National Aeronautics and Space Administration

Development Status of a 3-D Electron Fluid Model for Hall Thruster Plume Simulations

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July 11, 2018

53rd Joint Propulsion Conference

Background

- Simulation results are most often validated through comparisons with ground facility measurement data, while models need to predict on-orbit EP thruster operations
  - Predicting behavior of flight system requires deep understanding of components and system-level interaction (e.g., thruster, cathode, facility effects, backsputtering)
- While the background pressure effects have been extensively studied in the past,[1-6] we still have limited understanding electrical facility effects in vacuum chambers
  - The presence of conducting chamber walls provides alternate pathways for electrons to travel from the cathode and serve as a recombination site[7-8]
  - Need to better understand how electrons in Hall thruster plumes travel through and interact with the metallic walls of vacuum chambers

[1] Walker, *et.al.*, AIAA-4253, 2002.
[2] Walker, *et.al.*, Journal of Propulsion and Power, 2005.
[3] Walker, Ph.D. Dissertation, University of Michigan, 2005.
[4] Reid, Ph.D. Dissertation, University of Michigan, 2009.
[5] Kamhawi, *et.al.* AIAA-3707, 2014.
[6] Huang, W. *et.al.*, AIAA-3708, 2014.
[7] Frieman, *et.al.*, *Journal of Propulsion and Power*, 2015.
[8] Frieman, *et.al.*, *Journal of Propulsion and Power*, 2014.

Development of an Electron Plume Model

- In general, the Boltzmann relation is used to simulate electrons in EP thruster plumes
  - Simple and useful for isothermal, collisionless, & unmagnetized regions
  - Assumes zero-electron current everywhere
  - Unable to capture strong gradients in the near-field plume
- A detailed electron fluid model was developed by Boyd & Yim using a current conservation and Generalized Ohm's law[9-10]
  - Can predict plume characteristics better than the Boltzmann relation
  - Still could not capture steep gradients in the near-field plume
- This study was further improved by including a full electron mobility tensor to account for anisotropies in electron transport due to magnetic field effects (Choi 2016)
  - Similar approach in 3-D is being pursued at NASA GRC

[9] Boyd, I.D. and Yim, J.T, AIAA-3952, 2004.
[10] Cai, C and Boyd, I.D., AIAA-4462, 2005.
[11] Choi, M., Space Propulsion, 2016

3-D Electron Plume Model for TURF

- The Thermophysics Universal Research Framework (TURF) is developed by Air Force Research Laboratory (AFRL)
  - Many capabilities including but not limited to:
    - Vlasov, particle, fluid, and hybrid simulation;
    - Flexible geometry import;
    - Surface interaction and charging modules[12]
- This electron model is currently being implemented in TURF to perform:
  - Full vacuum facility simulations with magnetic field effects
  - Full spacecraft integration modeling with surface charging capability

[12] Araki, S.J., and Barrie, A.C., AIAA-4906, 2018.

Physical Model

- Generalized Ohm's law:

$$\boldsymbol{j}_e = \mu_e(\boldsymbol{j}_e \times \boldsymbol{B}) + \sigma_e\left(\boldsymbol{E} + \frac{1}{en_e}\nabla P_e\right)$$

- Steady-state current conservation:

$$\nabla \cdot (\boldsymbol{j}_e + \boldsymbol{j}_i) = 0$$

- Rearranging, we get:

$$\nabla \cdot (\bar{\bar{\mu}}^{-1}\sigma_e \nabla\phi) = \nabla \cdot \left(\bar{\bar{\mu}}^{-1}\frac{\sigma_e}{en_e}\nabla P_e\right) + \nabla \cdot \boldsymbol{j}_i$$

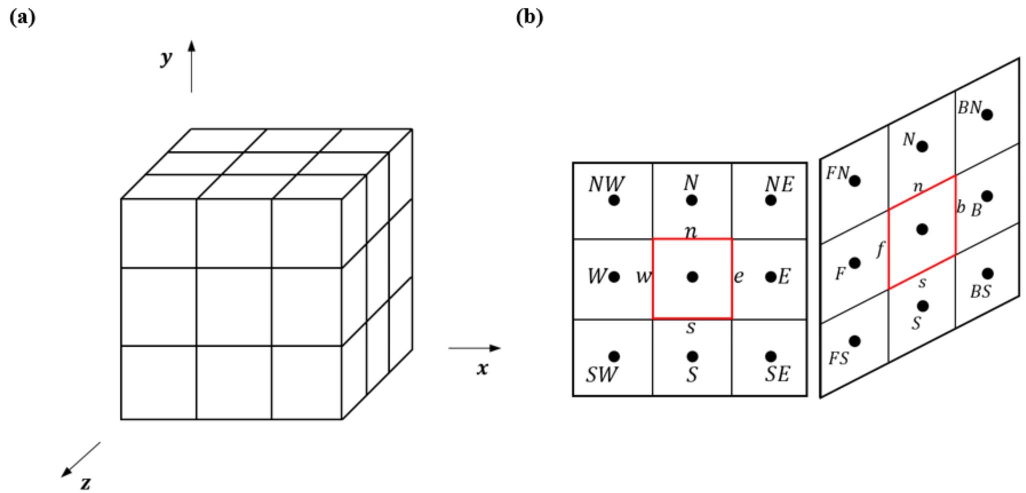
where the electron mobility is $\mu_e = \dfrac{q}{m_e \nu_{ce}}$ and the full tensor is defined as: $\bar{\bar{\mu}} = \begin{bmatrix} 1 & -\mu_e B_x & \mu_e B_z \\ \mu_e B_x & 1 & -\mu_e B_y \\ -\mu_e B_z & \mu_e B_y & 1 \end{bmatrix}$

Numerical Model

- 2nd order cell-centered finite volume approach is used

$$\int_V [\nabla \cdot (\bar{\bar{\mu}}^{-1}\sigma_e \nabla \phi)] dV = \int_V \left[\nabla \cdot \left(\bar{\bar{\mu}}^{-1}\frac{\sigma_e}{en_e}\nabla P_e + \boldsymbol{j}_i\right)\right] dV$$

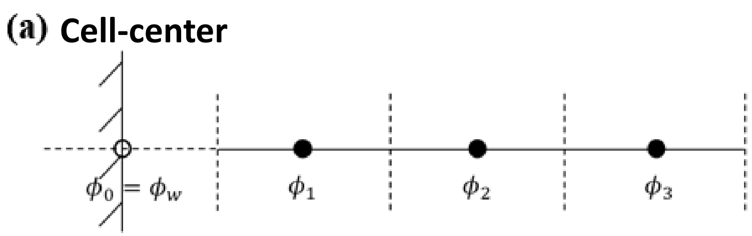
- Using the Green's theorem, the volume integral is transformed into surface integrals along all faces
- The final discretized equation becomes a linear system of equations that is solved explicitly ($A\boldsymbol{x} = \boldsymbol{b}$)
- FVM = FDM for this equation

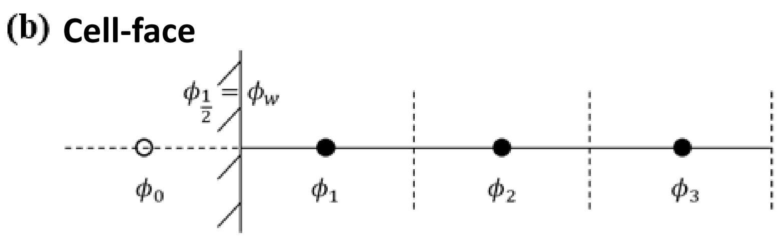


Boundary Treatment: Dirichlet Boundary

- (a) Cell-center: the simplest way -> use ghost cells
- (b) Cell-face:
  - FVM: $\left.\frac{\partial\phi}{\partial x}\right|_{\frac{1}{2}} = \frac{\phi_1 - \phi_{\frac{1}{2}}}{\left(\frac{\Delta x}{2}\right)}$
  - FDM: $\phi_0 = 2\phi_{\frac{1}{2}} - \phi_1$; O(h)

    $\phi_0 = \frac{8}{3}\phi_{\frac{1}{2}} - 2\phi_1 + \frac{1}{3}\phi_2$; O(h^2)
- Flux at the boundary for (b):
  - 0^{th} order: $(\sigma\bar{\bar{\mu}}^{-1})_{\frac{1}{2}} = (\sigma\bar{\bar{\mu}}^{-1})_1$
  - 2^{nd} order: $(\sigma\bar{\bar{\mu}}^{-1})_{\frac{1}{2}} = \frac{3(\sigma\bar{\bar{\mu}}^{-1})_1 - (\sigma\bar{\bar{\mu}}^{-1})_2}{2}$





Corner Boundaries (in 2-D Example)

- Need to approximate the value of the ghost cell ϕ_{00}
- For O(h):
  - 2-point distance-weighted averaging

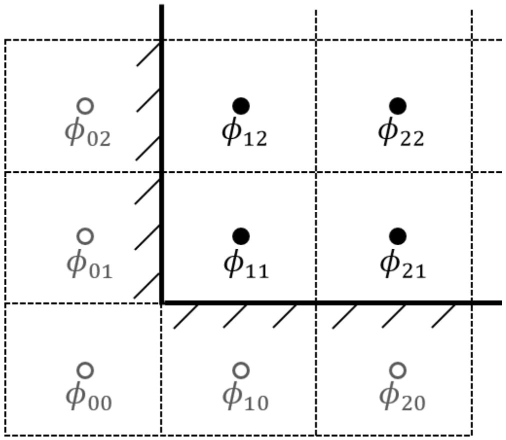
$$\phi_{00} = \frac{\phi_{10}\Delta y + \phi_{01}\Delta x}{\Delta x + \Delta y} \text{ which reduces to } \frac{\phi_{10}+\phi_{01}}{2} \text{ for } \Delta x = \Delta y$$

  - 3-point distance-weighted averaging

$$\phi_{00} = \frac{\phi_{10} r \Delta y + \phi_{01} r \Delta x + \phi_{11}\Delta x \Delta y}{\Delta x + \Delta y + r} \text{ where } r = \sqrt{\Delta x^2 + \Delta y^2}$$

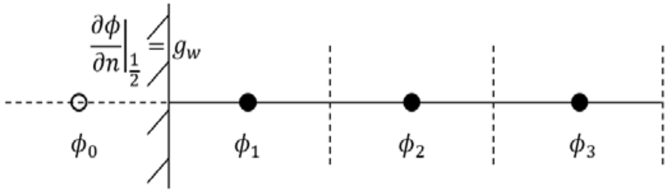
- For O(h^2):
  - $\phi_{00} = \frac{\phi_{10}+\phi_{01}}{2}$

$$= \frac{1}{2}\left[\frac{8}{3}\left(\phi_{\frac{1}{2}1} + \phi_{1\frac{1}{2}}\right) - 4\phi_{11} + \frac{1}{3}\left(\phi_{12} + \phi_{21}\right)\right]$$



Boundary Treatment: Neumann Boundary

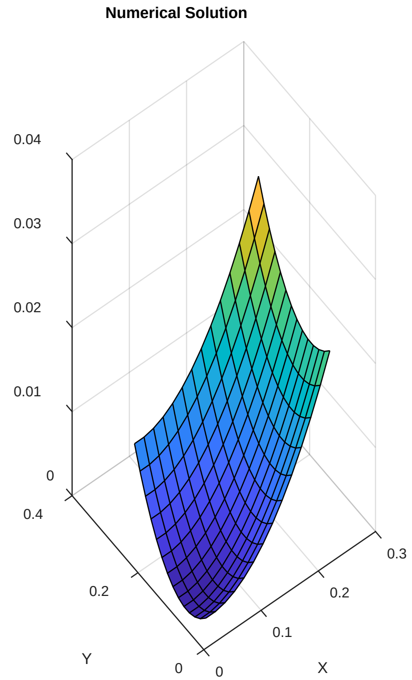
- Need to calculate cell-face values
  - $\phi_0 = \phi_1$ for zero gradient
  - $\phi_0 = \phi_1 + g_w \Delta x$ (1st order) for non-zero gradient

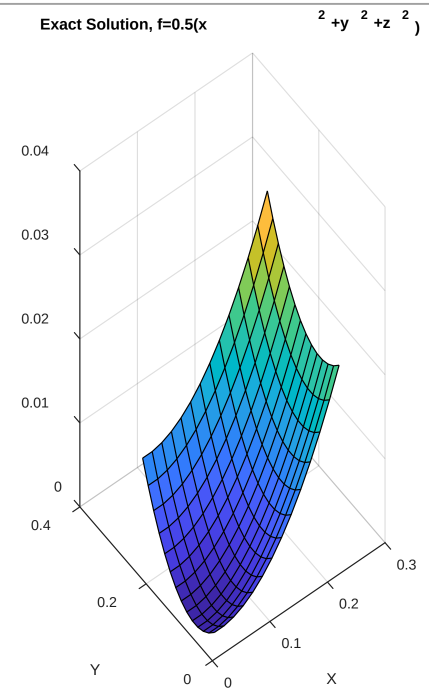


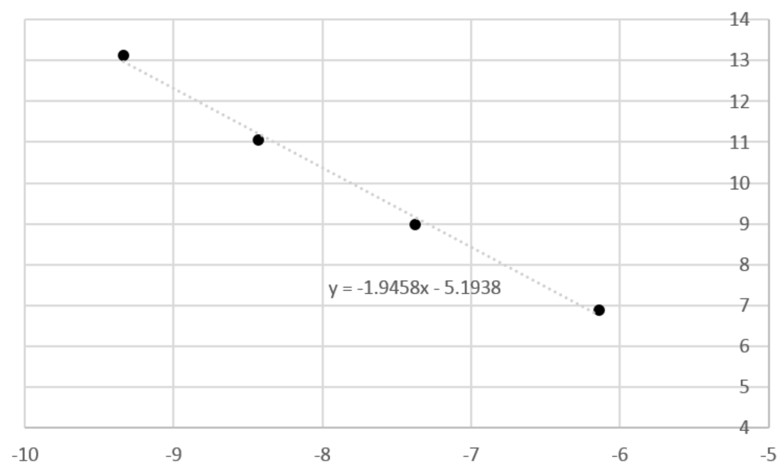
Verification Tests

- Method of Manufactured Solutions
  - $\phi_{exact} = \frac{K}{6}(x^2 + y^2 + z^2)$ where K is a constant
  - $F = \frac{K}{3}\sigma_e(\mu_{11} + \mu_{22} + \mu_{33})$
- Grid convergence study
  - $\|e\|_{L_2} = \sqrt{\sum_{k=1}^{N_k} \int_{\Omega_k} (\phi_h - \phi_{exact})^2 d\Omega_k}$

Manufactured Solution Result

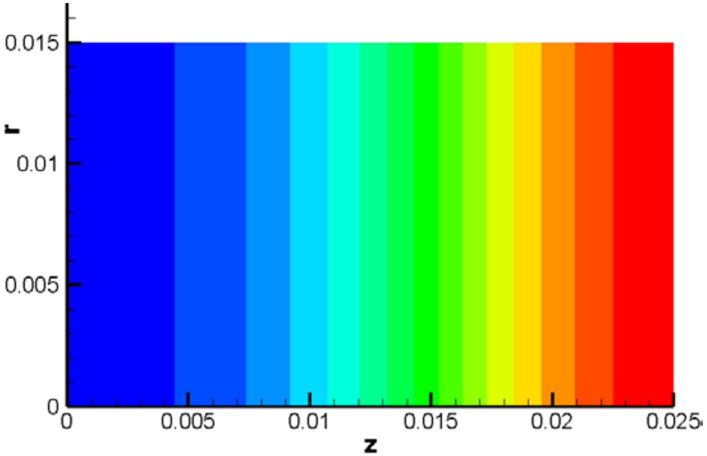






A Hall Thruster-Like Testcase

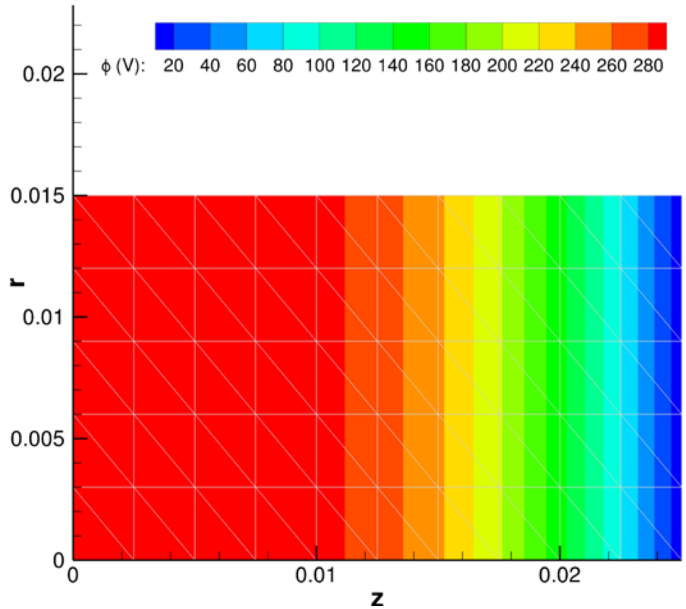
- Simple testcase made by Dragnea[13]
- Assumptions:
  - $T_e = 25\ eV, n_e = 1\text{e}17\ m^{-3}$; ideal gas law
  - $\boldsymbol{j}_i = 0$
  - 300 V on the left (anode) and 10 V on the right (cathode) surfaces
  - Zero-flux on the top and the bottom walls
  - Infinitely long plate in z-direction (periodic BC)
  - Magnetic field strength as shown below:

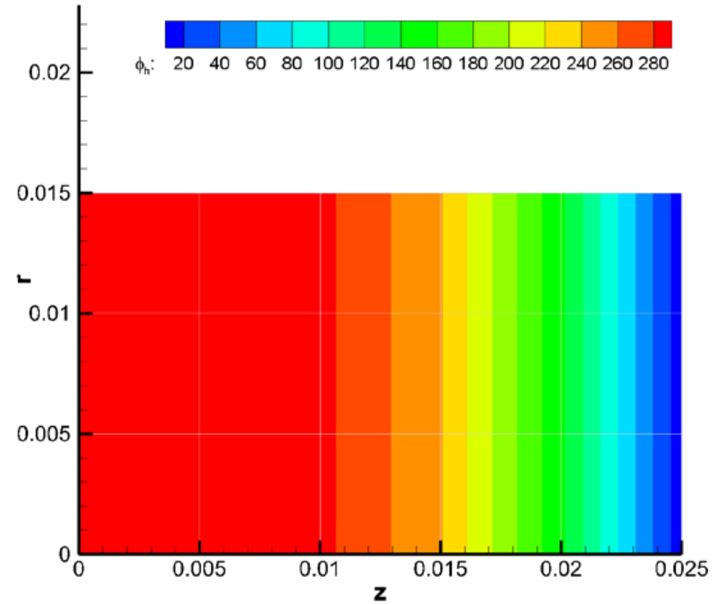


[13]Dragnea, *et.al.*, AIAA-4632, 2017.

Plasma Potential Calculation

- Qualitative comparison between the 2-D axisymmetric finite element model (left) and the 3-D finite volume model (right)



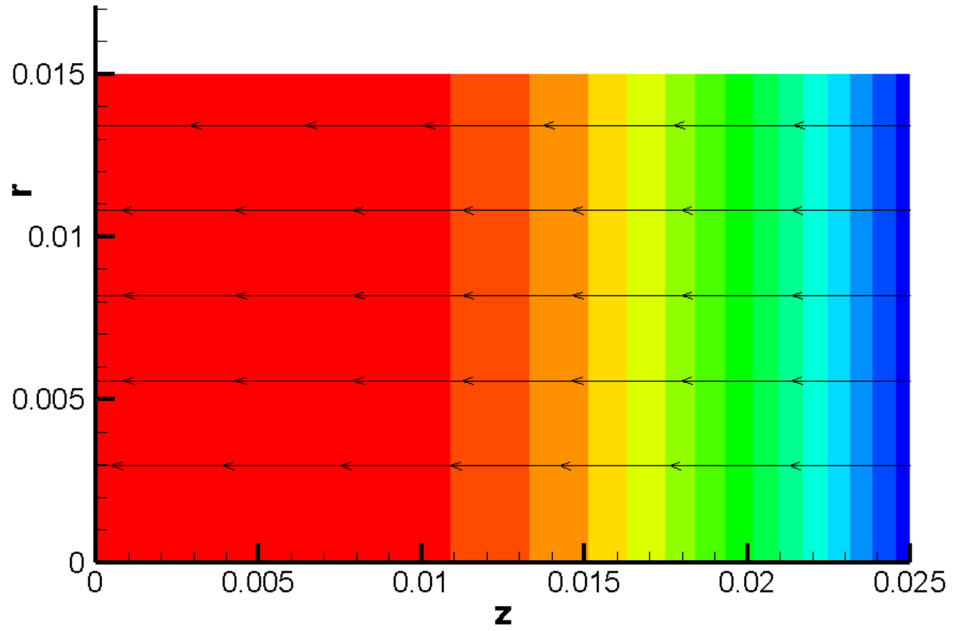


Electron Current Flow Calculation

- Use the plasma potential gradient in Generalized Ohm's law:

$$\boldsymbol{j}_e = \bar{\bar{\mu}}^{-1}\sigma_e\left(-\nabla\phi + \frac{1}{en_e}\nabla P_e\right)$$

- Electron current density streamline $\left(j_{e_z}, j_{e_r}\right)$:



Summary & Future Work

- A 3-D electron fluid model has been developed as a stepping stone to simulate
  - the electrical facility effects in a conducting vacuum chamber
  - predict electron flux values to solar arrays and other components of a spacecraft
- The model is also being implemented in TURF in collaboration with AFRL