Trajectory Design Leveraging Low-thrust, Multi-Body Equilibria and Their Manifolds

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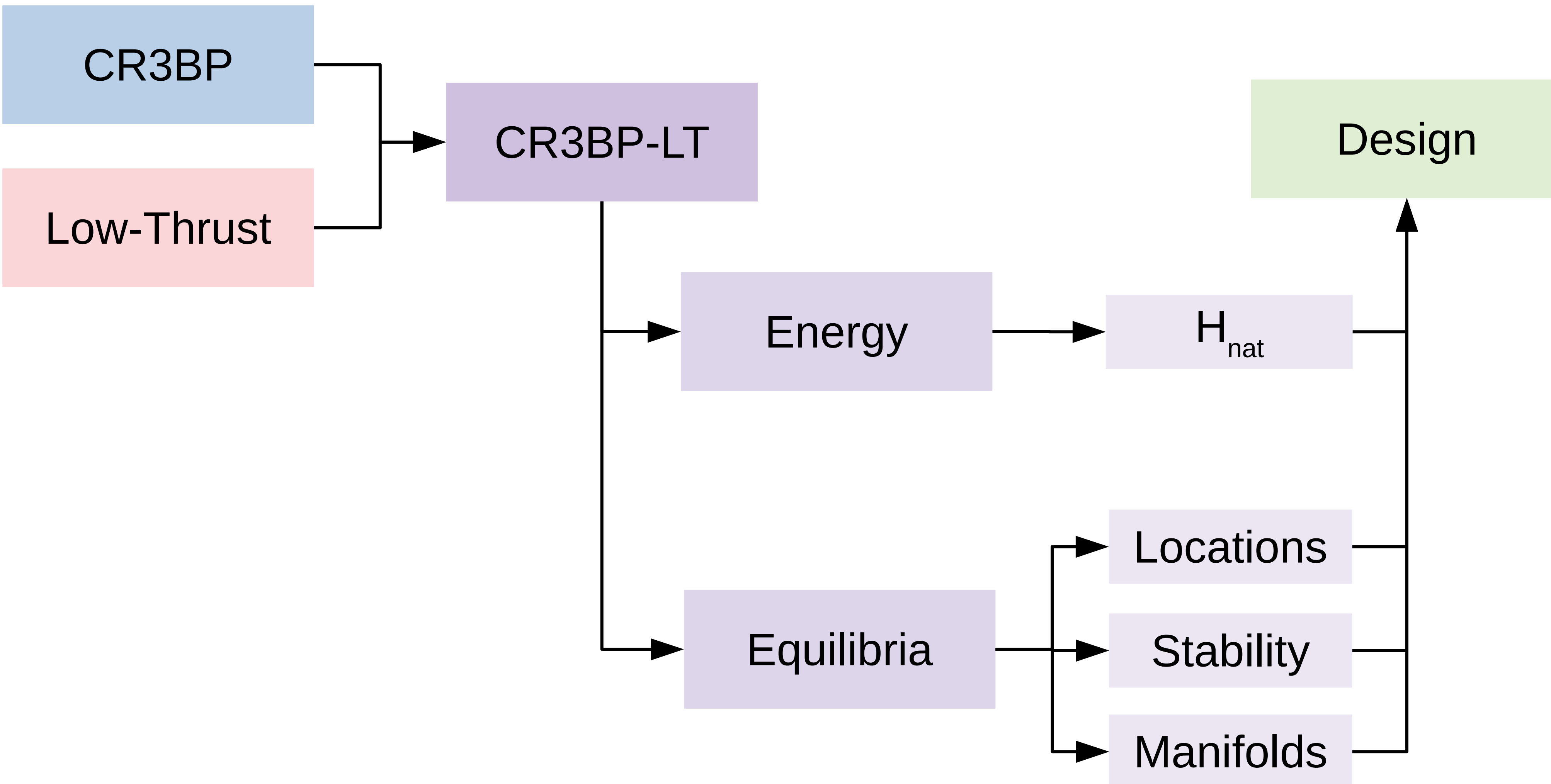
Motivation

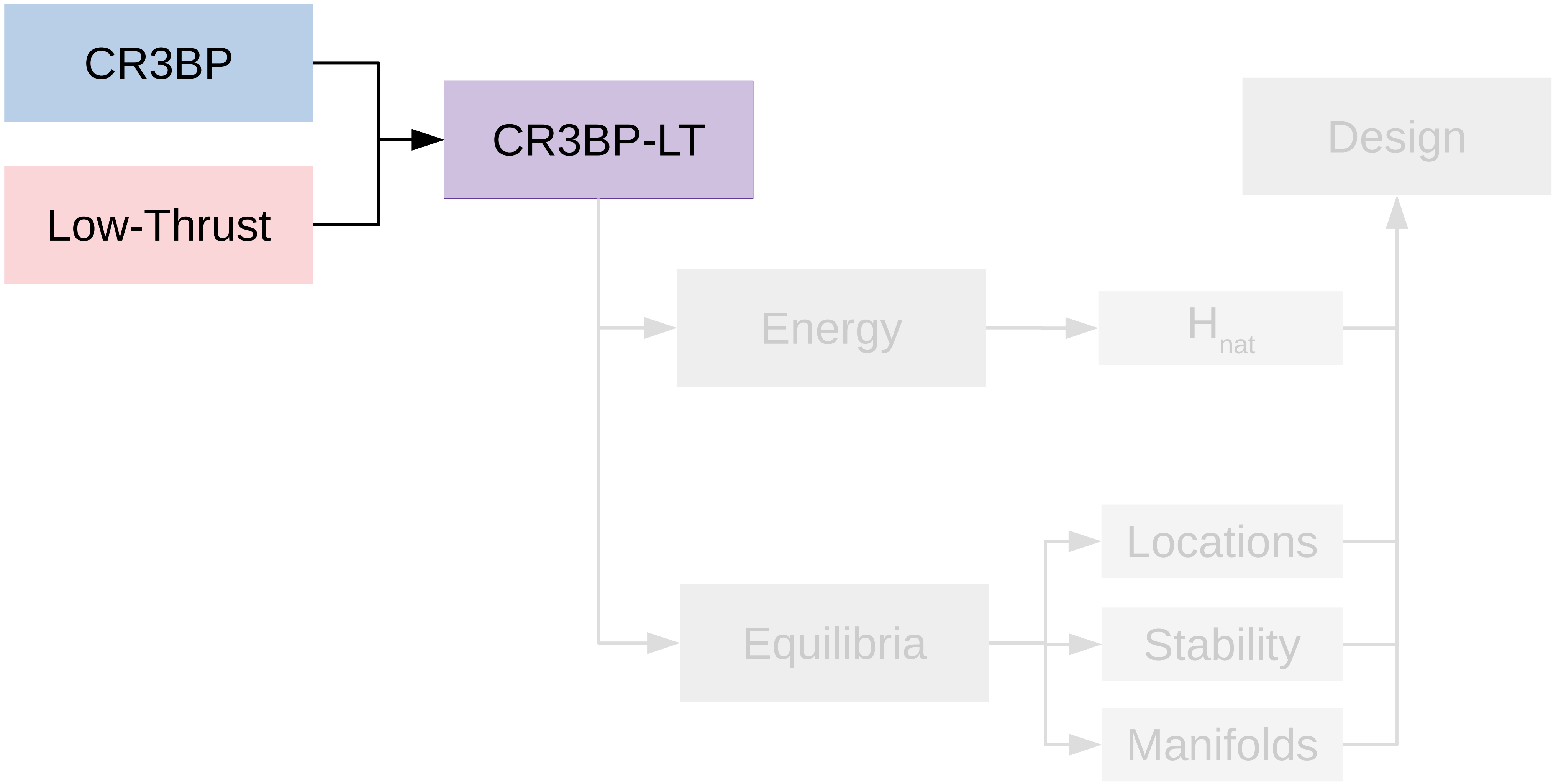
- Low-thrust trajectory design: require position, velocity, & control histories for initial guess
- Many current methods leverage optimization → point solution
- Chaotic multi-body regimes; initial guess may strongly bias result
- Ballistic designs benefit from available dynamical structures but supply no initial control history

Seek a more general understanding of low-thrust + multi-body dynamics

Roadmap


CR3BP
Low-Thrust
CR3BP-LT
Design
Energy
H_{nat}
Locations
Equilibria
Stability
Manifolds

Roadmap



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Combined Model

CR3BP + LT

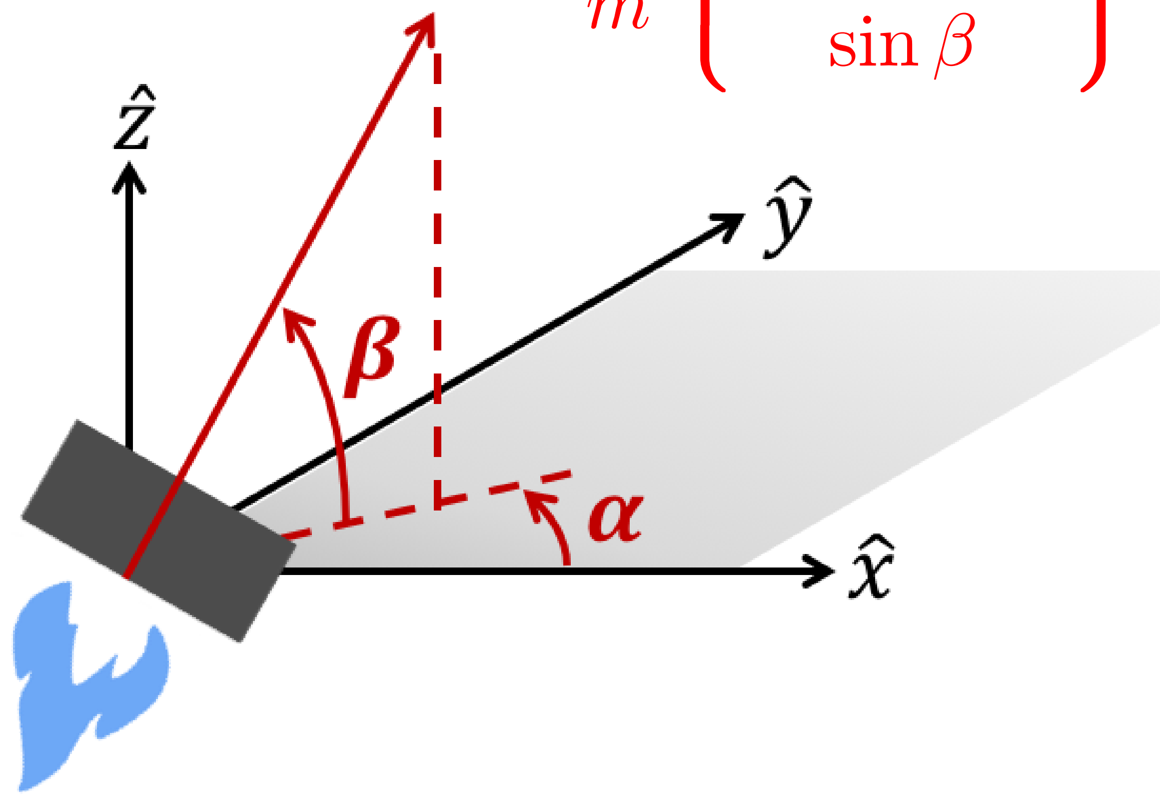
$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} + \vec{a}_{lt} \cdot \hat{x}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} + \vec{a}_{lt} \cdot \hat{y}$$

$$\ddot{z} = \frac{\partial \Omega}{\partial z} + \vec{a}_{lt} \cdot \hat{z}$$

$$\dot{m} = \text{const}$$

$$\vec{a}_{lt} = \frac{f}{m} \begin{Bmatrix} \cos\alpha\cos\beta \\ \sin\alpha\cos\beta \\ \sin\beta \end{Bmatrix}$$



Simplifications

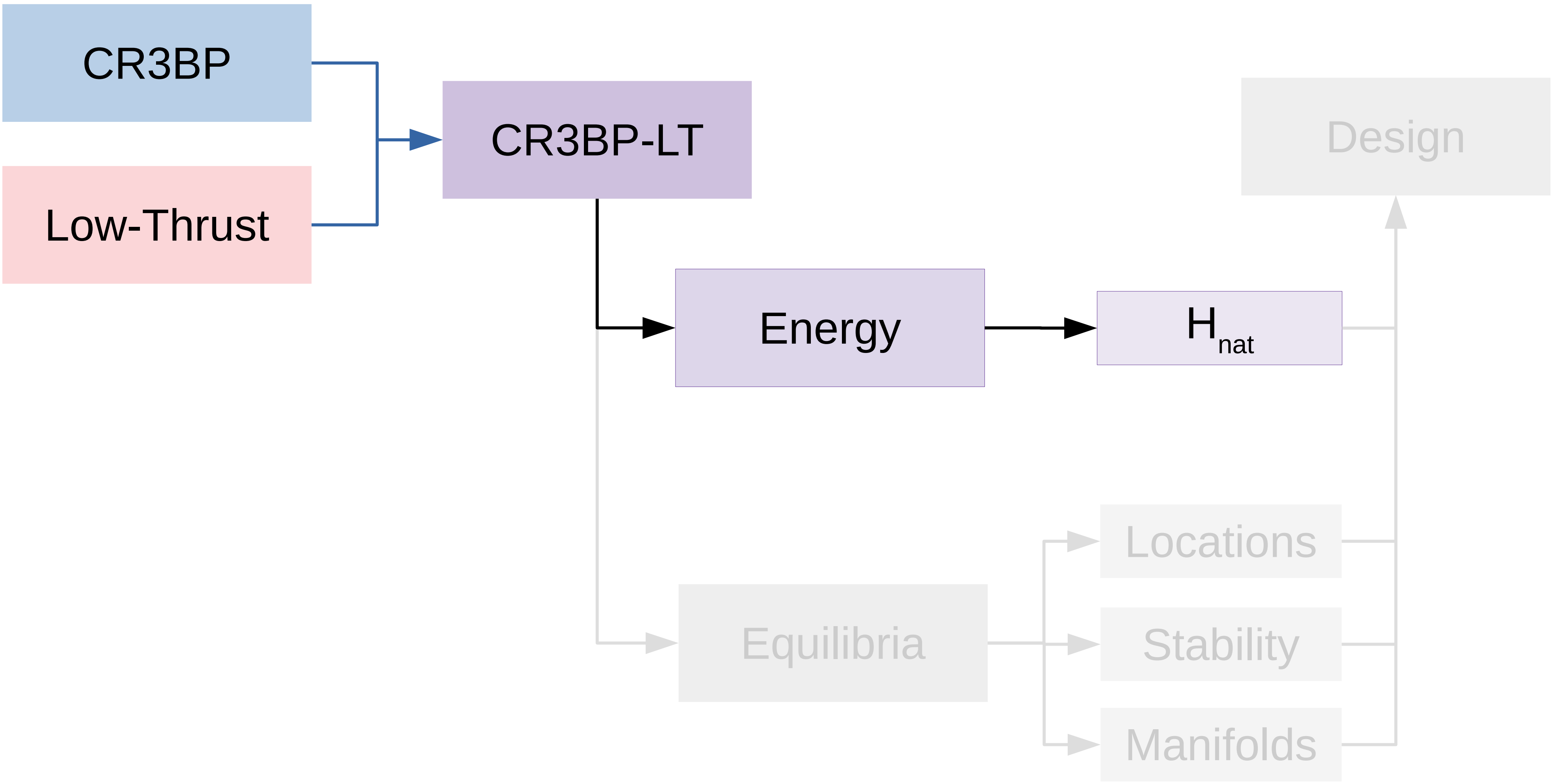
- $a_{lt} = \dfrac{f}{m} = \text{constant}$ (reasonable in Earth-Moon CR3BP-LT)
- Planar motion: $z = \dot{z} = 0, \;\beta = 0$

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} + a_{lt}\cos\alpha$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} + a_{lt}\sin\alpha$$

→ Conservative
Autonomous
Hamiltonian

Roadmap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Energy-Like Integral

Hamiltonian: $$H = \sum_{i=1}^{3} (p_i \dot{g}_i) - L(\vec{g},\ \dot{\vec{g}},\ \tau)$$

CR3BP (natural) system:

$$H_{nat} = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{1}{2}\left(x^2 + y^2\right) - \frac{1-\mu}{r_{13}} - \frac{\mu}{r_{23}} = -\frac{1}{2}C$$

Constant on ballistic arcs

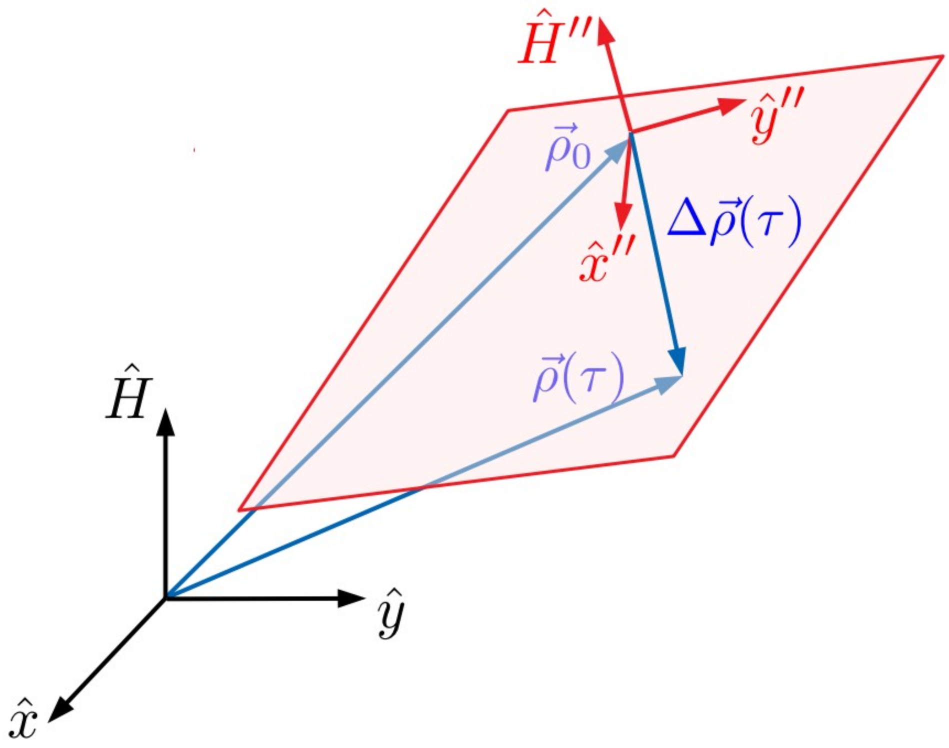
Varies on low-thrust arcs **independent of path**

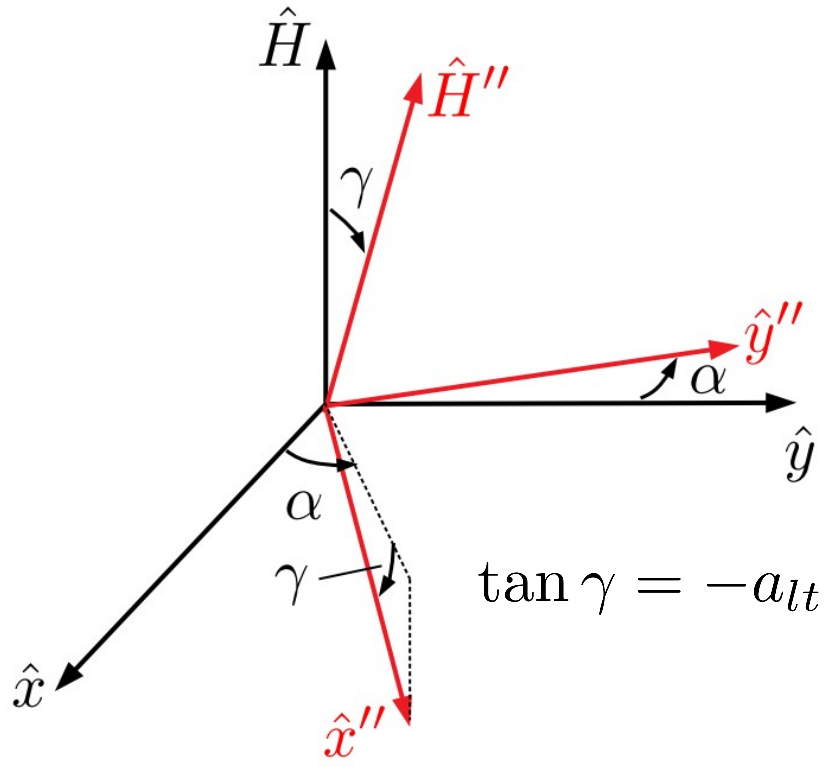
$$H_{nat}(t_f) - H_{nat}(t_0) = \left[\vec{r}(t_f) - \vec{r}(t_0)\right] \cdot \vec{a}_{lt}$$

Energy Plane

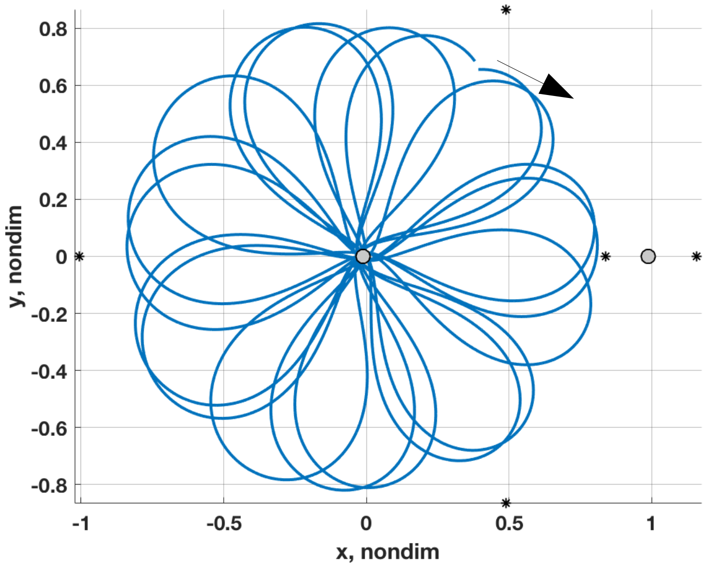
Control Point: $\vec{\rho} = \{x \quad y \quad H_{nat}\}$

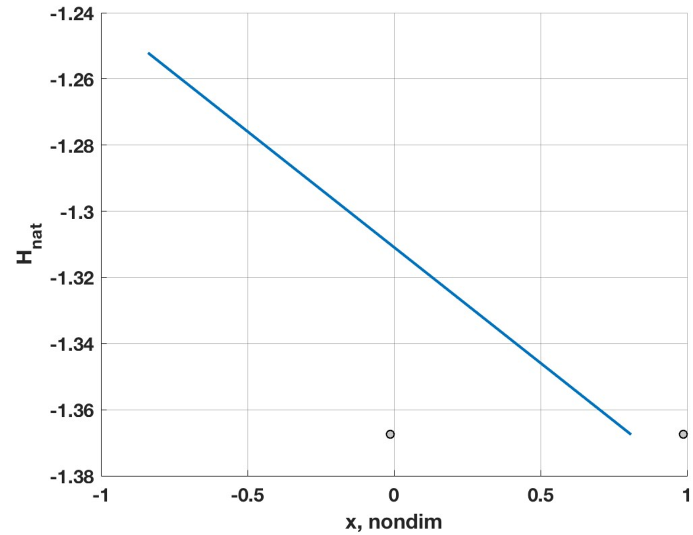
Every low-thrust arc (with single a_{lt} & α) is confined to an *energy plane*



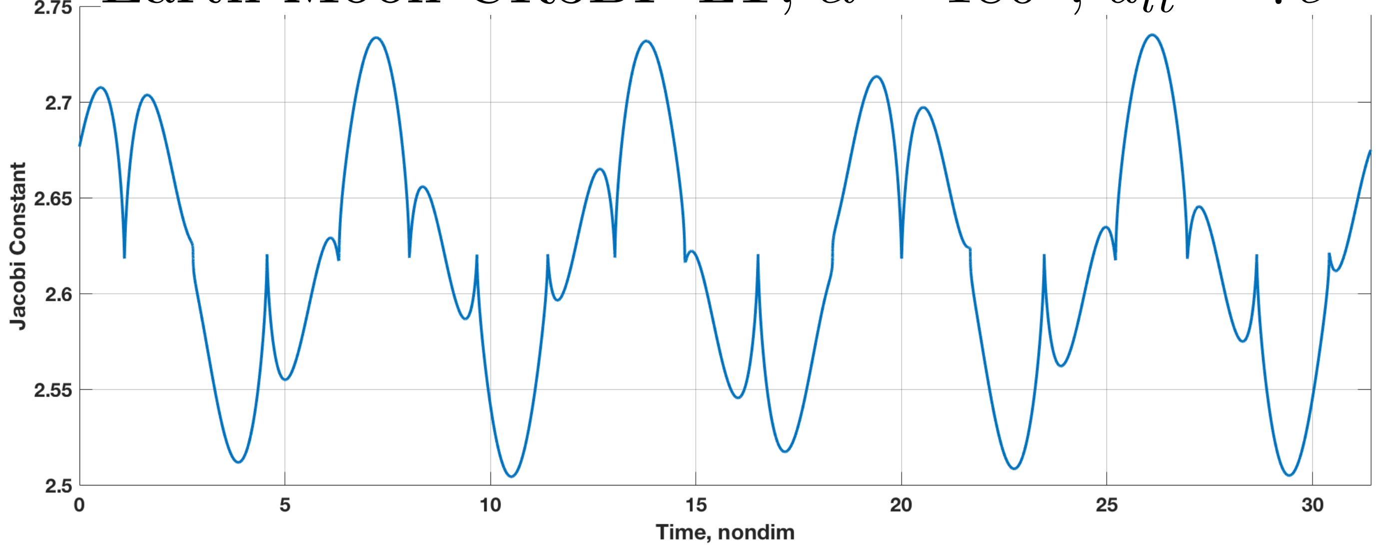


Energy Plane Example





Earth-Moon CR3BP-LT, $\alpha = 180°$, $a_{lt} = 7e-2$



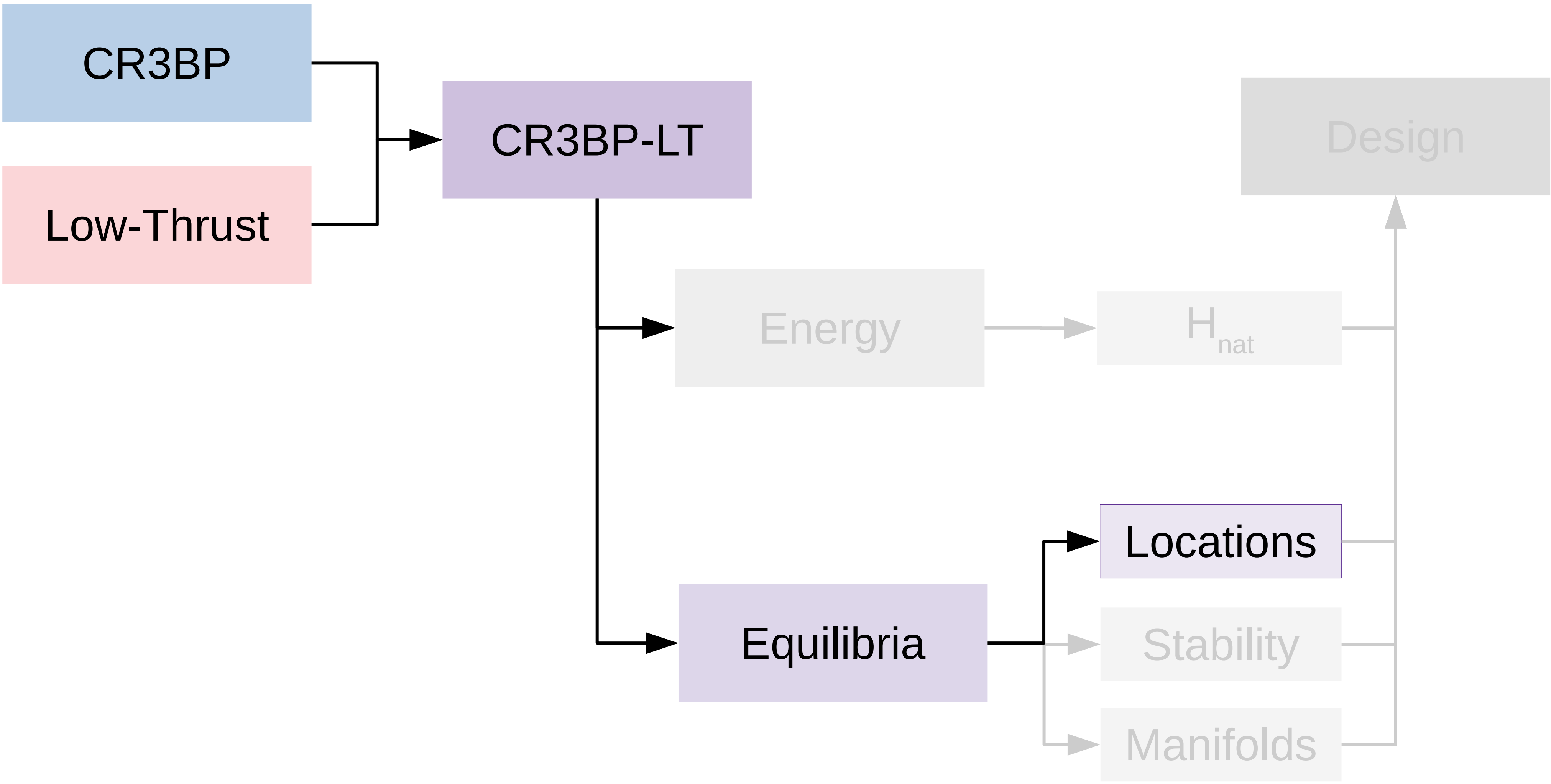
Simple

Intuitive

Complex

Unintuitive

Roadmap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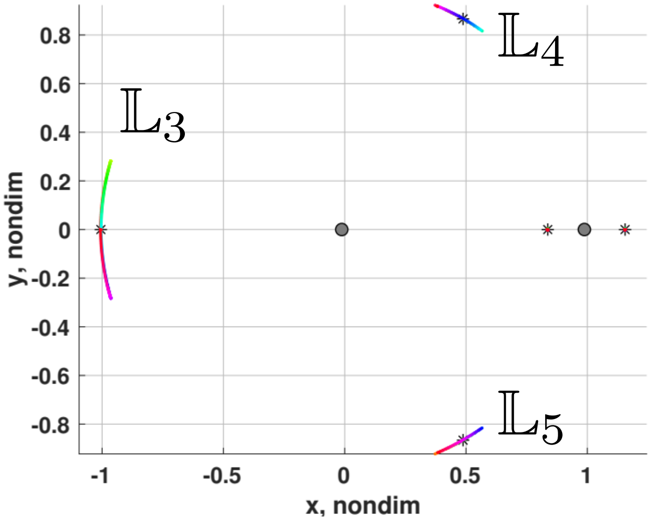
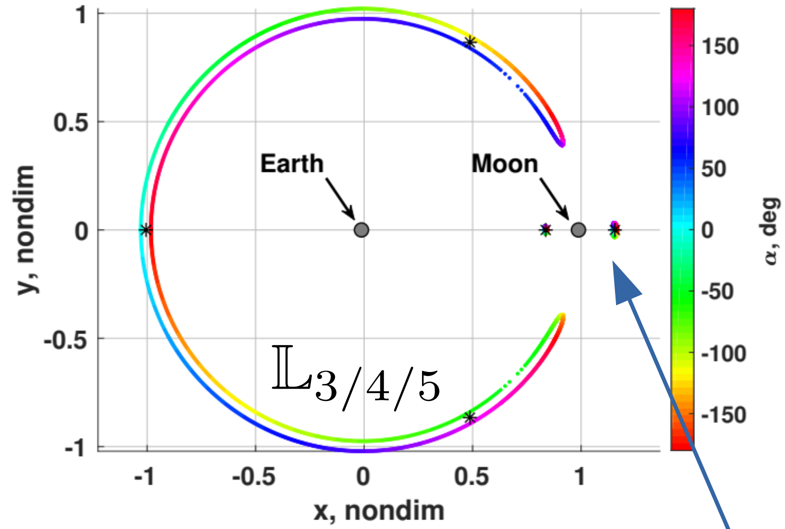
Planar Low-Thrust Equilibria



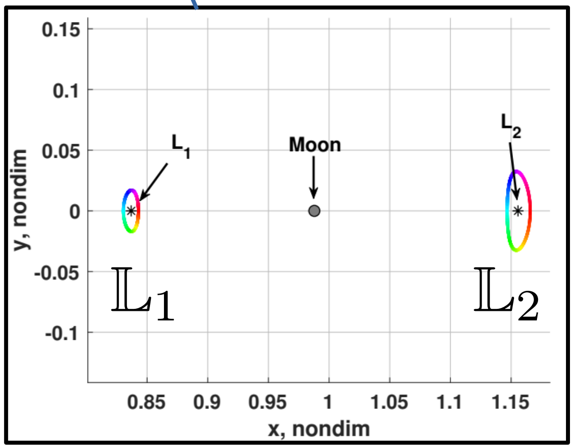
Small a_{lt} value

a_{lt} = 8.73e-3

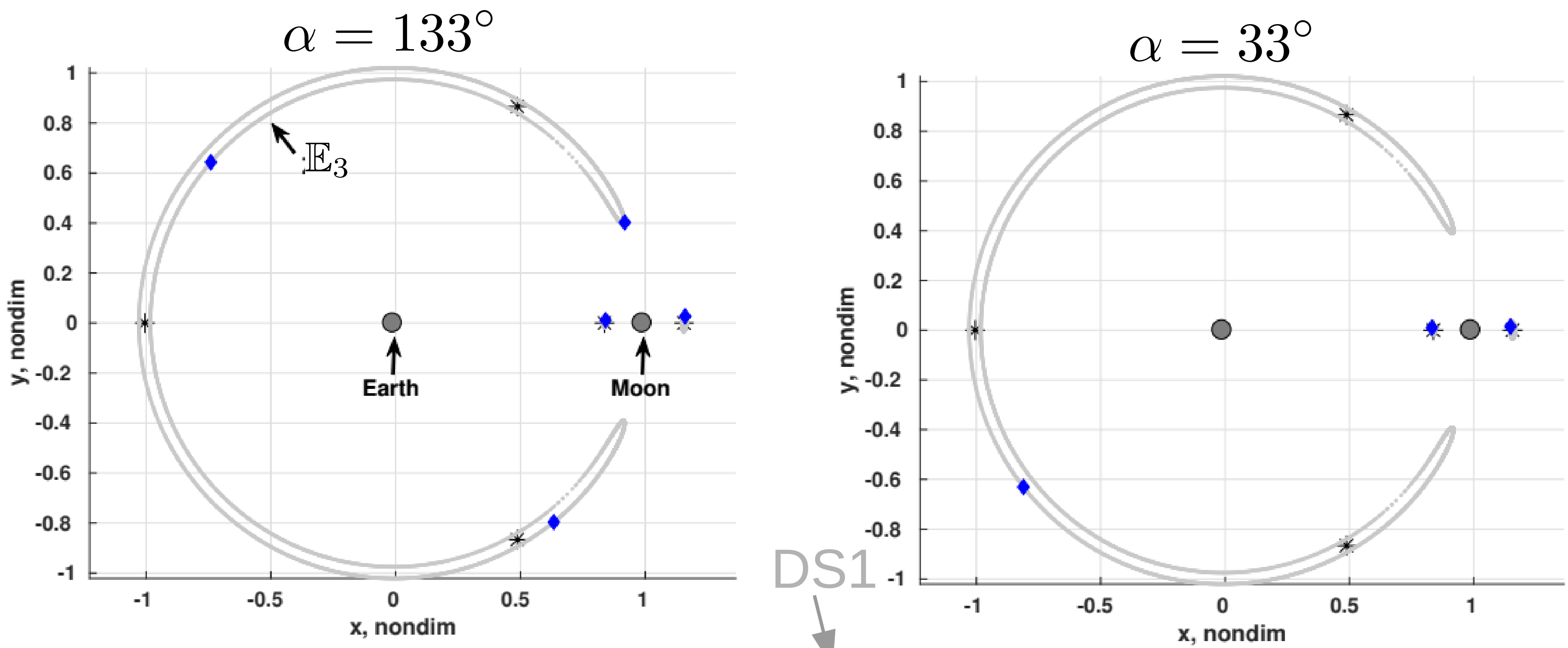


Larger a_{lt} value (DS1)

a_{lt} = 7.0e-2



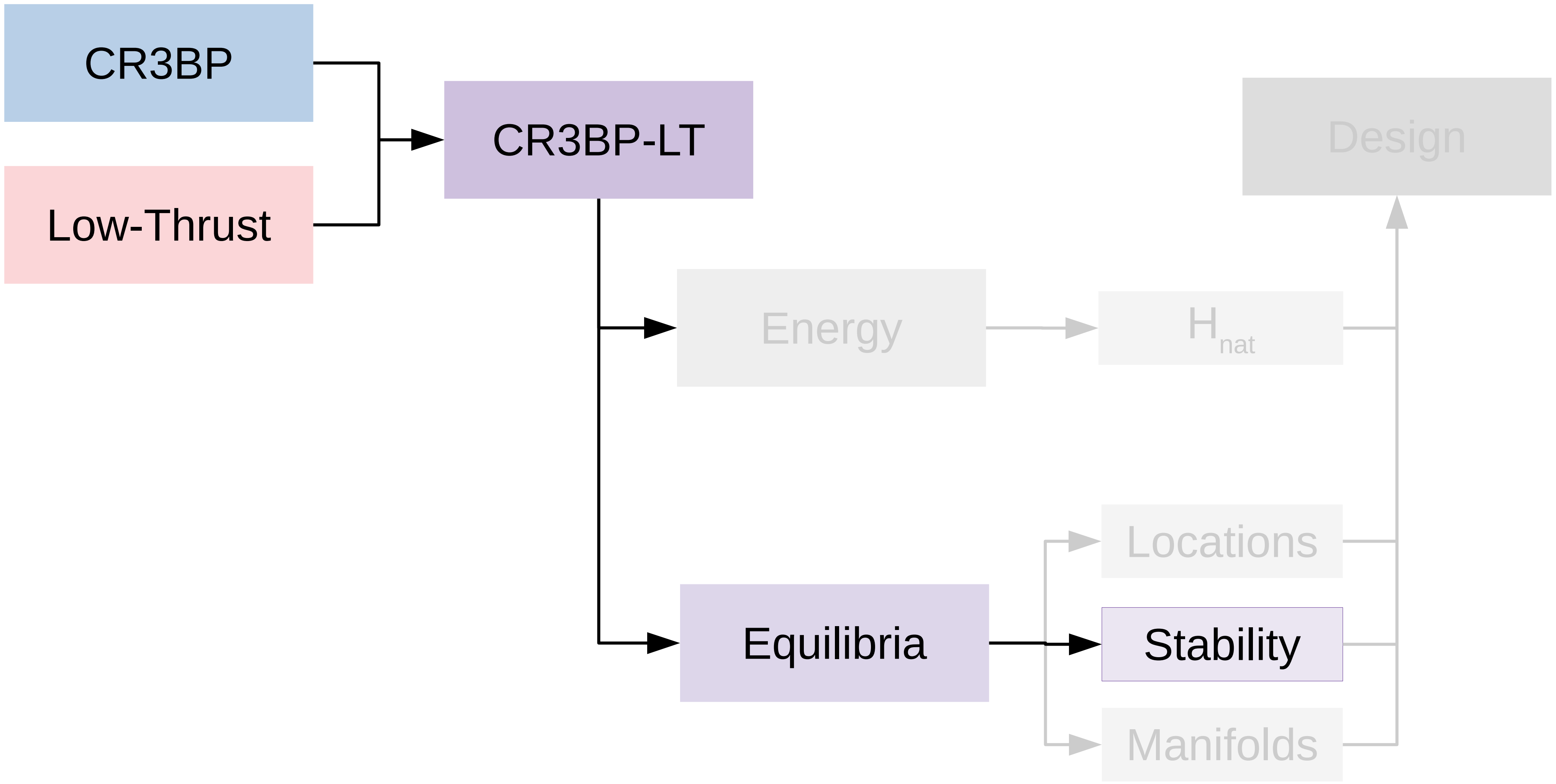
Distinct Equilibrium Points



Earth-Moon CR3BP-LT, $a_{lt} = 7.00\text{e-}2, \ \beta = 0$

Accel. Mag. and direction determine equilibria locations and count

Roadmap



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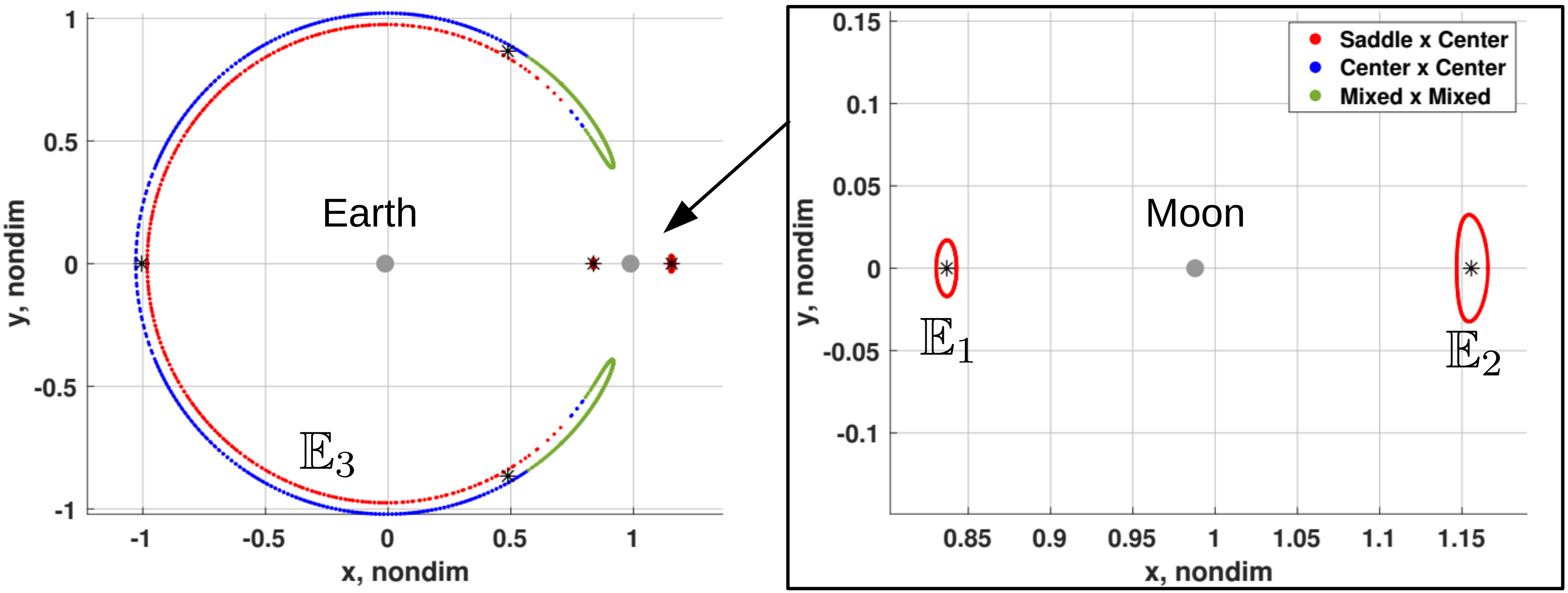
Equilibria Stability



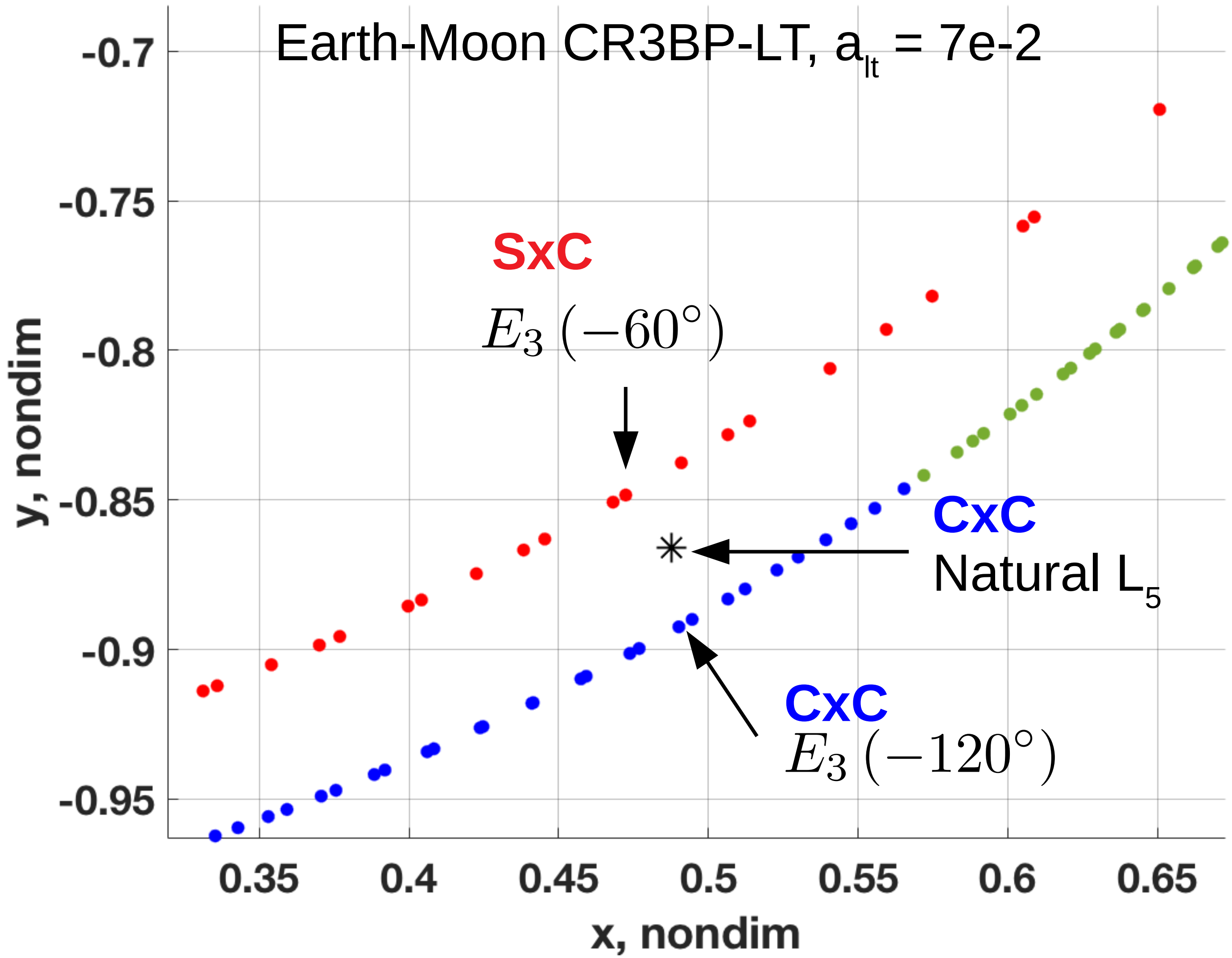
Natural CR3BP

- L_1, L_2, L_3 – **SxC**
- L_4, L_5 – **CxC**

CR3BP-LT for a_{lt} = 7e-2

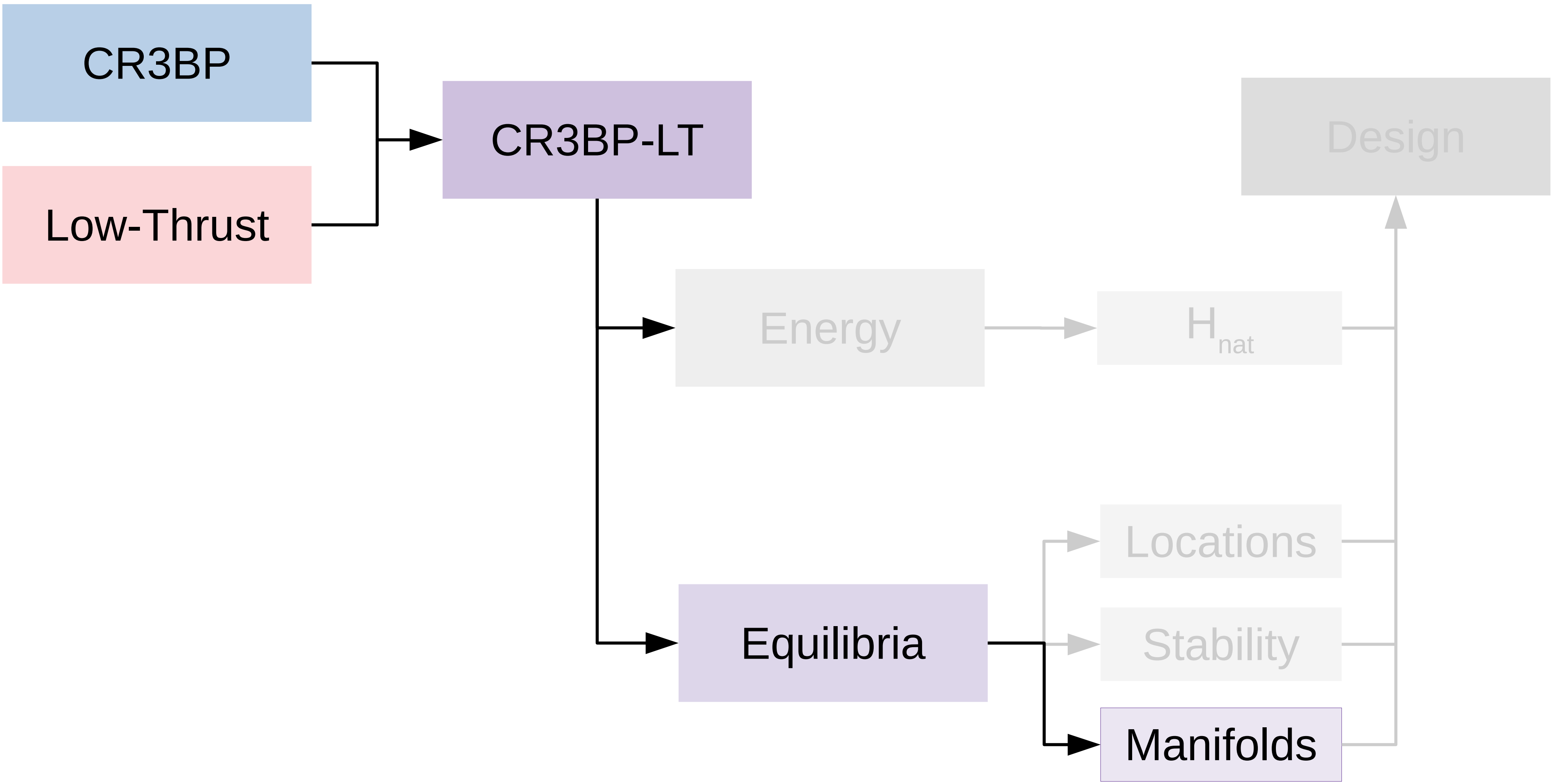
- $\mathbb{E}_1, \mathbb{E}_2$ – **SxC**
- $\mathbb{E}_3$ – **CxC**, **SxC**, & **MxM**

Near L_5



Earth-Moon CR3BP-LT, a_{lt} = 7e-2
SxC
$E_3\,(-60^\circ)$
CxC
Natural L_5
CxC
$E_3\,(-120^\circ)$
-0.7
-0.75
-0.8
-0.85
-0.9
-0.95
y, nondim
0.35
0.4
0.45
0.5
0.55
0.6
0.65
x, nondim

Roadmap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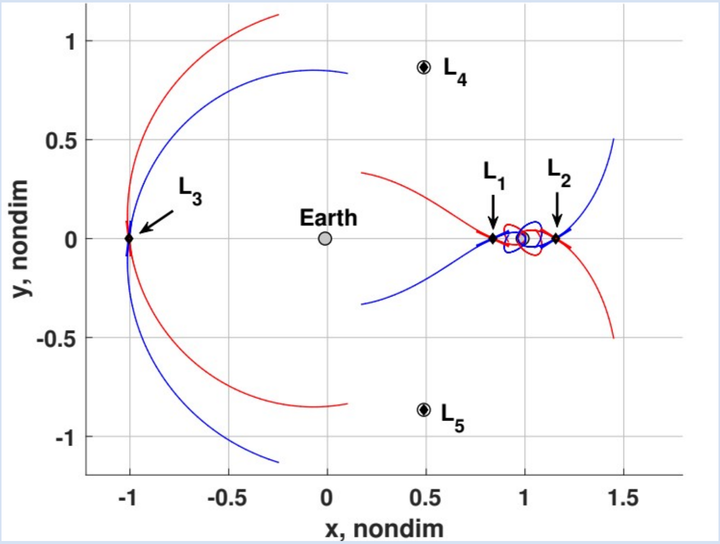
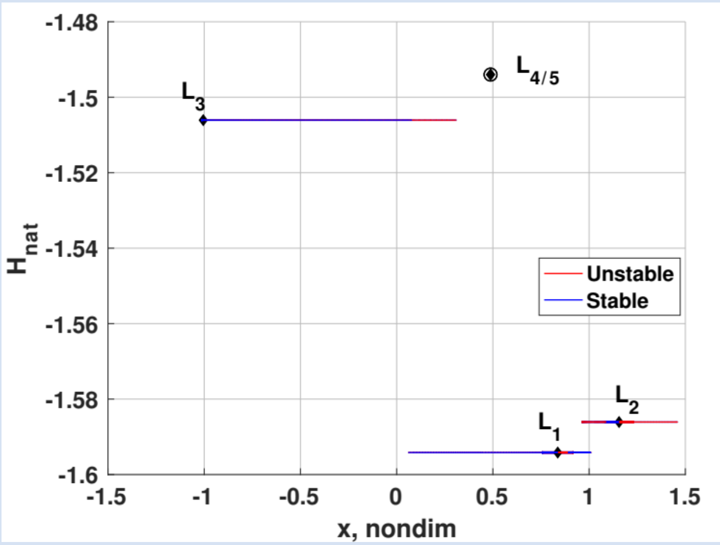
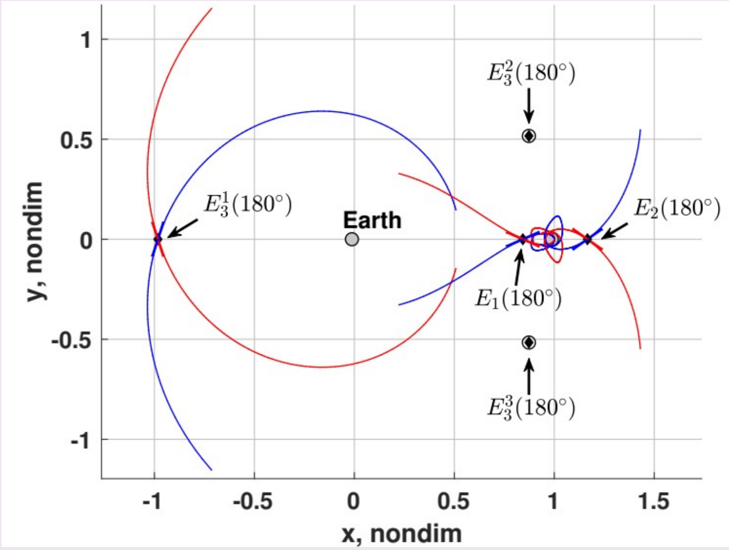
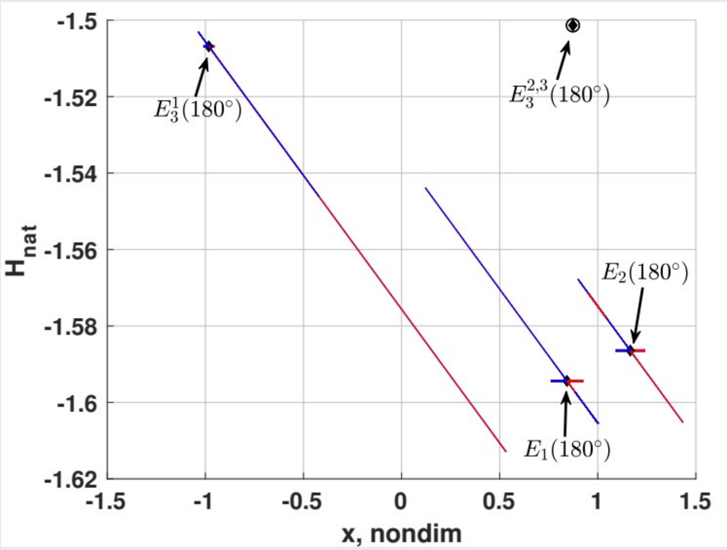
Equilibria Manifolds

Natural CR3BP





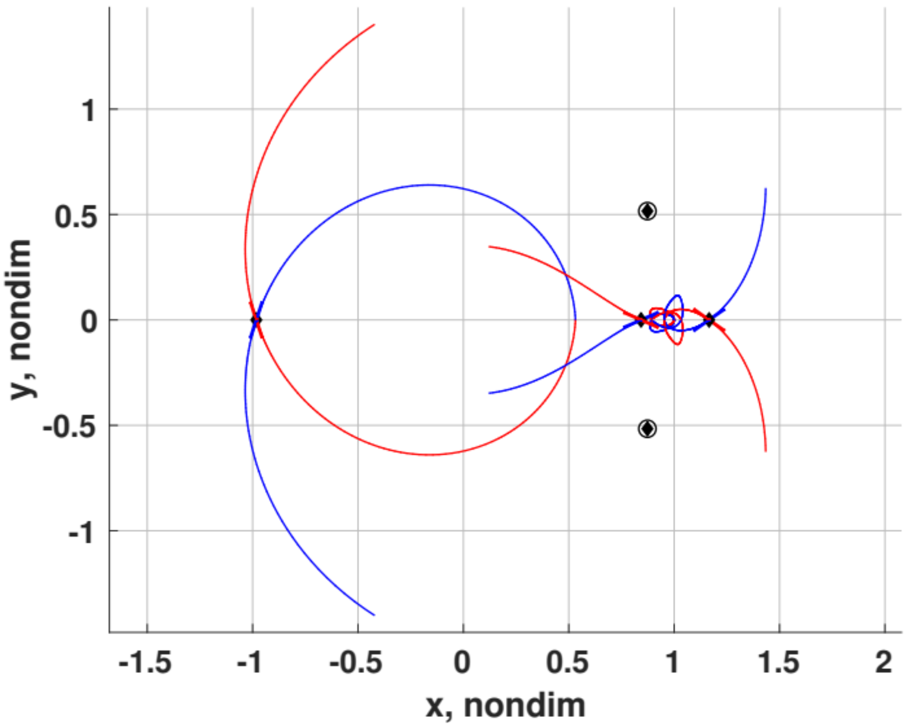




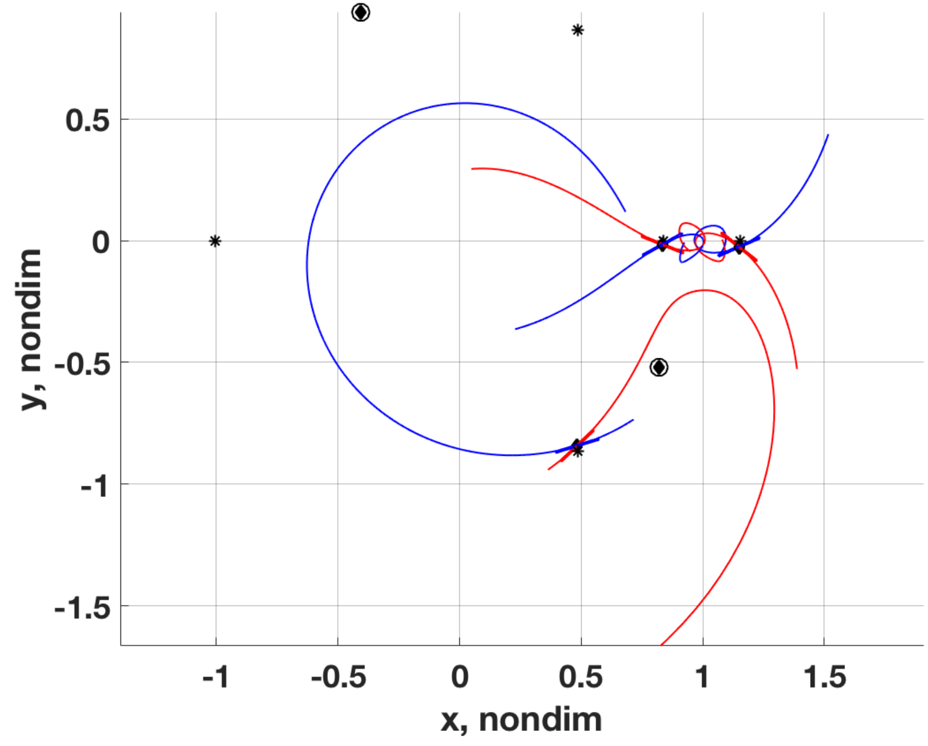
CR3BP-LT
a_{lt} = 7e-2
α = 180°

Equilibria Manifolds

Earth-Moon CR3BP-LT a_{lt} = 7e-2

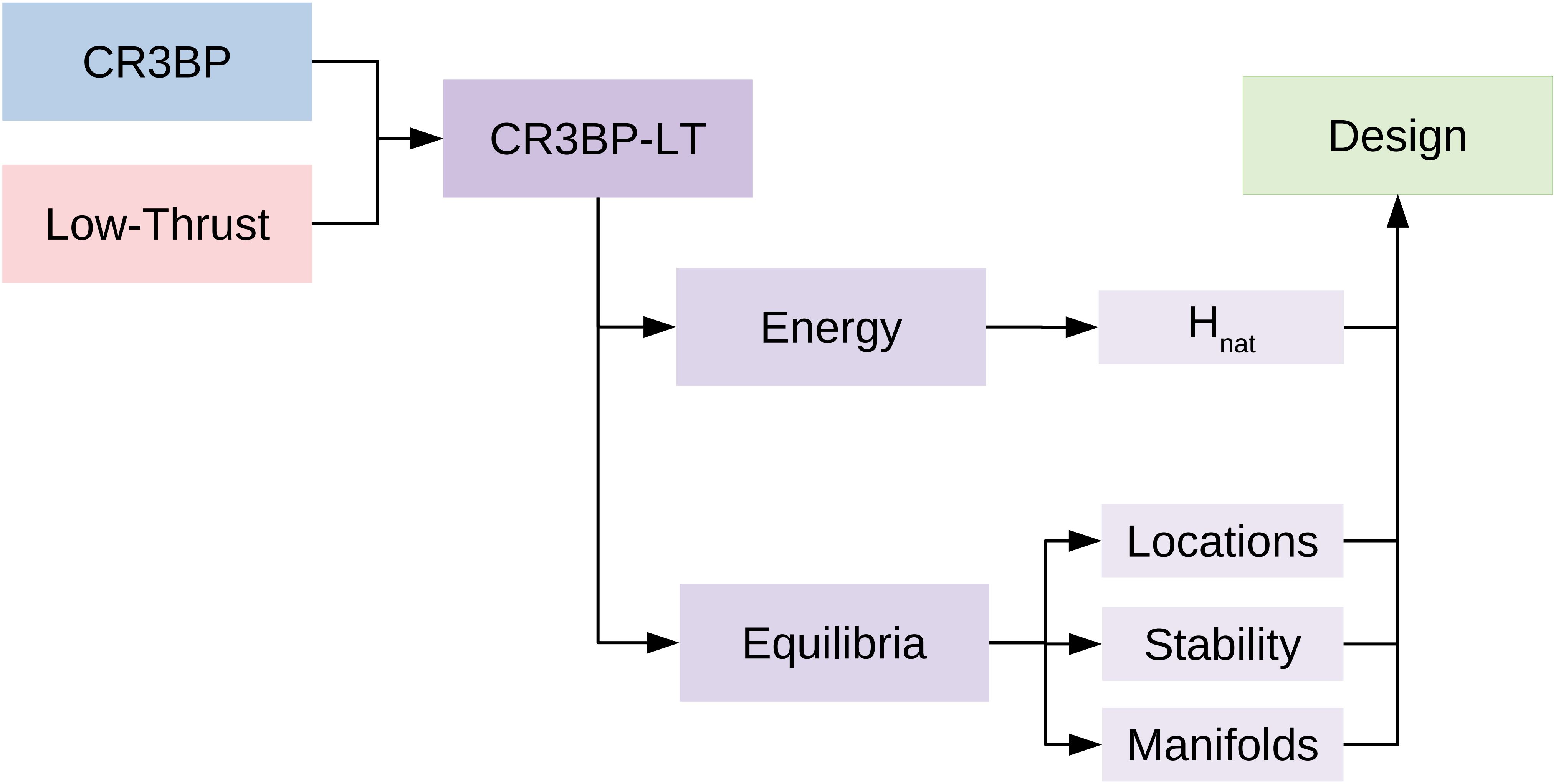


α = 180°



α = -60°

Roadmap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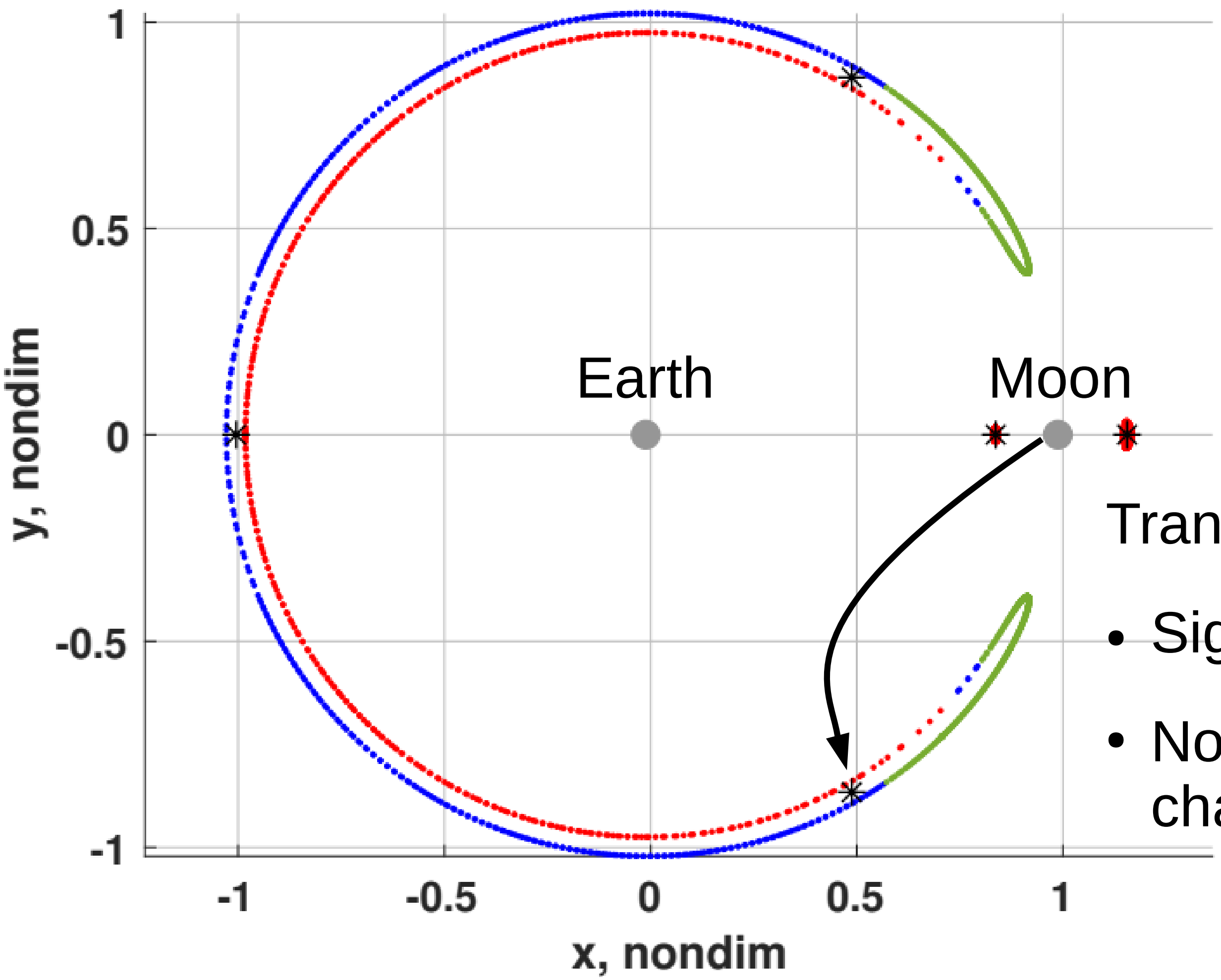
Transit Design: Setup

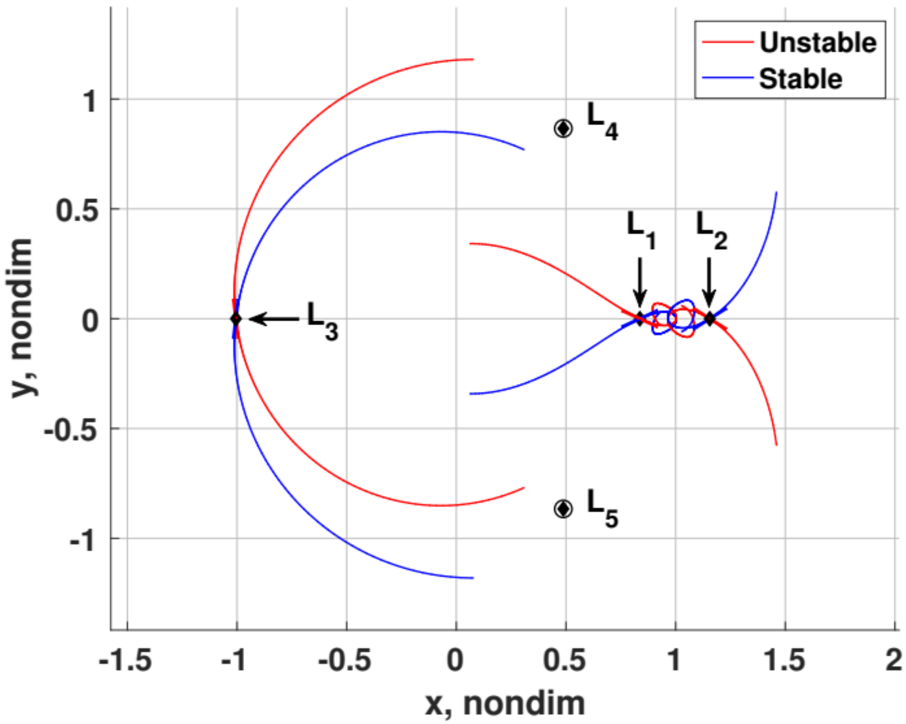
- Demonstrate use of dynamical structures and techniques

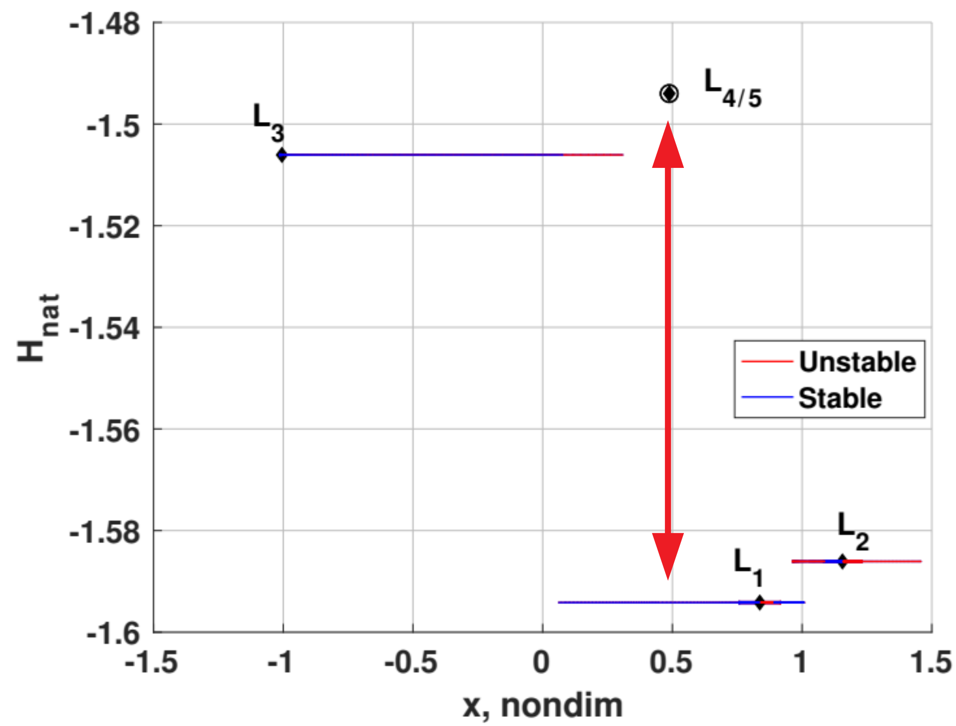


Transfer from Moon to L_5

- Significant ΔH_{nat}
- Nontrivial geometric change as well

Use Natural Structures?

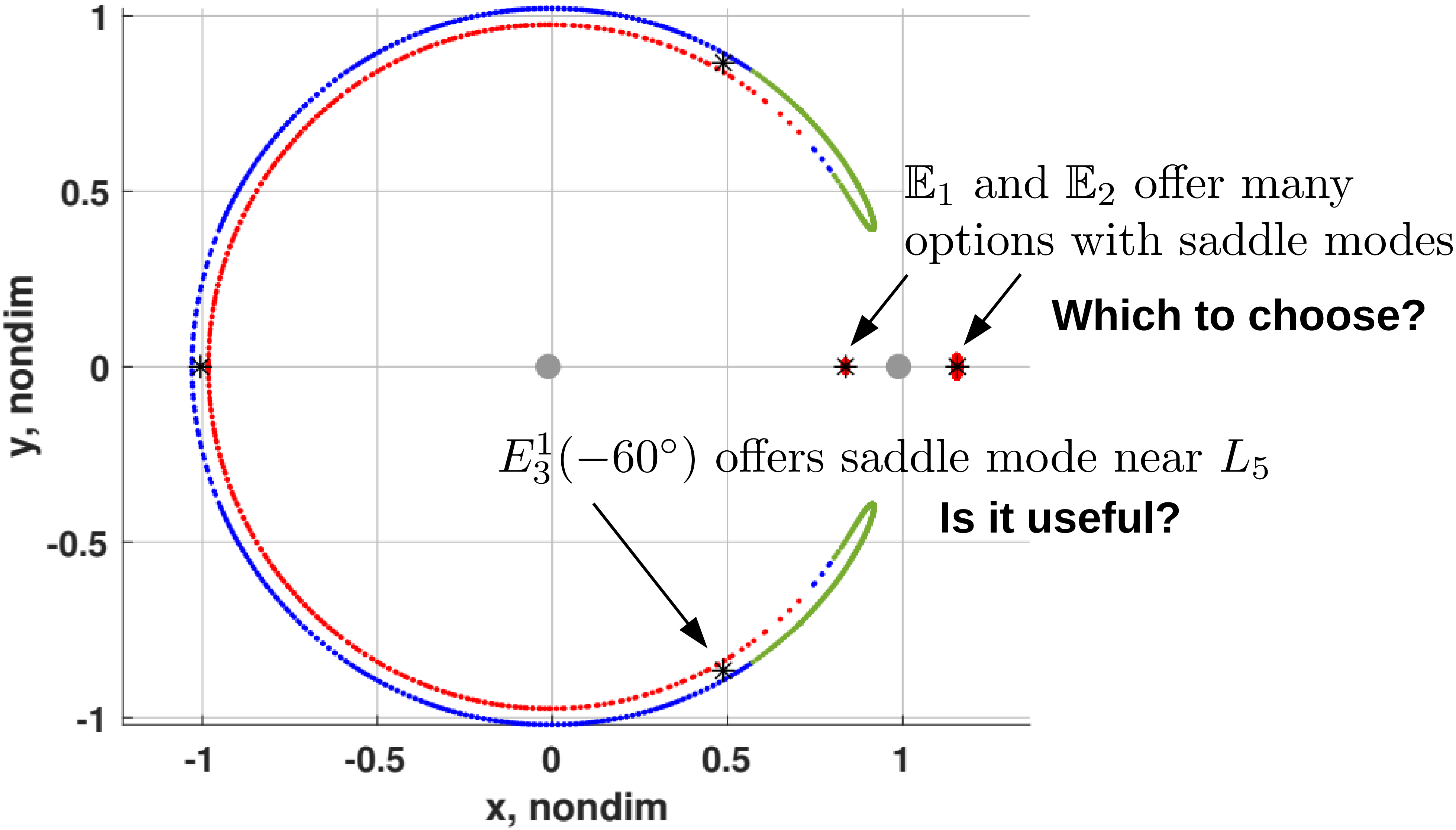




- L_1 and L_2 – saddle mode, offer access to moon
- L_5 – no saddle mode

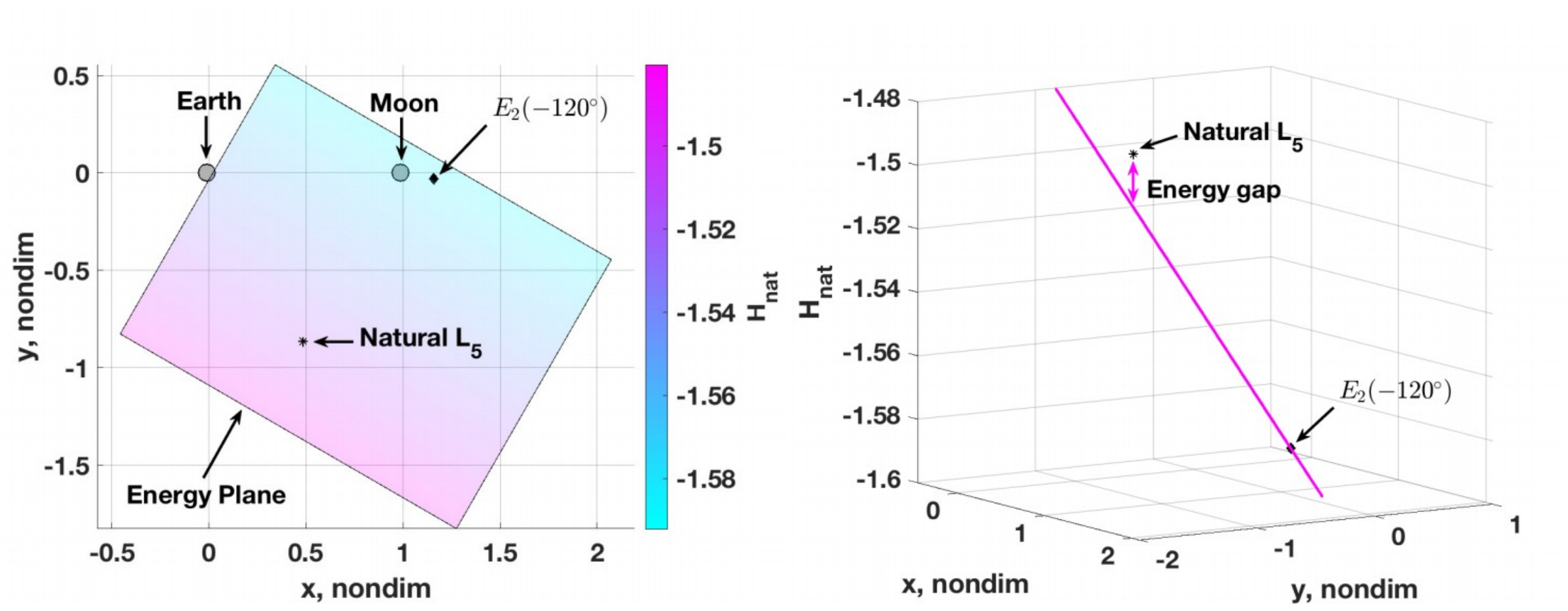
- Even with favorable geometry, energy difference is large
- Natural manifolds do not supply energy change

Use Low-Thrust Structures?



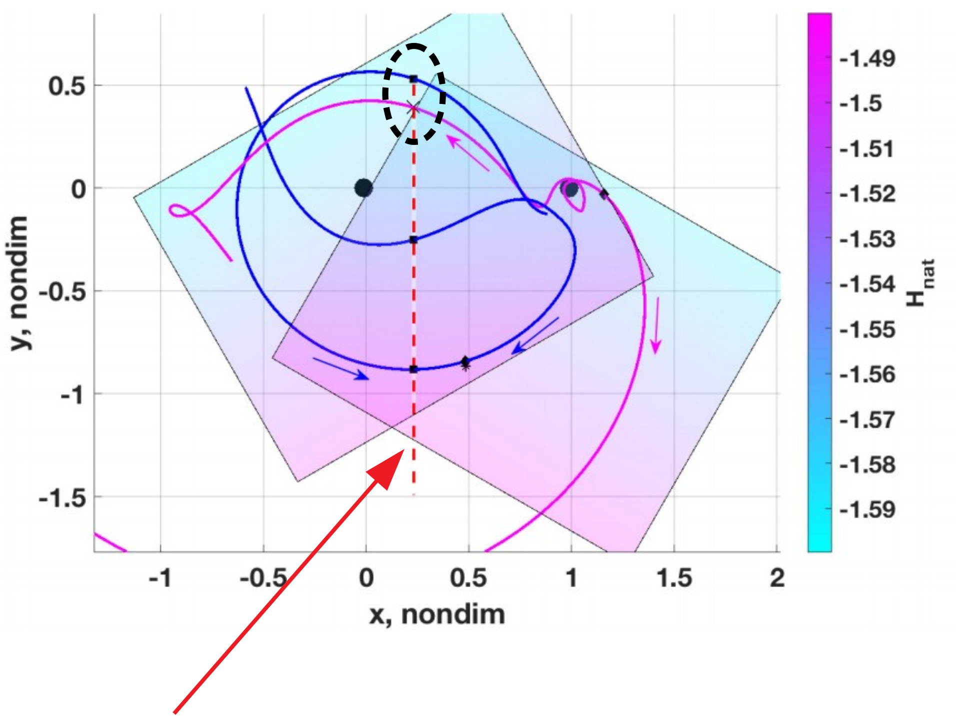
Low-Thrust Structures

1. Choose $\mathbb{E}_2$ (arbitrary)
2. Use energy plane to select equilibrium point with maximum H_{nat} increase

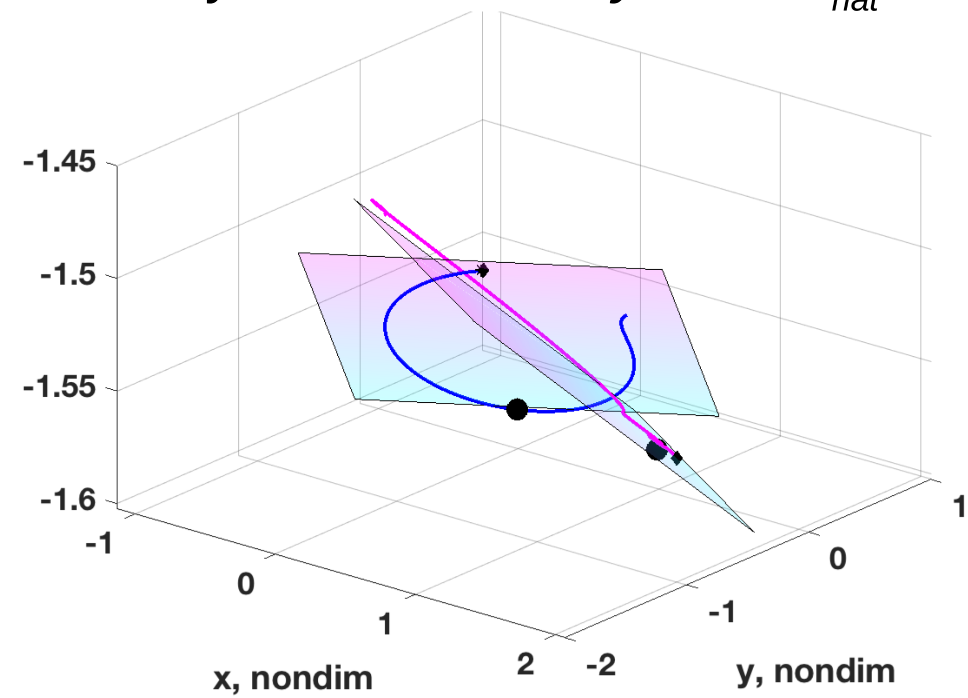


Additional Structures

3. Include $E_3^1(-60^\circ)$ stable manifolds (blue)

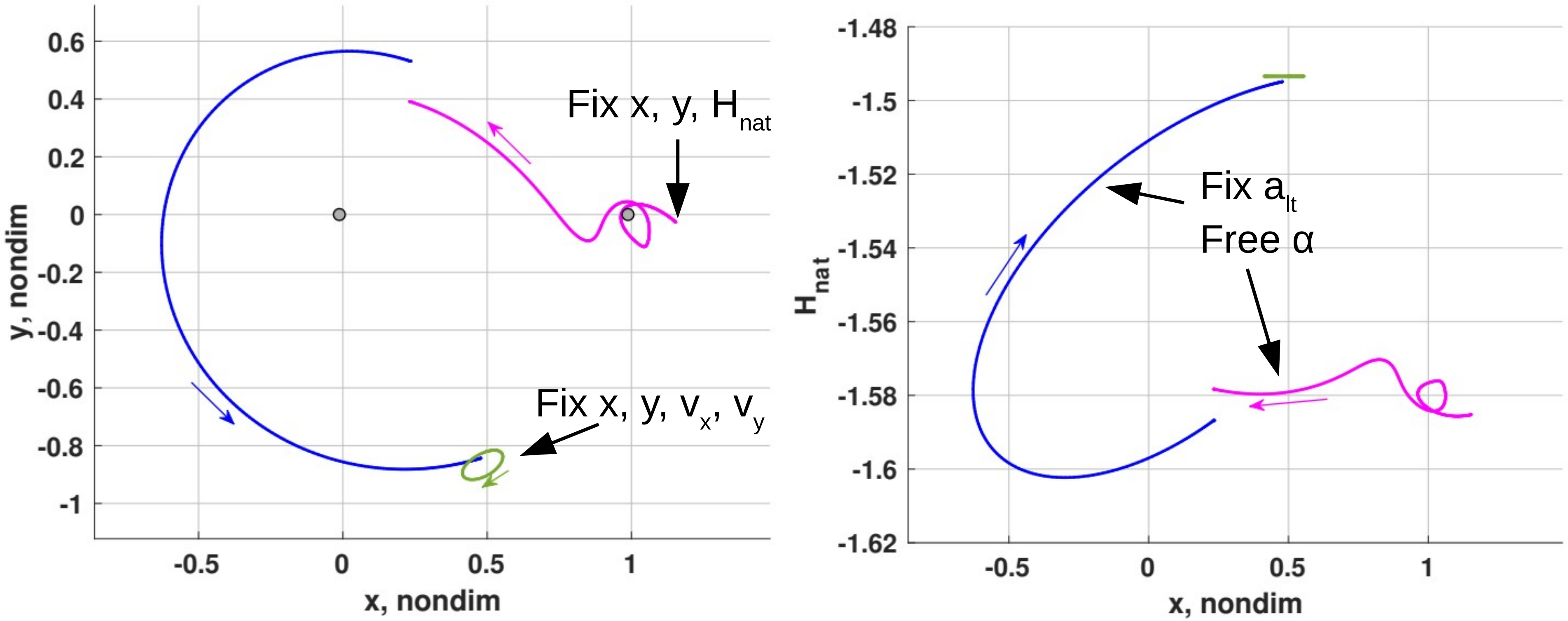


4. Select candidate arcs that nearly intersect in *x*, *y*, and H_{nat}



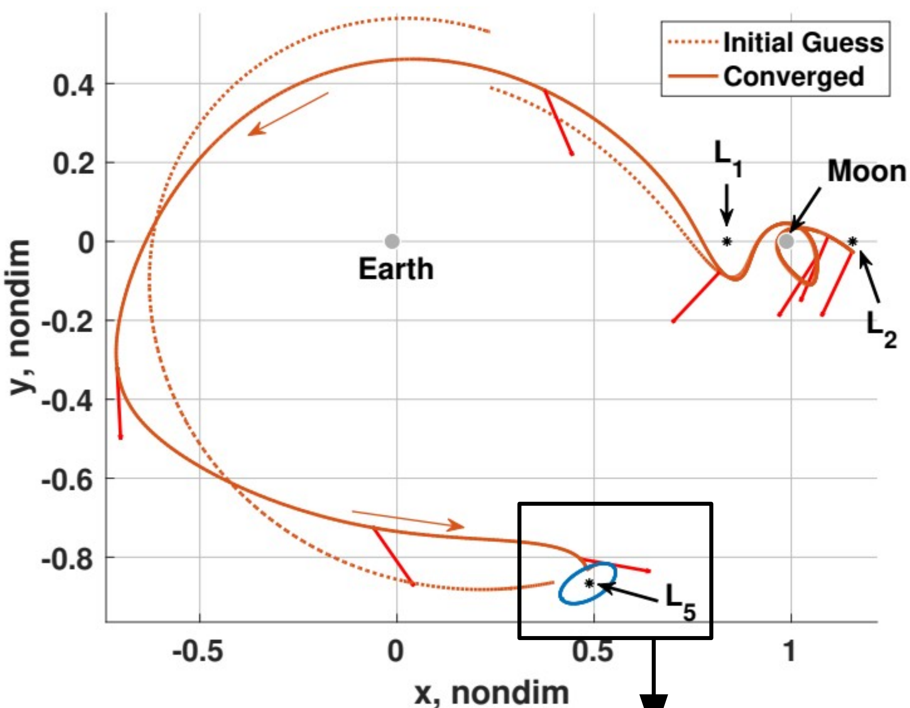
Intersection of two energy planes is convenient hyperplane: nearby points possess similar positions *and* energies

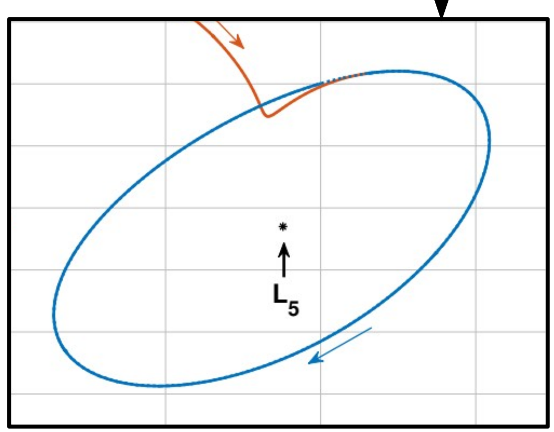
Sample Transfer: Initial Design

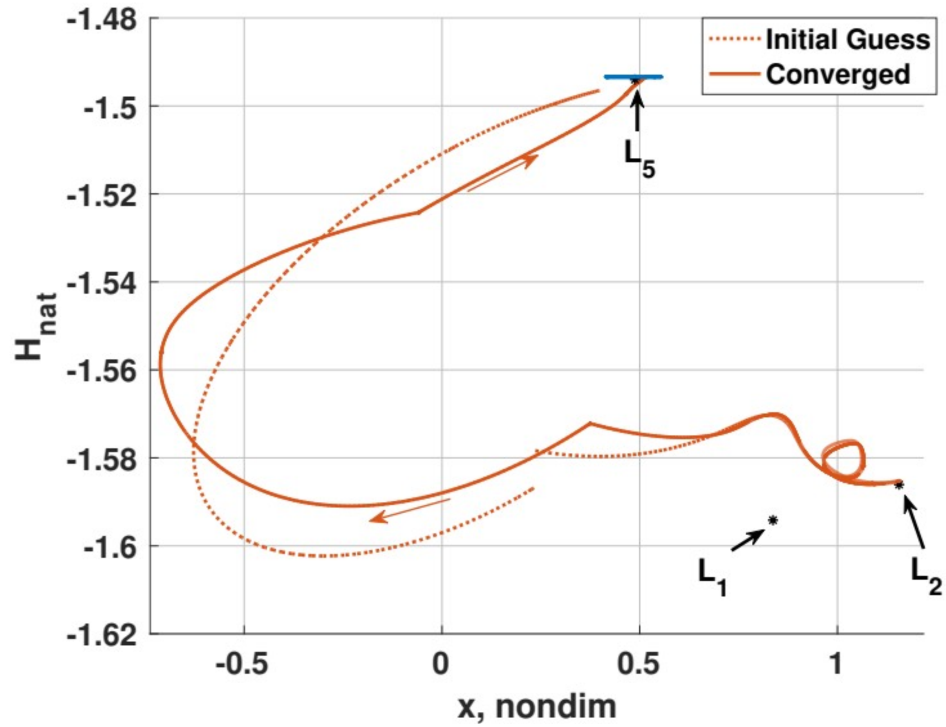


5. Include L_5 short period orbit (SPO) near destination to maintain proximity to L_5
6. Discretize into smaller arcs
7. Correct for continuity

Sample Transfer: Feasible Sol’n







- Rapid convergence
- Initial geometry and energy profile preserved (unsurprising; min-norm)

Conclusions

- Reasonable assumptions yield conservative, autonomous CR3BP-LT
- Low-thrust equilibria possess diverse locations & stability, function of thrust vector magnitude and orientation
- Energy along low-thrust arc confined to a plane oriented by thrust vector properties
- Links between thrust vector and arc geometry & energy facilitate initial design for thrust vector

Acknowledgements

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Backup Slides

Thrust Magnitude

$$f = \frac{F t_*^2}{1000 l_* M_{3,0}}$$

Spacecraft	$M_{3,0}$ kg	F mN	a_{lt} m/s^2	f (Earth-Moon) nondim	f (Sun-Earth) nondim
Deep Space 1[1]	486	92.0	1.893e-4	6.95e-2	3.19e-2
Hayabusa[2]	510	24.0	4.706e-5	1.73e-2	0.79e-2
Dawn[3]	1218	92.7	7.611e-5	2.79e-2	1.28e-2
Lunar IceCube[4]	14	1.0	7.143e-5	2.62e-2	1.20e-2

Nondimensional magnitude between 1e-2 and 1e-1 is “reasonable”

Hamiltonian Time Derivative(s)

$$H_{lt} = H_{nat} - \vec{r} \cdot \vec{a}_{lt}$$

$$\frac{\partial H_{nat}}{\partial \tau} = \vec{v} \cdot \vec{a}_{lt}$$

$$\frac{\partial}{\partial \tau}\left[\vec{r} \cdot \vec{a}_{lt}\right] = \vec{v} \cdot \vec{a}_{lt} - \vec{r} \cdot \dot{\vec{a}}_{lt}$$

$\hat{u} \perp \vec{v}$	H_{nat} = const.
$\hat{u} \parallel \vec{v}$	Maximize H_{nat} rate of change

If $\dot{\vec{a}}_{lt} = \vec{0}$

then $\frac{\partial}{\partial \tau}\left[\vec{r} \cdot \vec{a}_{lt}\right] = \vec{v} \cdot \vec{a}_{lt}$

and H_{lt} = const.*

*Demonstrated reasonable via Monte Carlo with variable a_{lt} in Earth-Moon system

Energy-Like Quantities

CR3BP-LT system:

$$H_{lt} = H_{nat} - \vec{r} \cdot \vec{a}_{lt}$$

- Constant on low-thrust arcs when $\mathbf{a}_{\mathbf{lt}}$ is const. (mag., angle)
- Rapidly changed by varying a_{lt} or α

Energy Plane Proof

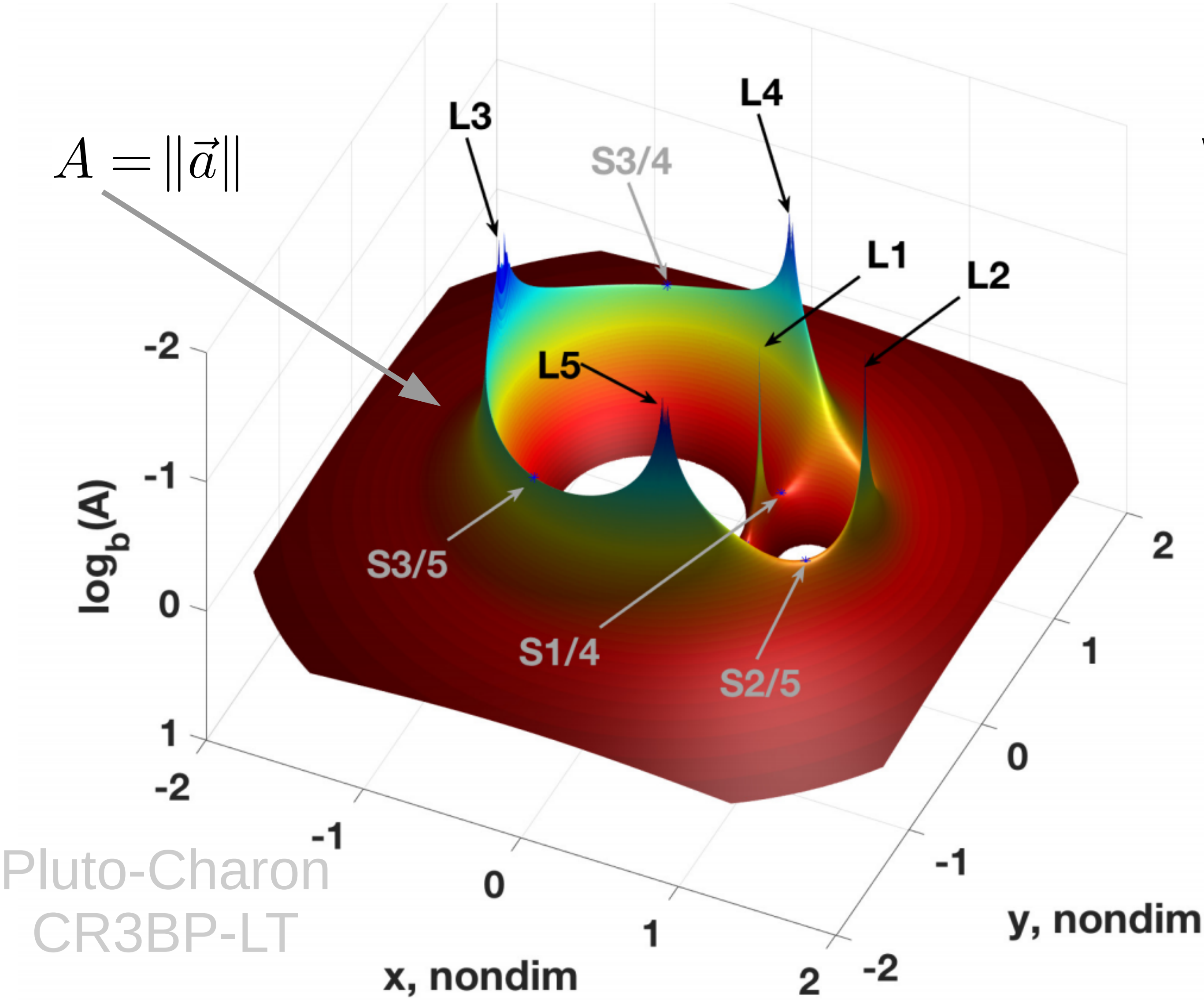
$$\Delta\vec{\rho} = \vec{\rho}(\tau_2) - \vec{\rho}(\tau_1) = (x_2 - x_1)\hat{x} + (y_2 - y_1)\hat{y} + (H_{nat,2} - H_{nat,1})\hat{H}$$
$$= \Delta x\hat{x} + \Delta y\hat{y} + \Delta H\hat{H}$$
$$= \left[\Delta x C_\alpha C_\gamma + \Delta y S_\alpha C_\gamma - \Delta H S_\gamma\right]\hat{x}'' + \left[\Delta y C_\alpha - \Delta x S_\alpha\right]\hat{y}'' +$$
$$\left[\Delta x C_\alpha S_\gamma + \Delta y S_\alpha S_\gamma + \Delta H C_\gamma\right]\hat{H}''$$

For in-plane motion: $\Delta\vec{\rho}\cdot\hat{H}'' = 0$

$$\Delta\vec{\rho}\cdot\hat{H}'' = 0 = \Delta x C_\alpha S_\gamma + \Delta y S_\alpha S_\gamma + \Delta H C_\gamma$$
$$= \Delta H + \tan\gamma\left[\Delta x C_\alpha + \Delta y S_\alpha\right]$$
$$= \Delta H - \vec{r}\cdot\vec{a}_{lt}$$

Substitute H_{nat} path invariance to yield zero on LHS

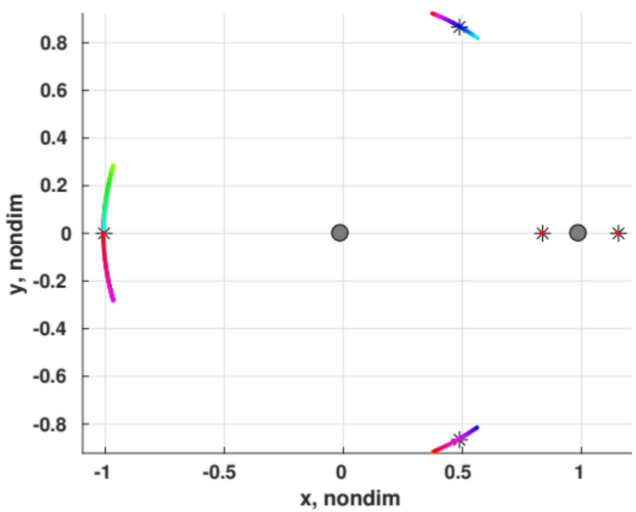
Low-Thrust Equilibria: Balance



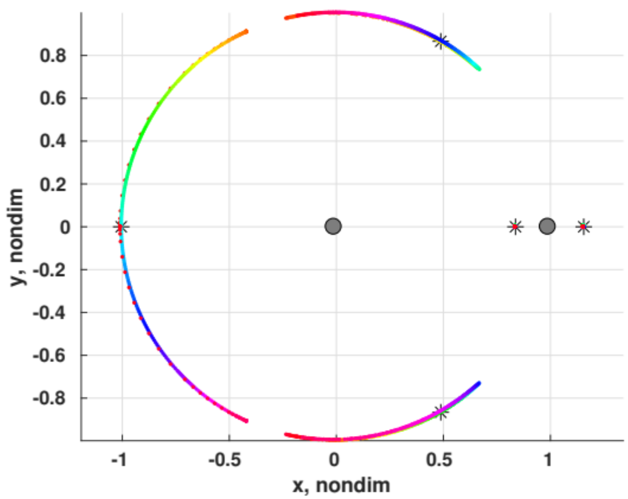
Low-thrust equilibria where natural and low-thrust accelerations balance: $\vec{a}_{lt} = -\vec{a}$

- Horizontal slice
- Equilibria shift with magnitude (a_{lt}) and orientation (α)

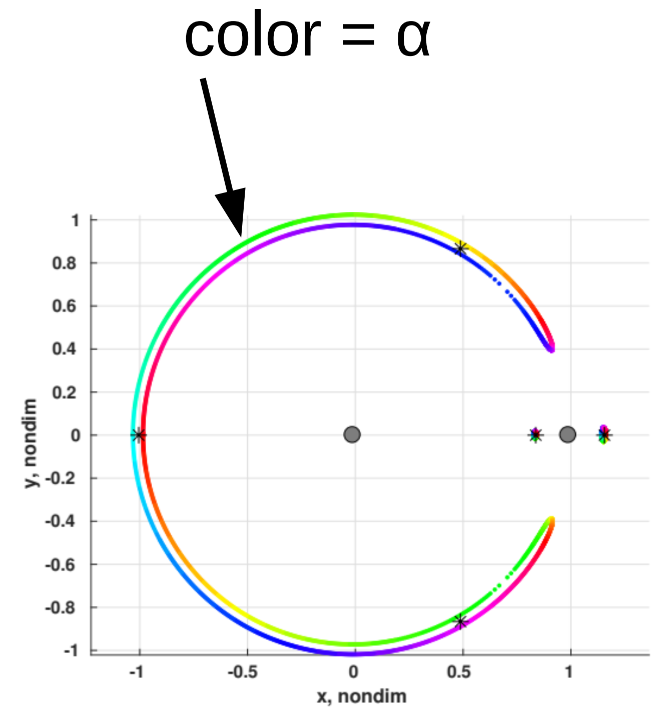
Equilibria, Cont'd



$a_{lt} = 3.00\text{e-}3$



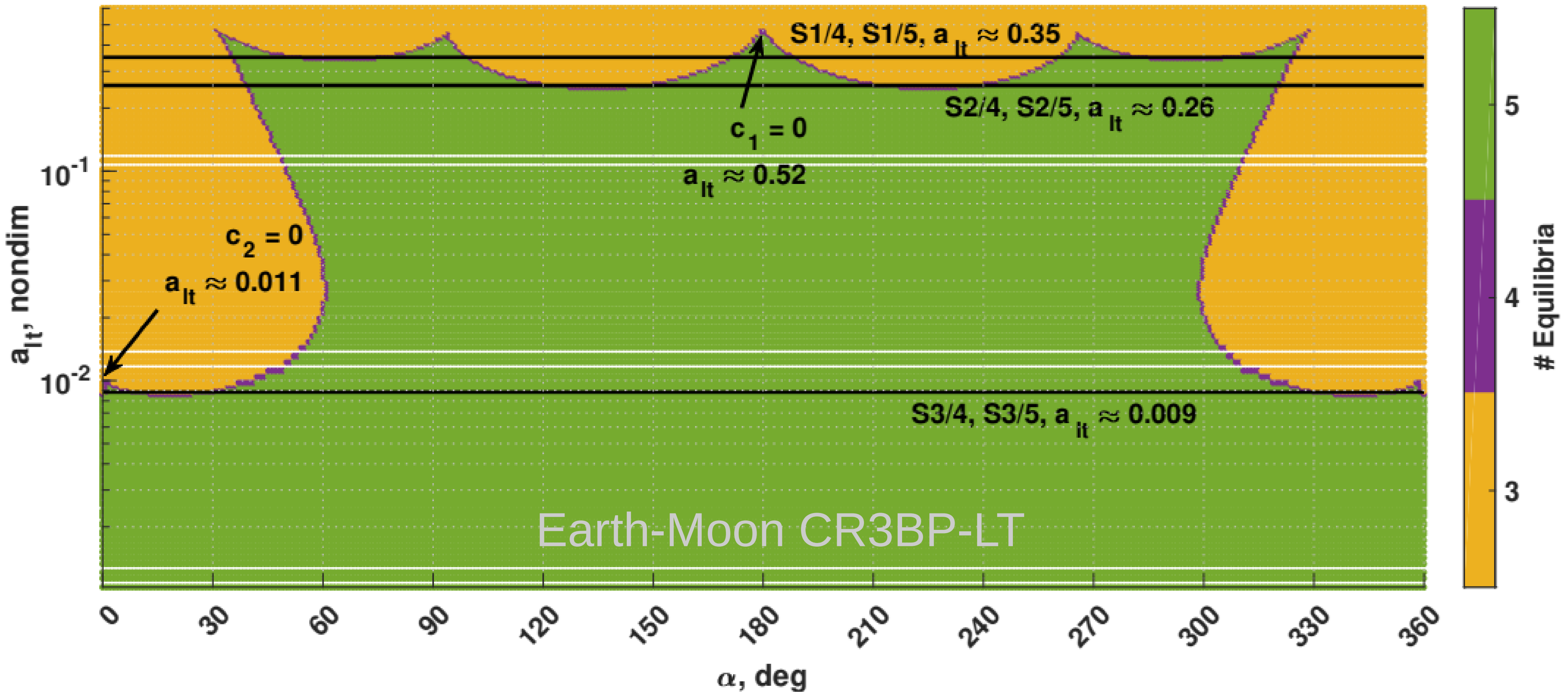
$a_{lt} = 8.73\text{e-}3$



$a_{lt} = 7.00\text{e-}2$

- At small a_{lt} values: equilibria remain near CR3BP L pts (asterisks)
- Larger a_{lt} values: equilibria form larger contours
  - "Zero Acceleration Contours" (ZACs) balance accelerations

Distinct Equilibria



Number of distinct solutions varies with a_{lt} and α