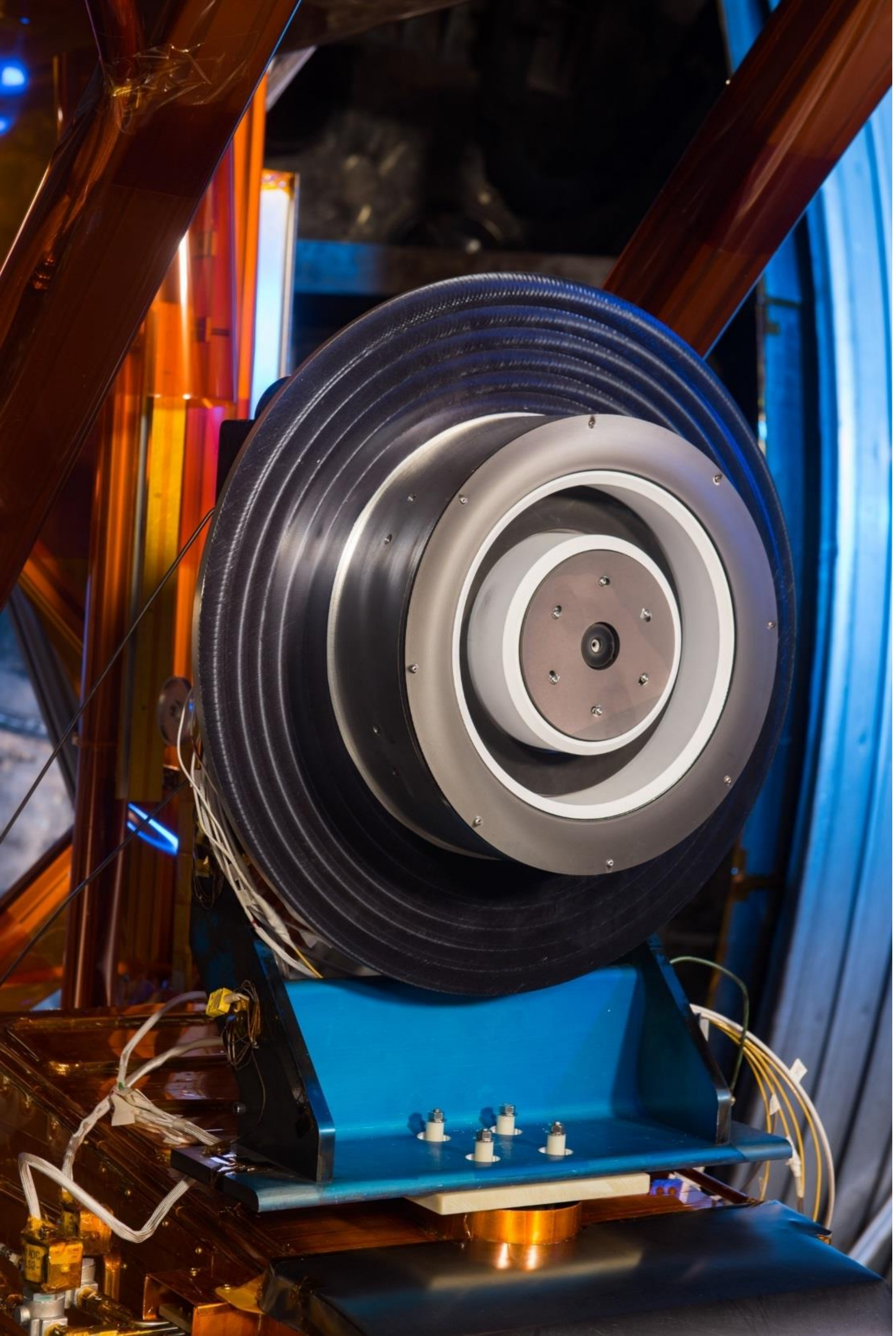

Uncertainty in Inverted Pendulum Thrust Measurements

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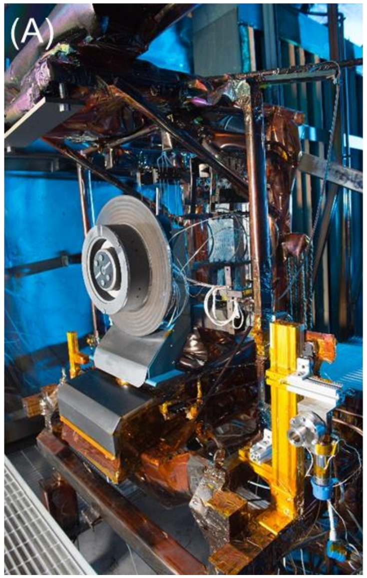
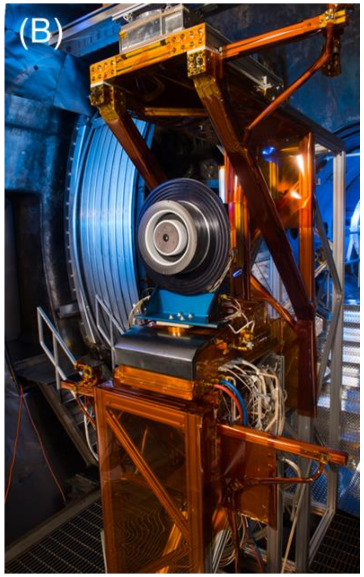
54th AIAA/SAE/ASEE Joint Propulsion Conference
9 July 2018, Cincinnati, Ohio

Introduction

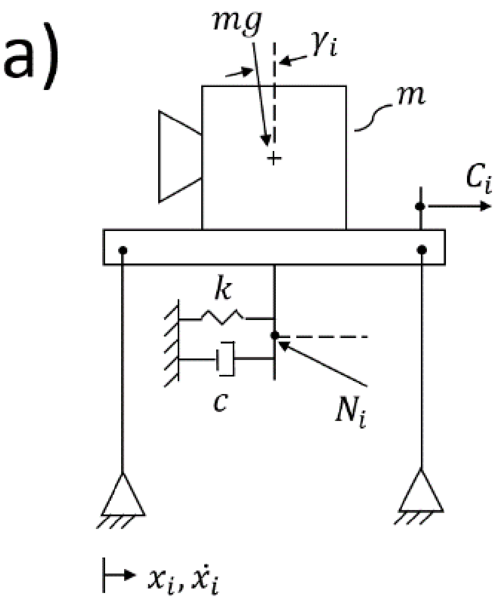
- Thrust measurement is critical to the characterization of electric propulsion (EP) systems.
  - Quantify: thrust, specific impulse, and efficiency.
  - Challenging because of low thrust to weight ratios of EP systems.
  - Sources of potential error must be well understood for reliable data.
  - Strong need for detailed uncertainty analysis.
- NASA GRC vacuum facility 5 (VF-5) and vacuum facility 6 (VF-6) have similar inverted pendulum thrust stands.
  - Several other NASA GRC thrust stands also exist with similar characteristics.



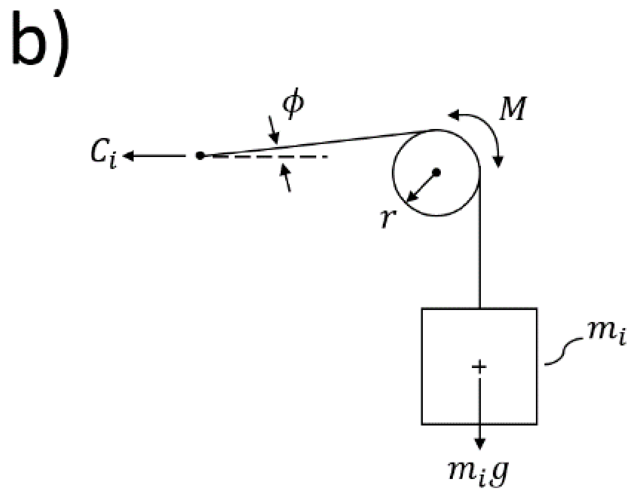


Model of an Inverted Pendulum Stand

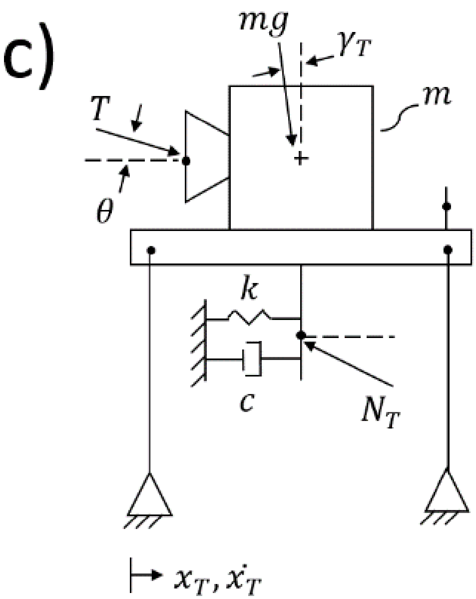
Calibration Mode



Calibration Details

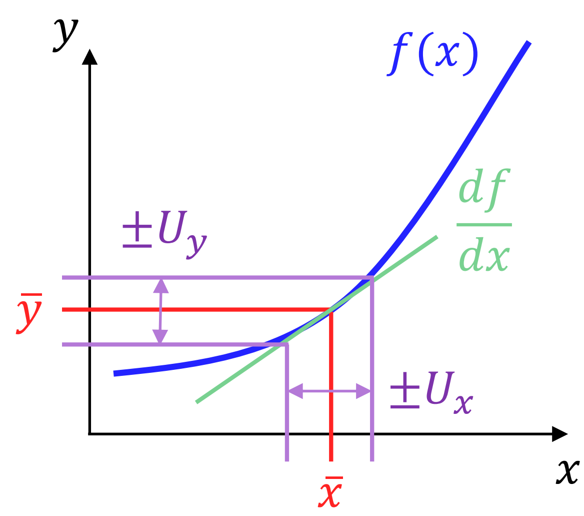


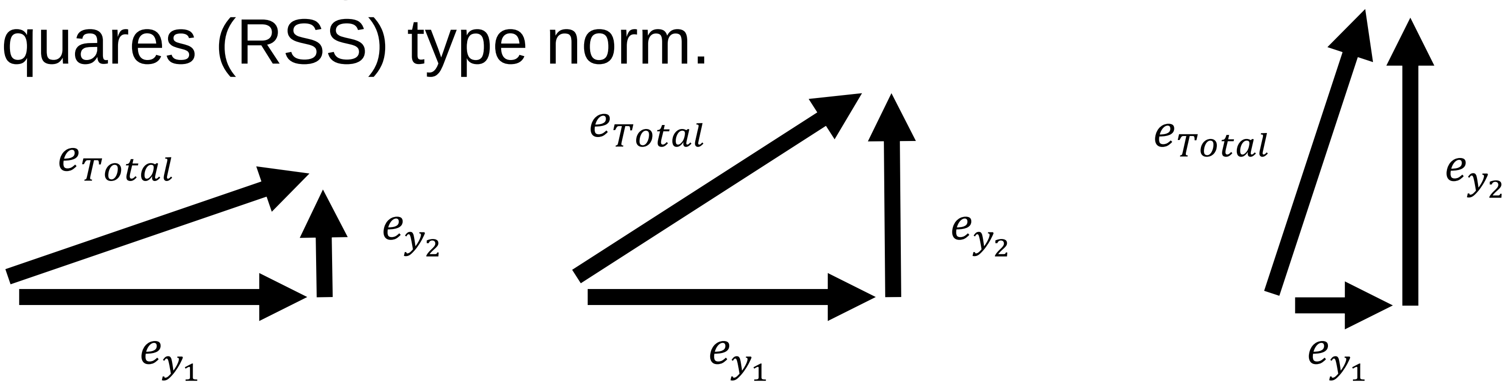
Thruster Mode



Methodology

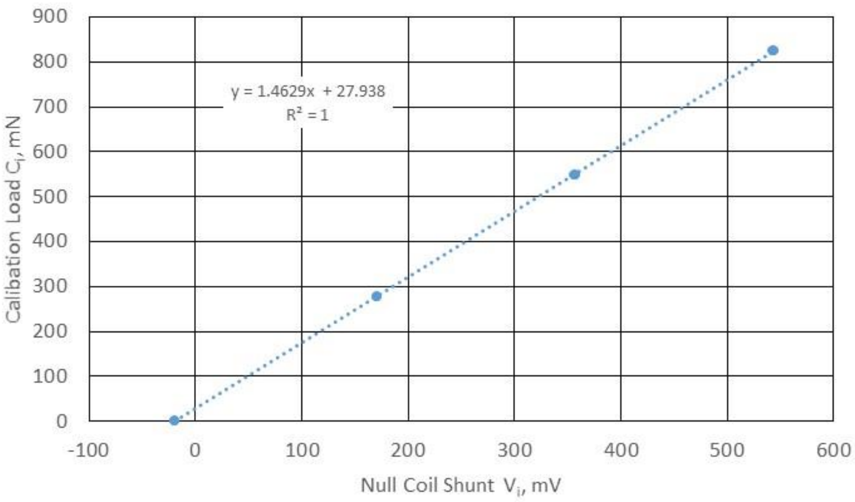
- Classic approach of Figliola and Beasley, Abernethy et al., or Moffat.
  - Design-stage, single-measurement
- Propagate error of independent sources using a truncated Taylor Series expansion.
- Combine normalized error sources using a root sum of squares (RSS) type norm.





Methodology (cont.)

- Thrust stands can be operated in one of two modes:
  - Displacement mode
    - Load vs. deflected position
  - Null coil mode
    - Position held constant with a restoring force provided by a "null coil"
    - Load vs. null coil current
- Example calibration dataset (right) can be reliably fit with a linear regression.
- Thrust can be estimated from calibration regression parameters and null coil shunt voltage.
- Objective of this work is to quantify the possible difference between a "measured" thrust and the "actual" thrust to some confidence level.



$$T = a + b\,V_T \pm U_T$$

$$U_T \; 95\% \; Confidence$$

Sources of Uncertainty

Source	Relative Uncertainty	Parameters of Interest	Parameter Description
Thrust vector	$e_\theta = 1 - \cos(\theta)$	θ	Thrust vector angle
Stand displacement drift	$e_x = \frac{k\lvert x_T - \tilde{x}_i \rvert}{\bar{T}}$	$\lvert x_T - \tilde{x}_i \rvert$, $k = \omega_n^2 m$	Stand position drift, Stiffness
Stand velocity drift	$e_{\dot{x}} = \frac{c\left\lvert \dot{x}_T - \tilde{\dot{x}}_i \right\rvert}{\bar{T}}$	$\left\lvert \dot{x}_T - \tilde{\dot{x}}_i \right\rvert$, c	Stand velocity drift, Damping coefficient
Stand inclination drift	$e_\gamma = \frac{mg \ \sin\lvert \gamma_T - \tilde{\gamma}_i \rvert}{\bar{T}}$	$\lvert \gamma_T - \tilde{\gamma}_i \rvert$, mg	Stand inclination drift, Thruster weight
Shunt thermal drift	$e_{shunt} = \frac{\alpha\lvert t_T - \tilde{t}_i \rvert}{\bar{R}_{shunt}}$	$\lvert t_T - \tilde{t}_i \rvert$, α, $\bar{R}_{shunt}$	Temperature drift, Thermal sensitivity, Nominal resistance
Calibration slope repeatability	$e_{slope} = \frac{S_b \overline{V_T}}{\bar{T}} = S_b \left(\frac{1}{b} - \frac{a}{b\bar{T}} \right)$	S_b	Calibration gain standard deviation
Calibration regression correlation	$e_{S_{xy}} = \frac{\sqrt{\frac{\sum (C_i - [a + bV_i])^2}{n-2}}}{\bar{T}}$	C_i, $a + bV_i$	Calibration force, Calibration regression
DAQ uncertainty	$e_{V_i} = \frac{U_{V_i}}{\overline{V_T}} = U_{V_i} \frac{\bar{T} - a}{b}$	U_{V_i}	Data acquisition uncertainty
Calibration uncertainty	$e_{C_i} = \frac{1}{\bar{T}} \frac{\partial T}{\partial C_i} U_{C_i}$	U_{C_i}	Calibration uncertainty, see table 2

Sources of Uncertainty

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Thrust vector	$e_\theta = 1 - \cos(\theta)$	θ	Thrust vector angle
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Stand velocity drift	[illegible]	[illegible]	Stand velocity drift, Damping coefficient
Stand inclination drift	[illegible]	[illegible]	Stand inclination drift, Thruster weight
Shunt thermal drift	[illegible]	[illegible]	Temperature drift, Thermal sensitivity, Nominal resistance
Calibration slope repeatability	e [illegible]	[illegible]	Calibration gain standard deviation
Calibration regression correlation	e [illegible]	[illegible]	Calibration force, Calibration regression
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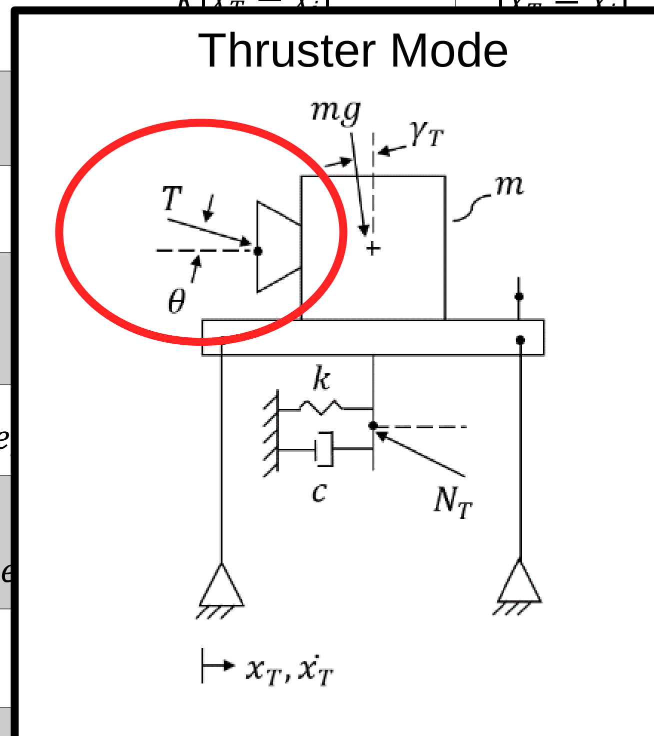

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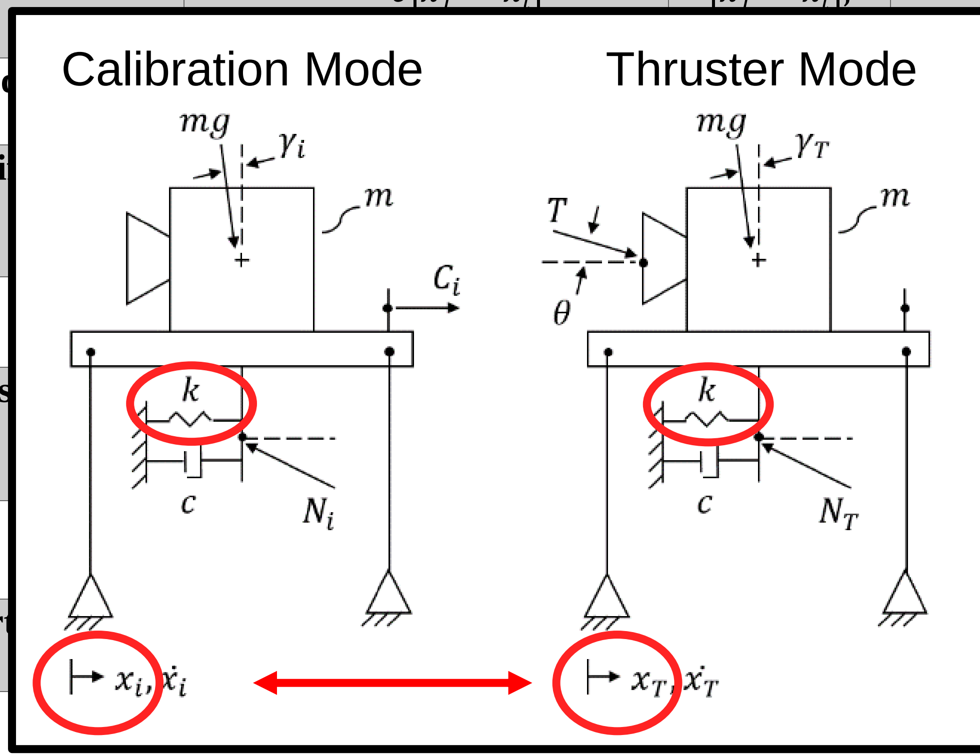

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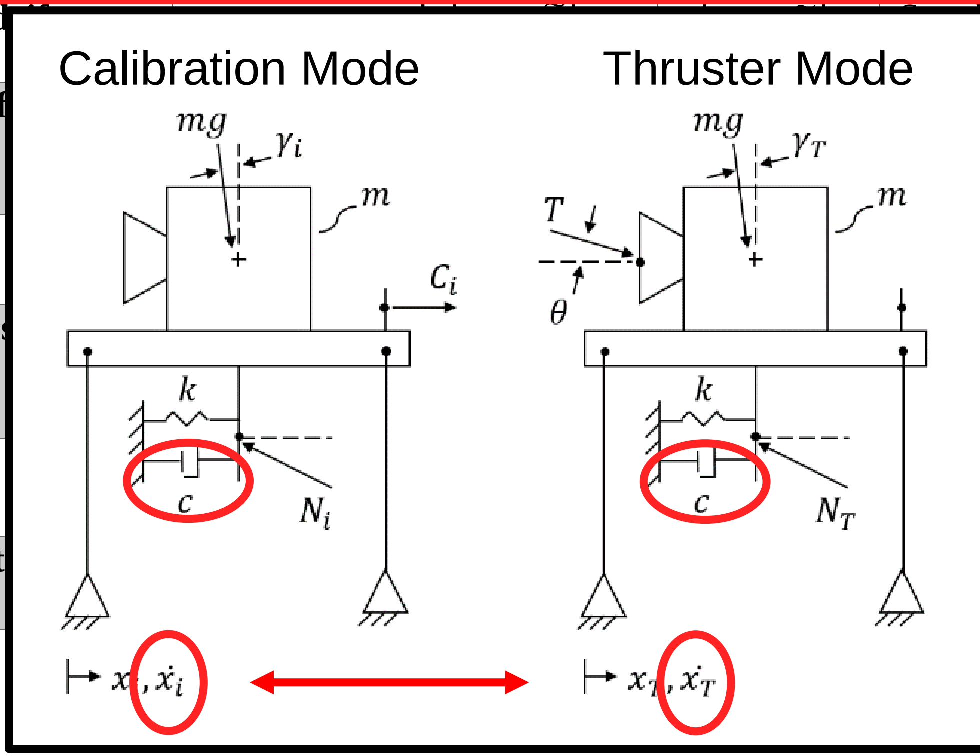

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Calibration slope repeatability			…tion gain …rd deviation
Calibration regress… correlation			…tion force, …tion regression
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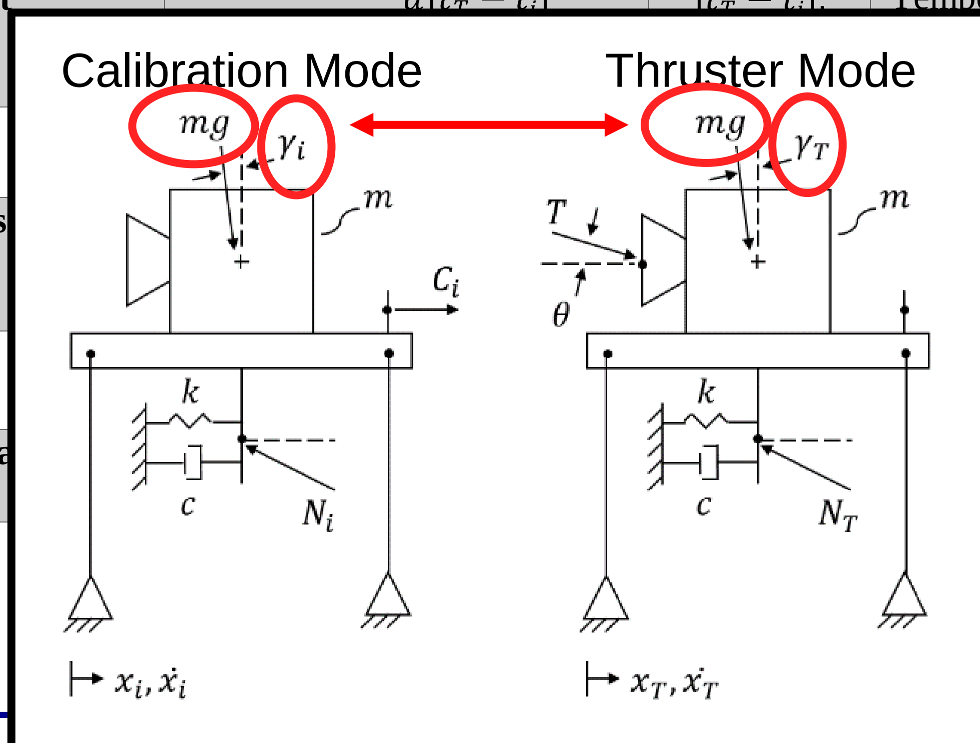

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Calibration slope repeatability	[illegible]	S_b	Calibration gain ...rd deviation
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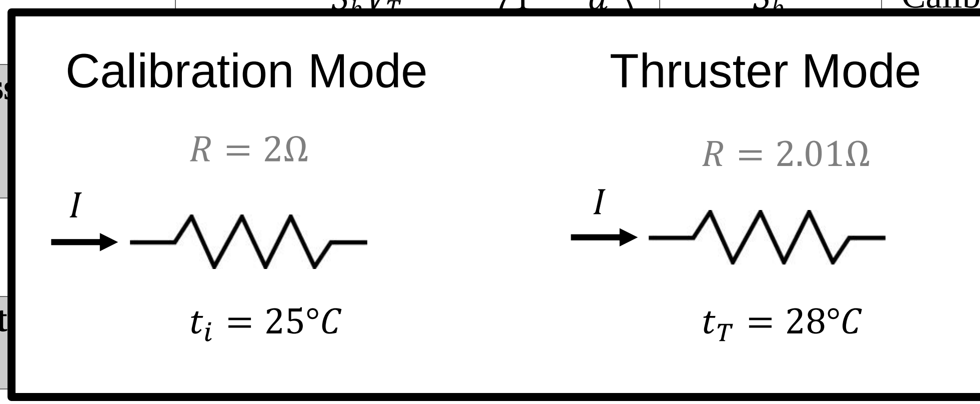

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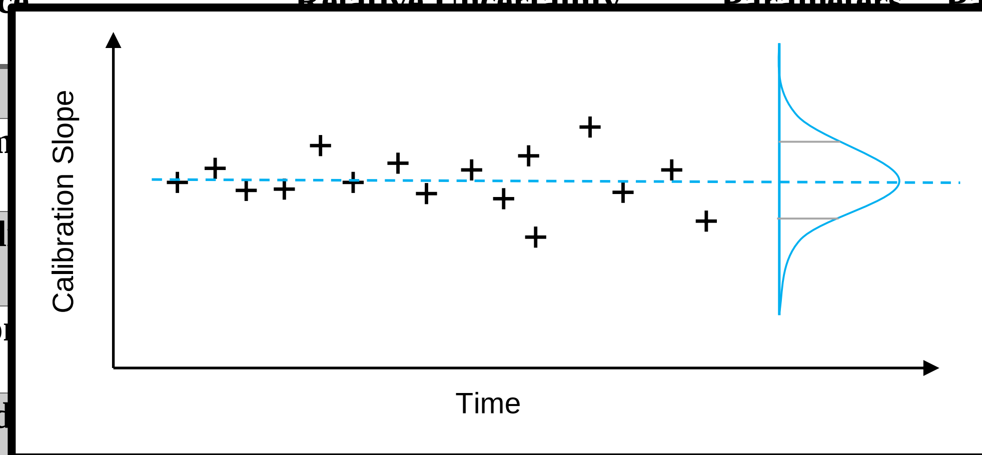

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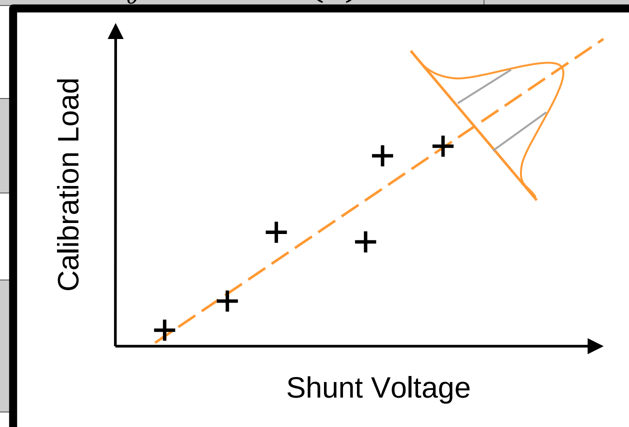

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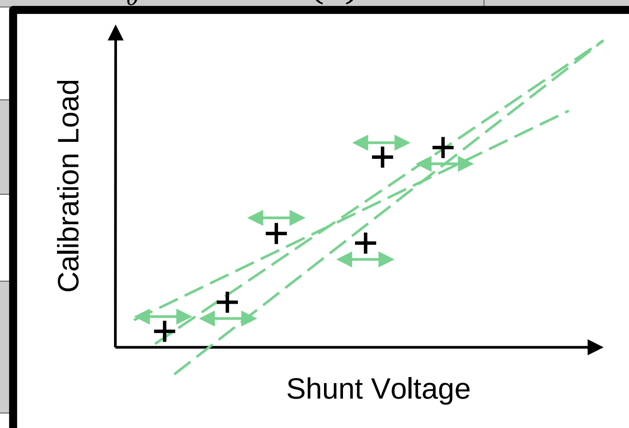

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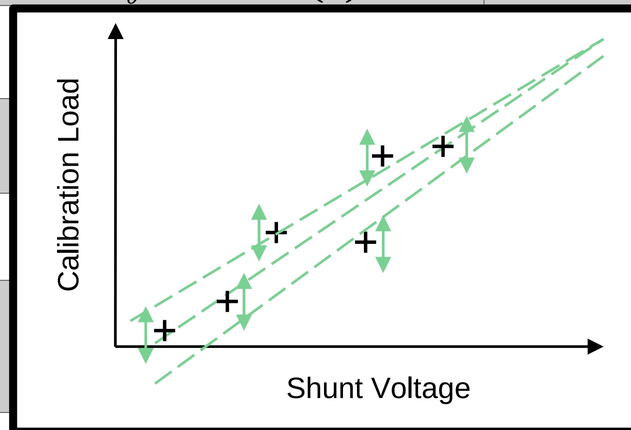

Calibration Sources of Uncertainty

Source	Relative Uncertainty	Parameters of Interest	Parameter Description
Calibration alignment	$e_\varphi = 1 - \cos(\varphi)$	φ	Calibration alignment angle
Calibration pulley moment	$e_M = \frac{M}{r\bar{C}_i}$	M	Calibration pulley moment
Calibration mass uncertainty	$e_{m_i} = \frac{U_{m_i} g}{\bar{C}_i}$	U_{m_i}	Calibration mass uncertainty
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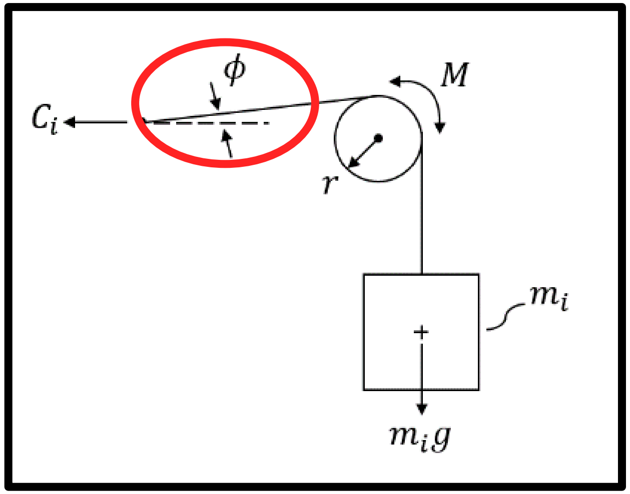

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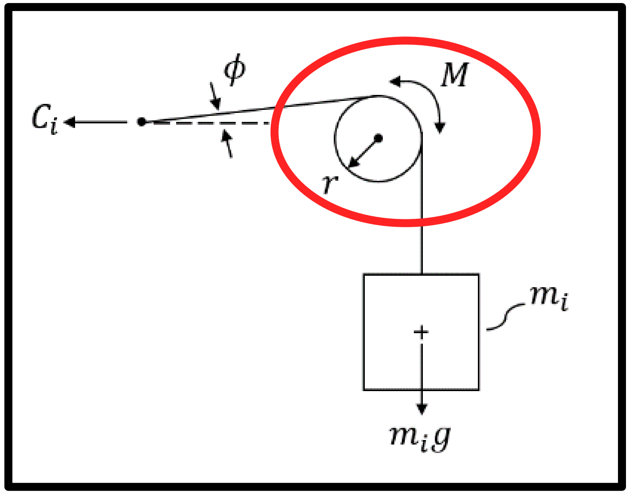

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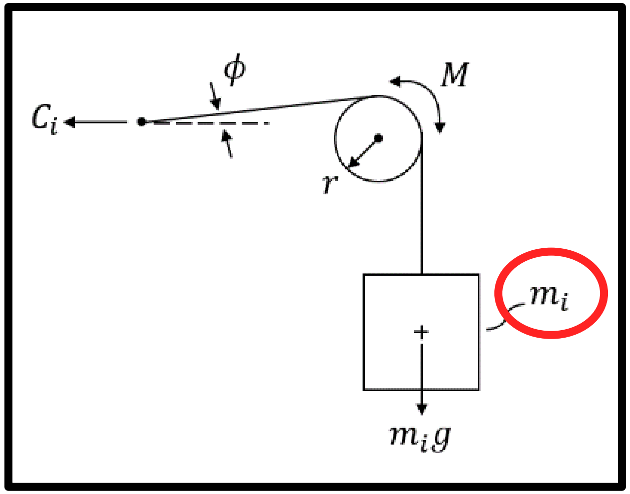

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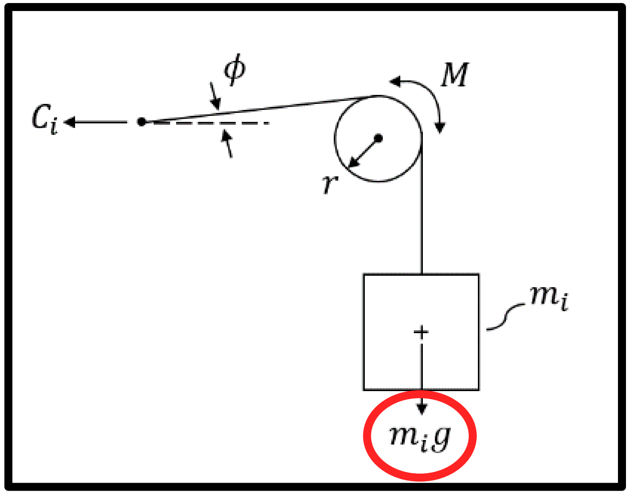
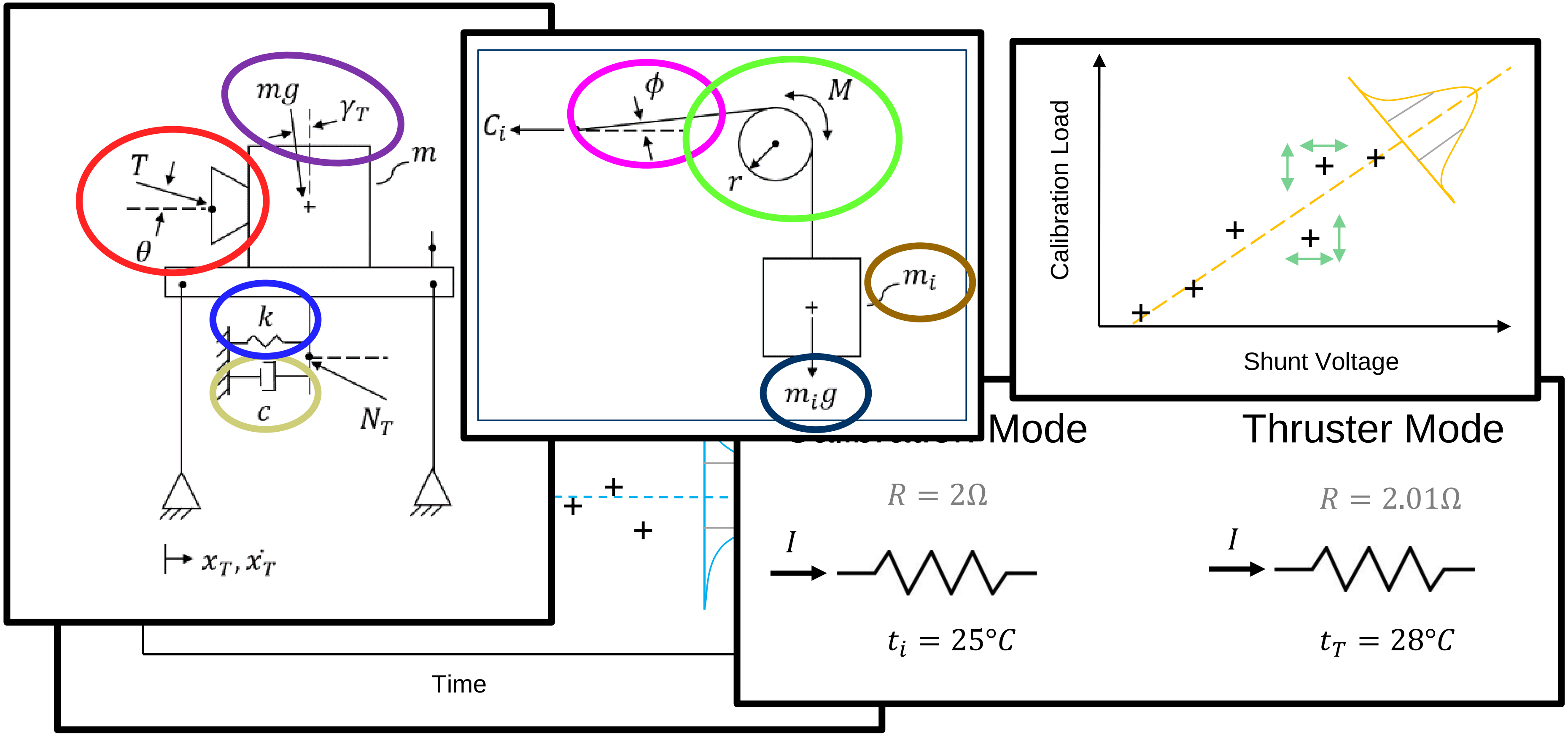


Combination of Sources

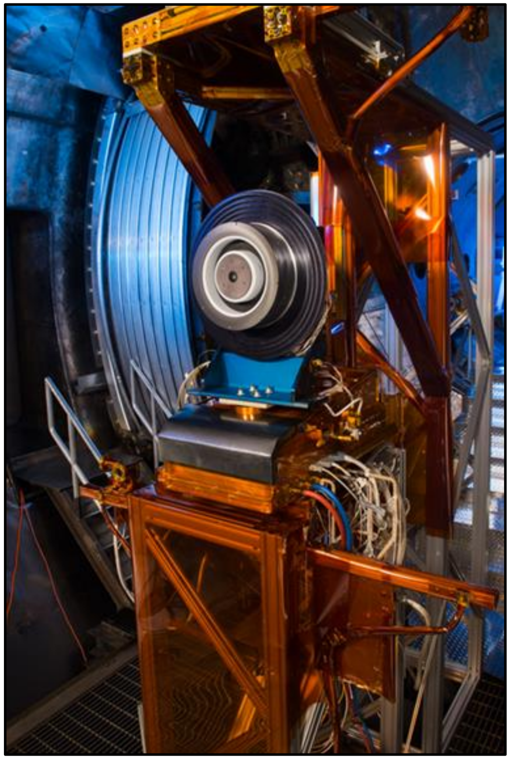
$$e_T = \frac{U_T}{\bar{T}} = \sqrt{e_\theta^2 + e_x^2 + e_{\dot{x}}^2 + e_\gamma^2 + e_{shunt}^2 + e_{slope}^2 + e_{S_{xy}}^2 + e_{V_i}^2\sqrt{n} + e_{C_i}^2\sqrt{n}}.$$

$$U_{C_i} = \bar{T}\sqrt{e_\varphi^2 + e_M^2 + e_{m_i}^2 + e_g^2}$$



Case Study NASA VF-6 Thrust Stand

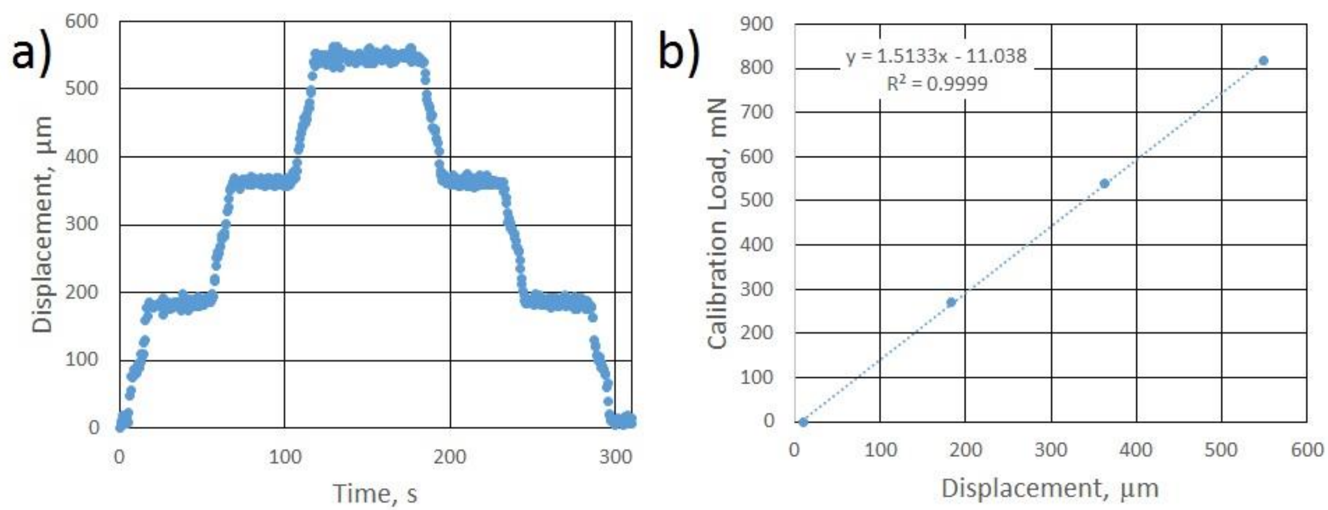
- VF-6 inverted pendulum thrust stand has been recently constructed with uncertainty quantification in mind.
  - Technology Development Unit (TDU) 12.5 kW Hall thruster for Advanced Electric Propulsion System (AEPS).
- Null and damper coils on PID feedback loops.
- Inclination control on PID feedback loop.
- Linear variable differential transformer (LVDT) position measurement for PID feedback.
- Laser triangulation sensor as secondary position measurement.
- Electrolytic inclinometer tilt sensor for PID feedback.
- Inertial inclinometer tilt sensor for secondary inclination measurement.
- In-situ calibration mechanism.
- Protective thermal/environmental shroud.



Stand Characterization

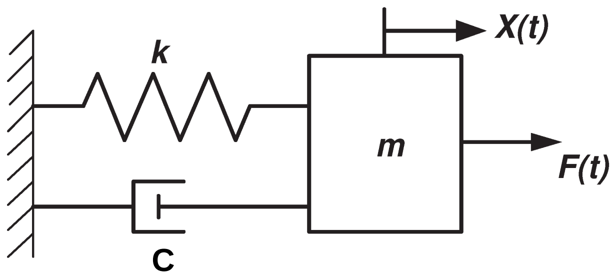
- Stand stiffness, damping coefficient, and total dynamic mass were calculated.
  – May change with different thrusters, different thrust stands, and different setup configurations.

Displacement Mode Operation

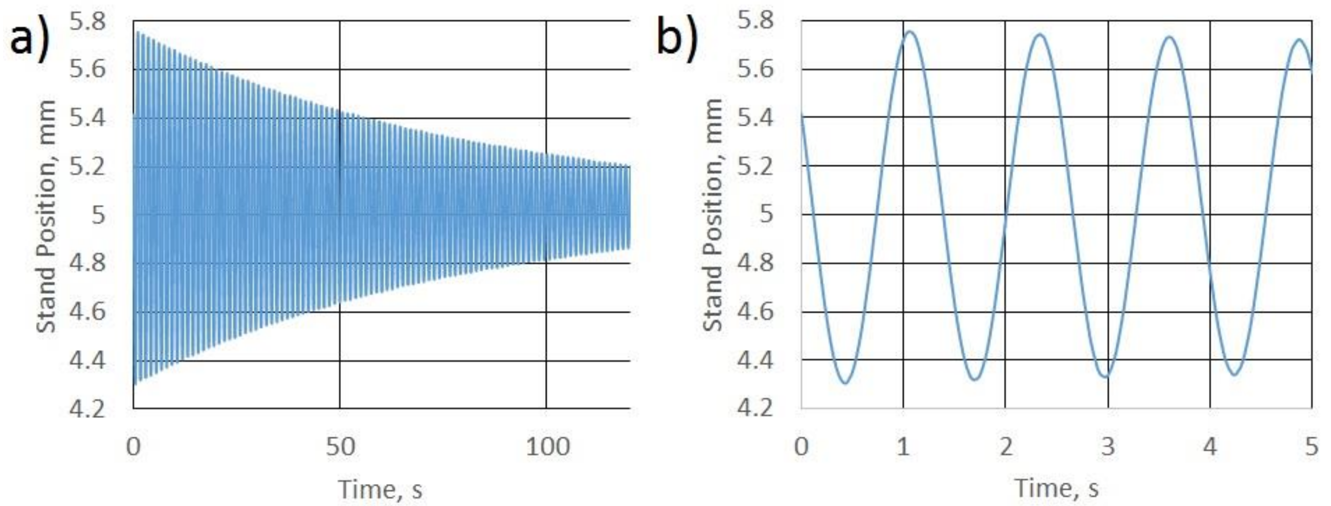


Stand Stiffness k=1.5 mN/µm

Stand Characterization (cont.)



"Free" Response to Impulse, No Active Coils

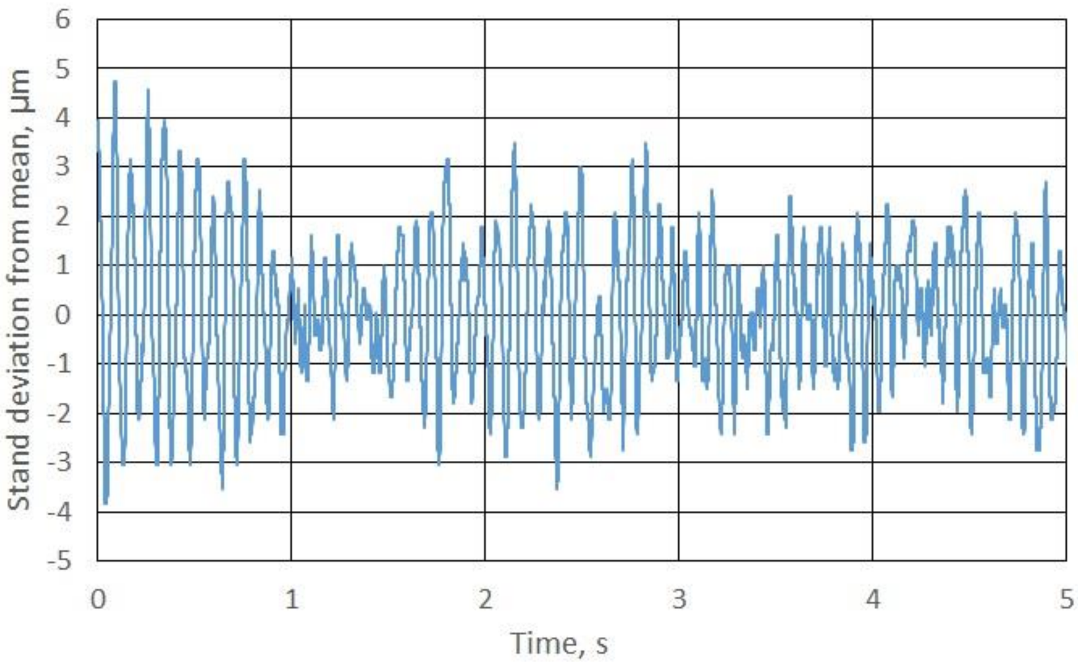


Stand Damping Coefficient c=1.5 kg/s (ξ=2.5e-3)

Stand Natural Frequency ω_n=0.788 Hz

Stand Characterization (cont.)

Nominal Quiescent Operation



Stand Velocity $\dot{x}$=0.3 mm/s

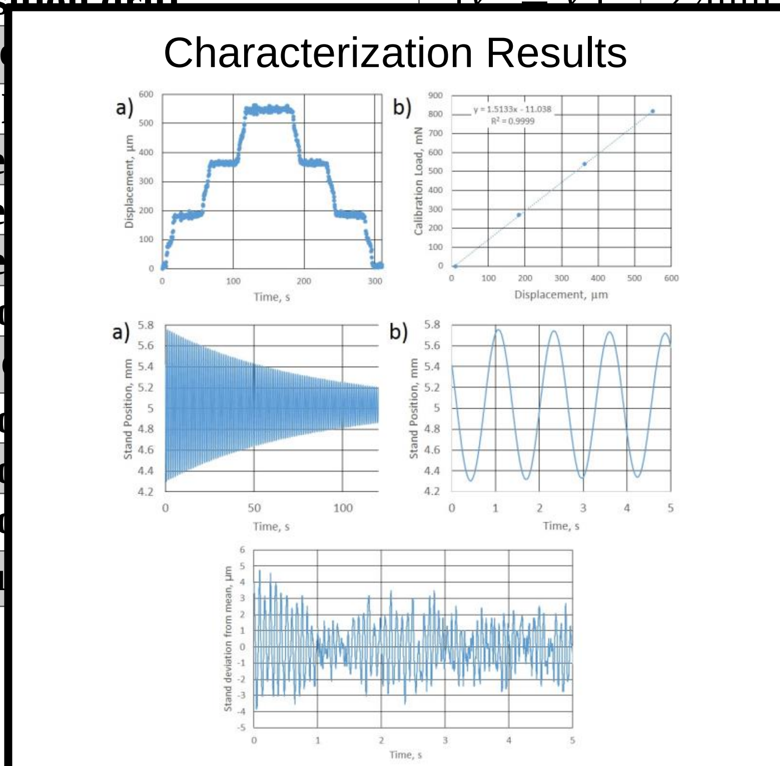
Total Mass $m_{stand} + m_{thruster}$=(14.9+46.7) kg

Summary of Assumed Values

Parameter	Term	Assumed Value
TDU Mass	m_{TDU}	46.7 kg
Stand Mass	m_{Stand}	14.9 kg
Total Weight	mg	604 N
Stand natural frequency	ω_n	0.788 Hz
Stand damping coefficient	c	1.5 kg/s
Stand position drift	$\lvert x_T - \widetilde{x}_i \rvert$	2.4μm
Stand velocity drift	$\lvert \dot{x}_T - \widetilde{\dot{x}}_i \rvert$	0.3 mm/s
Stand inclination drift	$\lvert \gamma_T - \widetilde{\gamma}_i \rvert$	2.0 arc seconds
Shunt thermal sensitivity	α	100 ppm/^{o}C
Shunt thermal drift	$\lvert t_T - \widetilde{t}_i \rvert$	10^{o}C
Thrust vector alignment angle	θ	2.0^{o}
Calibration slope repeatability	S_b	1.54 mN/V
DAQ uncertainty	U_{v_i}	600 μV
Calibration pulley moment	M	2.7e-6 Nm
Calibration alignment angle	φ	2.0^{o}
Calibration mass uncertainty	U_{m_i}	0.1 g
Gravity uncertainty	U_g	0.01 m/s^2

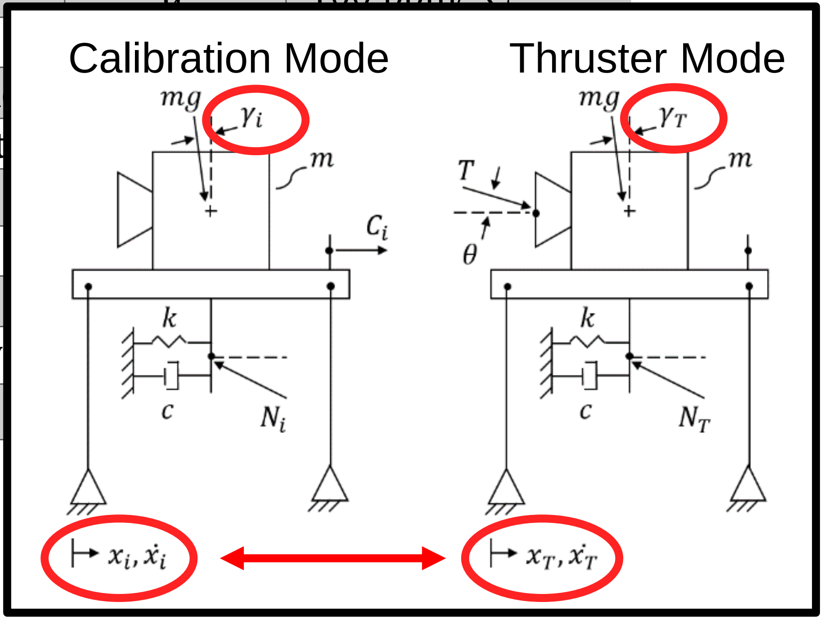
Summary of Assumed Values

Parameter	Term	Assumed Value
TDU Mass	m_{TDU}	46.7 kg
Stand Mass	m_{Stand}	14.9 kg
Total Weight	mg	604 N
Stand natural frequency	ω_n	0.788 Hz
Stand damping coefficient	c	1.5 kg/s
Stand position drift	$\lvert x - \tilde{x} \rvert$	2.4μm
Stand vel		/s
Stand inc		seconds
Shunt the		m/ºC
Shunt the		
Thrust ve		
Calibrati		N/V
DAQ unc		
Calibrati		Nm
Calibrati		
Calibrati		
Gravity u		s^2



Summary of Assumed Values

Parameter	Term	Assumed Value
TDU Mass	m_{TDU}	46.7 kg
Stand Mass	m_{Stand}	14.9 kg
Total Weight	mg	604 N
Stand natural frequency	ω_n	0.788 Hz
Stand damping coefficient	c	1.5 kg/s
Stand position drift	$\lvert x_T - \tilde{x}_i \rvert$	2.4μm
Stand velocity drift	$\lvert \dot{x}_T - \tilde{\dot{x}}_i \rvert$	0.3 mm/s
Stand inclination drift	$\lvert \gamma_T - \tilde{\gamma}_i \rvert$	2.0 arc seconds
Shunt thermal sensitivity	α	100 ppm/°C
Shunt thermal drift		
Thrust vector alignment angl		
Calibration slope repeatabilit		
DAQ uncertainty		
Calibration pulley moment		
Calibration alignment angle		
Calibration mass uncertainty		
Gravity uncertainty		

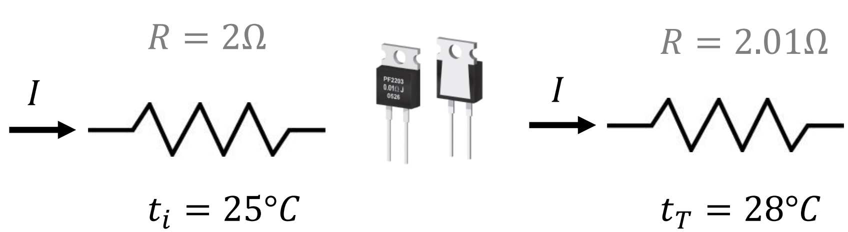


Summary of Assumed Values

Parameter	Term	Assumed Value
TDU Mass	m_{TDU}	46.7 kg
Stand Mass	m_{Stand}	14.9 kg
Total Weight	mg	604 N
Stand natural frequency	ω_n	0.788 Hz
Stand damping coefficient	c	1.5 kg/s
Stand position drift	$\lvert x_T - \tilde{x}_i \rvert$	2.4μm
Stand velocity drift	$\lvert \dot{x}_T - \tilde{\dot{x}}_i \rvert$	0.3 mm/s
Stand inclination drift	$\lvert \gamma_T - \tilde{\gamma}_i \rvert$	2.0 arc seconds
Shunt thermal sensitivity	α	100 ppm/ºC
Shunt thermal drift	$\lvert t_T - \tilde{t}_i \rvert$	10ºC
Thr		
Cali		
DAC		
Cali		
Cali		
Cali		
Gra		

Calibration Mode

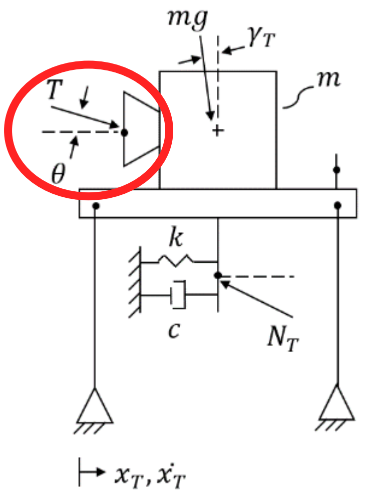
Thruster Mode



Type	Power Rating Heatsink[1]	Free Air[2]	Thermal Resistance	Resistance Range[3] Min	Max	Tolerances	Temperature Coefficients
PF2205	50W	1W	2.3°C/W	0.02Ω	51KΩ	±1% (R≥0.1Ω) ±5%	±100ppm/°C (0.1Ω ≤ R < 10Ω)
PF2203	35W	1W	3.3°C/W	0.01Ω	51KΩ	1% (R≥0.1Ω) ±5%	±50ppm/°C (R≥10Ω) ±100ppm/°C (0.1Ω ≤ R < 10Ω) ±250ppm/°C (R < 0.1Ω)
PF2202	20W	1W	5.9°C/W	0.02Ω	51KΩ	0.1%,0.25%,0.5%, (R±≥10 Ω) ±1% (R≥0.1Ω) ±5%	±50ppm/°C (R≥10Ω) ±100ppm/°C (0.1Ω ≤ R < 10Ω) ±250ppm/°C (R < 0.1Ω)

Summary of Assumed Values

Thruster Mode



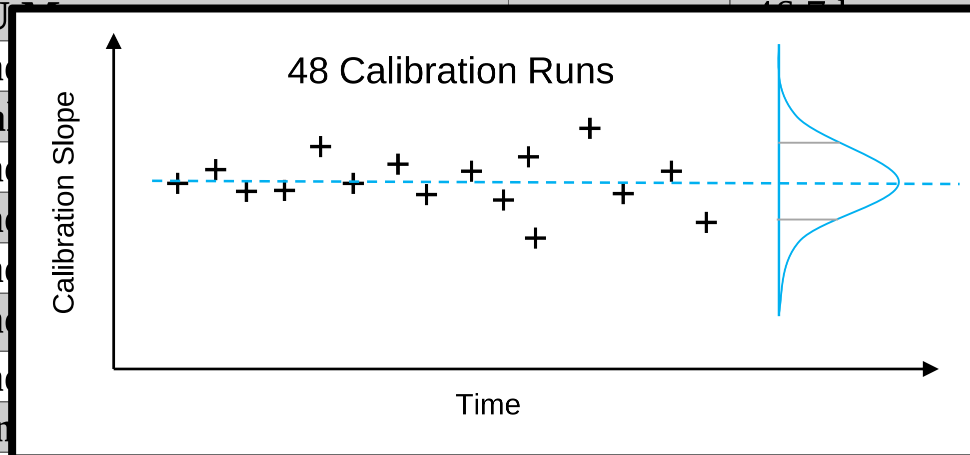
Diagnostic for Verifying the Thrust Vector Requirement of the AEPS Hall-Effect Thruster and Comparison to the NEXT-C Thrust Vector Diagnostic

Gabriel F. Benavides[1], Jonathan A. Mackey[2], Drew M. Ahern[3], and Robert E. Thomas[4]
NASA Glenn Research Center, Cleveland, OH, USA 44135

Shunt thermal drift	$\|t_T - \tilde{t}_i\|$	10ºC
Thrust vector alignment angle	θ	2.0º
Calibration slope repeatability	S_b	1.54 mN/V
DAQ uncertainty	U_{v_i}	600 µV
Calibration pulley moment	M	2.7e-6 Nm
Calibration alignment angle	φ	2.0º
Calibration mass uncertainty	U_{m_i}	0.1 g
Gravity uncertainty	U_g	0.01 m/s^2

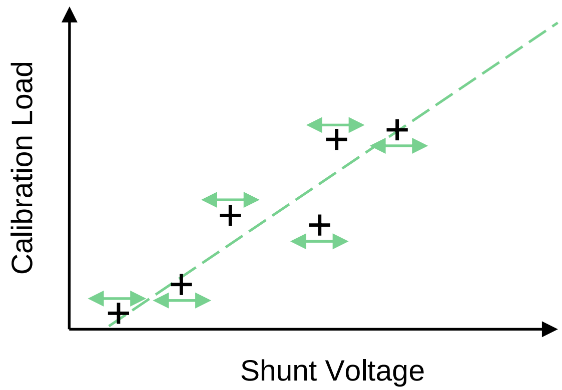
Summary of Assumed Values

Parameter	Term	Assumed Value
TDU		
Stan		
Total		
Stan		
Stan		
Stan		
Stan		
Stan		ds
Shun		
Shunt thermal drift	$\lvert t_T - \tilde{t}_i \rvert$	10ºC
Thrust vector alignment angle	θ	2.0º
Calibration slope repeatability	S_b	1.54 mN/V
DAQ uncertainty	U_{v_i}	600 μV
Calibration pulley moment	M	2.7e-6 Nm
Calibration alignment angle	φ	2.0º
Calibration mass uncertainty	U_{m_i}	0.1 g
Gravity uncertainty	U_g	0.01 m/s²



Summary of Assumed Values

Parameter	Term	Assumed Value
TDU Mass	m_{TDU}	46.7 kg
Shunt thermal drift	$\lvert t_T - \tilde{t}_i \rvert$	10ºC
Thrust vector alignment angle	θ	2.0º
Calibration slope repeatability	S_b	1.54 mN/V
DAQ uncertainty	U_{v_i}	600 µV
Calibration pulley moment	M	2.7e-6 Nm
Calibration alignment angle	φ	2.0º
Calibration mass uncertainty	U_{m_i}	0.1 g
Gravity uncertainty	U_g	0.01 m/s^2



34980A system specifications and characteristics

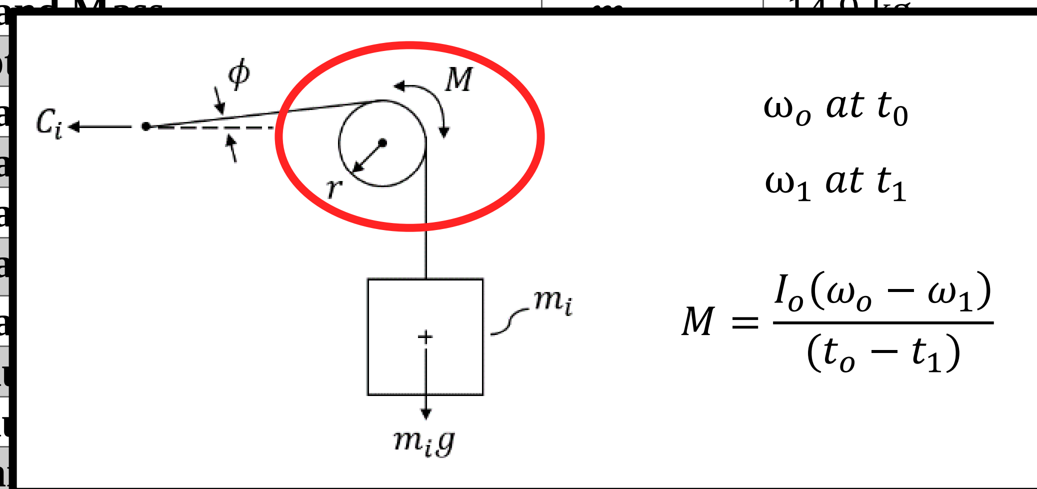
DMM accuracy ± (% of reading + % of range)

Includes measurement error, switching error, and transducer conversion error

Function	Range [4]	Frequency, etc.	24 hour [2,3] Tcal ± 1°C	90 days Tcal ± 5°C	1 year Tcal ± 5°C	Temperature coefficient/°C ›Tcal ± 5°C
			Measurement including switch error [1]			
DC voltage	100.0000 mV		0.0030 + 0.0035	0.0040 + 0.0040	0.0050 + 0.0040	0.0005 + 0.0005
(with 34921A/22A/	[illegible]		0.0020 + 0.0006	0.0030 + 0.0007	[illegible]	[illegible]
31A/32A) [10][11]	10.00000 V		0.0015 + 0.0004	0.0020 + 0.0005	0.0035 + 0.0005	0.0005 + 0.0001
Input impedance = Hi-Z	[illegible]		0.003 + 0.0006	0.0045 + 0.0006	[illegible]	[illegible]
10 V range and below	300.0000 V		0.003 + 0.0020	0.0045 + 0.0030	0.0055 + 0.0030	0.0005 + 0.0003

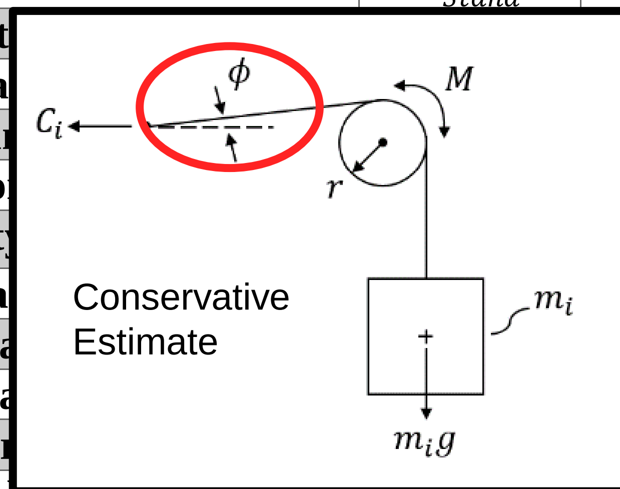
Summary of Assumed Values

Parameter	Term	Assumed Value
TDU Mass	m_{TDU}	46.7 kg
Stand Mass	[illegible]	14.0 kg
Tot		
Sta		
Sta		
Sta		
Sta		
Sta		
Shu		
Shu		
Thi		
Calibration slope repeatability	S_b	1.54 mN/V
DAQ uncertainty	U_{v_i}	600 µV
Calibration pulley moment	M	2.7e-6 Nm
Calibration alignment angle	φ	2.0º
Calibration mass uncertainty	U_{m_i}	0.1 g
Gravity uncertainty	U_g	0.01 m/s^2



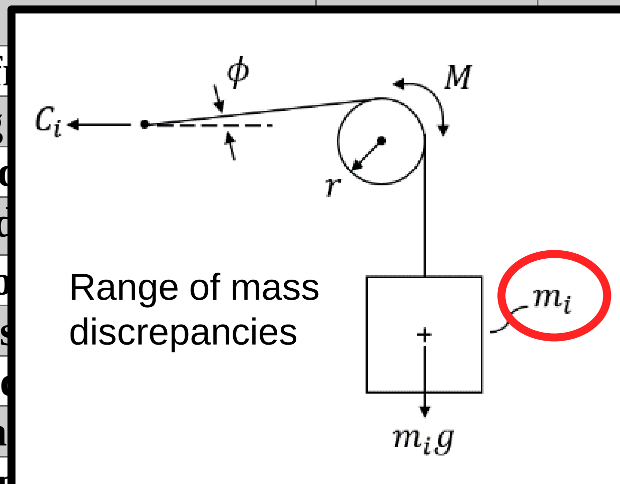
Summary of Assumed Values

Parameter	Term	Assumed Value
TDU Mass	m_{TDU}	46.7 kg
Stand Mass	m_{Stand}	14.9 kg
Total Weight		04 N
Stand natura		.788 Hz
Stand dampi		.5 kg/s
Stand positio		.4μm
Stand velocity		.3 mm/s
Stand inclina		.0 arc seconds
Shunt therma		00 ppm/ºC
Shunt therma		0ºC
Thrust vector		.0º
Calibration slope repeatability	S_b	1.54 mN/V
DAQ uncertainty	U_{v_i}	600 μV
Calibration pulley moment	M	2.7e-6 Nm
Calibration alignment angle	φ	2.0º
Calibration mass uncertainty	U_{m_i}	0.1 g
Gravity uncertainty	U_g	0.01 m/s^2



Summary of Assumed Values

Parameter	Term	Assumed Value
TDU Mass	m_{TDU}	46.7 kg
Stand Mass	m_{Stand}	14.9 kg
Total Weight		N
Stand natural fr		8 Hz
Stand damping		kg/s
Stand position d		m
Stand velocity d		mm/s
Stand inclinatio		arc seconds
Shunt thermal s		ppm/°C
Shunt thermal c		C
Thrust vector a		
Calibration slope repeatability		mN/V
DAQ uncertainty	U_{v_i}	600 μV
Calibration pulley moment	M	2.7e-6 Nm
Calibration alignment angle	φ	2.0°
Calibration mass uncertainty	U_{m_i}	0.1 g
Gravity uncertainty	U_g	0.01 m/s^2



Summary of Assumed Values

Parameter	Term	Assumed Value
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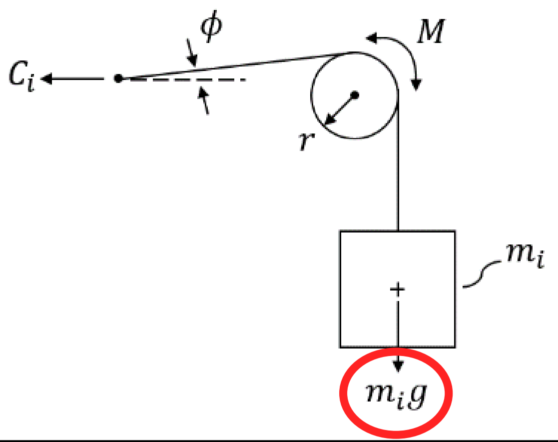
Stand Mass	m_{Stand}	14.9 kg
Calibration slope repeatability	S_b	1.54 mN/V
DAQ uncertainty	U_{v_i}	600 µV
Calibration pulley moment	M	2.7e-6 Nm
Calibration alignment angle	φ	2.0°
Calibration mass uncertainty	U_{m_i}	0.1 g
Gravity uncertainty	U_g	0.01 m/s^2



GEODETIC REFERENCE SYSTEM 1980

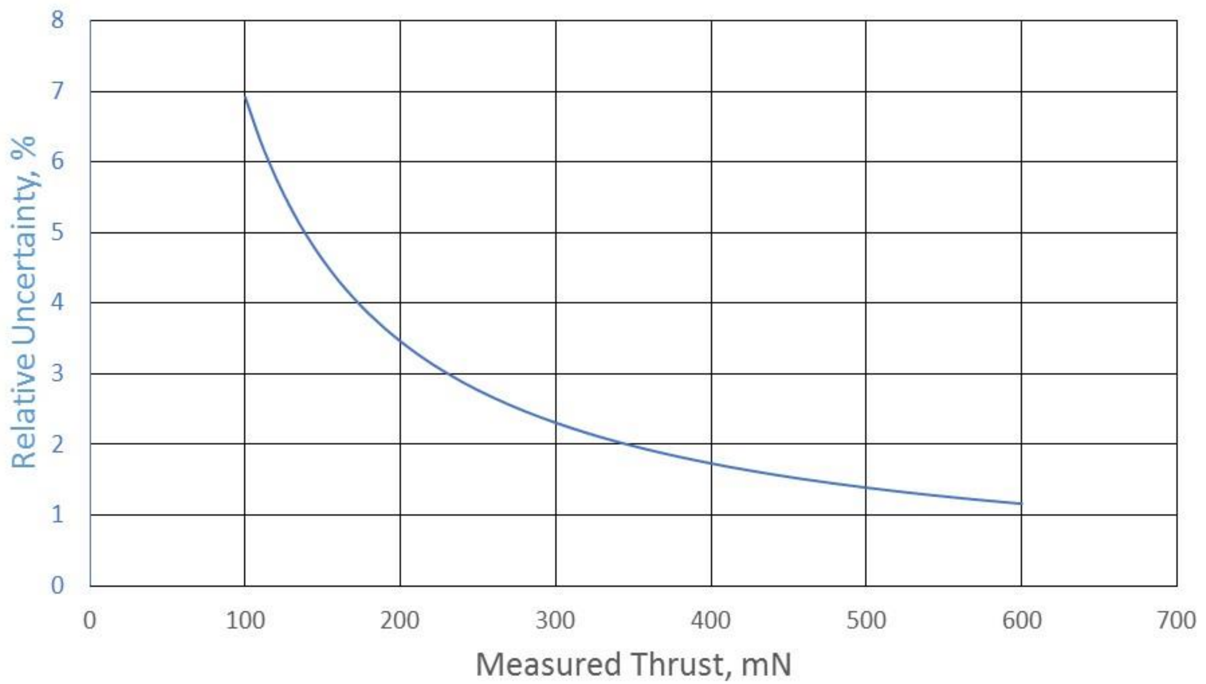
by H. Moritz

$$g(\varphi = 41.4161°N) = 9.80297\ m/s^2$$

$$g_{assumed} = 9.81\ m/s^2$$

Uncertainty Results

- Relative uncertainty ranges from 7 to 1% depending on thrust level.
  - TDU generally operates 400 to 600 mN, so realistic uncertainty is between 1 to 2%.
  - Higher thrust has lower relative uncertainty.
- Absolute uncertainty (also full scale uncertainty) is fairly constant over full range, ~6.9 mN.

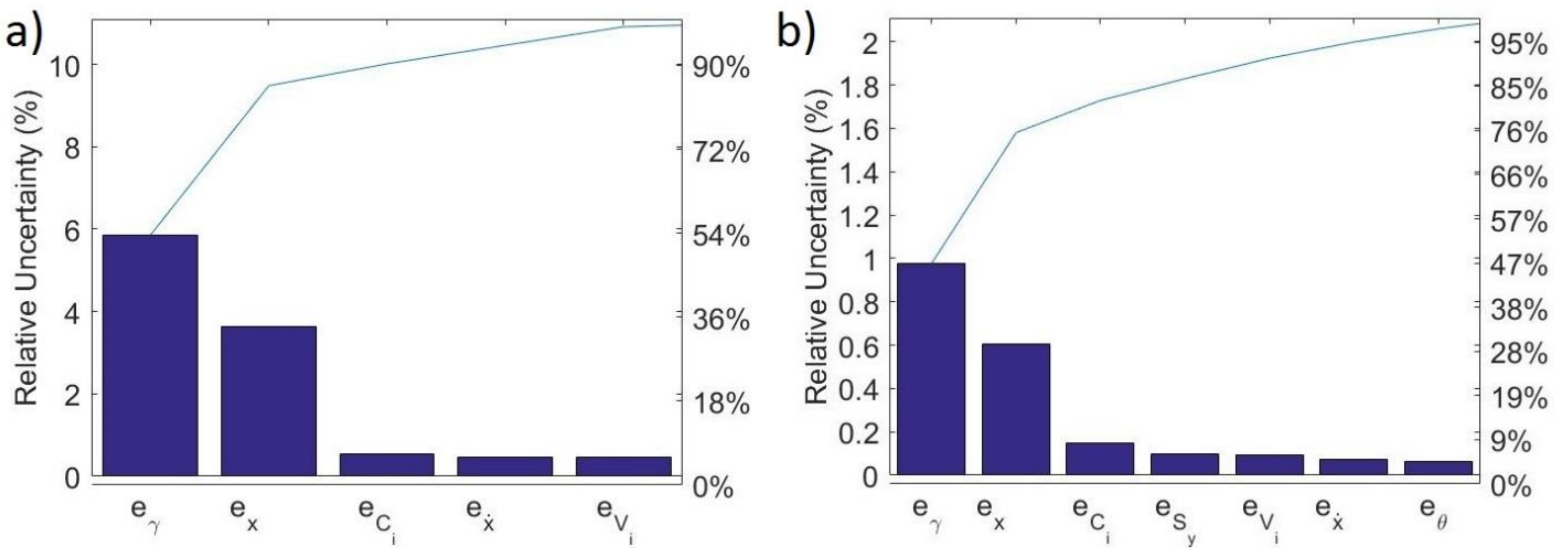


Uncertainty Sources

- Pareto plots highlight the leading sources of uncertainty for two cases of nominal thrust (100mN and 600mN).
- Inclination drift and displacement drift account for >70% of the uncertainty.
- Calibration uncertainty, slope repeatability, velocity drift, and DAQ uncertainty are the remaining significant sources.

100 mN

600 mN



Scaling Parameters

- Scaling results to other thrust stands, thrusters, and configurations will depend primarily on three scaling factors:
  - TDU in VF-6 stiffness to thrust ratio $k/T = 2.54\, mm^{-1}$
  - TDU in VF-6 damping to thrust ratio $c/T = 2.54\, s/m$
  - TDU in VF-6 weight to thrust ratio $mg/T = 1023$
- Considering uncertainty only, it is advisable to minimize these terms.
  - Temporal resolution of the stand may suffer for low stiffness ratio.
  - Structural integrity of the stand may suffer for low weight ratio.

Conclusions

- A 95% confidence uncertainty of ±6.9mN has been established for TDU in VF-6.
- The leading sources of uncertainty are stand inclination drift and stand displacement drift.
- A stand characterization method and a set of scaling parameters have been established and calculated for the case study.
- Future Work:
  - Various thrusters on VF-6 thrust stand, other inverted pendulum thrust stands, torsional thrust stands, and quantification of additional sources.

Acknowledgements

- Space Technology Mission Directorate in support of the Solar Electric Propulsion Technology Demonstration Mission Project for funding the joint NASA GRC and JPL development of the Advanced Electric Propulsion System.

Questions?

NASA