa_o}=\cos\theta^s=\frac{1}{1-M_\infty^2}\left(-M_\infty+\frac{\cos\theta}{\sqrt{1-M_\infty^2\sin^2\theta}}\right)$$

Radiation Angle θ^s vs. Observer Angle θ



- θ_{Max} : Maximum observer angle at which θ^s is a real number





Green's Function to Pridmore-Brown Eq.

- **Homogeneous PB Eq.** $V_n^{(j)}(k_1,r,\omega) = \dfrac{c(r)}{-\omega + U(r)k_1} g_n^{(j)}(k_1,r,\omega),$

$$\left(\frac{d^2}{dr^2} + \left(\frac{1}{r} + \frac{{c^2}'}{c^2} - \frac{2k_1 U'}{-\omega+Uk_1}\right)\frac{d}{dr} + \frac{(-\omega+Uk_1)^2}{c^2} - k_1^2 - \frac{n^2}{r^2}\right) g_n^{(j)}(k_1,r,\omega) = 0$$

- **Surface BC**

$$\begin{cases} g_n^{(1)}(k_1,r_o,\omega) = 1, \\ \dfrac{\partial}{\partial r} g_n^{(1)} - \psi_1 g_n^{(1)} = 0, \end{cases} \qquad r = r_s$$

$$\psi_1(k_1,r,\omega,\bar{Z}) = \frac{-i}{\cos\alpha}\left(k_1 \sin\alpha + \frac{\kappa_o}{\bar{Z}}\frac{c_\infty^2}{c^2}\right) \qquad \text{(no-slip boundary)}$$

- **Two forms of the Green's Function**

$$G(\vec{x},\vec{y};\omega) \sim \frac{-1}{4\pi^3}\frac{1}{c_\infty^3}\frac{e^{i\kappa_o R\Psi}}{\kappa_o R}\frac{(1-M_\infty\cos\theta^s)^2}{(1-M(r_o)\cos\theta^s)^3}\frac{1}{\sqrt{1-M_\infty^2\sin^2\theta}}$$
$$\times \sum_{n=0}^{\infty}\varepsilon_n e^{-i\frac{n\pi}{2}}\cos(n\phi) g_n^{(1)}(k_1^*,r_o,\omega)\,\Lambda(g_n^{(1)},H_n^{(1)})$$

$$G(\vec{x},\vec{y};\omega) \sim \frac{-1}{4\pi^3}\frac{1}{c_\infty^2\, c(r_o)}\frac{e^{i\kappa_o R\Psi}}{\kappa_o R}\frac{(1-M_\infty\cos\theta^s)}{(1-M(r_o)\cos\theta^s)^2}\frac{1}{\sqrt{1-M_\infty^2\sin^2\theta}}$$
$$\times \sum_{n=0}^{\infty}\varepsilon_n e^{-i\frac{n\pi}{2}}\cos(n\phi) V_n^{(1)}(k_1^*,r_o,\omega)\Lambda(V_n^{(1)},H_n^{(1)})$$

NUMERICAL RESULTS

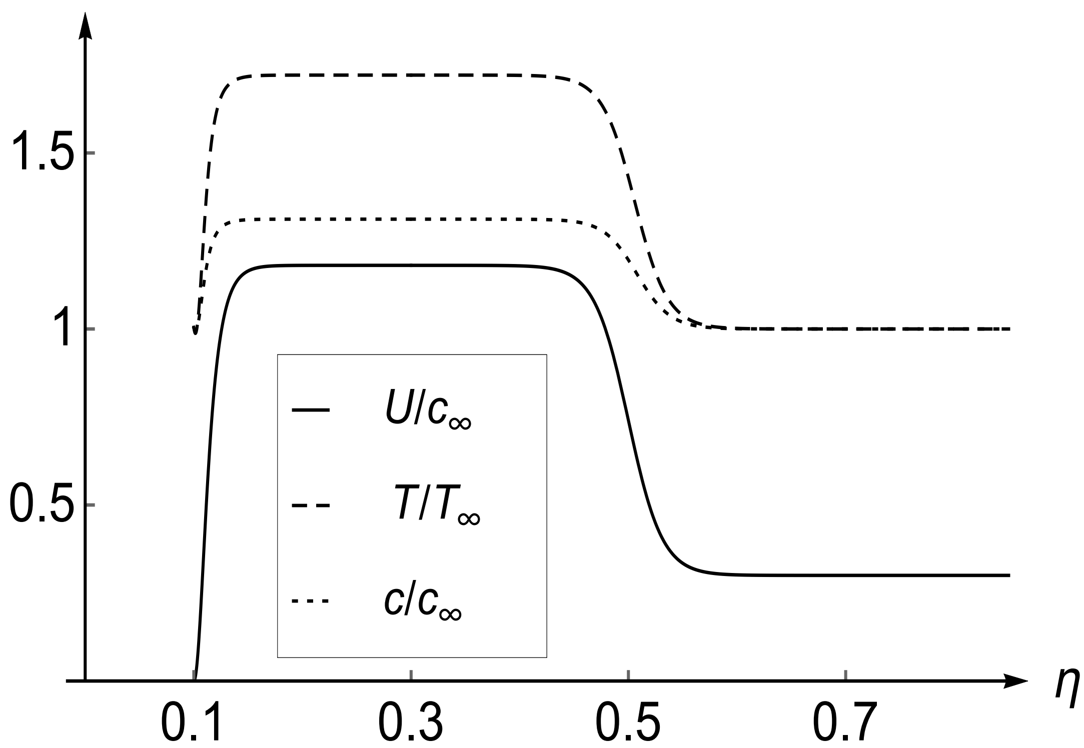
- **Mean velocity profile** $\eta \equiv r/D$

$$\frac{U(\eta)}{U_j} = \begin{cases} 1 - d_o \,\mathrm{sech}\left(\frac{\eta - \eta_s}{d_1}\right), & \eta_s \le \eta < 2\eta_s + \delta_w \\ \frac{1}{2}\left(1 + \frac{U_\infty}{U_j}\right) - \frac{1}{2}\left(1 - \frac{U_\infty}{U_j}\right)\tanh\big(d_2(\eta - 0.5)\big). & \eta \ge 2\eta_s + \delta_w \end{cases}$$

- **Mean temperature profile** $T(\eta) = T_1(\eta) + T_2(\eta)$

T_1 : Crocco-Busemann law

T_2 : Adjustment near the wall $\frac{T_2(\eta)}{T_\infty} = \frac{1}{2} - \frac{1}{2}\tanh\big(d_3(\eta - d_4)\big)$



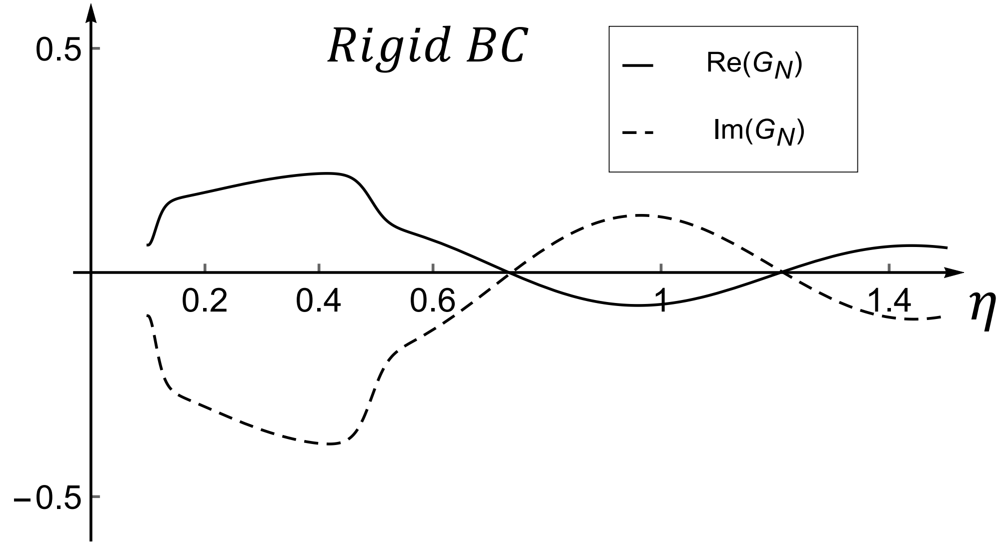
$M_j = 0.90$
$T_R = 2$
$U_j/c_\infty = 1.18$
$M_\infty = 0.30$
$\eta_s = 0.10$
$\alpha = \pi/6$

NUMERICAL RESULTS

$$G_N(\vec{x},\vec{y};\omega) \equiv G(\vec{x},\vec{y};\omega) \Big/ \left(\frac{1}{\pi^2 c_\infty^3}\frac{e^{i\kappa_o R\Psi}}{4\pi R/D}\right)$$

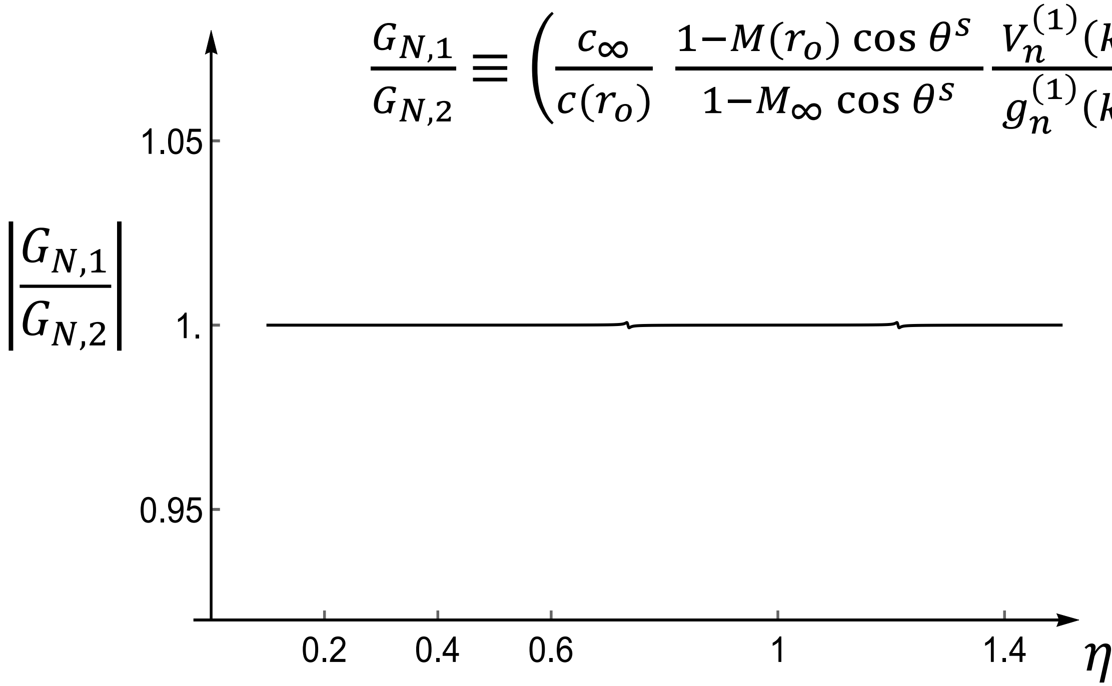
$$St = 1.0,\ \theta = \frac{\pi}{3},\ \Delta\phi = 0$$

Point Source GF, G_N, at mode *n* = 1



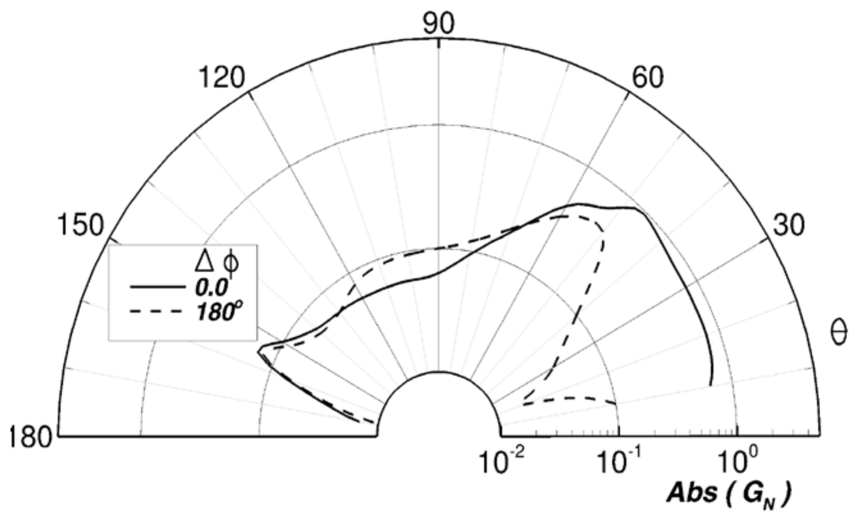
- **Ratio of two Green's functions**

$$\frac{G_{N,1}}{G_{N,2}} \equiv \left(\frac{c_\infty}{c(r_o)}\ \frac{1-M(r_o)\cos\theta^s}{1-M_\infty\cos\theta^s}\ \frac{V_n^{(1)}(k_1,r_o,\omega)}{g_n^{(1)}(k_1,r_o,\omega)}\right)\frac{\Lambda(V_n^{(1)},H_n^{(1)})}{\Lambda(g_n^{(1)},H_n^{(1)})}\Bigg|_{k_1=k_1^*}$$

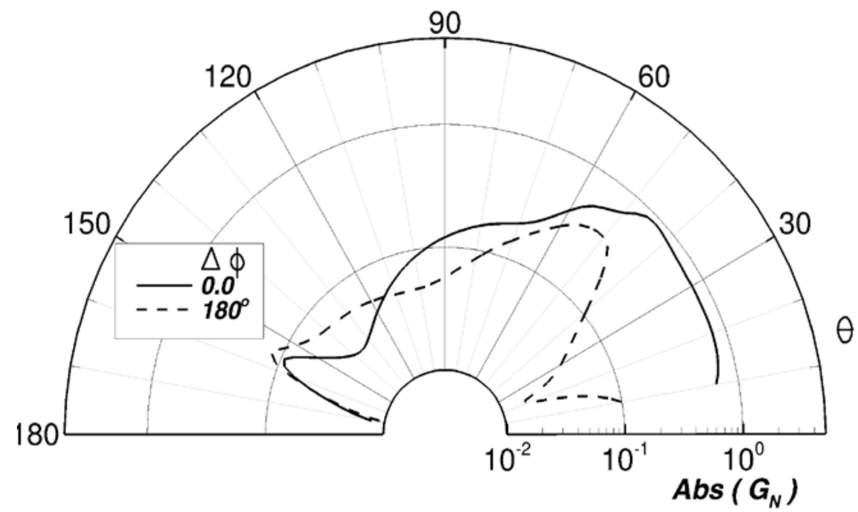


POINT SOURCE POLAR DIRECTIVITY $(St = 1.0,\ \ \eta_o = 0.40)$

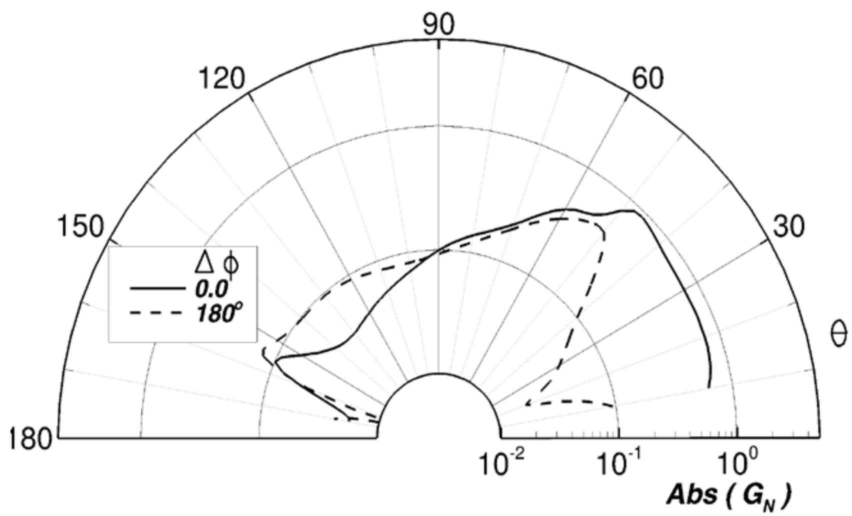
Center-body $\bar{Z}$:Rigid



Center–body $\bar{Z}$ = (0.50, 0.50)



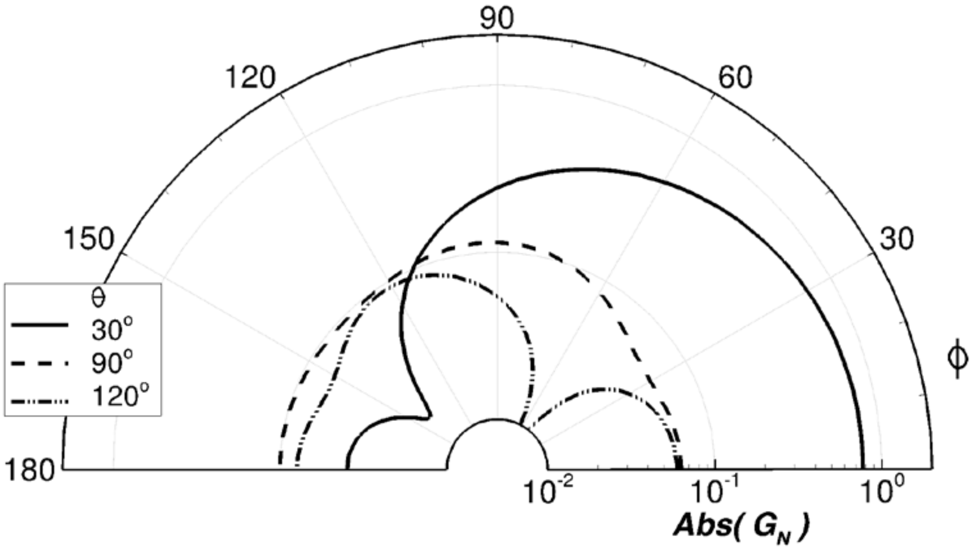
No Center-body



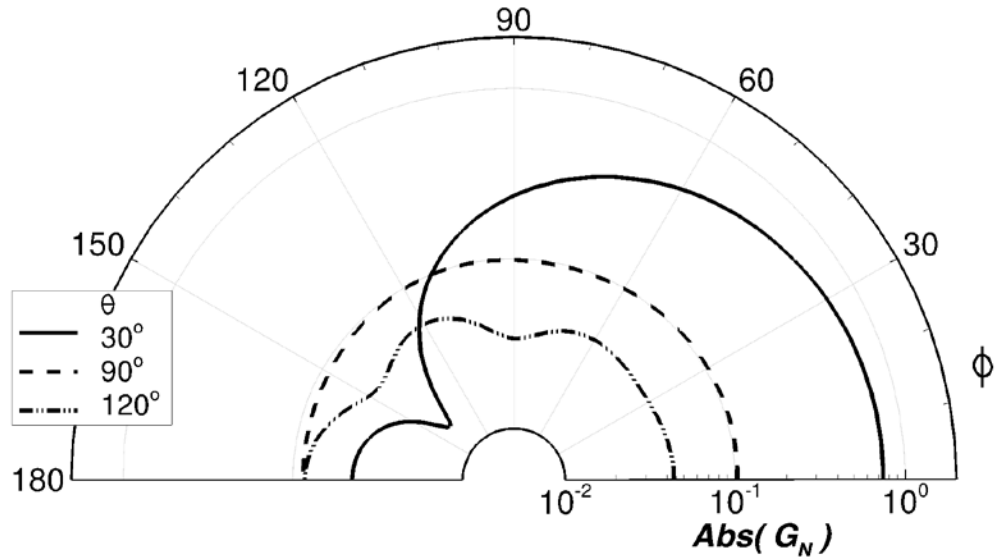
POINT SOURCE AZIMUTHAL DIRECTIVITY

$(St = 1.0, \ \ \eta_o = 0.40, \ \ \phi_o = 0)$

Rigid Center-body

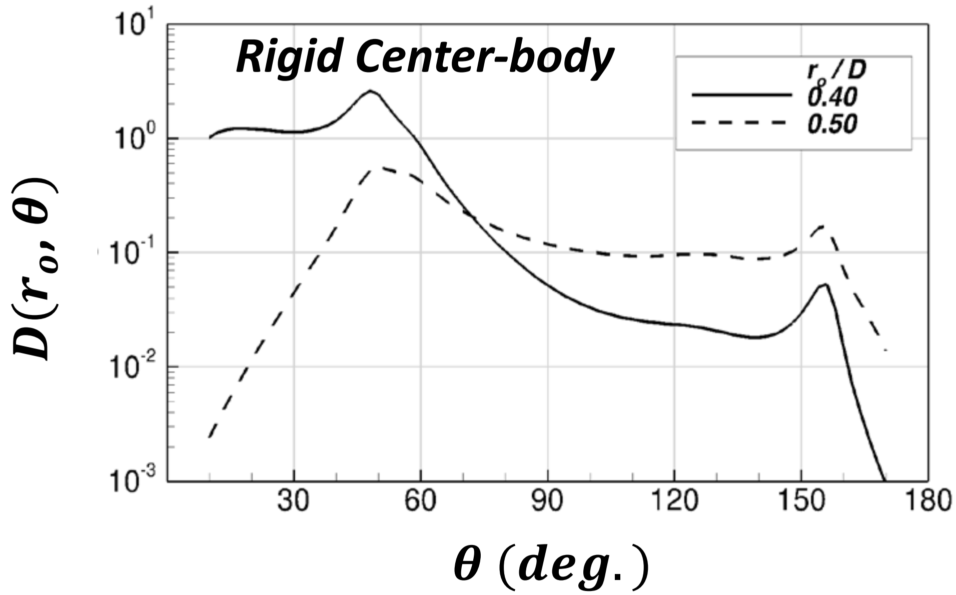


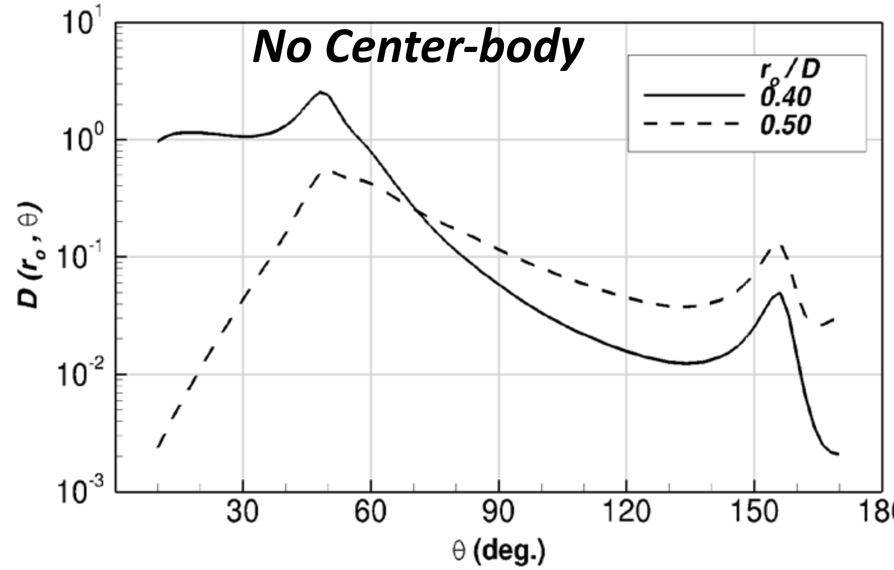
No Center-body



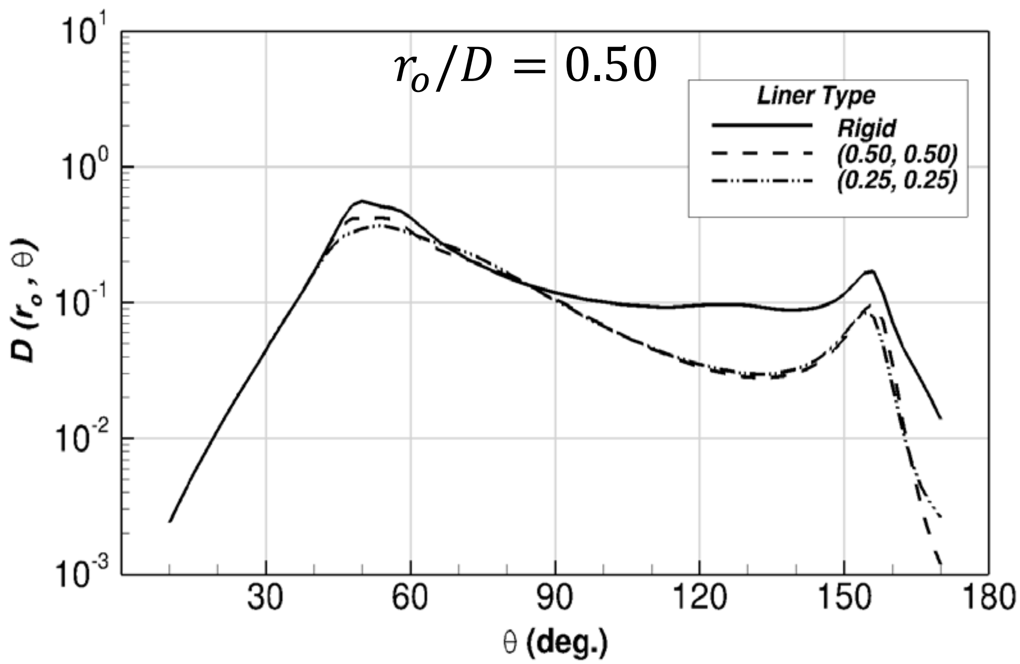
RING SOURCE GF VS. WALL CONDITION, $(St = 1.0)$

$$D(r_o, \theta) \equiv \int_0^{2\pi} G_N G_N^* \, d\phi_o$$





2dB reduction near 45°

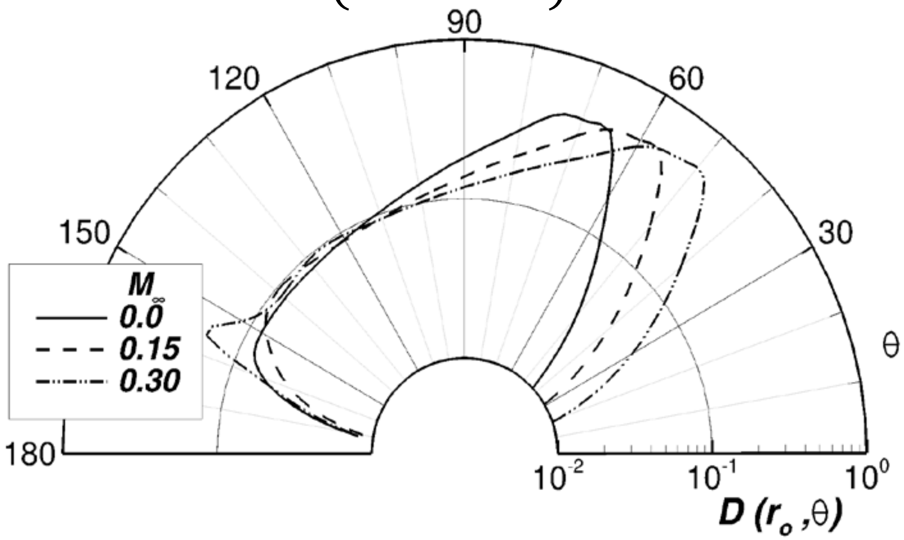


5-6 dB reduction near 130°

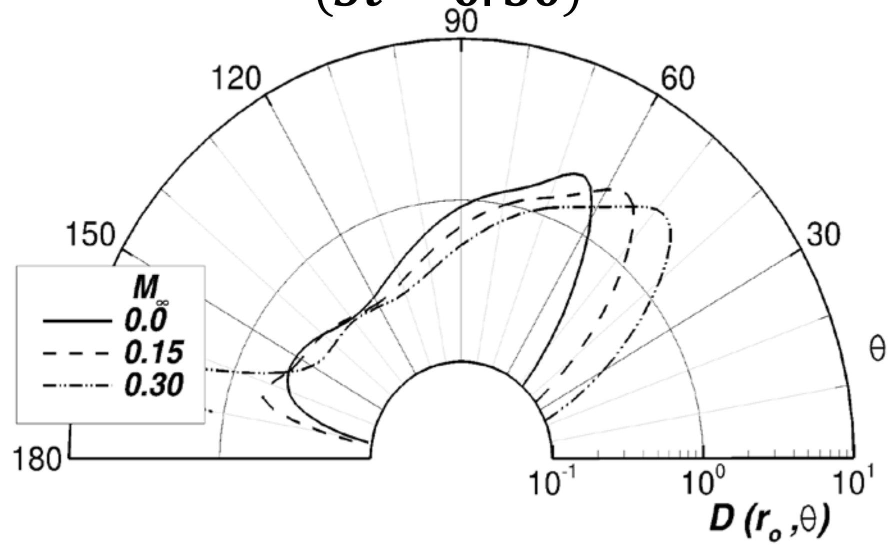
RING SOURCE DIRECTIVITY VS. FLIGHT MACH NO.

$(\eta_o = 0.50, \textit{Rigid Boundary})$

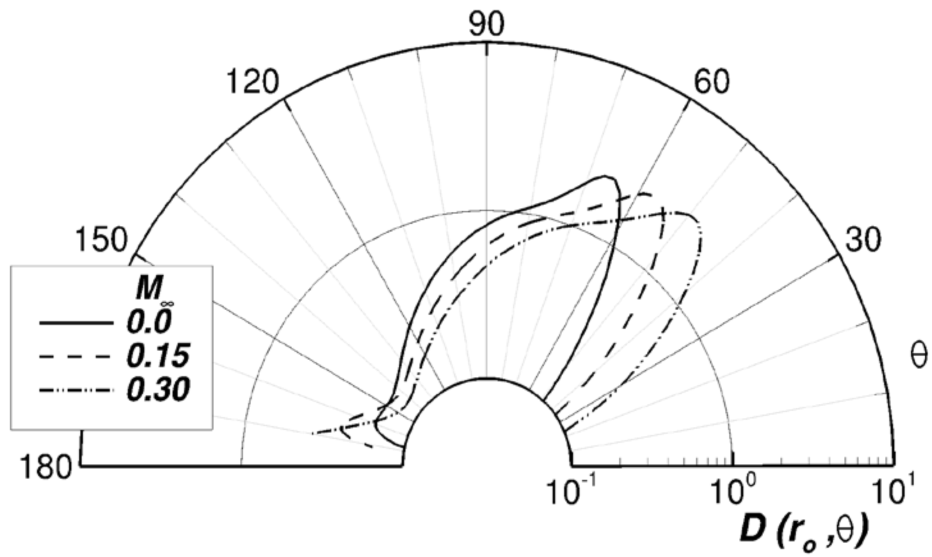
$(St = 1.0)$



$(St = 0.50)$



(No Center-body, $St = 0.50$)



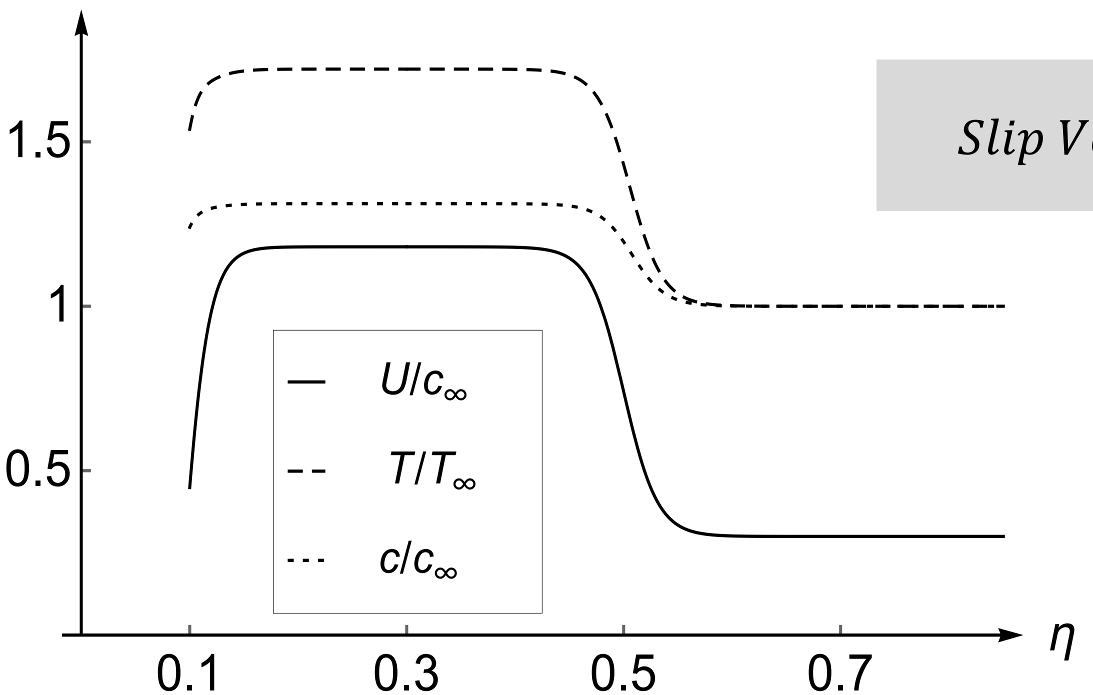
Surface Condition – Slip Boundary

- **Myers boundary condition**

$$v'_{\perp}(\omega) = -\frac{p'}{Z} + \frac{1}{i\omega Z}\tilde{\boldsymbol{v}}.\nabla p' - \frac{p'}{i\omega Z}\vec{n}.(\vec{n}.\nabla\tilde{\boldsymbol{v}})$$

$$\frac{\partial}{\partial r} g_n^{(1)} - \psi_1 g_n^{(1)} = 0, \qquad r = r_s$$

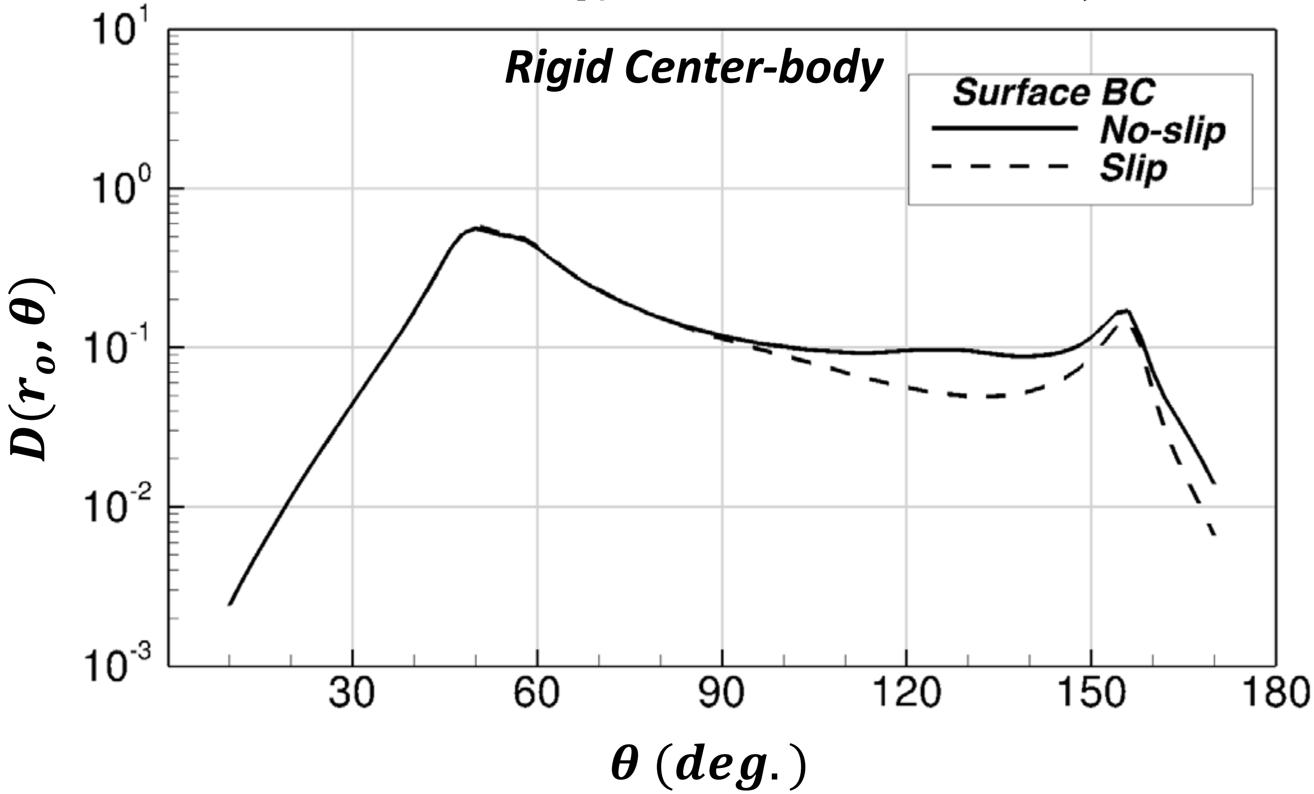
$$\psi_1(k_1, r, \omega, \bar{Z}) = \frac{\left(k_1 \sin\alpha + \frac{c_\infty^2}{c^2}(\kappa_o - k_1 M)/\bar{Z}_1\right)}{i\cos\alpha + \frac{\sin\alpha}{\kappa_o - k_1 M}\left(\frac{\partial M}{\partial r}\right)}, \qquad \frac{1}{\bar{Z}_1} = \frac{1}{\bar{Z}}\left(1 - \frac{1}{i\omega}\left(i\,k_1 U - \sin\alpha\ \cos\alpha\ \frac{\partial U}{\partial r}\right)\right)$$



$$Slip\ Velocity \quad \frac{U(\eta_s)}{c_\infty} = 0.45$$

RING SOURCE DIRECTIVITY– SLIP VS. NO-SLIP BOUNDARY

$(St = 1.0,\ \eta_o = 0.50,\ M_\infty = 0.30)$



Reduction of ~ 4.0dB at forward direction $\theta = 130^o$

Summary

- Point source GF with $\Delta\phi = 0$ dominates the one with $\Delta\phi = \pi$ at aft polar angles. At forward angles, the opposite is true.

- Presence of a rigid center-body amplifies the ring source GF at forward angles. The enhancement may be reduced ($\sim 5dB$) by appropriate impedance liner.

- An increase in flight Mach number sways the noise in direction of downstream axis, and amplifies the noise at large upstream angles (more so at low frequency).

- A slip condition on a rigid CB could attenuate noise by as much as $4dB$ in forward direction (near $\theta = 130^o$) relative to a no-slip condition. Aft angle GF remain relatively insensitive.

QUESTIONS ?