On the Minimum Induced Drag of Wings

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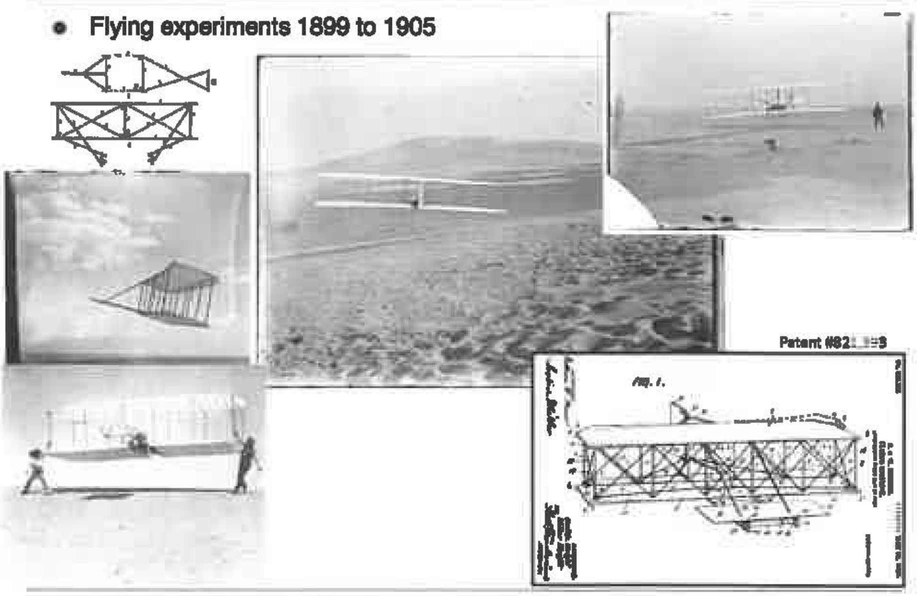
Breguet Range Equation

Range = $\eta_{propulsion}$ * η_{aero} * log (TOGW / LWT) * constant

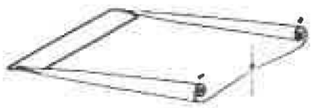
Birds

Wilbur & Orville Wright

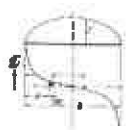
- Flying experiments 1899 to 1905



Prandtl Lifting Line Theory



- Prandtl's "vortex ribbons"



- Elliptical spanload for a given span (1920)
- "the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift." y = c

Ludwig Prandtl

This is the accepted theory and the standard for the minimum drag of wings. But what is a wing? Is it only aerodynamic? What about the structure?

Wingtip Vortices and Elliptical Spanload

Fundamental Assumptions

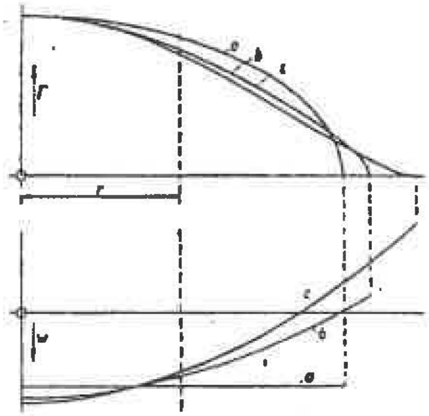
For a long time it was difficult to find suitable functions to express the distribution of lift, from which a plausible distribution of w would be obtained by equation (37). After various attempts it was found that a distribution of lift over the span according to a half ellipse gave the desired solution. According to this, if the origin of coordinates is taken at the center of the wing,

$$\Gamma = \Gamma_0 \sqrt{1 - \left(\frac{x}{b/2}\right)^2}, \text{ hence } \frac{d\Gamma}{dx} = \frac{-\Gamma_0 x}{\frac{b}{2}\sqrt{\left(\frac{b}{2}\right)^2 - x^2}}$$

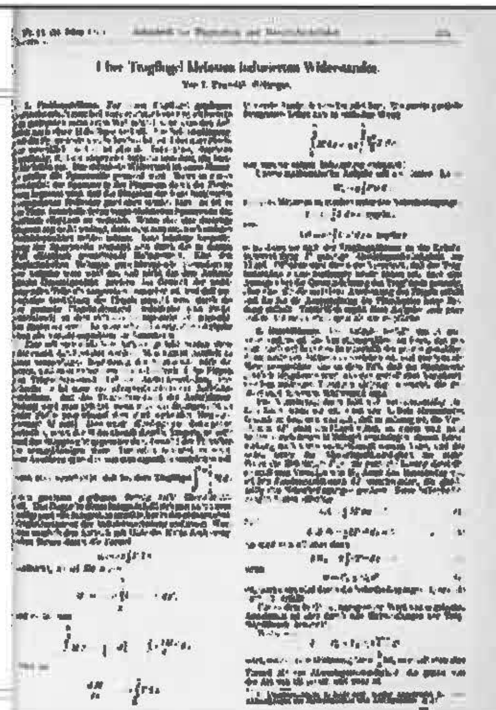
(4) The most important of Betz's theorems, from a practical standpoint, furnishes the complete analogy to Munk's theorem concerning the wing system having the least drag, and, corresponding perfectly to the statements in sections 27 and 28, may be expressed thus: The flow behind a propeller having the least loss in energy is as if the screw surfaces passed over by the propeller blades were solidified into a solid figure and this were displaced backward in the nonviscous fluid with a given small velocity. The potential difference between the front and rear sides of a screw surface at one and the same point furnishes, then, again the circulation Γ of the corresponding point of the propeller blade.

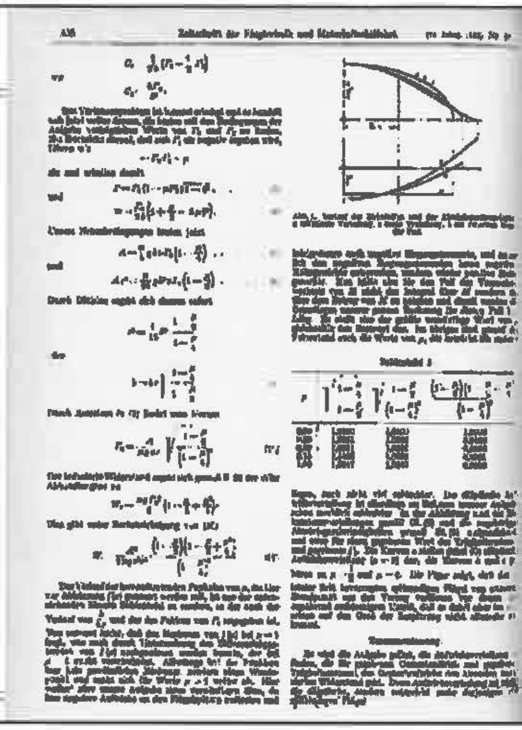
Birds

Minimum Induced Drag & Bending Moment



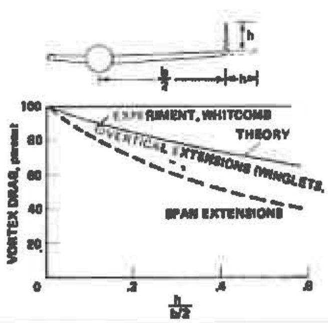
- Prandtl (1933)
  Constrain minimum induced drag
  Constrain integrated wing bending moment
  22% increase in span with 11% decrease in induced drag

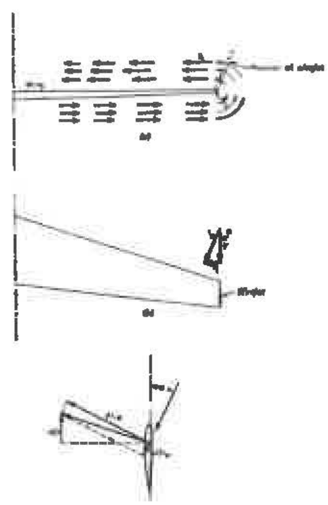




Winglets

- Richard Whitcomb's Winglets
  - induced thrust on wingtips
  - induced drag decrease is about half of the span "extension"
  - reduced wing root bending stress



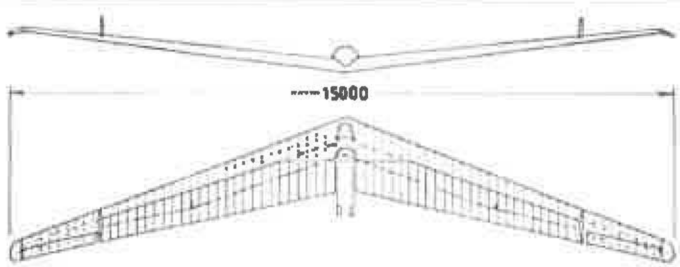


Whitcomb's Winglets

KC-135A in flight - winglet study

KC-135A in flight - closeup of winglet with attached tufts

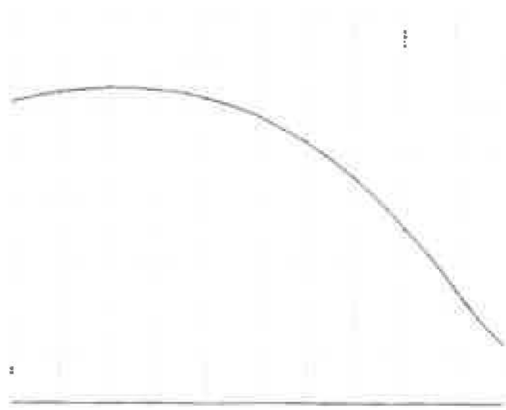
Prandtl Wing



- 24 degree leading edge sweep angle
- Chord:
  root – 15.75 inches
  tip – 3.875 inches
- Span: 147.6 inches

← 3750 →

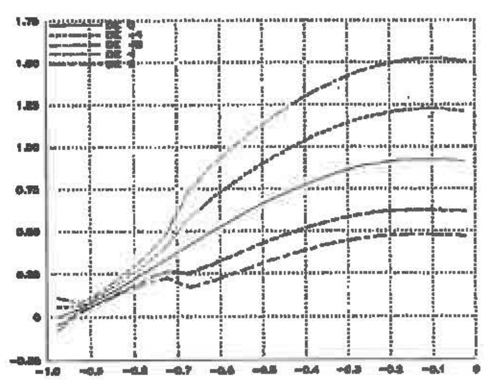
Twist



R0	8.3274
R1	8.5524
R2	8.7259
R3	8.8441
R4	8.9030
R5	8.8984
R6	8.8257
R7	8.6801
R8	8.4565
R9	8.1492
R10	7.7522
R11	7.2592
R12	6.6634
R13	5.9579
R14	5.1362
R15	4.1927
R16	3.1253
R17	1.9394
R18	0.6589
R19	-0.6417
R20	-1.6726

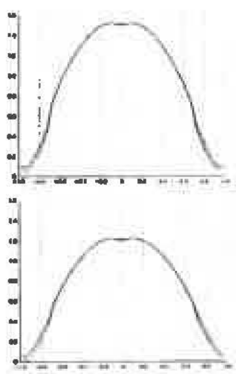
Symmetrical Spanloads

- Elevon Trim
- CG Location

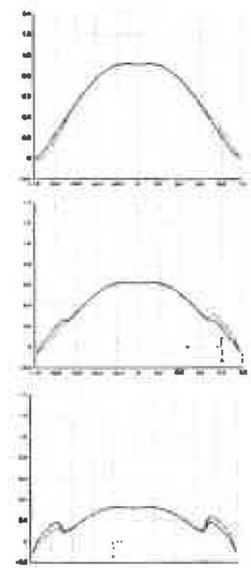


Asymmetrical Spanloads

- Cl∂a (roll due to aileron)
- Cn∂a (yaw due to aileron)
  induced component
  profile component
  change with lift
- Cn∂a/Cl∂a
- CL(Lift Coefficient)
  Increased lift:
  increased Clβ
  increased Cnβ*
  Decreased lift:
  decreased Clβ
  decreased Cnβ*



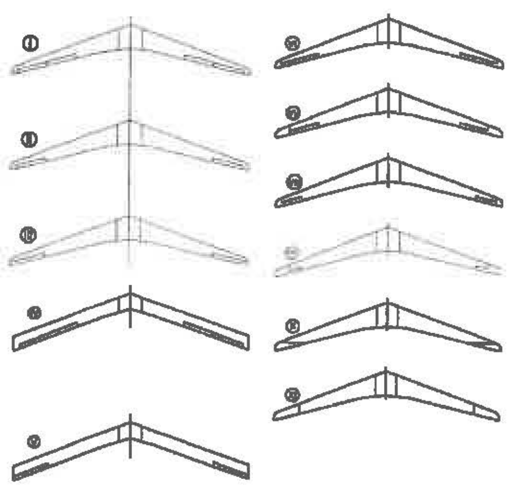
CL	Cl	Cn
.966	.01384	.00055
.774	.01384	.00037
.582	.01345	.00021
.390	.01384	.00003
.198	.01345	-.00015



Dr Edward Uden's Results

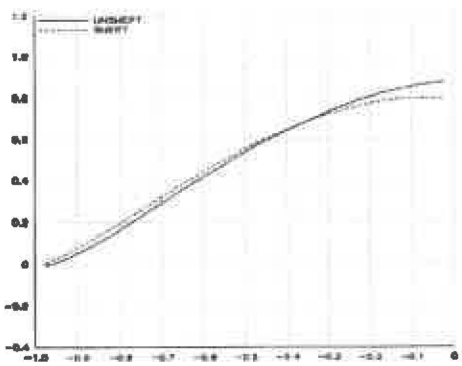
- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

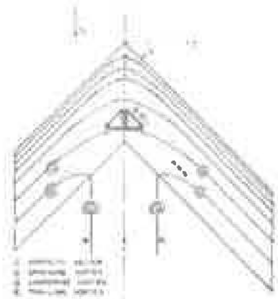
Elevon Config	Cn∂a	Spanload
I	-.002070	bell
II	.001556	bell
III	.002788	bell
IV	-.019060	elliptical
V	-.015730	elliptical
VI	.001942	bell
VII	.002823	bell
VIII	.004529	bell
IX	.005408	bell
X	.004132	bell
XI	.005455	bell

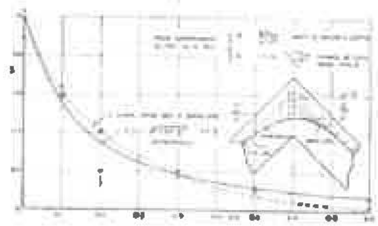


Kucheman

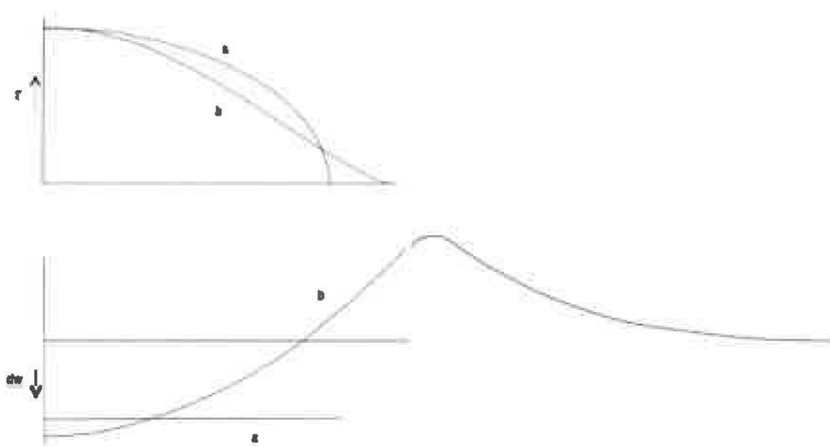
- Effect on spanloads
  increased load at tips
  decreased load near centerline
- Upwash due to sweep unaccounted for
- Residual (Kucheman)







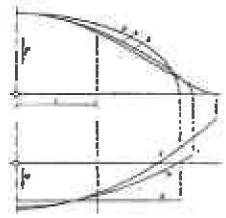
Prandtl's Bell Spanload



Prandtl's Spanload

$$sl = (1 - x^2)^{3/2}$$

$$w = 3/2\,(x^2 - \tfrac{1}{2})$$

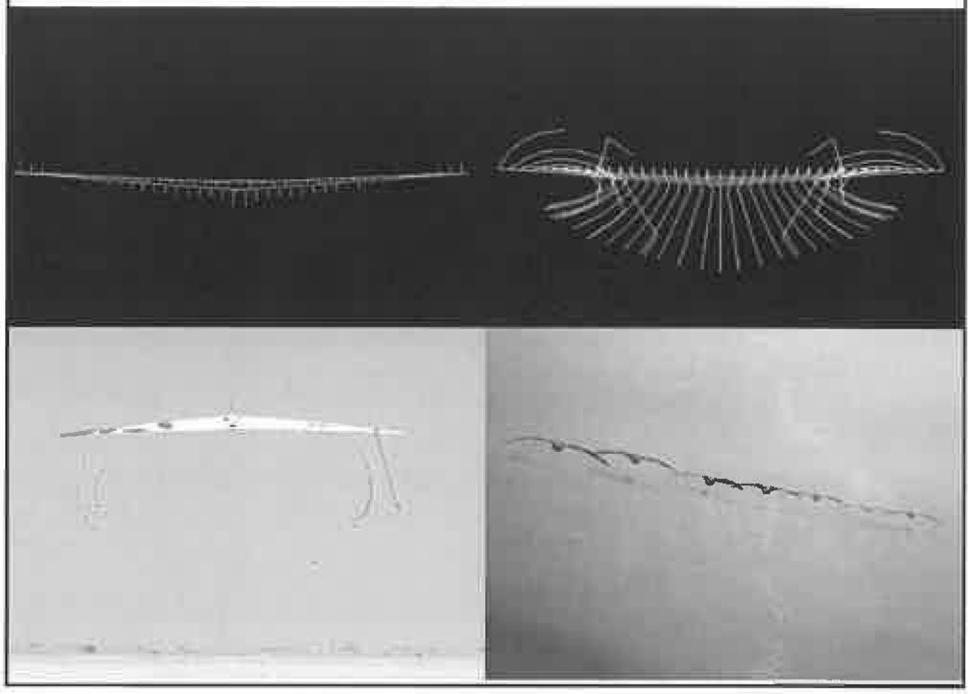


$$\lim_{x:\,0 \to b/2} L(x) = 0 \qquad (1)$$

$$\lim_{x:\,0 \to b/2} \frac{dL(x)}{dx} = 0 \qquad (2)$$

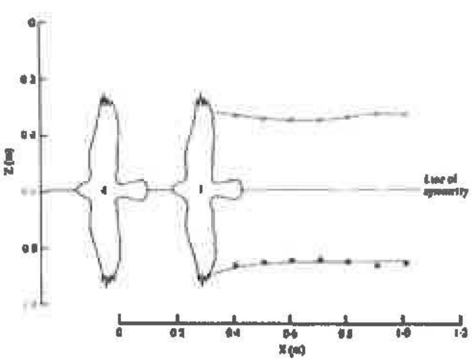
$$\lim_{x:0 \to b/2} \frac{d\,DW(x)}{dx} = \lim_{x:\infty \to b/2} \frac{d\,DW(x)}{dx} \qquad (3)$$

Mike Allen

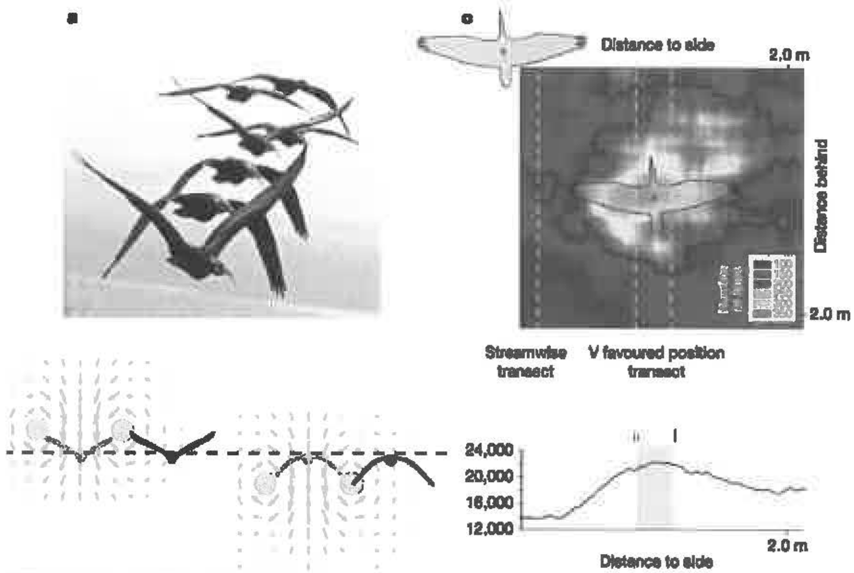


Spedding's Gliding Falcon

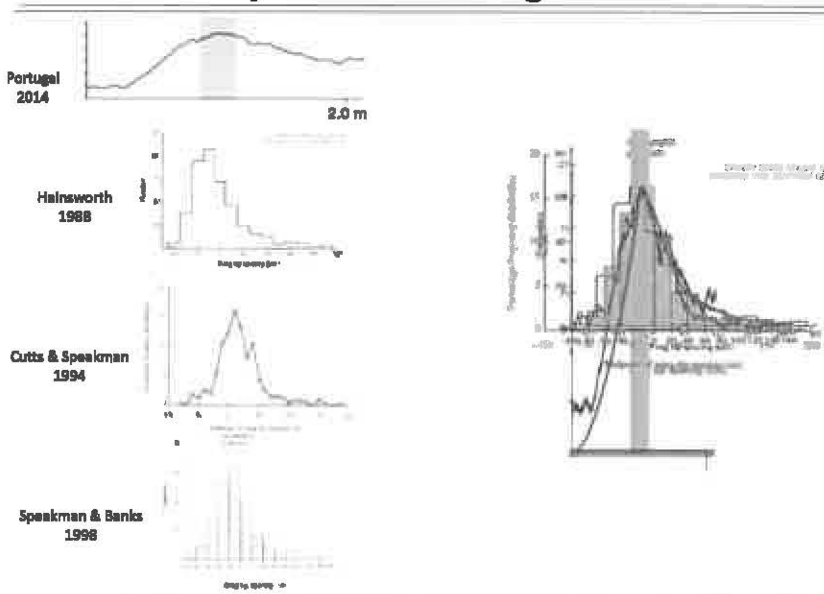
- Spedding photograpgh's a gliding falcon's wake with He bubbles
- Vortex cores are 0.76 b apart
- Elliptical spanload is assumed, so the vortex cores are assumed to come from the wingtips



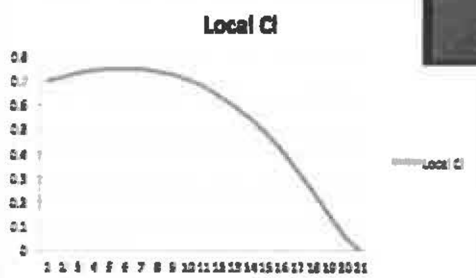
Portugal, et al 2014 (Nature)



Upwash and Wing Beats

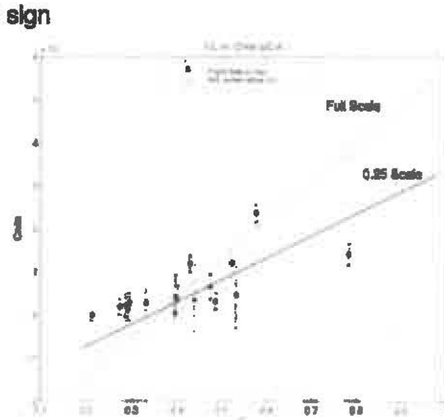


Pointy Wings and Wing Stall



Proverse Yaw

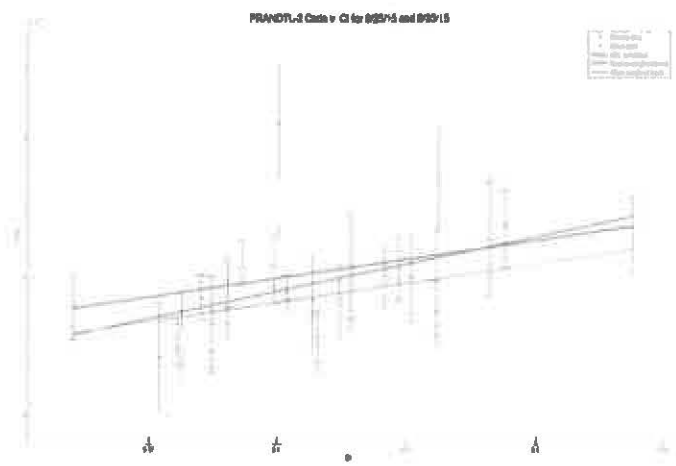
- ...until June 26th, 2013
- Roll and Yaw are the same sign
- From Uden: Cnda is +ve
- uncertainty



Inertias; configuration changes, turbulence, and slope of Cnda

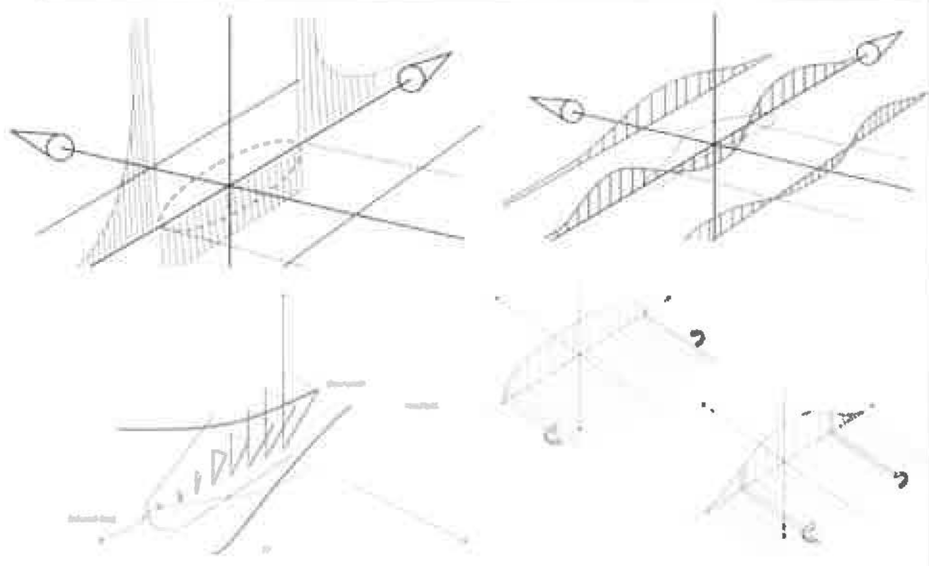
Kristyn Kadala

Revalidated Proverse Yaw



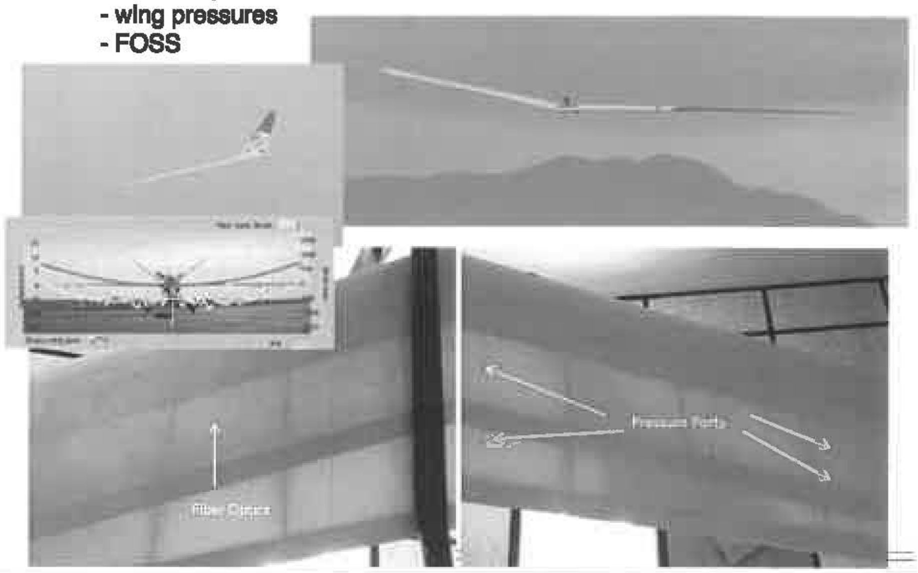
Loren Newton & Keenan Albee

Elliptical and Bell shaped Spanloads



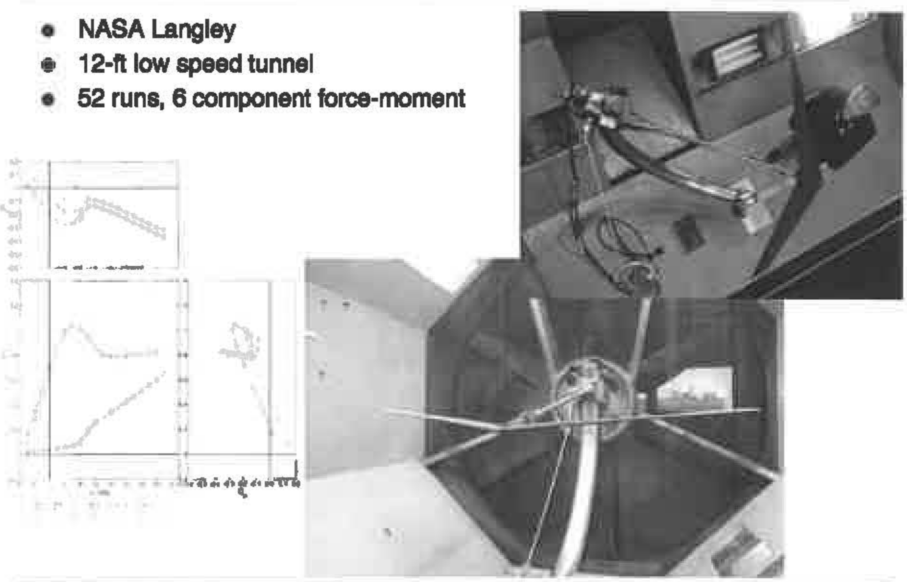
Prandtl 3

- The aerodynamic testbed
  - wing pressures
  - FOSS

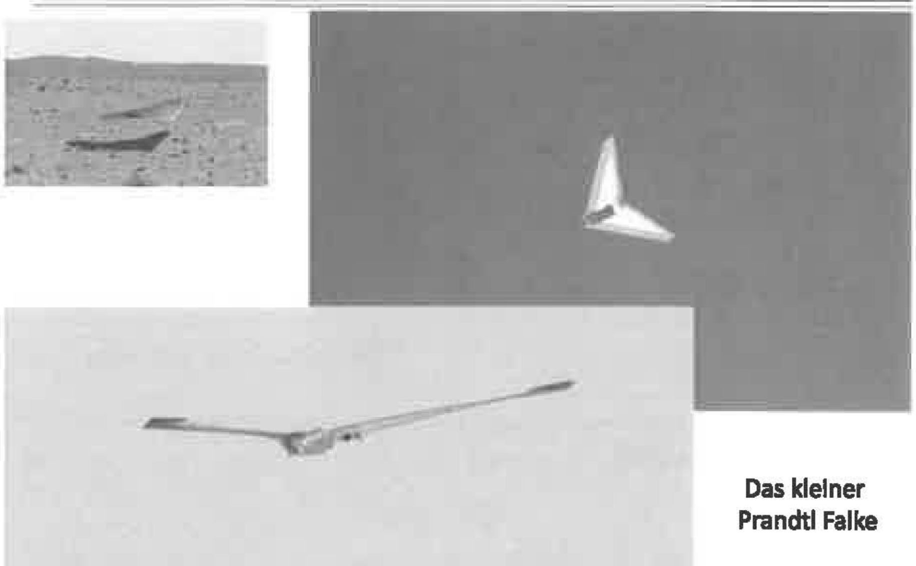


Wind Tunnel Test

- NASA Langley
- 12-ft low speed tunnel
- 52 runs, 6 component force-moment

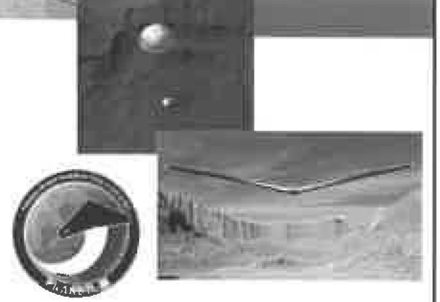


The Future: Prandtl-M on Mars?



Das kleiner
Prandtl Falke

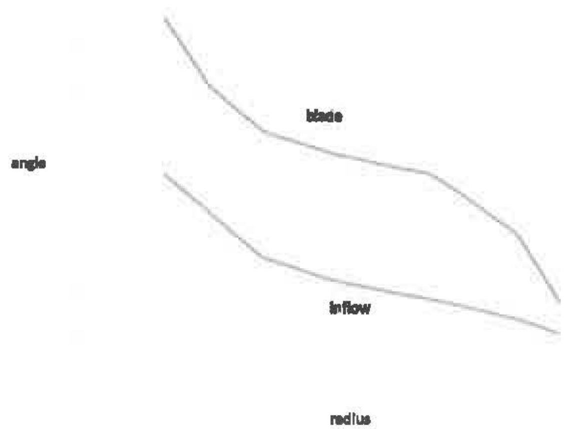
The Mars Mission



- Deploy cubesat from large Mars rover
- Use Exobrake parachute for Mars atmosphere entry
- Deploy Prandtl-m (2 lbm, 2 ft span) from cubesat at 15,000 ft agl
- Glide 5 mins, 22 miles, Mach 0.6
- Crash land on Mars
- Transmit images & data back

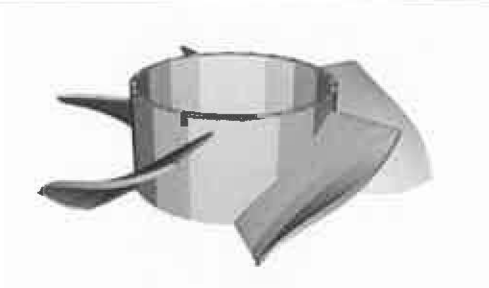
Prandtl Propulsion

- Propulsion systems currently use "minimum induced loss"
- What if we switched to minimum torque for a given thrust? +15.4%

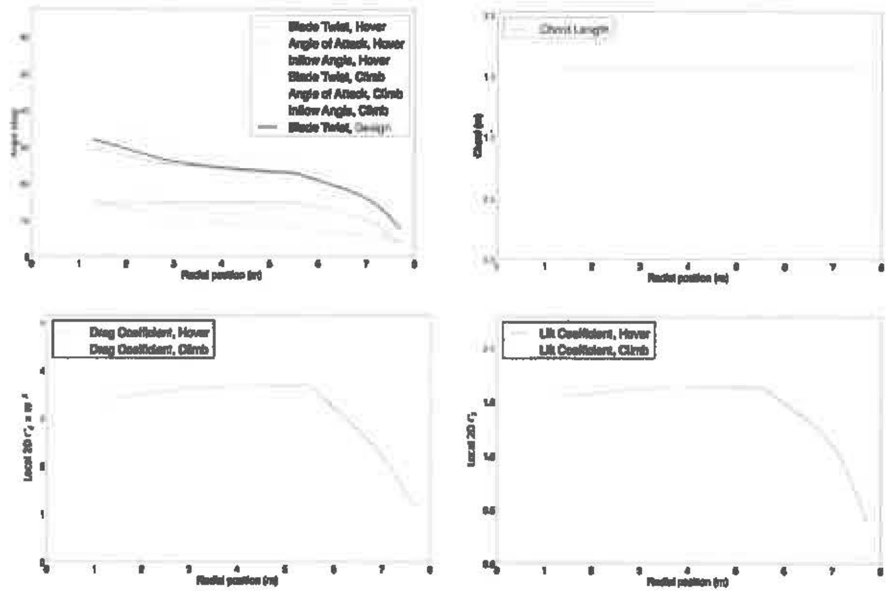


Prandtl Fan

Same voltage
Same diam
24% increase in flow
88% reduction in noise

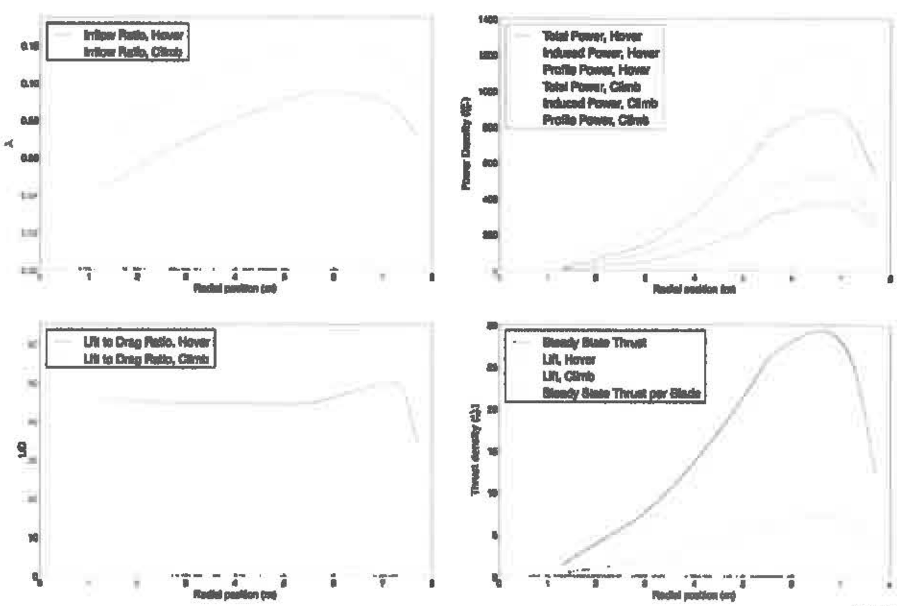


Prandtl Prop 1



Cody Karcher

Prandtl Prop 2



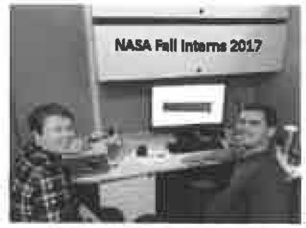
Cody Karcher

Prandtl 3 Team

Prandtl Interns Spring 2016

2016 Interns

2017 & 2018 Interns



PRANDTL-D

- Videos

  - TEDxNASA 2011
  http://www.youtube.com/watch?v=223OmaQ9uLY

  - NASA Aero Academy 2013
  http://www.youtube.com/watch?v=Hr0I6wBFGpY

Red Jensen: pilot, engineer

Red Jensen, Justin Hall, & Derek Abramson

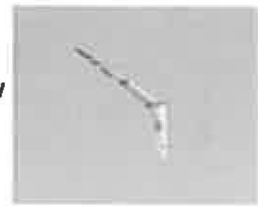
Control of Yaw

- You Have Three Choices:
- 1/ drag a vertical tail around with you all the time to create a yawing moment
- 2/ manipulate drag at the wing tips to control yaw

} Current Design Options

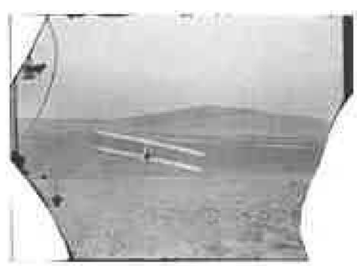
-OR-

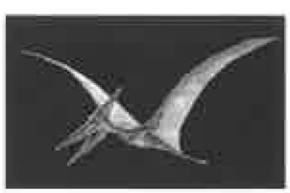
- 3/ manipulate **THRUST** at the wing tips to control yaw
- Biological vs Mechanical Flight

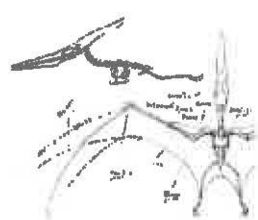


Biological Flight

- Mechanical Flight (110 yrs)
- Vertabrate Flight (128 My)







Efficiency

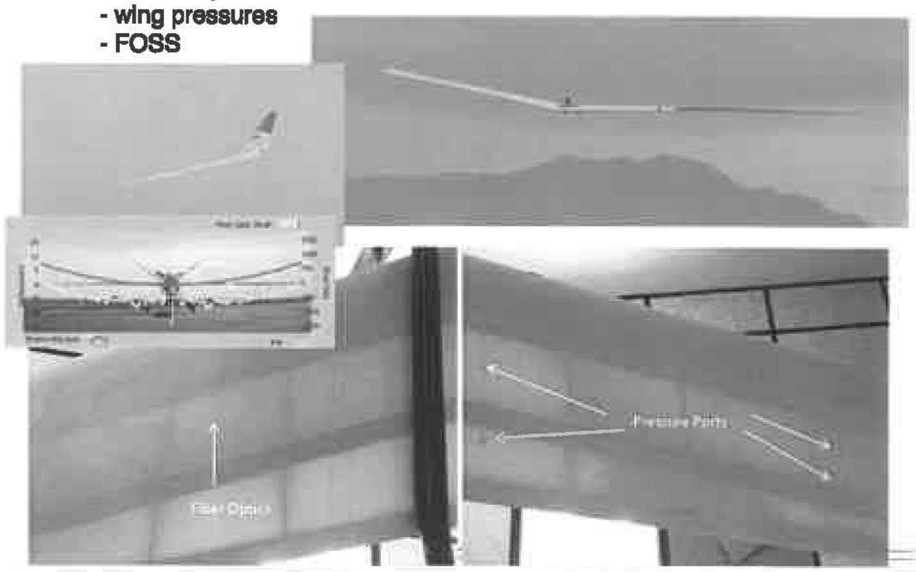
- Efficiency: 12.5% increase in wing efficiency
- 20-30% increase in efficiency by eliminating the tail
- 15.4% increase in propulsive efficiency
- TOTAL EFFICIENCY INCREASE: 69%

- CY2011: world jet fuel consumption $134B
- $55B in jet fuel saved

- CY2011 World GDP: $69.7T
- World power production: $12.0T
- $1.85T savings in world power production

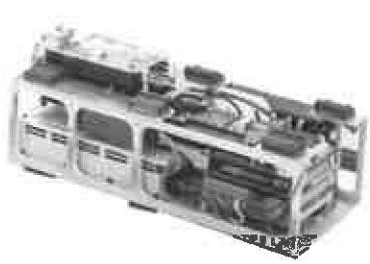
Prandtl P3

- The aerodynamic testbed
  - wing pressures
  - FOSS



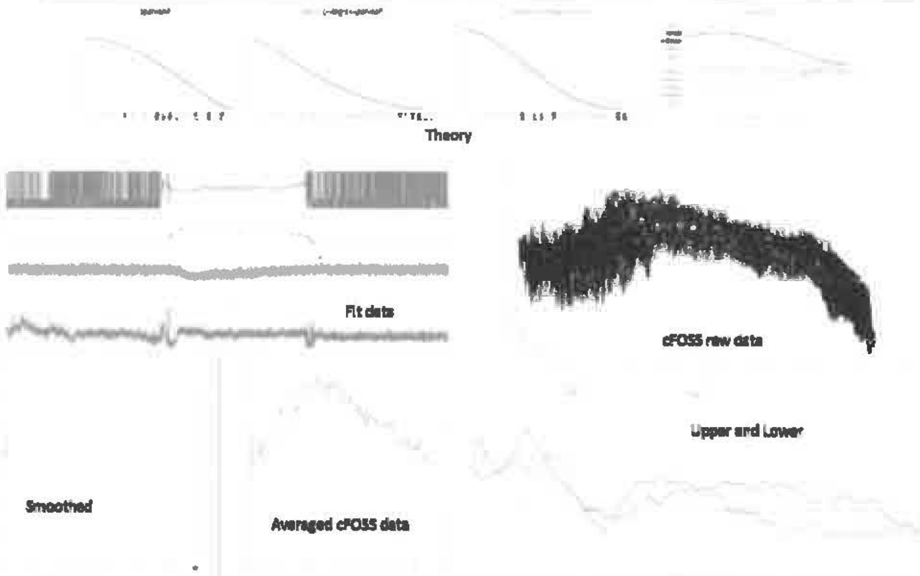
Prandtl P3c

cFOSS System



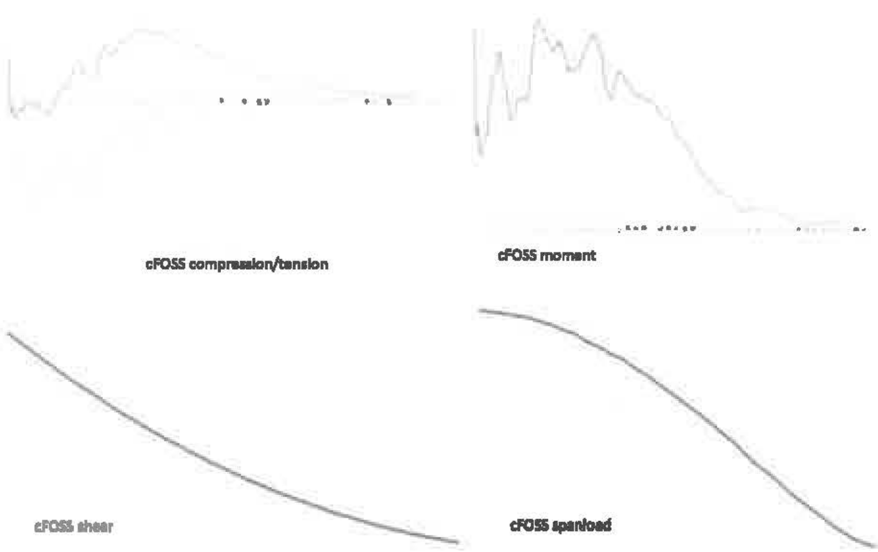
cFOSS computer

Prandtl 3c cFOSS



Deborah Jackson

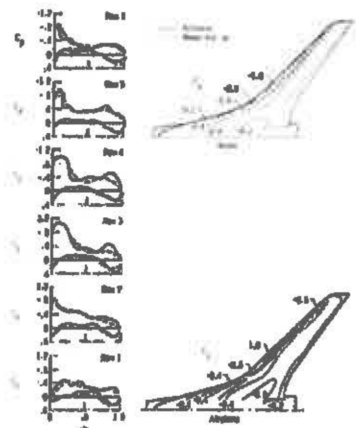
cFOSS Results



Deborah Jackson, Ryan Dunphy, & Macie Kowalski

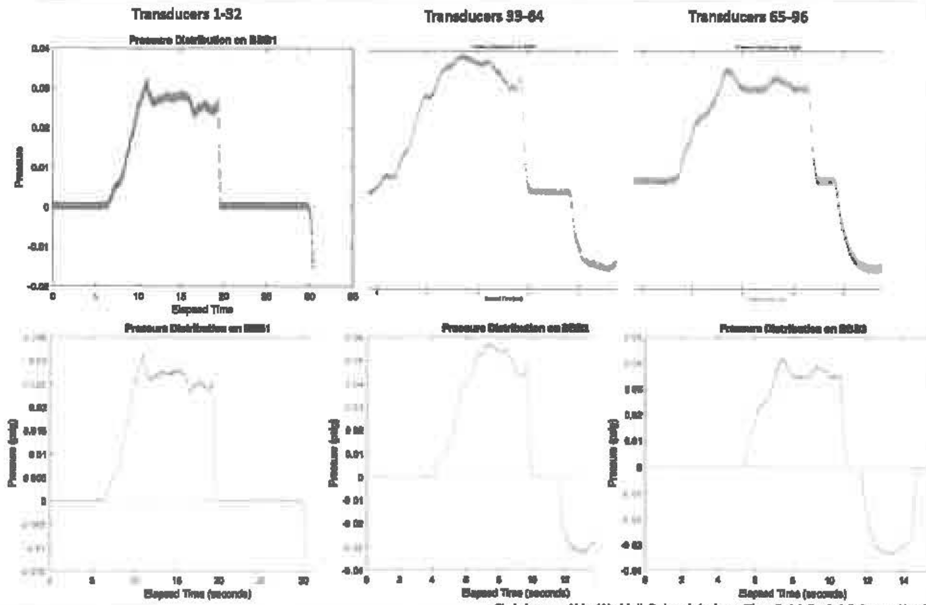
F-8U Supercritical Wing (SCW)

Dryden Flight Research Center E73-3468 Photographed 1973
F-8 modified with Supercritical Wing. NASA photo



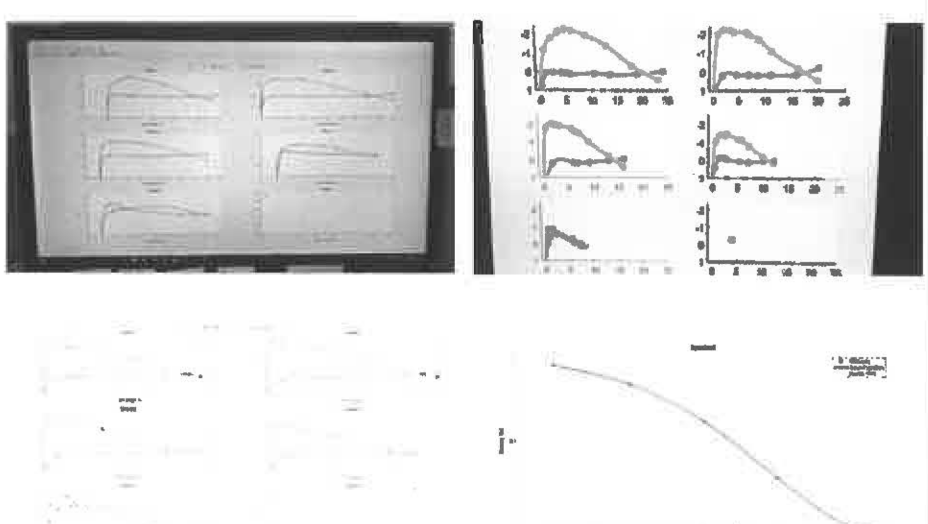
Noah Edwards, Abbey Waddell, Deborah Jackson & Tori Hawkins

Electronic Pressure Measurement



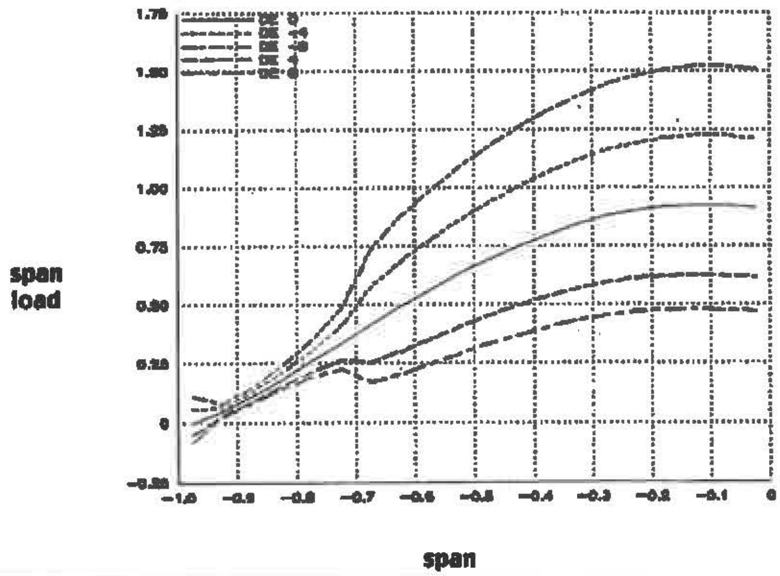
Chris Jensen, Abby Waddell, Deborah Jackson, Than Boisjolie-Gair& Steven Harris

EPM

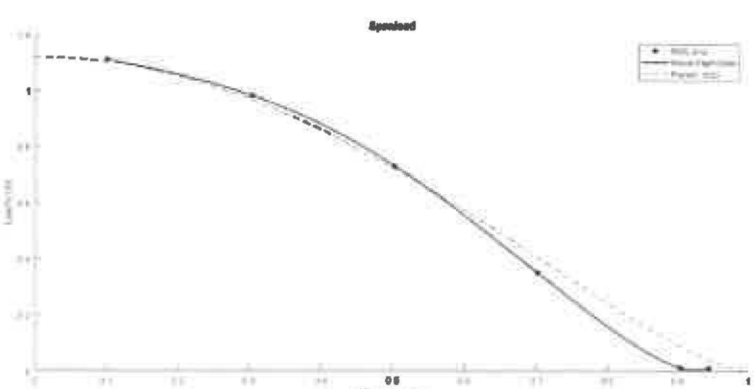


Deborah Jackson, Abby Waddell, Tori Hawkins, Rachel Suitor, Lydia Hantsche, Noah Edwards

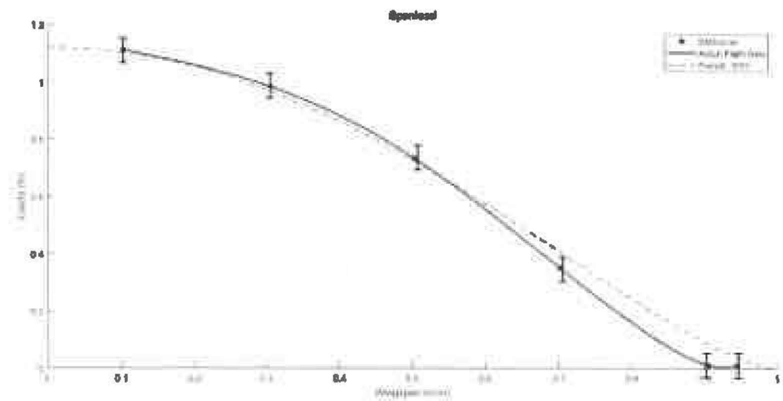
Predicted Spanload



Spanload



Spanload



Deborah Jackson, Abby Waddell, Rachel Sultor, Tori Hawkins

NASA Interns & Others

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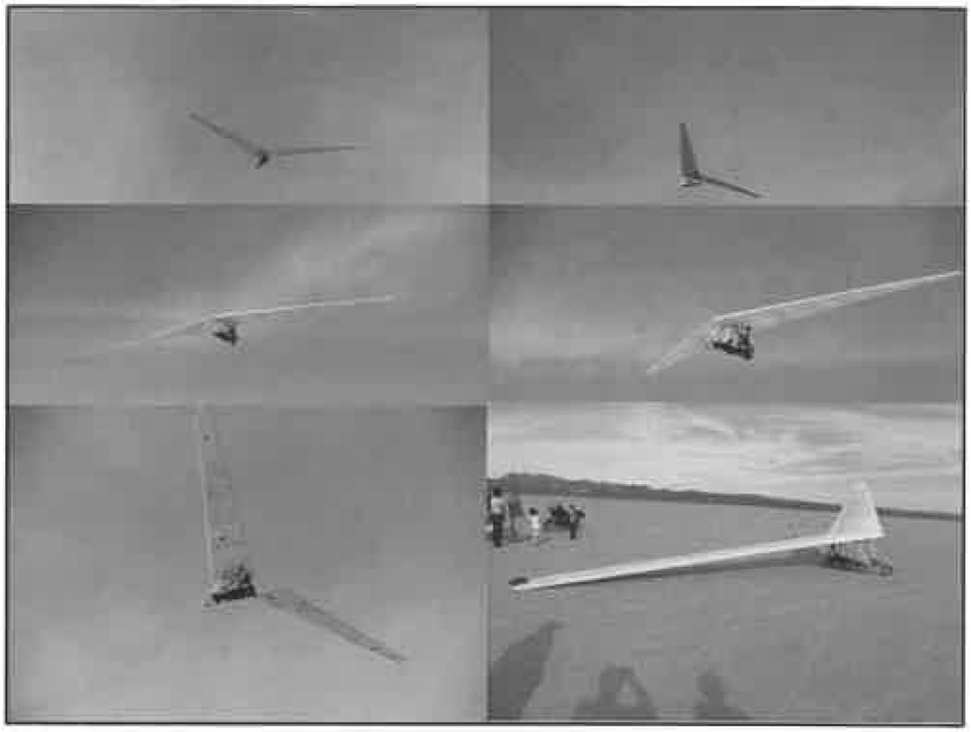
References

- Prandtl, Ludwig: "Applications of Modern Hydrodynamics to Aeronautics"; NACA Report No. 116; 1921.
- Munk, Max M.: "The Minimum Induced Drag of Aerofoils"; NACA Report No. 121, 1923.
- Nickel, Karl; and Wohlfart, Michael; with Brown, Eric M. (translator): "Tailless Aircraft in Theory and Practice"; AIAA Education Series, AIAA, 1994.
- Prandtl, Ludwig: "Uber Tragflugel kleinsten induzierten Widerstandes"; Zeitschrift fur Flugtecknik und Motorluftschiffahrt, Jg 24 Nr 11 pg 305-306 1933; Munchen, Deustchland.
- Horten, Reimar; and Selinger, Peter; with Scott, Jan (translator): "Nurflugel: the Story of Horten Flying Wings 1933 - 1960"; Weishapt Verlag; Graz, Austria; 1985.
- Horten, Reimar; unpublished personal notes.
- Uden, Edward; unpublished personal notes.
- Jones, Robert T.; "The Spanwise Distribution of Lift for Minimum Induced Drag of Wings Having a Given Lift and a Given Bending Moment"; NACA Technical Note 2249, Dec 1950.
- Klein, Armin and Viswanathan, Sathy; "Approximate Solution for Minimum Induced Drag of Wings with a Given Structural Weight"; Journal of Aircraft, Feb 1975, Vol 12 No 2, AIAA.
- Whitcomb, R.T.; "A Design Approach and Selected Wind Tunnel Results at High Subsonic Speeds for Wing-Tip Mounted Winglets," NASA TN D-8260, July 1976.
- Jones, Robert T; "Minimizing Induced Drag."; Soaring, October 1979, Soaring Society of America.
- Koford, Carl; "California Condor"; Audobon Special Report No 4, 1950, Dover, NY.
- Hoey, Robert; "Research on the Stability and Control of Soaring Birds"; AIAA Report 92-4122-CP, AIAA, 1992
- Lee, Russell; "Only the Wing: Reimar Horten's Epic Quest to Stabilize and Control the All-Wing Aircraft," Smithsonian Institution Scholarly Press (Rowman & Littlefield), Washington D.C., 2011.
- Spedding, G. R., The wake of a kestrel (*falco tinnunculus*) in gliding flight, *J. exp. Biol.*,127, 45-57 (1987).
- Portugal, S. J. et al., Upwash exploitation and downwash avoidance by flap phasing in ibis formation flight, Nature 505, 399-402 (2014).



If you want to build a ship, don't drum up people to collect wood and don't assign them tasks and work, but rather teach them to long for the endless immensity of the sea...

- *Antoine de Saint-Exupery*