S-band Fence Study Phase 1: Estimating the Increased Level of Screening Volume Incursions

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Agenda

- Motivation, objectives and methodology
- Estimating the satellite population revealed by the S-Band Fence
- Estimating the rate that satellites make incursions into CARA's screening volumes
- Comparing incursion rates before and after the deployment of the S-Band Fence
- Conclusions

Overall Motivation and Objectives

- **Motivation: CARA assesses collision risk for a set of high-value satellites. Deploying the S-band Fence (SBF) radar system will significantly increase the number of conjunctions to process, possibly overloading the current CARA system.**

- **Phase 1 study objective: Estimate changes in *screening volume incursion rates* caused by the SBF deployment**

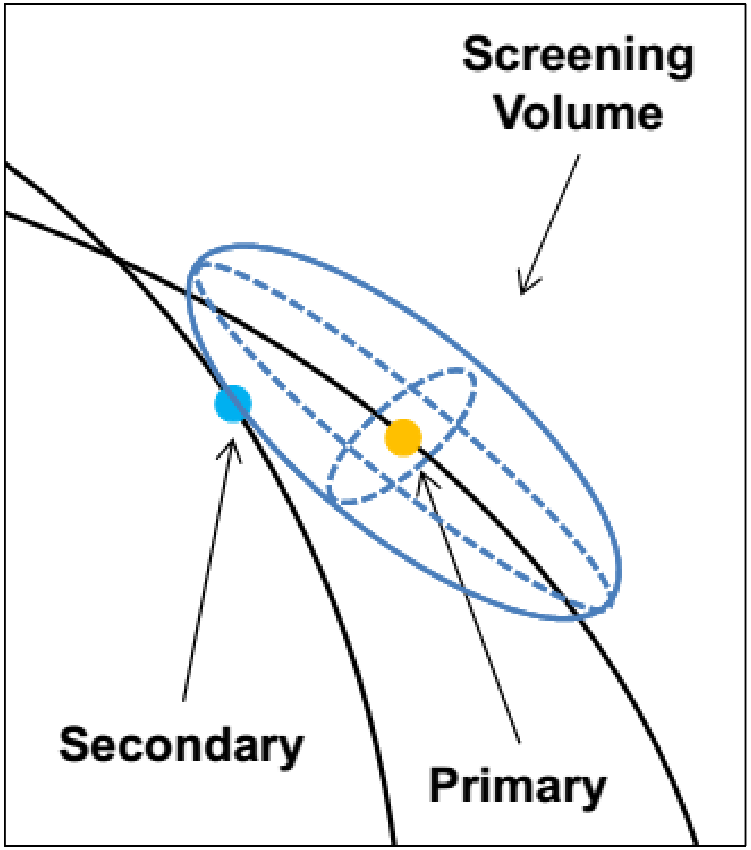
- **Phase 2 study objective: Estimate associated changes in *serious conjunction rates* (e.g., Pc > 10^{-4})**

- **Phase 3 study objective: Develop methods for *filtering and prioritizing the increased tasking*, as required**

- **All to be discussed today are Phase I results**

Phase 1 Study Overview

- **CARA assesses collision risk for a set of high-value, primary satellites**
  – Among the larger population of cataloged, secondary satellites
- **Only potentially serious conjunctions are analyzed**
  – An ellipsoidal screening volume centered on each primary must be predicted to be penetrated by a secondary to initiate the process

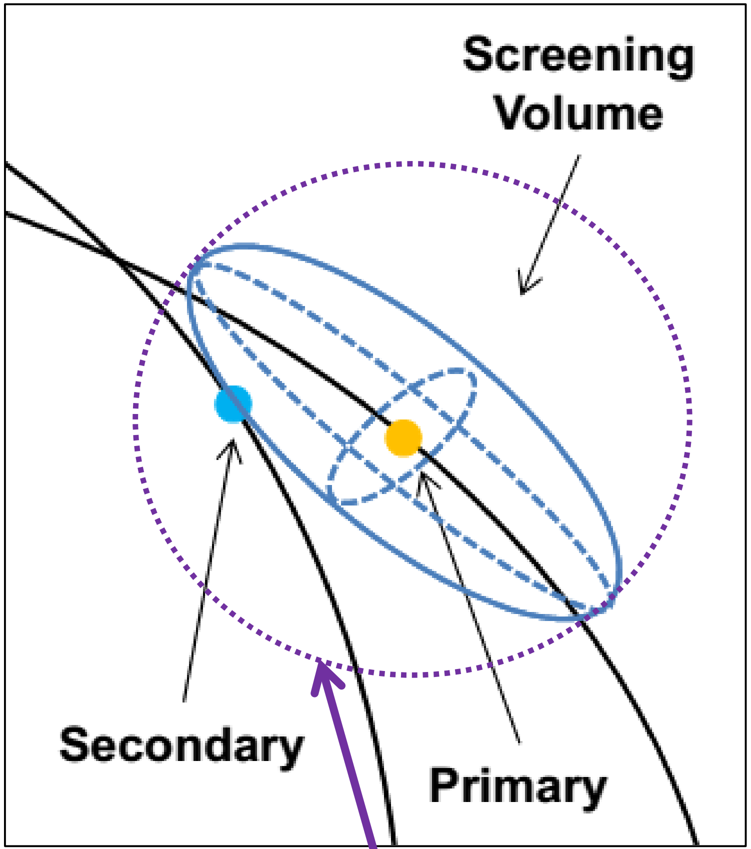


<u>GOAL</u>: Estimate the increased rate of incursions into CARA's screening volumes due to deployment of the SBF

Phase 1 Study Methodology

Two questions must be addressed to estimate the increased screening volume incursion rates:

1. **How many new secondaries could be detected?**
   – Addressed in simulations performed by the S-Band Fence project
2. **How often will these penetrate CARA's screening volumes?**
   – Addressed by the CARA analysis team using both semi-analytical and Monte Carlo methods



Incursion rates for spherical volumes can be estimated semi-analytically

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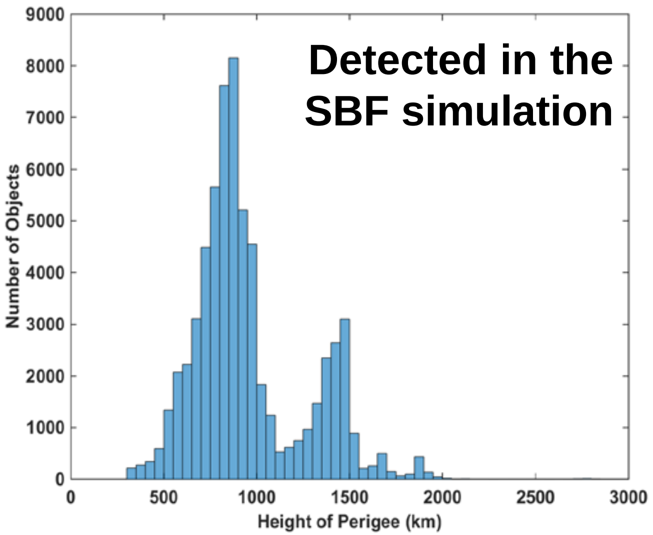
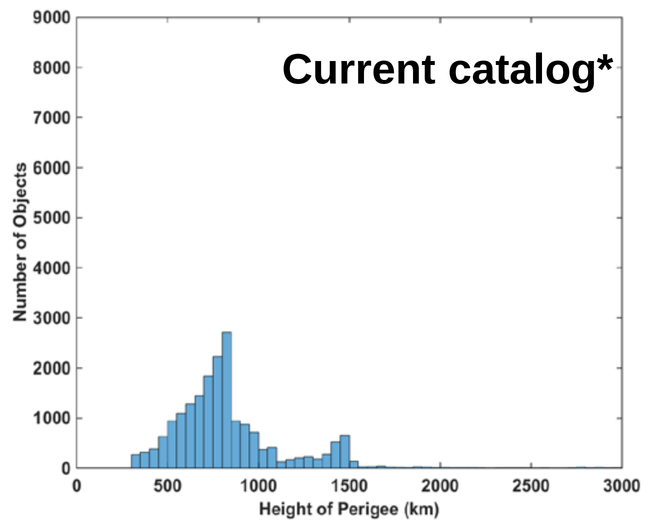
Estimating the Population Detected by the SBF

- **The SBF project has analyzed a long-term forecast orbital population model**
  - Based on a NASA Orbital Debris Program Office (ODPO) model generated several years ago, with an epoch of 2030
  - More recent and advanced ODPO models with different epochs could also be analyzed
- **A 5-day simulation of the SBF in operation indicates that 65,237 of the original 150,014 ODPO objects could be detected and maintained well**
  - Restricted to perigee altitudes below 3000 km
  - About a factor of 3.3 more than the 19,946 objects below 3000 km in the current catalog*
  - However, even this relatively large post-SBF model catalog seems to under-represent segments of the current catalog, especially for altitudes below about 550 km (more on this later)





***Current catalog = SATF 2016 Day 274**

Pre-SBF and Post-SBF Model Populations Used For the Incursion Rate Analysis

Given the 5-day operation simulation results provided by the SBF project, two methods have been used to model the required pre- and post-SBF satellite populations:

1. Use the current catalog* for the pre-SBF model, but retain the 2030 catalogue for a post-SBF model
2. Use the current catalog* for the pre-SBF model, and augment the original post-SBF model with the current catalog* to ensure adequate representation of objects that are currently orbiting

This analysis compares results for both models

***Current catalog = SATF 2016 Day 274**

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The Direct Time Averaging Approach

One conceivable way to estimate screening volume incursion rates would be to use a "direct propagation with time averaging" approach:

1. Propagate Two Line Elements (TLEs) for each CARA primary over an "averaging time" past the deployment of the SBF
2. Similarly propagate the pre-SBF and post-SBF secondary populations
3. Calculate the close approach events and count the screening volume incursion events
4. Divide the number of events by the averaging time to obtain an estimate for the screening volume incursion rate (events day^{-1})
5. Compare the screening volume incursion rates for the pre-SBF and post-SBF secondary populations

This approach is not feasible for many reasons!
Solution: Use a "phase space averaging" approach.

Some Reasons Why the Direct Time Averaging Approach Is Not Feasible

- **Specify initial orbits using Keplerian elements (a,e,i,Ω,ω,M)**
- **None of the secondary satellites detected in the SBF 5-day operation simulation have known (Ω,ω,M) values at any epoch**
  - ODPO's long-term population models really only provide (a,e,i) values
- **Also, predicting (Ω,ω,M) for the CARA primaries is not feasible**
  - The SBF deployment date is uncertain by several months, or even longer
  - Typically, (Ω,ω,M) vary cyclically over shorter time scales than this, so these should be considered "fast" angular variables
- **Incursion rates must be estimated using only (a,e,i) values**
  - The angular variables (Ω,ω,M) can be considered to be uniformly distributed between 0° and 360° (for most objects)

Donald Kessler developed a semi-analytical approach to solve a very similar problem: estimating long-term average collision rates between Jupiter's outer moons

Phase Space Averaging

- **"Phase space averaging" has wide applicability**
  - Especially in statistical mechanics and particle kinetic theory
- **The method relies on the "ergodic hypothesis"**
  - Averaging over the phase space variables is equivalent to averaging over long time periods
- **For the current problem, the phase space comprises the three Keplerian orbital elements (Ω,ω,M)**
  - This is because these "fast" variables vary so quickly that they explore the entire phase space over the averaging timescale

However, this approach still requires numerically finding close approach events, as well as any associated screening volume incursions

COMBO-based Close Approach and Incursion Event Analysis

- **Program *COMBO* provides an efficient means to calculate close approach events among satellites**
  - Available as part of the *AstroStandards* software package

- **When used in general perturbations propagation mode (*i.e.*, driven with TLEs), *COMBO* has some restrictions:**
  - It only calculates close approaches; incursions into non-spherical screening volumes must be calculated separately
  - It will not propagate beyond ~60 days of a TLE's epoch

- **This last restriction prevents using long duration *COMBO* runs to estimate time-averaged rates**
  - Solution: Average a Monte Carlo series of many 7-day or 10-day *COMBO* runs to approximate long-term average rates
  - Perturbations in (a,e,i) are usually negligible over these short periods

COMBO-based Monte Carlo Algorithm to Estimate Screening Volume Incursion Rates

1. For each CARA primary, repeat step 2 for many Monte Carlo iterations
2. Use *COMBO*-based processing to perform the following:
   a) Propagate the primary's TLE over a 7-day propagation time, resetting its epoch to the SBF-deployment time, and randomly varying (Ω,ω,M) for each iteration
   b) Similarly propagate the pre-SBF and post-SBF secondary populations, resetting their epochs and randomizing their (Ω,ω,M) values as well
   c) Calculate the close approach events; then calculate and accumulate the screening volume incursion events
3. Divide the number of incursions by the number of Monte Carlo iterations and the propagation time to estimate the incursion rate (events day^{-1})
4. Compare the screening volume incursion rates for the pre-SBF and the post-SBF secondary populations

This approach is both feasible and the implementation can be validated using Kessler's semi-analytical method (demonstration in backup section)

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Pre-SBF and Post-SBF Model Populations Used For the Incursion Rate Analysis

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2. Use the current catalog* for the pre-SBF model, and augment the original post-SBF model with the current catalog* to ensure adequate representation of objects that are currently orbiting

***Current catalog = SATF 2016 Day 274**

Pre-SBF and Post-SBF Model Populations Used For the Incursion Rate Analysis

Models 1 and 2 naturally span a range of one:
ρ_1 = post-SBF/SATF ρ_2 = (post-SBF+SATF)/SATF = ρ_2+1

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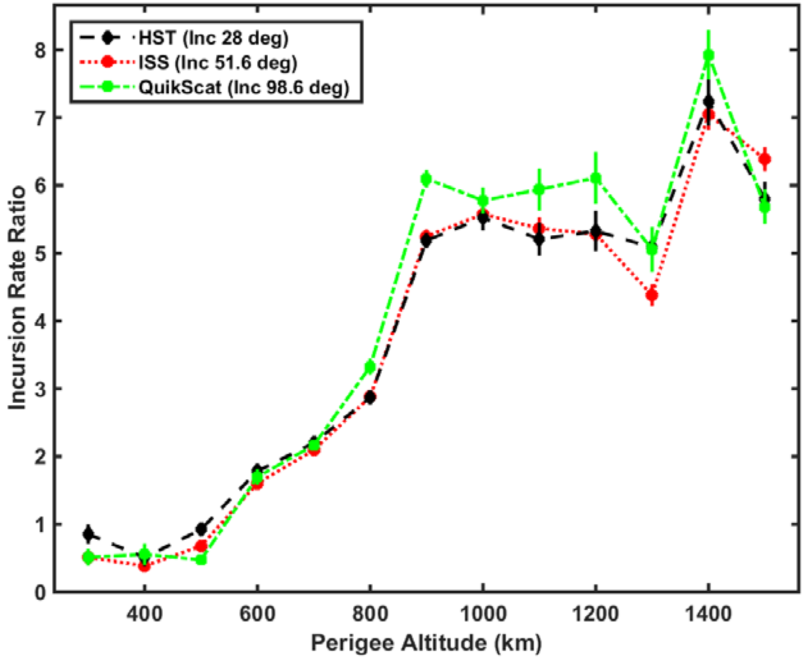
Post-SBF/Pre-SBF Screening Volume Incursion Rate Ratios for 14 Selected CARA Objects

Orbit Regime	RIC Ellipsoid (km)	Satellite (SCN)	Models 1 & 2 Incursion Rate Ratio*
LEO #1	2×44×51	Grace-1 (27391)	(0.8 to 1.8) ± 0.01
		ISS (25544)	(0.5 to 1.5) ± 0.01
		WorldView-1 (32060)	(0.9 to 1.9) ± 0.01
LEO #2	0.5×17×20	FGST (33053)	(1.2 to 2.2) ± 0.01
		HST (20580)	(1.4 to 2.4) ± 0.01
		Hinode (29479)	(2.2 to 3.2) ± 0.02
		Aqua (27424)	(2.0 to 3.0) ± 0.14
LEO #3	0.5×12×10	QuickScat (25789)	(3.3 to 4.3) ± 0.02
		Metop-A (29499)	(3.4 to 4.4) ± 0.03
LEO #4	0.5×2×2	TOPEX/Pos. (22076)	(10.1 to 11.1) ± 0.4
		Jason-3 (41240)	(9.3 to 10.3) ± 0.4
HEO #1	40×77×107	IMAGE (26113)	(3.5 to 4.5) ± 0.02
HEO #2	40×77×107	MMS-1 (40482)	(7.1 to 8.3) ± 0.06
		Meridian-3 (37212)	(4.7 to 5.7) ± 0.04

*Quoted uncertainties represent ±1σ estimates due to Monte Carlo counting variations only

Post-SBF/Pre-SBF Rate Ratios for LEOs: Variation with Perigee Altitude and Inclination

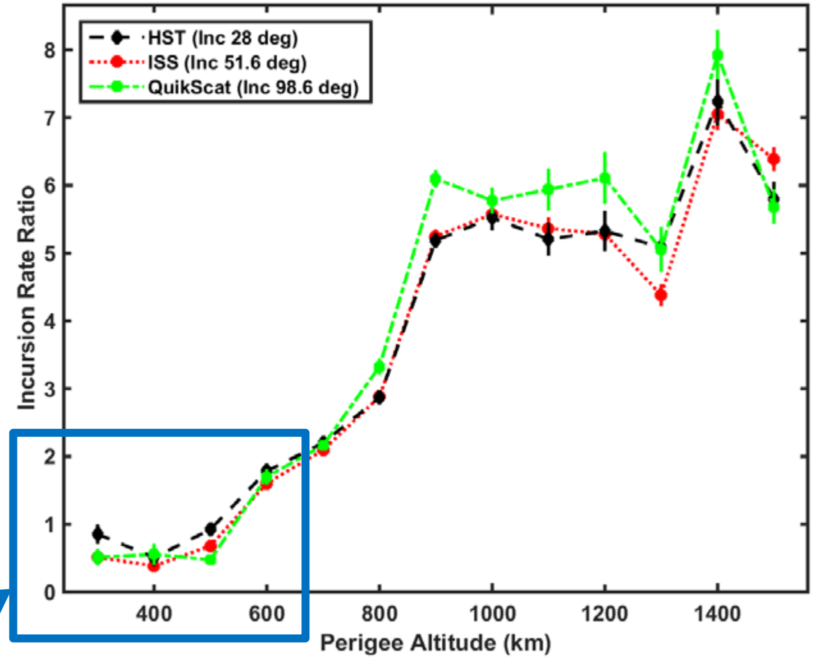
Population Model 2



General trend: Average screening volume incursion rates for CARA's LEOs increase significantly more for perigee altitudes above 1000 km than for lower altitudes, with relatively weak dependence on inclination. HEOs must be considered individually.

Post-SBF/Pre-SBF Rate Ratios for LEOs: Under-representation of the Current Catalog

Population Model 1



Model 1 post-SBF/pre-SBF ratios for altitudes below ~550 km can be less than one. This indicates that the original post-SBF model under-represents the current catalog at these altitudes. To ensure adequate representation, model 2 augments the post-SBF population with the current catalog.

Overall Expected Effect on CARA Loading: Expected CARA Missions through 2022

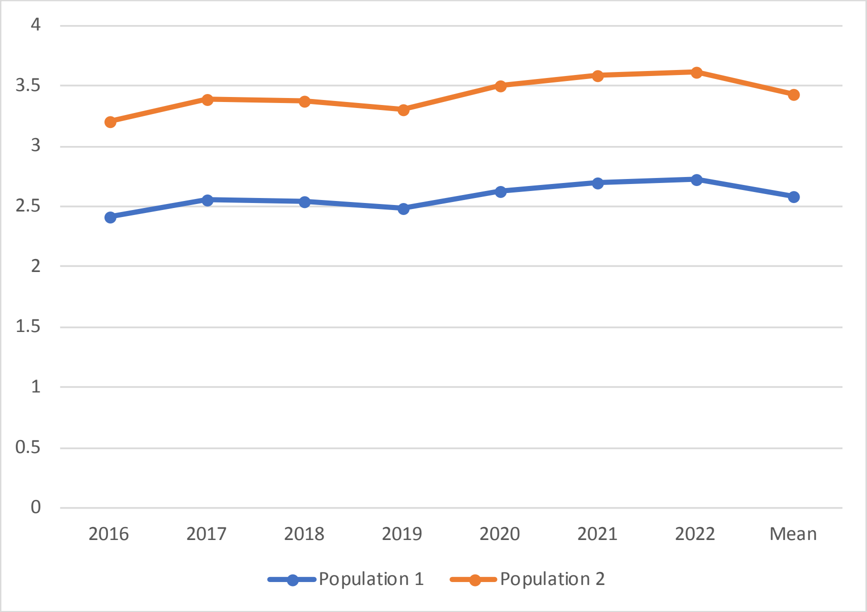
Regime	Common Name	Perigee	Apogee	Inc	2016	2017	2018	2019	2020	2021	2022
LEO 1-2	GRACE-1	352	352	89.0	Yes	Yes	Yes	No	No	No	No
	GRACE-2	348	356	89.0	Yes	Yes	Yes	No	No	No	No
	GRACE FO	490	490	89.0	No	Yes	Yes	Yes	Yes	Yes	Yes
LEO 1-3	ISS	407	417	51.6	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	GPM	400	414	65.0	Yes	Yes	Yes	Yes	Yes	Yes	Yes
LEO 2-2	HST	572	585	28.5	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	TIMED	612	612	74.1	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	RHESSI	483	498	38.0	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	SWIFT	555	571	20.6	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	AIM	535	542	98.0	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	NEOWISE	487	492	97.5	Yes	Yes	No	No	No	No	No
	NuSTAR	605	621	6.0	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	IRIS	622	661	97.9	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	ICON	575	575	27.0	No	Yes	Yes	Yes	Yes	Yes	Yes

Table of future mission for CARA's LEO 1-2, 1-3 and 2-2 orbital regimes. (Similar models exist for the other orbital regimes, but are not shown here.)

Overall Expected Effect on CARA Loading: Screening Volume Incursions per Day

- **Estimate observed incursion rate for each present mission**
  - Estimated using median over last six months
- **Model anticipated missions for future years through 2022**
- **Estimate postSBF/preSBF incursion rate scale factors for each present and future mission type**
  - Taken from actual mission estimates when possible
  - Taken from the trends calculated for perigee height and inclination for LEOs
  - "Nearest neighbor" assignments necessary for some missions
- **Estimate the expected future incursion rate for each mission**
  - Product of current incursion rate and the postSBF/preSBF scale factor
- **Sum over all missions to get future total incursion rates**
- **Divide future total incursion rates by current averages to estimate overall yearly growth factors**

Overall Expected Effect on CARA Loading: Results



	2016	2017	2018	2019	2020	2021	2022	Mean
Population 1	2.42	2.55	2.54	2.49	2.63	2.7	2.73	2.58
Population 2	3.21	3.39	3.38	3.31	3.5	3.58	3.62	3.43

Population 2 is recommended CARA solution for planning purposes: For 2017-2022, the overall growth factor is roughly 3.5

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Conclusions from the SBF Phase 1 Study: Satellite Population Models

1. **The 5-day operation simulation provided by the SBF project indicates that the catalog after the SBF deployment could contain ~65,000 objects with perigee altitudes below 3000 km**
   – Substantially more than contained in the current catalog
   – Based on an older NASA ODPO model, which could be updated
2. **The post-SBF model catalog provided by the SBF project may under-represent segments of the currently cataloged population**
   – Prompts augmenting the original post-SBF model with the current catalog as an alternative post-SBF model

Conclusions from the SBF Phase 1 Study: Analysis Methodology

1. **The “direct time averaging” approach is not feasible for estimating average screening volume incursion rates**
   – In part, because the original ODPO population model only provides the first three Keplerian orbital elements (a,e,i)
2. **“Phase space averaging” is a feasible method for estimating the screening volume incursion rates**
   – The “fast” angular variables (Ω,ω,M) define the 3D phase space
   – Implemented using a *COMBO*-based Monte Carlo algorithm
3. **Kessler’s semi-analytical theory can be used to test the software, and bound the screening volume incursion rates**
   – It also indicates that the “post-SBF/pre-SBF rate ratio” provides a natural measure of the increased level of screening volume incursions for each CARA primary

Conclusions from the SBF Phase 1 Study: Post-SBF vs. pre-SBF Incursion Rates

1. **Screening volume incursion rates have been estimated explicitly for fourteen CARA high-value satellites**
   - Selected to represent different inclinations, as well as CARA's multiple orbital regimes each of which has its own screening ellipsoid dimensions
   - In all cases, the incursion rates were bracketed by the upper- and lower-limit bounds calculated using Kessler's method
2. **The post-SBF/pre-SBF incursion rate ratios for the fourteen satellites vary considerably**
   - From about 1 up to 11 for satellite population models 1 and 2
3. **The post-SBF/pre-SBF incursion rates for LEO satellites generally increase as a function of altitude**
   - Those with perigee altitudes above about 1000 km will increase the most
4. **For 2017-2022 CARA's overall, mission-summed screening volume incursion rate will be about 3.5 times larger than present**

BACKUP SLIDES

Kessler's Semi-Analytical Method

- **Kessler's collision rate estimation method* has wide applicability**
  - Used for moon, asteroid, ring and debris population evolution studies
- **Key assumptions in Kessler's "particle kinetic theory" approach**
  1. The spherical particles are small relative to all other orbital dimensions, and are characterized by a cross section $\sigma_p = \pi(R_p)^2$
  2. Perturbations create uniform (Ω,ω,M) distributions over sufficiently long times
  3. The ergodic hypothesis applies: averaging over (Ω,ω,M) yields valid estimates for long-term averages
- **Kessler's method does not fit the current problem perfectly**
  - CARA's screening volumes are not well modeled as "particles"
  - They're ellipsoidal, not spherical
  - They're large, with sizes up to ~100km, which can significantly exceed the perigee-to-apogee distances of some CARA primaries

Kessler's theory is still useful for testing and bounding incursion rates

*D.J. Kessler, "Derivation of the Collision Probability Between Orbiting Objects: The Lifetimes of Jupiter's Outer Moons", *Icarus*, 48, 39-48, 1981.

Kessler's Rate Equations Applied to Spherical Screening Volumes

Cross section for a primary's spherical screening volume (m^2): $\sigma_p = \pi (R_p)^2$

Average rate for a primary + secondary pair (events s^{-1}): $(\dot{P}_c)_{p,s} = \sigma_p \, F_{p,s}$

Flux (events m^{-2} s^{-1}): $F_{p,s} = F(a_p, e_p, i_p, a_s, e_s, i_s) =$ 2D numerical integral

Total average rate (events s^{-1}): $(\dot{P}_c)_p = \sigma_p \Sigma_s (F_{p,s}) \propto (R_p)^2$

Rate ratio: $\rho_p = \dfrac{\Sigma_s (F_{p,s})}{\Sigma_{s'} (F_{p,s'})}$ for two secondary populations (s, s')

Kessler's Rate Equations
Key Trends in Rates and Rate Ratios

For spherical volumes, Kessler's theory indicates the following:

1) Incursion rates vary in proportion to the square of the screening volume size

2) The ratio of incursion rates between two secondary populations is independent of the screening volume size

- So a "post-SBF/pre-SBF rate ratio" provides a natural measure of the increased level of screening volume incursions

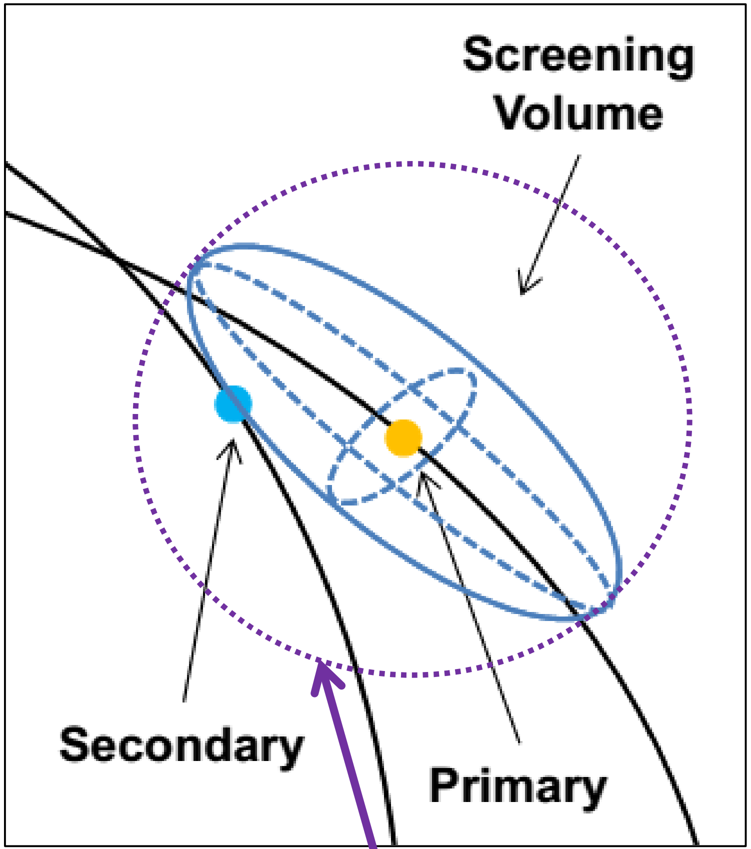
$$\text{Total average rate (events s}^{-1}\text{)}: \ (\dot{P}_c)_p = \sigma_p \Sigma_s (F_{p,s}) \propto (R_p)^2$$

$$\text{Rate ratio}: \rho_p = \frac{\Sigma_s (F_{p,s})}{\Sigma_{s'} (F_{p,s'})} \quad \text{for two secondary populations } (s, s')$$

Using Kessler's Method for Testing the Software and Bounding the Incursion Rates

- **The *COMBO*-based Monte Carlo software should work for both ellipsoidal and spherical screening volumes**
  - For spherical volumes, it must be able to reproduce results from Kessler's method
- **Kessler's method also provides upper- and lower-limit bounds for the estimated incursion rates**
  - The collision cross section for an ellipsoid with **semi-principal axes $a \leq b \leq c$** is bounded to the following range:

$$\pi(ab) \leq \sigma \leq \pi(bc)$$



Rates for circumscribed spheres can be estimated using Kessler's method

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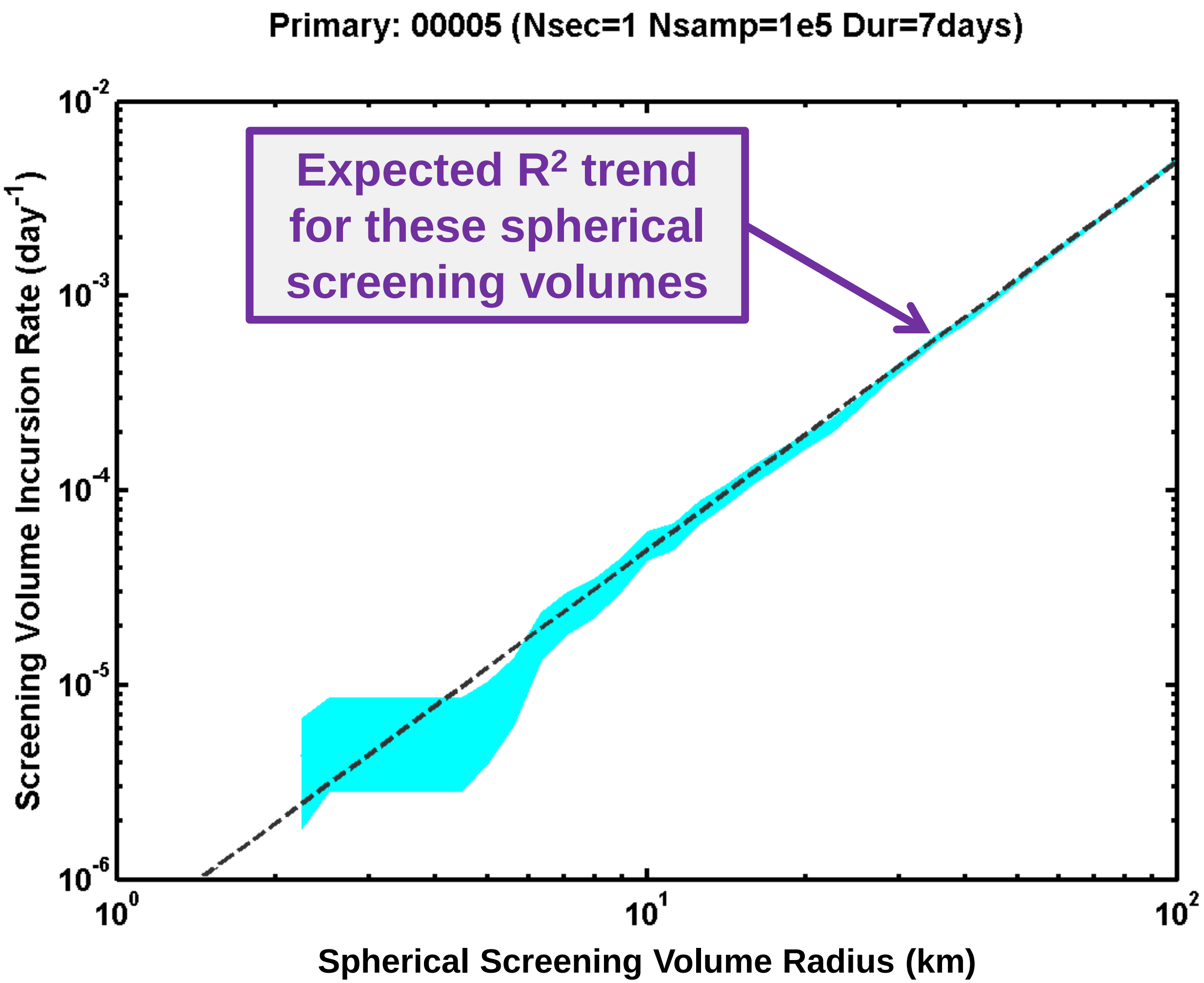
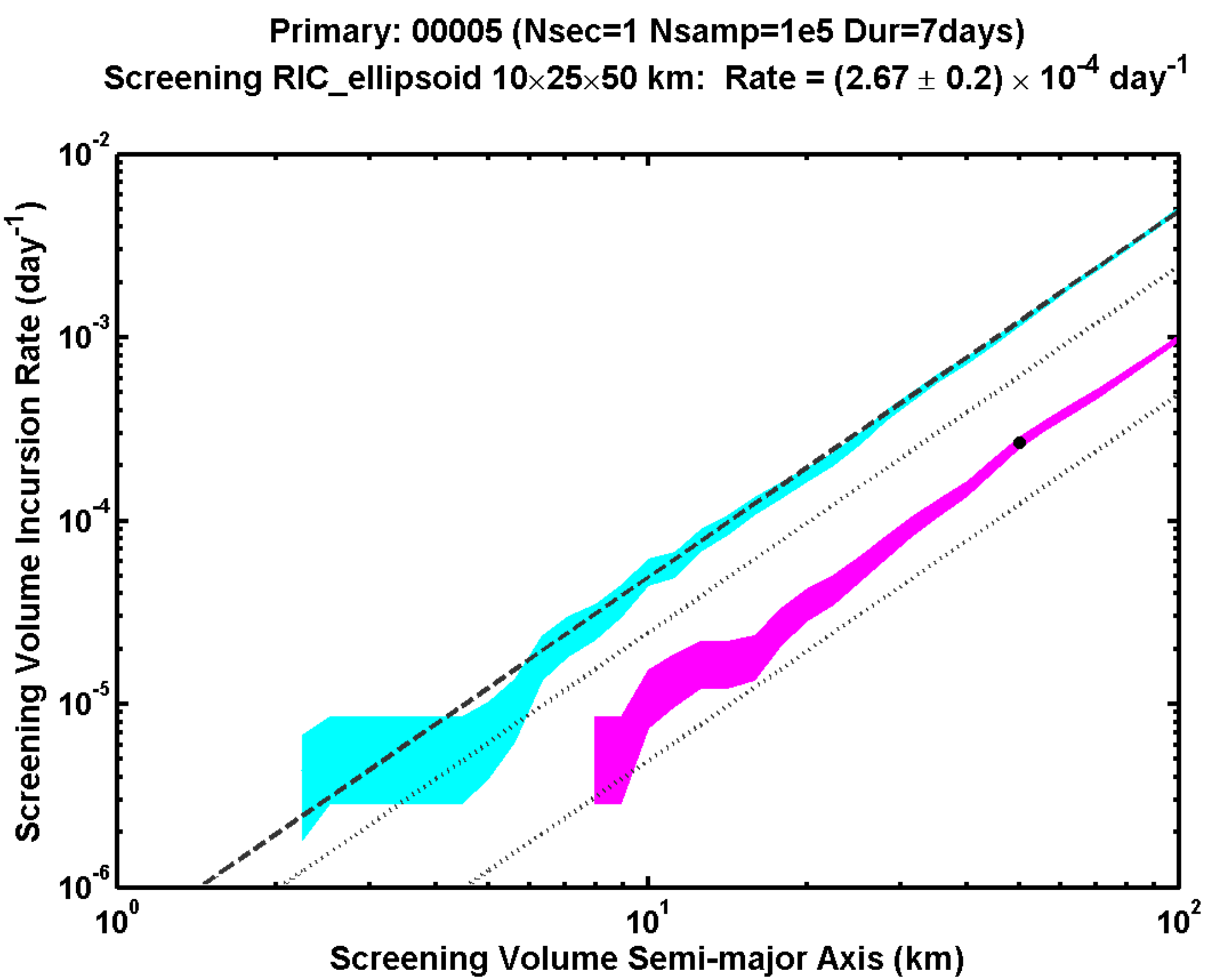
Testing and Validation: Spherical Screening Volume Incursion Rates for Vangaurd-1 and Vanguard-2

- The first test of the *COMBO*-based incursion rate software used two high-altitude LEOs
  - Vanguard 1 (00005)
  - Vanguard 2 (00011)
- Plotted results:
  - Cyan band: *COMBO*-based Monte Carlo analysis $N_{MC} = 10^5$ ($\pm 1\sigma$ range)
  - Dashed black line: Kessler's method



Testing and Validation: Ellipsoidal Screening Volume Incursion Rates for Vangaurd-1 and Vanguard-2

- **Spherical screening:**
  - Cyan band: *COMBO*-based Monte Carlo analysis (±1σ range)
  - Dashed black line: Kessler's method
- **Ellipsoidal screening:**
  - Magenta band: *COMBO*-based Monte Carlo with $N_{MC} = 10^5$ (±1σ range)
  - Dotted black lines: Kessler bounds
  - Black dot: Rate for the actual screening ellipsoid



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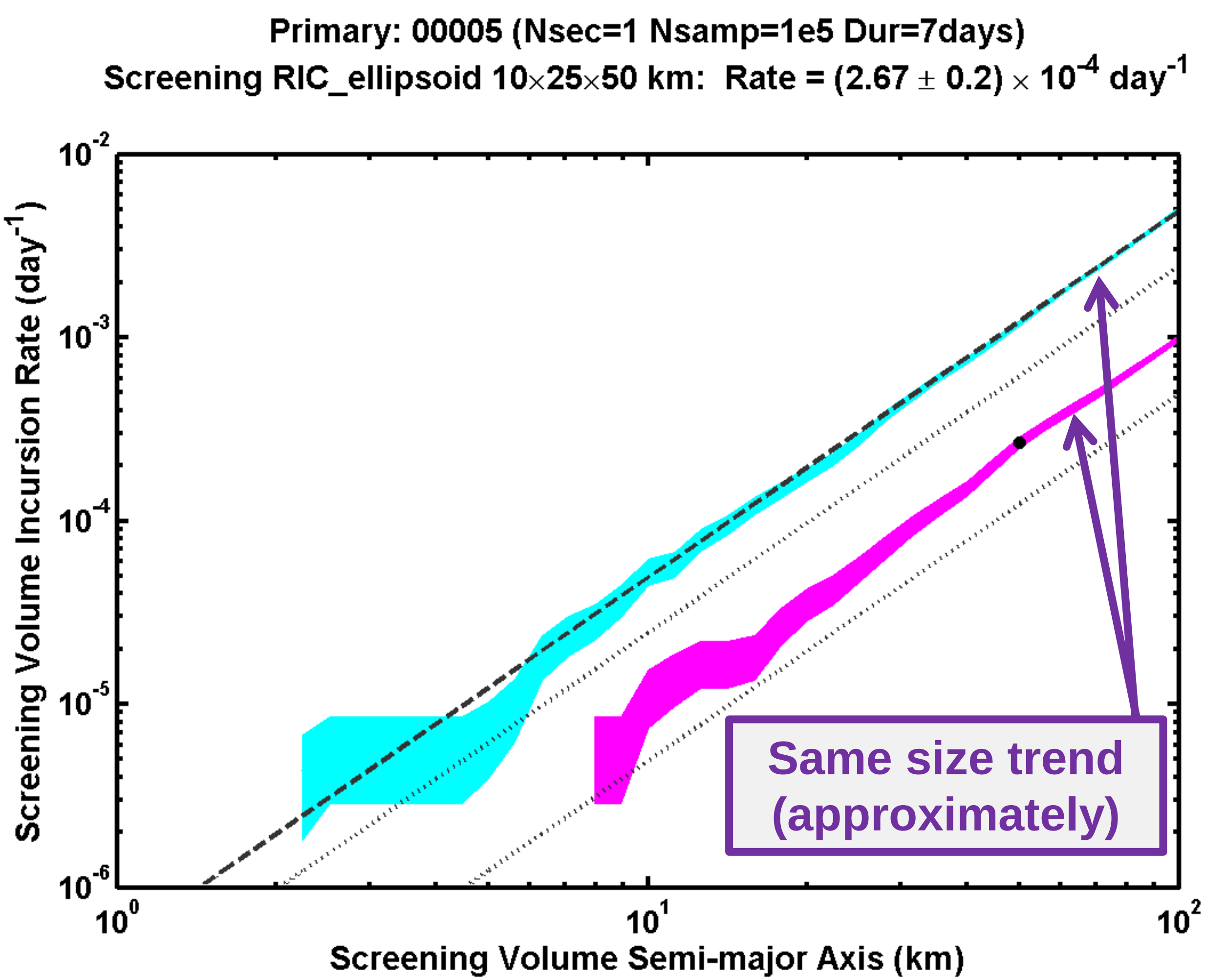

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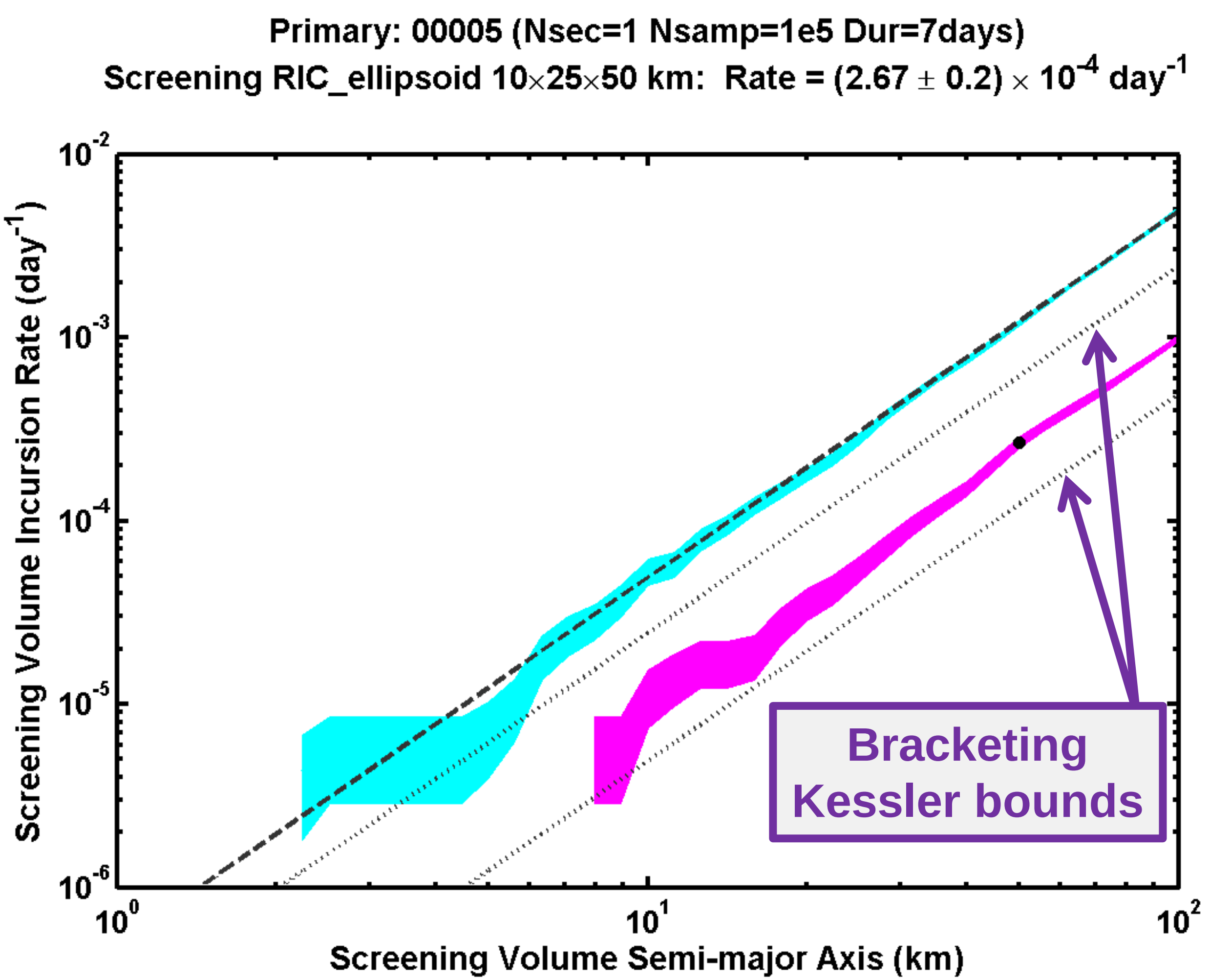

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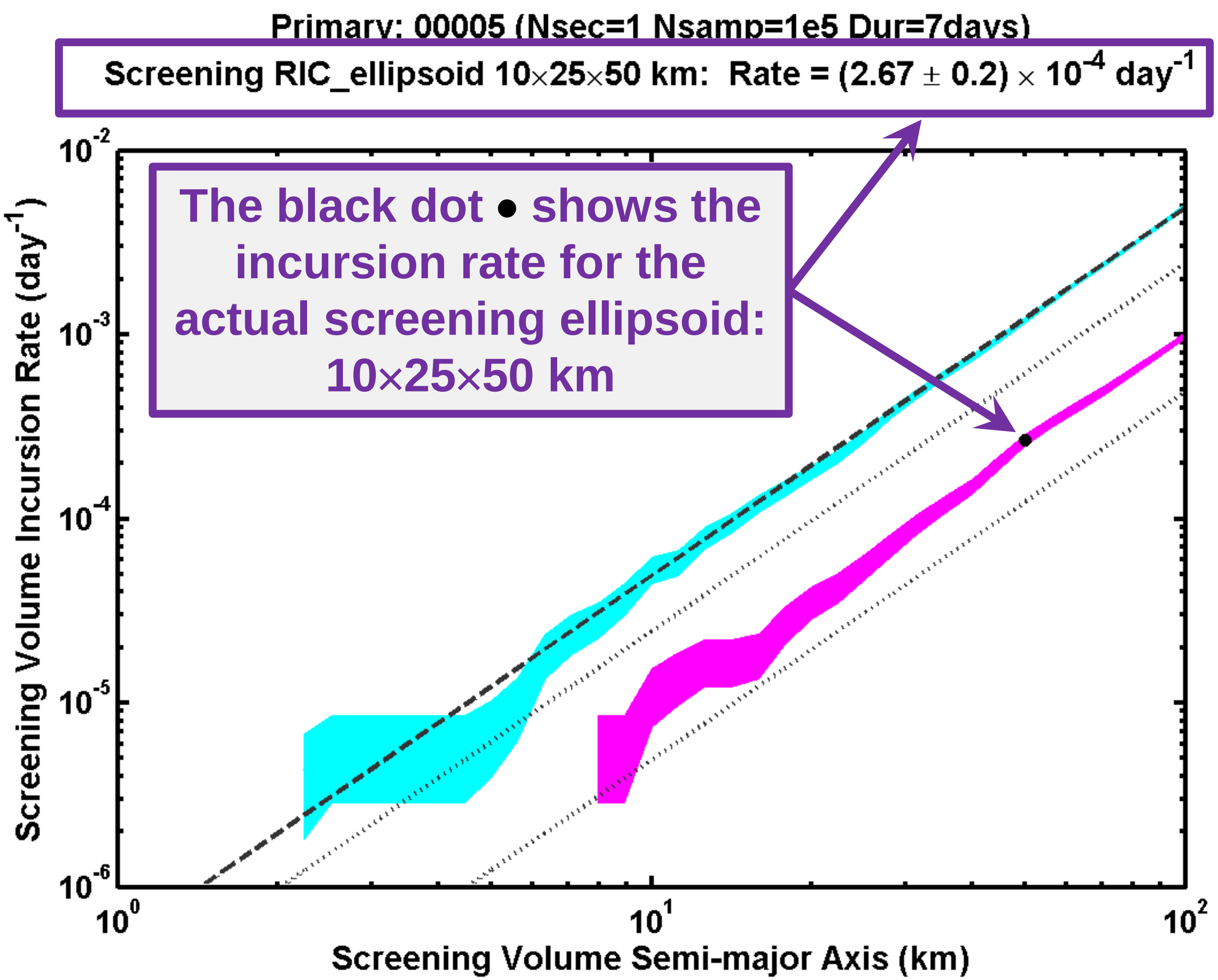

Total Incursion Rate for the MMS-1 Satellite (SCN 40482) Using the Current Catalog

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  - Cyan band: *COMBO*-based Monte Carlo analysis ($\pm 1\sigma$ range)
  - Dashed black line: Kessler's method
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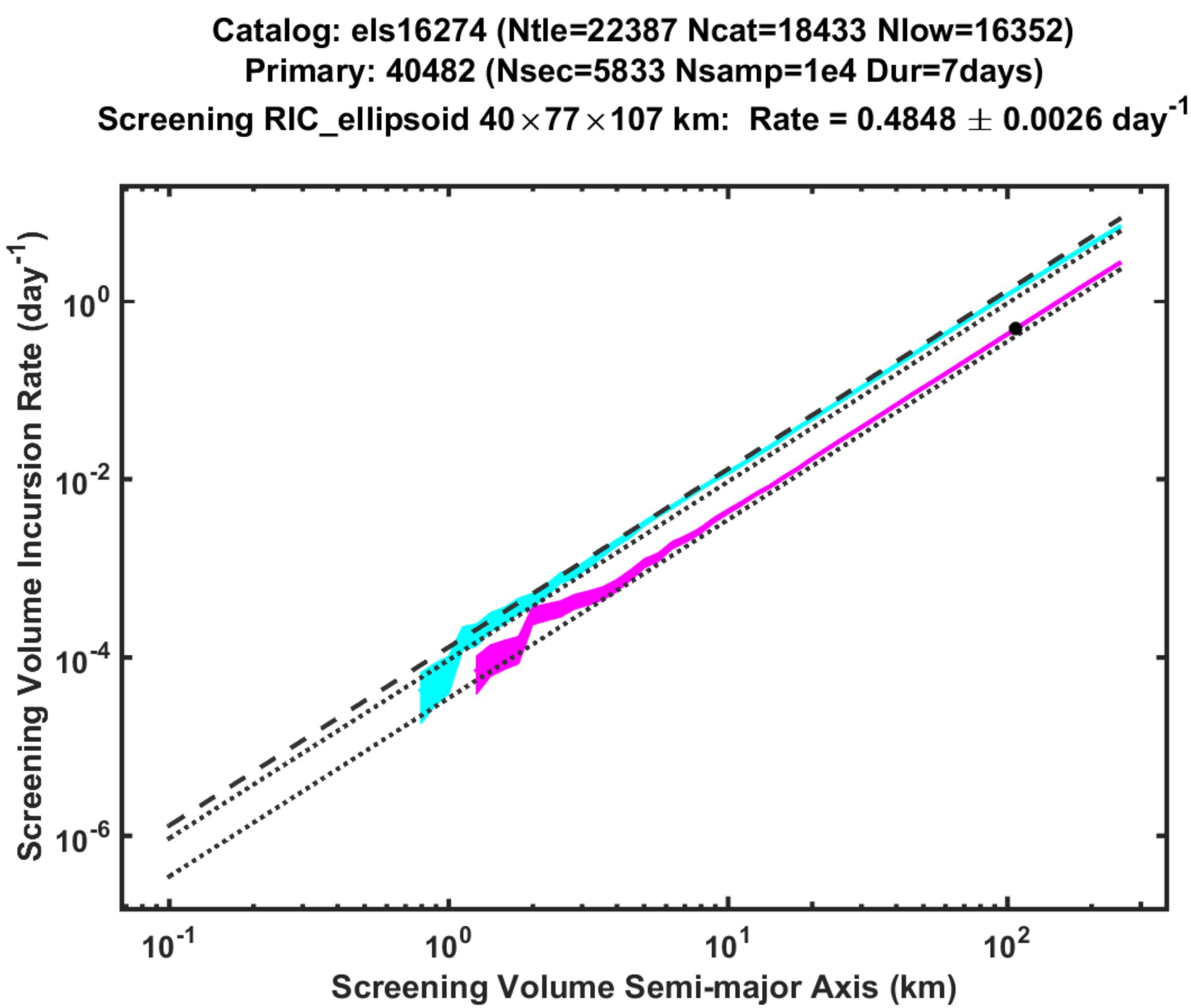

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