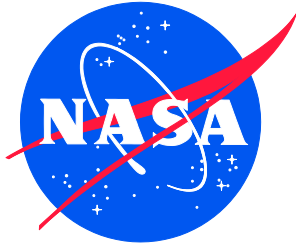


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An Examination of Launch Vehicle Loads Reanalysis Techniques

*Daniel C. Kammer and Paul A. Blelloch
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September 2018

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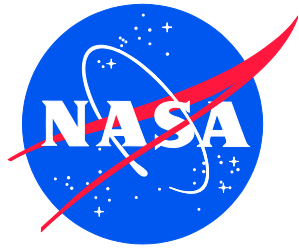
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Abstract

The typical approach to calculating dynamic launch loads in aerospace applications is coupled loads analysis (CLA). A component mode model of the launch vehicle is coupled with a component mode model of a payload, system modes are calculated, forcing functions are applied, and the dynamic responses are computed. This approach requires the explicit knowledge of the component models for the launch vehicle and payload, as well as the forcing functions. In many situations, the launch vehicle and forcing functions do not change from one analysis to the next only the payload is different. For this type of application, a method called reanalysis was developed to compute the response of a modified payload on the same launch vehicle. If the launch forcing functions are also the same, the approach eliminates the need for the launch vehicle model and the forcing functions and dramatically reduces the computation time.

This work investigates the application and accuracy of three previously developed reanalysis methods using a typical launch vehicle and two different payloads. All three methods are based on knowledge of system modes from the original CLA, and Hurty/Craig-Bampton (HCB) models of the original and new payloads. The first method was developed at JPL and is often referred to as substitution. It is a frequency-domain method, which requires transformation of time signals to and from the frequency domain. The other two methods are time-domain methods that more closely mimic the CLA process.

The three methods were applied to two simple examples and a more complex one. The results indicate that the time-domain methods are considerably more robust with respect to modal truncation and other numerical errors than the frequency-domain JPL method.

1.0 Introduction and Background

The typical approach to calculating dynamic launch loads in aerospace applications is coupled loads analysis (CLA). A component mode model of the launch vehicle is coupled with a component mode model of a payload, usually in the form of a Hurty/Craig-Bampton (HCB) representation [1]. System modes are calculated, forcing functions are applied, and the dynamic responses are computed. This approach can be computationally intensive and requires the explicit knowledge of the component models for the launch vehicle and payload, as well as the forcing functions. If it is necessary to simulate many launch systems, representing multiple payloads, the computational burden may become prohibitive.

However, in many situations, the launch vehicle and forcing functions do not change from analysis to analysis; only the payload is different. For this type of application, a method called reanalysis can be used to compute the response of a modified payload on the same launch vehicle. If the launch forcing functions are also the same, this approach eliminates the need for the launch vehicle model and forcing functions.

The work presented here was funded by the NASA Engineering Safety Center (NESC) to examine “Fast Coupled Loads Analysis Methods.” The primary focus of the NESC effort is a frequency-domain method referred to as Norton-Thevenin Receptance Coupling (NTRC). This method assumes knowledge of the interface accelerance¹ of both the original and modified payloads at the interface degrees of freedom (DOF) as well as the accelerance of the unloaded launch vehicle at the coupled DOF. This is effectively a frequency response function (FRF) synthesis method along

¹ The accelerance for an N DOF interface is the $N \times N$ transfer function of accelerations per force/moment. It is related to other frequency-dependent quantities such as impedance, which is force/moment per velocity.

the lines of refs. [2], [3], and [4], which uses FRFs of components to calculate the frequency response of a coupled system.

The purpose of the effort presented here was to investigate the application and accuracy of three previously developed reanalysis methods that have been used to solve the same problem. The methods presented here are not new and are being reprised for the purpose of supporting the primary NTRC effort. These methods all differ from NTRC in the sense that they are based on modal representations of an original and an updated payload, and system modes from a CLA using the original payload. All three methods use HCB models of the payloads, and CLA system mode shape coefficients at the payload interface. The first method was used at JPL until approximately 2000 and is referred to as substitution analysis. As implemented at JPL, this was a frequency-domain method, though Trubert and Peretti [5] indicate how it could be used in the time domain by taking a fast Fourier transform (FFT) of the input time histories, performing the substitution analysis in the frequency domain, and taking an inverse FFT (IFFT) of the results to return modified time-domain responses. As well as the HCB models of the original and modified payloads, the JPL method requires system modal frequencies and mode shape coefficients at the payload interface and payload interface time histories from a CLA using the original payload. The second two methods [6], [7] are variations on a time-domain reanalysis technique and are referred to here as time-domain reanalysis Method 1 and Method 2. Both methods were motivated by the JPL method [5], but the equations were formulated directly in the time domain without any intermediate use of FFTs to convert to the frequency domain. The difference between Method 1 and Method 2 is in the input data required. As well as HCB models of the original and modified payloads, Method 1 requires system modal frequencies and mode shape coefficients for all the original payload DOF (interface and modal) as well system modal generalized force time histories from the original CLA. Method 2 requires the same data, except that the system modal generalized force time histories are replaced by the time histories of the original payload DOF (interface and modal). Both these methods require more input data than the JPL substitution analysis, since the original payload modal as well as interface responses are required.² However, at the time of their development they were both found to be significantly more robust and accurate than the JPL method, particularly when used to replace a time-domain CLA. All three methods presented here have been documented previously along with sample problems. Two examples are presented in Trubert and Peretti [5]. The first example is a Shuttle/Centaur model with a single-node payload interface. The second is a stick model of a Galileo spacecraft, also with a single-node payload interface with a Centaur upper stage. Examples of time-domain reanalysis Method 2 are presented in Blelloch and Flanigan [6]. The first example is an expendable launch vehicle (ELV) booster engine cutoff (BECO) simulation with eighty system modes and two very different payloads using a single-node interface. Though the responses differed significantly between the two payloads, the reanalysis prediction was extremely accurate. The second example was a Space Shuttle with an indeterminate (7 DOF) interface and over 500 system modes. It also proved to be very accurate. The *Reanalysis User's Guide* [7] adds one additional example based on Method 1. In this case, the example involved changes in a component at the tip of a complex superelement tree with a statically indeterminate interface. Reanalysis reduced the time required to evaluate the response of a single model change from close to 40 hours to about 5 minutes,³ and while the results were not

² In theory the modal responses can be calculated by integrating the HCB equations using the interface motion as an input. However, these results will be different from those calculated from the system-level analysis which truncates modes, so using the results directly is preferred.

³ These computation times were on a VAX 6420 and are not applicable to today's computing environments.

quite as accurate as the previous examples, critical loads were still within 3% of the “exact” values. These references can be provided if desired.

In this report, the theory of each of the three methods is presented first in section 2.0, followed by results for three example models in section 3.0. These examples are used to illustrate the strengths and weaknesses and relative accuracy of these methods. In section 4.0, the results are summarized with conclusions and recommendations.

2.0 Theory

The mathematical background behind the three reanalysis methods studied in this report is presented here. The first is the frequency-domain method, sometimes called substitution analysis, developed by JPL and used for many years. The second is the time-domain “Method 1” approach, which requires the original generalized force time histories. The third is the time-domain “Method 2” approach, which only requires the original component time histories (physical and modal).

2.1 Reanalysis—JPL Method

For many years, JPL has used a reanalysis methodology that they often refer to as substitution analysis. The method is documented in Trubert and Peretti [5]. This approach is fundamentally a frequency-domain method, though it can be applied to time-domain data by transforming the dynamic response from the time domain to frequency and then back again. Both time-domain methods discussed in sections 2.2 and 2.3 were originally based on the JPL approach but have all calculations performed in the time domain without requiring any transformations to the frequency domain.

The derivation of the JPL method follows that in Trubert and Peretti [5]. The nominal system equations of motion, assuming mass-normalized modes, can be written in the frequency domain as

$$[-\omega^2 + j\omega 2\zeta_q \omega_q + \omega_q^2]Q(\omega) = B^T F(\omega) \quad (2-1)$$

in which capital letters represent the Fourier transform of their time-domain counterparts, and

- ω = forcing frequency
- ζ_q = diagonal modal damping matrix for the nominal system
- ω_q = diagonal modal frequency matrix for the nominal system
- $Q(\omega)$ = vector of modal degrees of freedom as function of frequency
- B = mode shape coefficients at input locations
- $F(\omega)$ = applied loads as a function of frequency

It should be noted that while Trubert and Peretti [5] assume diagonal damping of the system modes, this is not necessary and the diagonal matrix $2\zeta_q \omega_q$ can be replaced with a fully coupled damping matrix. ATA’s implementation does not assume diagonal damping, though the solution takes advantage of diagonal damping to accelerate the solution time if it is specified.

Making the substitution $H_q(\omega) = [-\omega^2 + j\omega 2\zeta_q\omega_q + \omega_q^2]^{-1}$, this can be written as

$$Q(\omega) = H_q(\omega)B^T F(\omega) \quad (2-2)$$

If we remove Payload 1 and replace it with Payload 2, we get a modified set of equations based on the force applied by the system to Payload 1 ($F_{t1}(\omega)$) and the force applied by the system to Payload 2 ($F_{t2}(\omega)$).⁴

$$Q'(\omega) = H_q(\omega)B^T F(\omega) + H_q(\omega)\Phi_t^T(F_{t1}(\omega) - F_{t2}(\omega)) \quad (2-3)$$

Subtracting Eq. (2-1) from (2-3) eliminates the input forces and gives

$$Q'(\omega) = Q(\omega) + H_q(\omega)\Phi_t^T(F_{t1}(\omega) - F_{t2}(\omega)) \quad (2-4)$$

Assume that the payloads are represented by HCB models in the frequency domain as follows:

$$\left(-\omega^2 \begin{bmatrix} m_{tt1} & m_{tq1} \\ m_{qt1} & I \end{bmatrix} + j\omega \begin{bmatrix} c_{tt1} & 0 \\ 0 & 2\zeta_1\omega_1 \end{bmatrix} + \begin{bmatrix} k_{tt1} & 0 \\ 0 & \omega_1^2 \end{bmatrix}\right) \begin{Bmatrix} \ddot{U}_{t1}(\omega) \\ Q_1(\omega) \end{Bmatrix} = \begin{Bmatrix} F_{t1}(\omega) \\ 0 \end{Bmatrix} \quad (2-5)$$

$$\left(-\omega^2 \begin{bmatrix} m_{tt2} & m_{tq2} \\ m_{qt2} & I \end{bmatrix} + j\omega \begin{bmatrix} c_{tt2} & 0 \\ 0 & 2\zeta_2\omega_2 \end{bmatrix} + \begin{bmatrix} k_{tt2} & 0 \\ 0 & \omega_2^2 \end{bmatrix}\right) \begin{Bmatrix} U_{t2}(\omega) \\ Q_2(\omega) \end{Bmatrix} = \begin{Bmatrix} F_{t2}(\omega) \\ 0 \end{Bmatrix} \quad (2-6)$$

in which

- ζ_1 = diagonal modal damping matrix for Payload 1 fixed-interface modes
- ω_1 = diagonal modal frequency matrix for Payload 1 fixed-interface modes
- ζ_2 = diagonal modal damping matrix for Payload 2 fixed-interface modes
- ω_2 = diagonal modal frequency matrix for Payload 2 fixed-interface modes.

Note again that Gordis [4] assumes diagonal damping of the component modes. In general, diagonal damping of component modes is inconsistent with diagonal damping of system modes. If diagonal damping is assumed at component level, then the system damping should be consistent with this and will typically not be diagonal. This is not addressed in Gordis [4], which assumes diagonal damping at both component and system level. This assumption works fairly well for lightly damped systems, but the equations could easily be extended to more heavily damped systems by replacing the diagonal damping matrices by fully coupled consistent equivalents. For the purposes of the examples presented in this report, we just assume light (1%) diagonal damping at both the component and system level.

Solving the substructure Eqs. (2-5) and (2-6) for the interface force partitions (t -set) in the frequency domain results in

$$F_{t1}(\omega) = \left[m_{tt1} - j\frac{c_{tt1}}{\omega} - \frac{k_{tt1}}{\omega^2}\right] \ddot{U}_{t1}(\omega) - \omega^2 m_{tq1} Q_1(\omega) \quad (2-7)$$

⁴ Sign convention is important here. F_{t1} and F_{t2} are forces applied by the launch vehicle to the payload and these are opposite the forces applied by the payloads to the launch vehicle.

$$F_{t2}(\omega) = \left[m_{tt2} - j \frac{c_{tt2}}{\omega} - \frac{k_{tt2}}{\omega^2} \right] \ddot{U}_{t2}(\omega) - \omega^2 m_{tq2} Q_2(\omega) \quad (2-8)$$

where an explicit dependence on the interface accelerations $\ddot{U}_t(\omega)$ rather than displacements is maintained. Substructure Eqs. (2-5) and (2-6) can also be partitioned to the payload fixed-interface modes, giving the expressions

$$[-\omega^2 + j\omega 2\zeta_1\omega_1 + \omega_1^2]Q_1(\omega) = -m_{qt1}\ddot{U}_{t1}(\omega) \quad (2-9)$$

$$[-\omega^2 + j\omega 2\zeta_2\omega_2 + \omega_2^2]Q_2(\omega) = -m_{qt2}\ddot{U}_{t2}(\omega) \quad (2-10)$$

Defining the two payload fixed-interface modal displacement frequency response matrices as

$$H_1(\omega) = [-\omega^2 + j\omega 2\zeta_1\omega_1 + \omega_1^2]^{-1}$$

and

$$H_2(\omega) = [-\omega^2 + j\omega 2\zeta_2\omega_2 + \omega_2^2]^{-1}$$

Eqs. (2-9) and (2-10) become

$$Q_1(\omega) = -H_1(\omega)m_{qt1}\ddot{U}_{t1}(\omega) \quad (2-11)$$

$$Q_2(\omega) = -H_2(\omega)m_{qt2}\ddot{U}_{t2}(\omega) \quad (2-12)$$

Substituting the expressions for $Q_1(\omega)$ and $Q_2(\omega)$ into the right sides of Eqs. (2-7) and (2-8), and defining the terms

$$M_{s1}(\omega) = \left[m_{tt1} - j \frac{c_{tt1}}{\omega} - \frac{k_{tt1}}{\omega^2} + \omega^2 m_{tq1} H_1(\omega) m_{qt1} \right]$$

and

$$M_{s2}(\omega) = \left[m_{tt2}(\omega) - j \frac{c_{tt2}}{\omega} - \frac{k_{tt2}}{\omega^2} + \omega^2 m_{tq2} H_2(\omega) m_{qt2} \right]$$

and setting the interface response of the two payloads in the modified system to be the same, $\ddot{U}_{t1} = \ddot{U}_{t2} = \ddot{U}_t$, allows the difference in the payload interface forces to be written as

$$F_{t1}(\omega) - F_{t2}(\omega) = (M_{s1}(\omega) - M_{s2}(\omega))\ddot{U}_t(\omega) \quad (2-13)$$

Substituting Eq. (2-13) into Eq. (2-4) results in the expression

$$Q'(\omega) = Q(\omega) + H_q(\omega)\Phi_t^T(M_{s1}(\omega) - M_{s2}(\omega))\ddot{U}_t \quad (2-14)$$

The nominal and modified system payload interface accelerations can be written as

$$\ddot{U}_{t0}(\omega) = -\omega^2 \Phi_t Q(\omega)$$

and

$$\ddot{U}_t(\omega) = -\omega^2 \Phi_t Q'(\omega)$$

Therefore, multiplying Eq. (2-14) through by $-\omega^2 \Phi_t$ gives

$$\ddot{U}_t(\omega) = \ddot{U}_{t0}(\omega) + \omega^2 \Phi_t H_q(\omega) \Phi_t^T (M_{s2}(\omega) - M_{s1}(\omega)) \ddot{U}_t(\omega) \quad (2-15)$$

Rearranging terms gives the result

$$\ddot{U}_t(\omega) = [I - \omega^2 \Phi_t H_q(\omega) \Phi_t^T (M_{s2}(\omega) - M_{s1}(\omega))]^{-1} \ddot{U}_{t0}(\omega) \quad (2-16)$$

Eq. (2-16) is the substitution equation at the heart of the JPL method. It takes the vector of interface accelerations from the nominal payload as a function of frequency and transforms it into the vector of interface accelerations for the updated payload. This is an FRF synthesis method and would be exact if all the transfer functions were exact. Since the payloads are already reduced HCB models, their response and therefore $M_{s1}(\omega)$ and $M_{s2}(\omega)$ are effectively exact. However, $H_q(\omega)$ is based on a truncated set of modes and is therefore not exact. In addition, since damping is applied at both the component and the system level, the method assumes that these are equivalent. In reality they are not, and the system-level damping will in general depend on the launch vehicle damping and will not be diagonal. If one were to use a consistent damping matrix when calculating $H_q(\omega)$, this presumably would be accounted for, but the JPL substitution method, as documented in Trubert and Peretti [5], does not appear to do this. For the purposes of the examples presented here, 1% damping was assumed on both system and component damping, though it is recognized that this is not consistent.

The JPL method recovers the interface accelerations and then uses these as input to the HCB model of the updated payload. In general, because of modal truncation at the system level, the response of the HCB model of a payload to an input acceleration is not the same as the system-level response of that payload. This will introduce another difference when comparing internal payload responses with system-level CLA results.

It is also important to note that to avoid computational difficulties with the Fourier transform, only the elastic portion of $\ddot{U}_{t0}$ is used in Eq. (2-16). The rigid body response of the nominal system interface is then added once the inverse transform of $\ddot{U}_t$ is taken to recover the time-domain response of the modified system interface. This approach assumes that the rigid body response of the payload interface is the same in the nominal and modified systems. This will be approximately correct if the rigid body properties of the two payloads are not dramatically different relative to the overall system mass properties.

In summary, the frequency-domain JPL method was used for many years, though we are not aware of the time-domain version being used extensively. The method is an FRF synthesis method, which tends to be very sensitive to the accuracy of the underlying FRFs. It makes some assumptions associated with diagonal damping at both the component and system level which are not true in general, and it requires that the updated payload internal responses be recovered using a base shake. In addition, when used to simulate a time-domain CLA, it requires transformation of the time-domain signals to the frequency domain and then back to the time domain, which can introduce further errors. The time-domain methods presented in the next two sections were developed specifically to address these deficiencies.

2.2 Reanalysis—Method 1

The second reanalysis method studied during this project is an alternative time-domain approach proposed by Blelloch and Flanigan at SDRC at the time and incorporated in a Nastran DMAP-

based software product called Reanalysis. The published version of the method [6] uses the nominal payload accelerations as input, but an intermediate method was also developed that uses the original system-level forcing functions as input. Both methods were found to be useful, and both were implemented and described in the software documentation [7]. The method that uses the system-level forcing function is referred to as Method 1 and the one that uses the original payload accelerations as Method 2. We begin by documenting Method 1.

The nominal system modal equations of motion in the time domain are given by the expression

$$\hat{M}\ddot{q}(t) + \hat{C}\dot{q}(t) + \hat{K}q(t) = B^T f(t) \quad (2-17)$$

in which

$$\begin{aligned} \hat{M} &= \text{modal mass matrix} \\ \hat{C} &= \text{modal damping matrix} \\ \hat{K} &= \text{modal stiffness matrix} \\ q(t) &= \text{modal displacements} \\ B &= \text{generalized force distribution matrix} \\ f(t) &= \text{external forces as a function of time} \end{aligned}$$

The equations of motion for the modified system in terms of the nominal system modal coordinates can be written in the form

$$\hat{M}\ddot{q}'(t) + \hat{C}\dot{q}'(t) + \hat{K}q'(t) = B^T f(t) + \Phi_t^T (f_{t1}(t) - f_{t2}(t)) \quad (2-18)$$

where

$$\begin{aligned} q'(t) &= \text{response of the modified system in the nominal modal coordinates} \\ f_{t1}(t) &= \text{interface force applied by the launch vehicle to the original payload (Payload 1)} \\ f_{t2}(t) &= \text{interface force applied by the launch vehicle to the modified payload (Payload 2)} \\ \Phi_t &= \text{nominal system modes row partitioned to the interface locations} \end{aligned}$$

Assuming an HCB substructure representation for each payload, their corresponding equations of motion can be written in the forms

$$M_{aa1}\ddot{u}_{a1}(t) + C_{aa1}\dot{u}_{a1}(t) + K_{aa1}u_{a1}(t) = f_{a1}(t) \quad (2-19)$$

$$M_{aa2}\ddot{u}_{a2}(t) + C_{aa2}\dot{u}_{a2}(t) + K_{aa2}u_{a2}(t) = f_{a2} \quad (2-20)$$

The payload displacement and forcing vectors and the nominal system modes at the payload DOF can be partitioned into interface (t -set) and fixed-interface modal (q -set)

$$u_a = \begin{Bmatrix} u_t \\ q \end{Bmatrix} \quad f_a = \begin{Bmatrix} f_t \\ 0 \end{Bmatrix} \quad \Phi_a = \begin{Bmatrix} \Phi_t \\ \Phi_q \end{Bmatrix}$$

with the corresponding mass, damping, and stiffness matrices having the form

$$M_{aa} = \begin{bmatrix} M_{tt} & M_{tq} \\ M_{qt} & M_{qq} \end{bmatrix} \quad c_{aa} = \begin{bmatrix} C_{tt} & 0 \\ 0 & C_{qq} \end{bmatrix} \quad k_{aa} = \begin{bmatrix} K_{tt} & 0 \\ 0 & K_{qq} \end{bmatrix}$$

Equations (2-19) and (2-20) can be solved for the interface forces and substituted into Eq. (2-18) to produce the reanalysis equations of motion:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{q}' \\ \ddot{u}_{q2} \end{Bmatrix} + \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \begin{Bmatrix} \dot{q}' \\ \dot{u}_{q2} \end{Bmatrix} + \begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix} \begin{Bmatrix} q' \\ u_{q2} \end{Bmatrix} = \begin{Bmatrix} f_1 \\ 0 \end{Bmatrix} \quad (2-21)$$

in which

$$\begin{aligned} M_{11} &= \hat{M} - \Phi_a^T M_{aa1} \Phi_a + \Phi_t^T M_{tt2} \Phi_t \\ M_{12} &= \Phi_t^T M_{tq2} = M_{21}^T \\ M_{22} &= M_{qq2} \\ C_{11} &= \hat{C} - \Phi_a^T C_{aa1} \Phi_a + \Phi_t^T C_{tt2} \Phi_t \\ C_{22} &= C_{qq2} \\ K_{11} &= \hat{K} - \Phi_a^T K_{aa1} \Phi_a + \Phi_t^T K_{tt2} \Phi_t \\ k_{22} &= K_{qq2} \\ f_1 &= B^T f \end{aligned}$$

It can be shown that the mass matrix partition M_{11} in Eq. (2-21) is in general only positive semi-definite because all of the mass associated with the original payload component modes is being removed, resulting in massless DOF. These massless DOF must be removed prior to integration of the coupled equations of motion using a singular value decomposition of the M_{11} mass matrix partition

$$M_{11} = \begin{bmatrix} U_0 & U_1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & \Sigma \end{bmatrix} \begin{bmatrix} U_0^T \\ U_1^T \end{bmatrix} = U_1 \Sigma U_1^T \quad (2-22)$$

where Σ is a diagonal matrix of nonzero⁵ singular values associated with the DOF having mass, and U_1 is the corresponding matrix of singular vectors. The massless DOF can then be reduced out of the problem using the coordinate transformation

$$\{p'\} = [U_1]\{q'\}, \{q'\} = [U_1]^T\{p'\} \quad (2-23)$$

producing the equations of motion

$$M' \begin{Bmatrix} \ddot{p}'(t) \\ \ddot{u}_{q2}(t) \end{Bmatrix} + C' \begin{Bmatrix} \dot{p}'(t) \\ \dot{u}_{q2}(t) \end{Bmatrix} + K' \begin{Bmatrix} p'(t) \\ u_{q2}(t) \end{Bmatrix} = \begin{Bmatrix} U_1 f_1(t) \\ 0 \end{Bmatrix} \quad (2-24)$$

where

$$M' = \begin{bmatrix} \Sigma & U_1 M_{12} \\ M_{21} U_1^T & M_{22} \end{bmatrix} \quad C' = \begin{bmatrix} U_1 C_{11} U_1^T & 0 \\ 0 & C_{22} \end{bmatrix} \quad K' = \begin{bmatrix} U_1 K_{11} U_1^T & 0 \\ 0 & K_{22} \end{bmatrix}$$

⁵ “Nonzero” is a relative term. Since most of the singular values will be 1.0, a cut-off value of 1E-5 has been found to give satisfactory results.

Mass matrix M' is now positive definite, and Eq. (2-24) can either be diagonalized using a generalized eigenvalue solution, or it can be integrated directly. The modified payload interface response can then be recovered using $u_t(t) = \Phi_t U_1^T p'(t)$.

In this project, for simplicity, component damping was ignored, and both the nominal system and the system represented by (2-17) were assumed to have 1% modal damping. This is an advantage of the time-domain method, since it is still relatively common in CLA to apply damping at the system rather than the component level, and the time-domain method does not require any assumption on damping at the component level.

Further details on the derivation of Method 1 can be found in the *Reanalysis User's Guide* [7].

2.3 Reanalysis—Method 2

Method 1 proved to work well when the system-level forcing functions were known, but it was recognized that this is not often the case. Method 2 was developed to address this. It does this in a manner that is analogous to the JPL method.

Start by subtracting the equations of motion of the nominal system (2-17) from the equations of motion for the updated system (2-18) to arrive at a set of equations of motion in change in modal response (Δ) as follows:

$$\hat{M}\ddot{\Delta}(t) + \hat{C}\dot{\Delta}(t) + \hat{K}\Delta(t) = \Phi_t^T(f_{t1}(t) - f_{t2}(t)) \quad (2-25)$$

in which

$$\Delta(t) = \text{difference between the modified and original modal response, } (\Delta(t) = q'(t) - q(t))$$

Again assuming an HCB substructure representation for each payload ((2-19) and (2-20)), the modified interface response and modified response of the Payload 1 fixed-interface modes can be written in terms of the change in modal response Δ as follows:

$$u_t(t) = u_{t0}(t) + \Phi_t \Delta(t) \quad (2-26)$$

and

$$u_{q1}(t) = u_{q10}(t) + \Phi_{q1} \Delta(t) \quad (2-27)$$

where $u_{t0}(t)$ and $u_{q10}(t)$ are the nominal interface and Payload 1 fixed-interface mode responses, respectively.

Making the same substitutions as Method 1, this results in the following coupled equations of motion between the change in modal response and the modal degrees of freedom of Payload 2:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{\Delta}(t) \\ \ddot{u}_{q2}(t) \end{Bmatrix} + \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \begin{Bmatrix} \dot{\Delta}(t) \\ \dot{u}_{q2}(t) \end{Bmatrix} + \begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta(t) \\ u_{q2}(t) \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix} \quad (2-28)$$

in which

$$M_{11} = \hat{M} - \Phi_a^T M_{aa1} \Phi_a + \Phi_t^T M_{tt2} \Phi_t$$

$$M_{12} = \Phi_t^T M_{tq2} = M_{21}^T$$

$$M_{22} = M_{qq2}$$

$$\begin{aligned}
C_{11} &= \hat{C} - \Phi_a^T C_{aa1} \Phi_a \\
C_{22} &= C_{qq2} \\
K_{11} &= \hat{K} - \Phi_a^T k_{aa1} \Phi_a + \Phi_t^T K_{tt2} \Phi_t \\
k_{22} &= K_{qq2} \\
F_1(t) &= \Phi_a^T (M_{aa1} \ddot{u}_{a10}(t) + C_{aa1} \dot{u}_{a10}(t) + K_{aa1} u_{a10}(t)) - \Phi_t^T (M_{tt2} \ddot{u}_{t0}(t) + K_{tt2} u_{t0}(t)) \\
F_2(t) &= -M_{tq2} \ddot{u}_{t0}(t)
\end{aligned}$$

Note that the damping associated with the interface (C_{tt}) has been assumed to be zero in this case, though it can easily be added back in. Details of the derivation can be found in Blueloch and Flanigan [6].

Performing the same singular value decomposition of M_{11} as Eq. (2-22) results in the transformed equations of motion:

$$M' \begin{Bmatrix} \ddot{\Delta}' \\ \ddot{u}_{q2} \end{Bmatrix} + C' \begin{Bmatrix} \dot{\Delta}' \\ \dot{u}_{q2} \end{Bmatrix} + K' \begin{Bmatrix} \Delta' \\ u_{q2} \end{Bmatrix} = T^T \begin{Bmatrix} U_1 F_1 \\ F_2 \end{Bmatrix} \quad (2-29)$$

where

$$M' = \begin{bmatrix} \Sigma & U_1 M_{12} \\ M_{21} U_1^T & M_{22} \end{bmatrix} \quad C' = \begin{bmatrix} U_1 C_{11} U_1^T & 0 \\ 0 & C_{22} \end{bmatrix} \quad K' = \begin{bmatrix} U_1 K_{11} U_1^T & 0 \\ 0 & K_{22} \end{bmatrix}$$

Note that the equations of motion for Method 2 are identical to those for Method 1, except for the forcing terms, which are now in terms of the original payload time history. Because of the way that the time-domain method is formulated, the full a-set motion ($u_{a1}(t)$, $\dot{u}_{a1}(t)$, and $\ddot{u}_{a1}(t)$) of the original payload are required, rather than just the interface motion. Given $\ddot{u}_{a1}(t)$, $\dot{u}_{a1}(t)$ and $u_{a1}(t)$ can be calculated by integrating, and given $\ddot{u}_{t1}(t)$, $\dot{q}_1(t)$ can be calculated by integrating the HCB equations of motion for the original component.⁶

The fundamental difference between Method 1 and Method 2 is that while Method 1 directly calculates the modified system response for Payload 2, Method 2 calculates the difference in response, which then needs to be added to the original response. Eq. (2-26) is used to calculate the modified system response.

As in the case of Method 1, it is not necessary to make any assumption of damping on component modes, and 1% system damping was assumed for this study.

3.0 Example Problems

Three different example problems are presented here. The first example studied was the QLV model presented in section 3.3. Difficulties in achieving acceptable levels of accuracy, particularly for the JPL substitution method, led to the development of two simpler cases. The first, and simplest, case is a three-mass, two-spring/damper example used to demonstrate the JPL method at a very simple level. The second case uses realistic payloads with a simple stick model of the launch

⁶ It is preferred to get $\ddot{q}_1(t)$ directly from the system simulation, since the response calculated from integrating the HCB equations will differ from the system solution due to truncation of system modes.

vehicle. The third case is the most complex, using a generic but realistic 3-D launch vehicle model that includes fluid slosh and hundreds of modes in the frequency range of interest.

3.1 Three-Mass Example

Because of difficulties achieving good accuracy with the JPL method on the realistic launch vehicle/payload example, a very simple example was developed based on three masses and two spring/dampers, as illustrated in Figure 3-1.

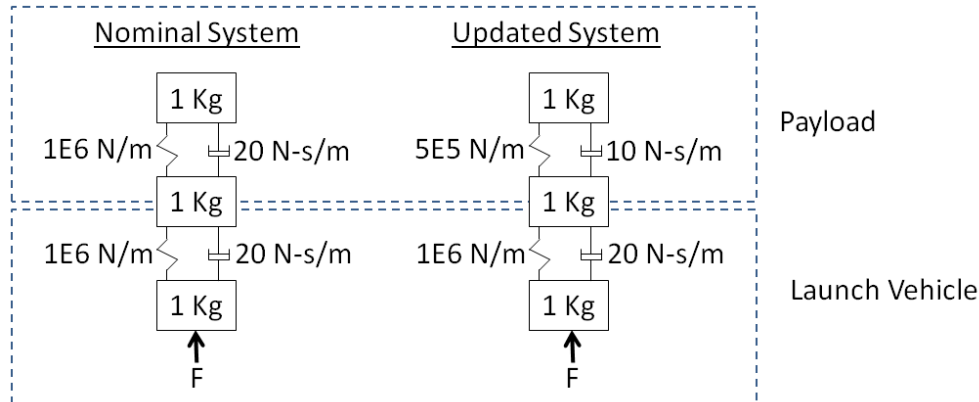


Figure 3-1. Simple three-mass example developed to illustrate JPL method.

The nominal system has modes at 159 and 256 Hz, and the updated system at 127 and 245 Hz. The viscous dampers are proportional to the springs so that the system-level damping is diagonal in both cases. The upper mass and half the middle mass are treated as the payload, and the lower mass and the other half of the middle mass are treated as the launch vehicle.

The system was solved by reducing the two “payload” models to HCB representations with a single mode each. The first system has a component mode at 159 Hz,⁷ and the second system at 125 Hz. The HCB payload representations were coupled with the launch vehicle model, and three system modes were calculated for each case. A unit frequency-domain input was then applied to the base, and the response of the middle mass was recovered from 50 to 500 Hz for both systems. A purely frequency-domain implementation of the JPL method was then applied to nominal acceleration responses. The resulting responses are plotted in Figure 3-2. Note that for this case the JPL frequency-domain method is very accurate and exactly captures the relatively large change in frequency response due to replacing the payload, suggesting that the equations were implemented correctly.

⁷ This is a special case where the component frequency happens to equal the system frequency because of the symmetry of the model.

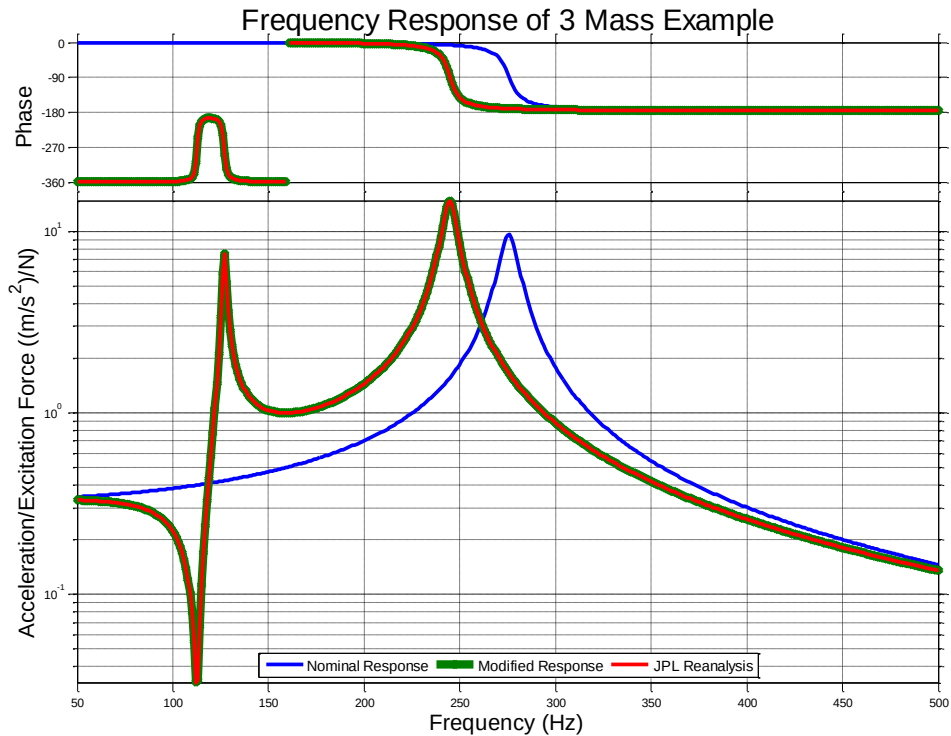


Figure 3-2. Frequency responses for three-mass example shows the initial response in blue and the modified response in green which is exactly matched by the JPL reanalysis in red.

To demonstrate the effect of modal truncation on the JPL method, this process was repeated except that the system modes were truncated at 200 Hz, so only the first two modes of the nominal system (rigid body plus 159 Hz) were used in the reanalysis calculation. The resulting response from 50–200 Hz is illustrated in Figure 3-3. Note that the mode of the nominal system at 159 Hz is symmetric, so it does not involve any motion of the middle mass. The nominal system, therefore, has no resonant response when truncating the third mode. For the case with modal truncation, the reanalysis calculation is very far off, even below the 200 Hz truncation frequency.

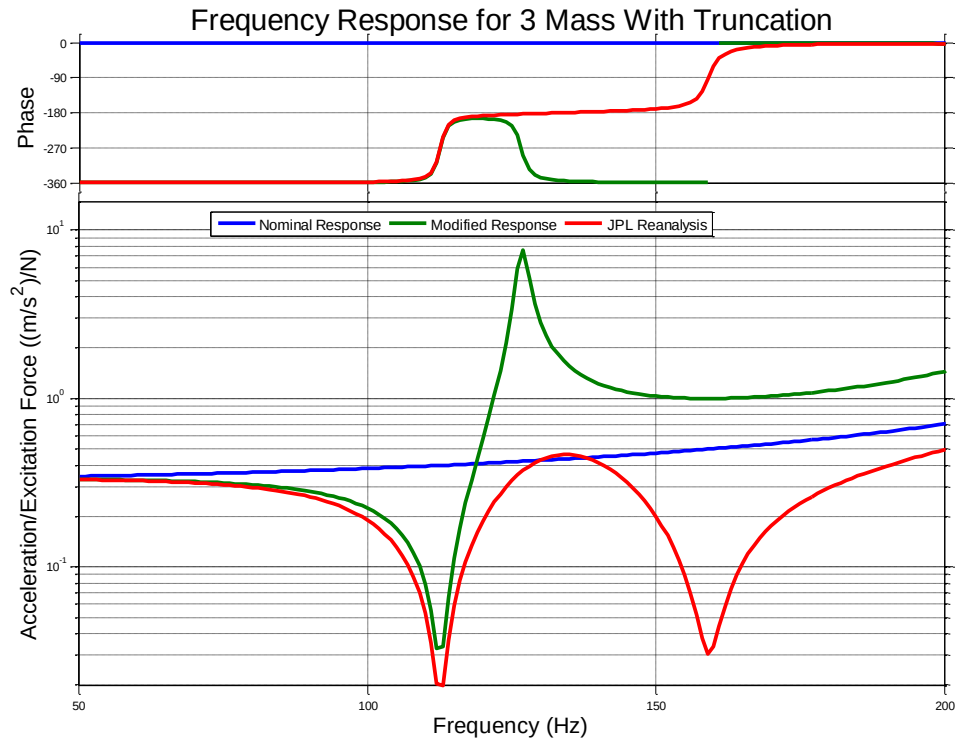


Figure 3-3. Frequency responses for three-mass example with modes truncated at 200 Hz shows large errors in reanalysis in red as compared with the true modified response in green.

These results are consistent with our previous experience using FRF synthesis methods. If the FRFs are “exact,” the results are also exact since there are no approximations involved other than numerical issues associated with inverting transfer functions. However, when errors are introduced into the transfer function due to modal truncation, or any other reason, the results can be very inaccurate.

3.2 Stick Launch Vehicle Example

The next two examples use realistic payload models. The nominal payload is called QSAT,⁸ and the modified payload is called ISAT,⁹ as illustrated in Figure 3-4.

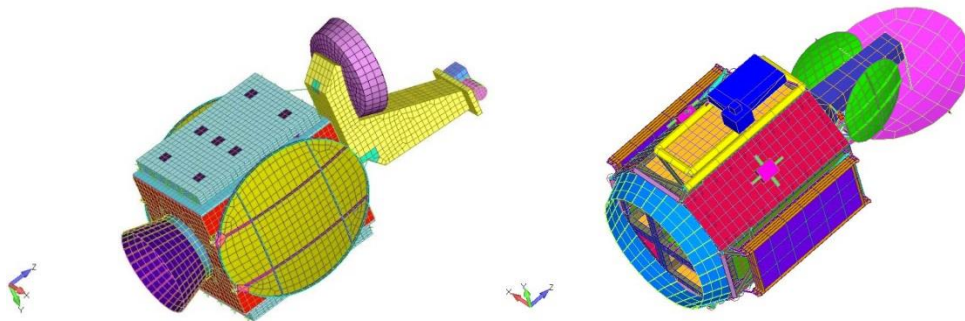


Figure 3-4. QSAT and ISAT finite element models.

⁸ The QSAT model was provided by Chris Flanigan, Quartus Engineering, Inc., San Diego, California.

⁹ ISAT is a modified version of a generic spacecraft model previously used by ATA. It was doubled in size and modified to include a 32 node adapter cone to be consistent with the QSAT/QLV interface.

QSAT has a weight of 3,426 lbs, while ISAT weighs 2,878 lbs, representing a 16% decrease in payload mass from nominal to modified, though this is negligible when coupled to the launch vehicle. Two cases were considered during this project. The first was a simplification in which the payload/launch vehicle interface was reduced to a single node (34000) with six DOF. In the second, more typical case, the interface consisted of 32 nodes, or 192 DOF. The 32 interface nodes are labeled 510101–510132 in the finite element models.

There are significant differences in the modal characteristics of the QSAT and ISAT payload FEMs. The modal effective mass for the 68 QSAT fixed-interface modes below 100 Hz is illustrated in Figure 3-5. The primary lateral modes are at approximately 10 Hz, with a primary torsion mode at approximately 30 Hz and a primary axial mode at approximately 45 Hz.

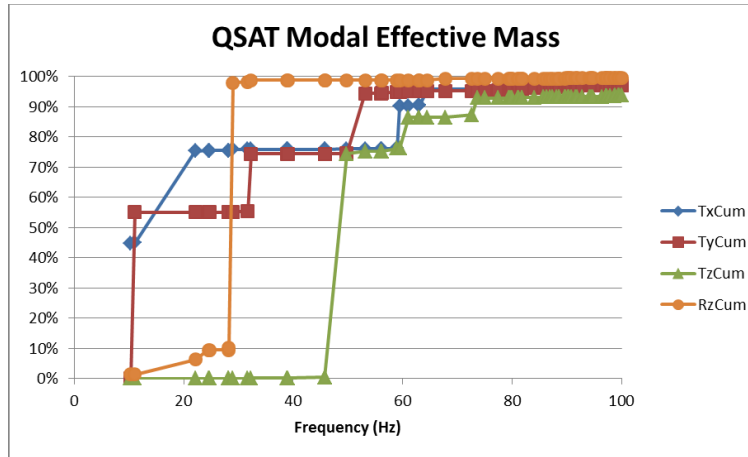


Figure 3-5. Cumulative modal effective mass for the QSAT payload FEM to 100 Hz.

The modal effective mass for the 37 ISAT fixed-interface modes below 100 Hz is illustrated in Figure 3-6. The primary lateral modes are at approximately 20 Hz, with a primary torsion mode at approximately 70 Hz and a primary axial mode at approximately 85 Hz.

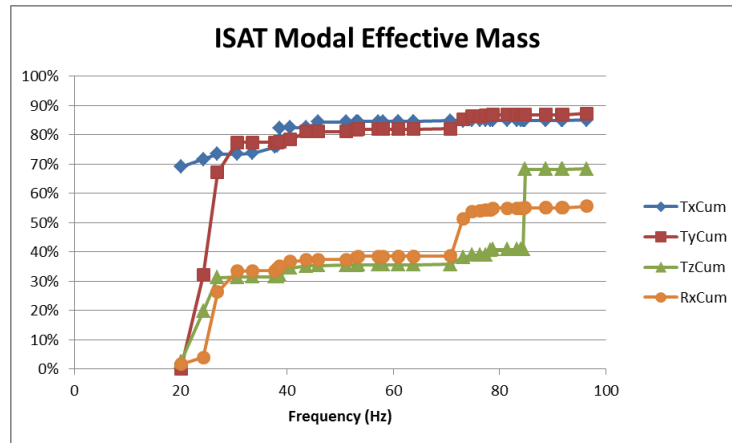


Figure 3-6. Cumulative modal effective mass for the ISAT payload FEM to 100 Hz.

For the first example using the two payloads, we chose a very simple “stick” model of a launch vehicle illustrated in Figure 3-7. This is a 1200" long, 120" diameter, 1" thick aluminum tube with

weight density increased to 0.75 lb/in³ to get representative modal and mass properties. The stick is modeled in Nastran with ten CBAR elements.¹⁰



Figure 3-7. Stick launch vehicle model.

The weight of the stick is 336,465 lbs, which is similar to the more realistic launch vehicle presented in the next section. With the nominal QSAT payload, the first bending model of this model is at 6.88 Hz, and the first axial mode is at 30.34 Hz. These shift to 7.04 Hz and 30.78 Hz with the ISAT payload.

The two payload models were coupled to the stick model and all system modes were calculated. This was a total of 120 modes for the QSAT and 89 modes with ISAT.

3.2.1 Reanalysis of Stick Launch Vehicle Model Using Frequency-Domain JPL Method

As in the first example, a frequency-domain implementation of the JPL reanalysis from 0 to 100 Hz was applied with a unit axial input at the base of the launch vehicle and no modal truncation. Since the JPL method requires that damping be applied at both the system and component levels, 1% diagonal modal damping was applied in both cases. The nominal, modified, and reanalysis predicted responses in the axial direction at the payload interface are plotted in Figure 3-8.

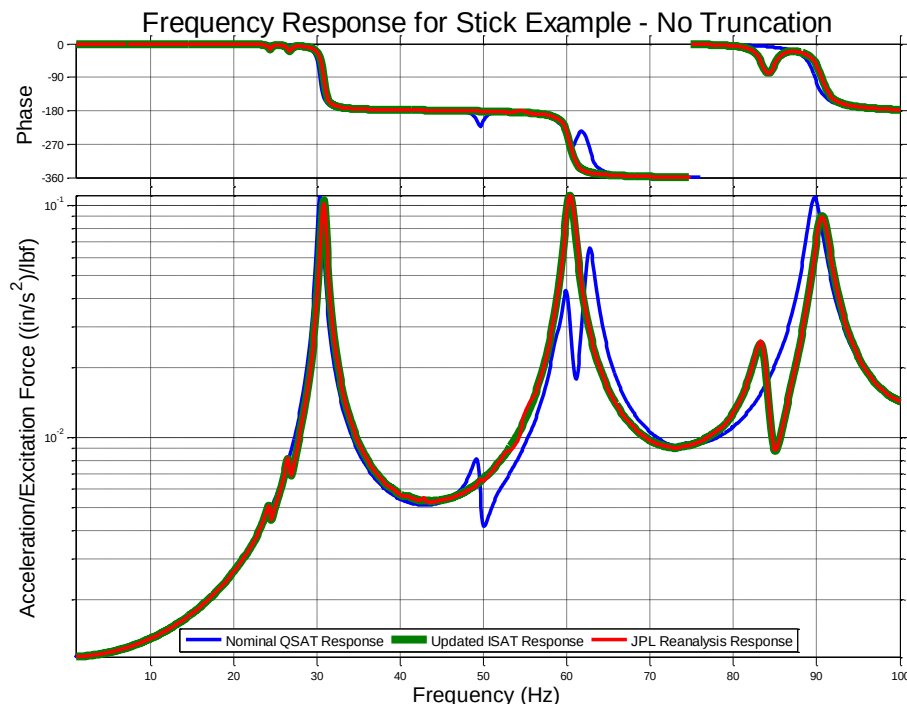


Figure 3-8. Axial frequency responses for stick example with no modal truncation shows that the reanalysis result in red exactly matches the updated response in green.

¹⁰ Lumped masses with rotational inertias were also added to capture torsional modes.

As in the case of the three-mass example, the JPL reanalysis is effectively exact. However, it is not feasible in the general case to retain all the modes of the system. The analysis was therefore repeated using just the system modes up to 100 Hz. These results are plotted in Figure 3-9.

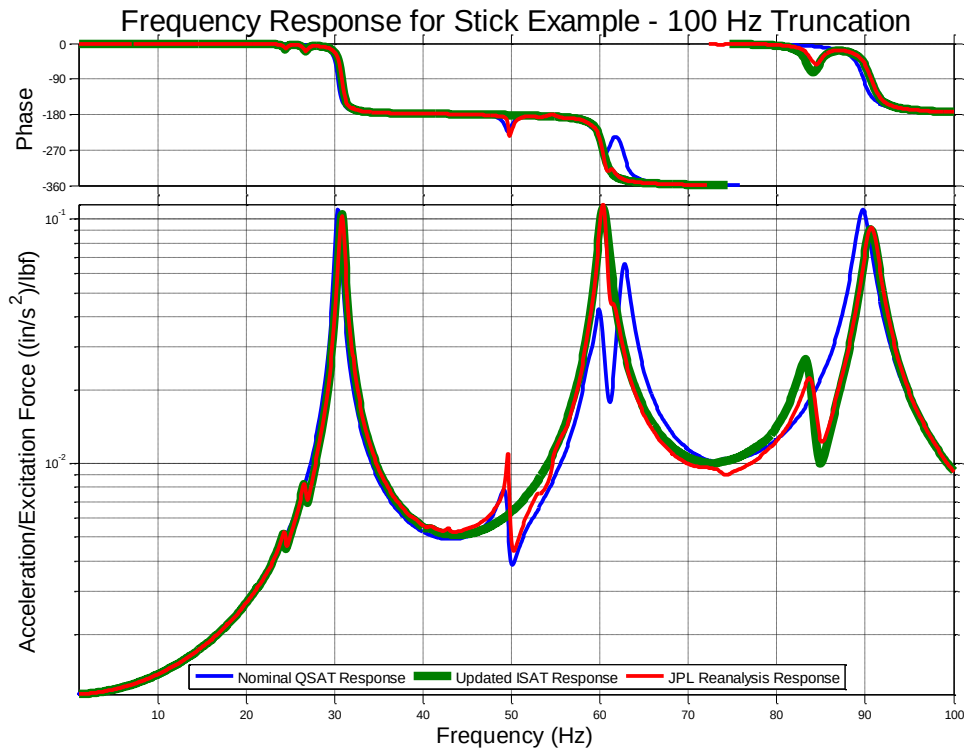


Figure 3-9. Axial frequency responses for stick example with modes truncated to 100 Hz shows that the reanalysis in red no longer exactly matches the actual result in green

In this case the reanalysis captures the response at the primary axial resonances fairly well, but the errors introduced from modal truncation are clearly evident, particularly around 50 Hz. It should be noted that the errors due to modal truncation are not necessarily greatest near the truncation frequency, but in this case at approximately half that frequency. This indicates that errors associated with modal truncation can show up far below the truncation frequency.

3.2.2 Reanalysis of Stick Launch Vehicle Using Time-Domain JPL Method

In the JPL method of reanalysis presented in Trubert and Peretti [5], the time-domain response of the nominal spacecraft is transformed to the frequency domain using an FFT. The reanalysis is performed in the frequency domain using the approach outlined in section 2.1, and then the predicted responses are transformed back to the time domain. In this example, a step thrust of 489,600 lbs was applied at the top of the launch vehicle first-stage nozzle along the +Z-axis for 1.0 second, providing a rigid body acceleration of approximately 500.0 in/s². One-percent modal damping was assumed for both systems, and one-percent modal damping was also assumed for the fixed-interface modes in each payload. System modes were restricted to 100 Hz. Response of the nominal system was simulated in the time domain to generate the nominal payload interface acceleration response required as input for the reanalysis method. The modified system was also simulated in the time domain to get the correct response of the modified system interface for comparison with the predicted results. Figure 3-10 shows the actual acceleration of the payload interface along the Z thrust direction compared to the response predicted using the JPL method.

The agreement is fairly good for this direction. Figure 3-11 illustrates a comparison of the actual and predicted interface acceleration in the X direction, transverse to the thrust. The agreement is not as good, especially in the first two tenths of a second. The remaining coordinate direction responses are not as accurately predicted by the JPL method with truncated modes.

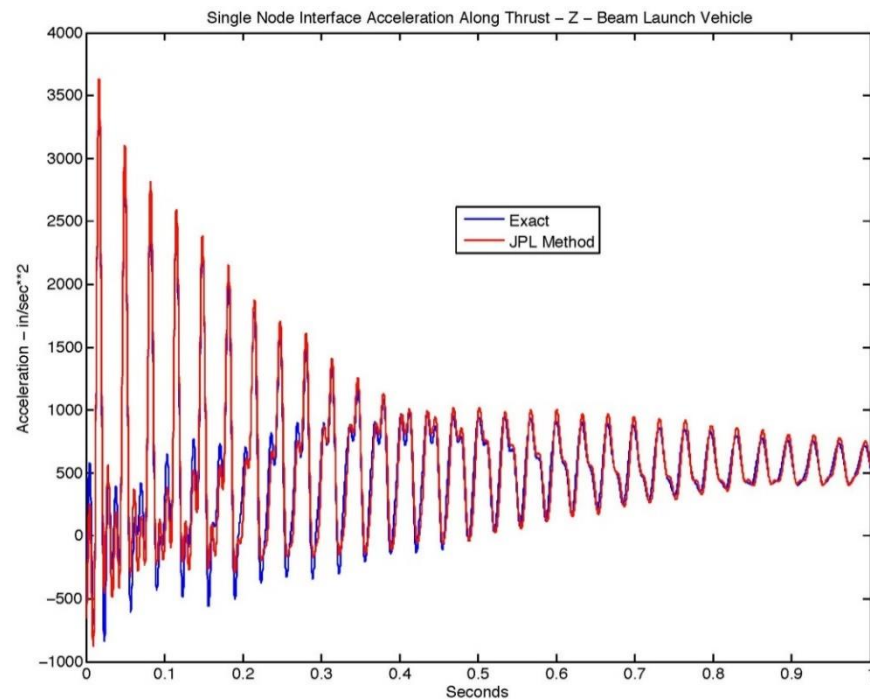


Figure 3-10. Reanalysis prediction for Payload 2 interface acceleration along thrust (Z) direction closely matches the exact results.

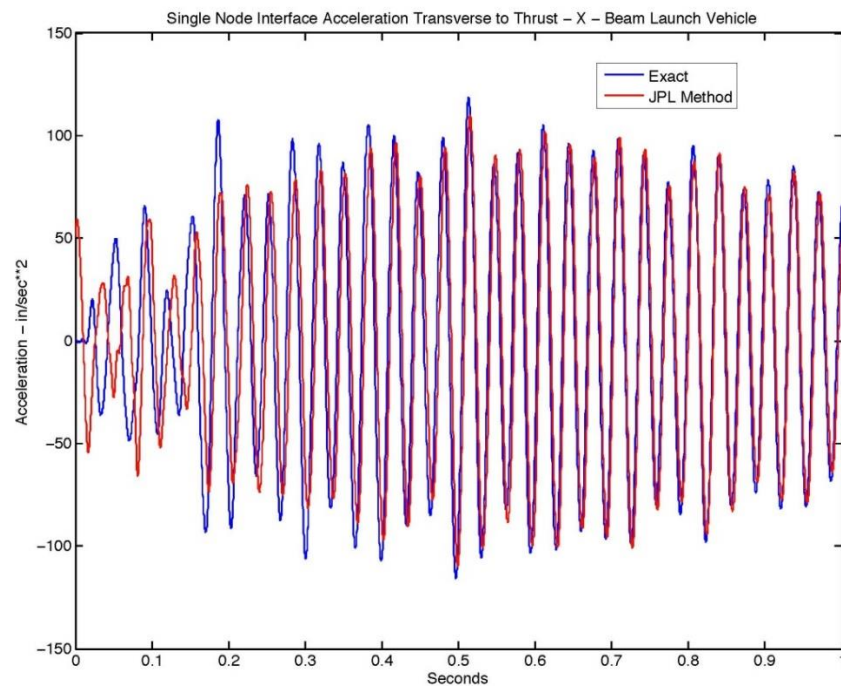


Figure 3-11. Reanalysis prediction for Payload 2 interface acceleration transverse to thrust (X) direction shows significant errors, particularly in the first 0.2 second.

3.2.3 Reanalysis of Stick Launch Vehicle Using Time-Domain Method 1

Method 1, described in section 2.2, was next applied to the stick launch vehicle reanalysis problem. In past work, it has been found that an acceptable cutoff value for the singular values of mass submatrix M_{11} in Eq. (2-21) is 1.E-5. In this example application, there are 62 values below this cutoff that are reduced out of the simulation using the singular value decomposition process presented in Eqs. (2-22) and (2-23). An eigenvalue solution was performed for the reduced mass and stiffness matrices in Eq. (2-24) to decouple the undamped system, and then 1% modal damping was added. Method 1 allows for a more consistent damping formulation, but this simple modal damping model was used to simplify the analysis. Seventy reduced system modes were used in the time-domain reanalysis response simulation, because there are seventy modified system modes with frequencies less than 100.0 Hz. Figure 3-12 illustrates a comparison between the reanalysis prediction and the exact solution for the interface node acceleration in the thrust direction. As shown, the predicted result is very accurate—and much more accurate than the JPL method with the same level of modal truncation.

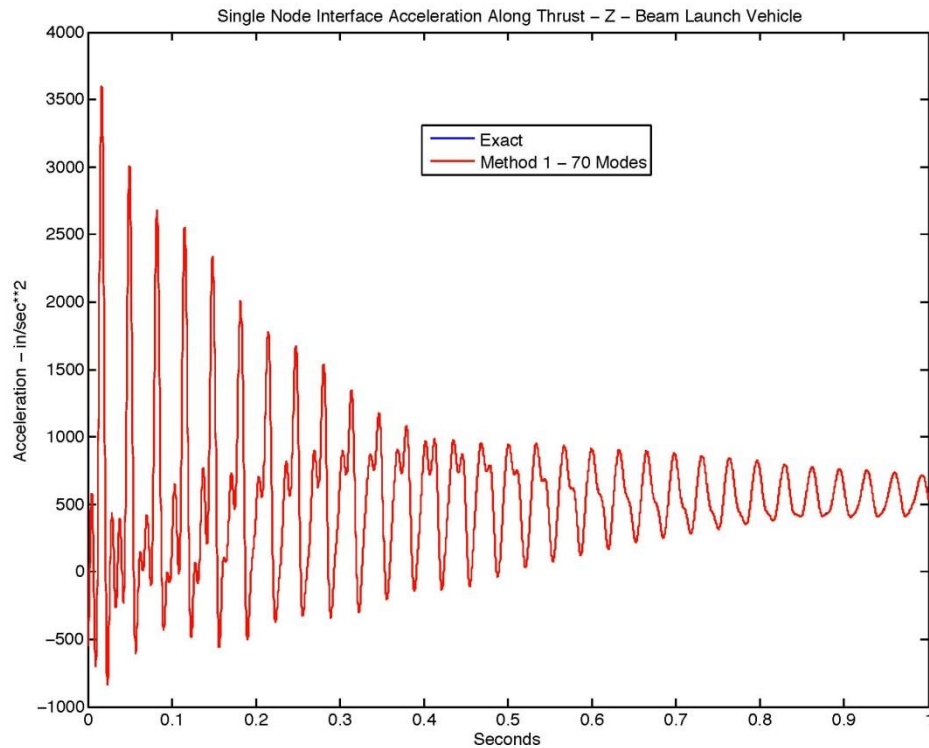


Figure 3-12. Time-domain reanalysis Method 1 predictions for Payload 2 interface acceleration along thrust (Z) direction matches exact results.

3.2.4 Reanalysis of Stick Launch Vehicle Using Time-Domain Method 2

Finally, Method 2, described in section 2.2, was applied to the stick launch vehicle reanalysis problem. As in Method 1, there are nine massless DOF that must be reduced from the reanalysis equations of motion. An eigenvalue solution was performed for the reduced mass and stiffness matrices in Eq. (2-29) to decouple the undamped system, and then 1% modal damping was added. Method 2 also allows for a more consistent damping formulation, but this simple modal damping model was again used to simplify the analysis.

As in Method 1, seventy reduced system modes were used in the time-domain reanalysis response simulation because there are seventy modified system modes with frequencies less than 100.0 Hz. Figure 3-13 illustrates the Method 2 prediction of the acceleration of the interface along the thrust direction. It also compares well with the actual response, though not quite as closely as Method 1. The prediction of the acceleration along the thrust direction of the ISAT center of mass is pictured in Figure 3-14. The response of the payload net center of gravity (CG) acceleration is a good overall measure of the accuracy of the reanalysis prediction of the modified payload response because it depends on both the interface and the component fixed-interface modes. In this case, the Method 2 prediction is again very accurate. Overall, for the stick launch vehicle example, the time-domain Method 1 and Method 2 do a very good job of predicting the modified payload response, both at the interface and for the net CG motion, while the frequency-based JPL method predictions are much less accurate.

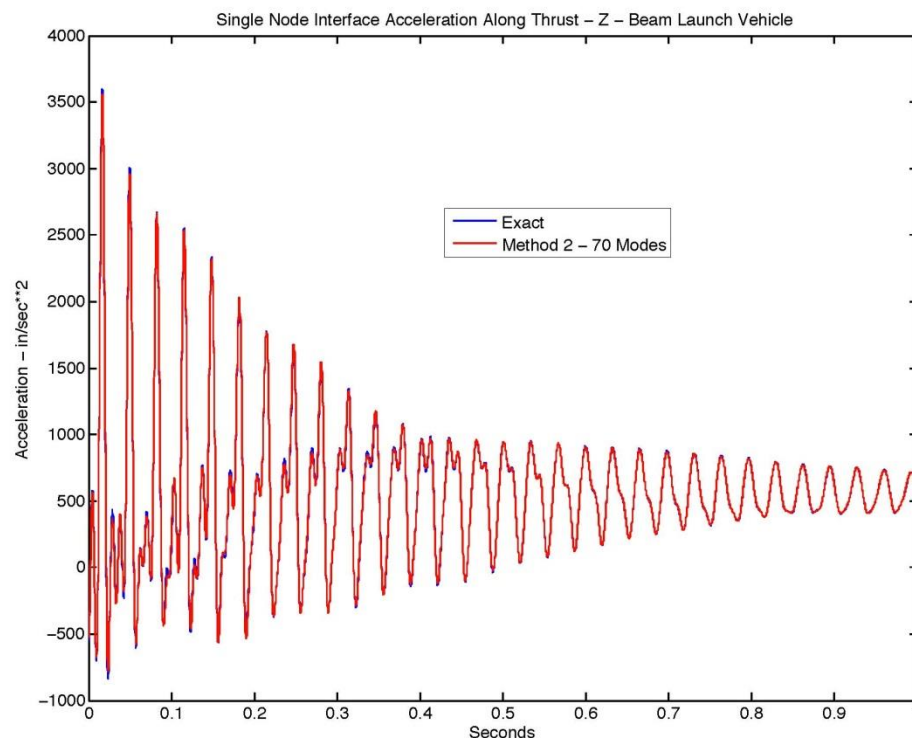


Figure 3-13. Time-domain reanalysis Method 2 predictions for Payload 2 interface acceleration along thrust (Z) direction match well with the exact results but not quite as well as those of Method 1.

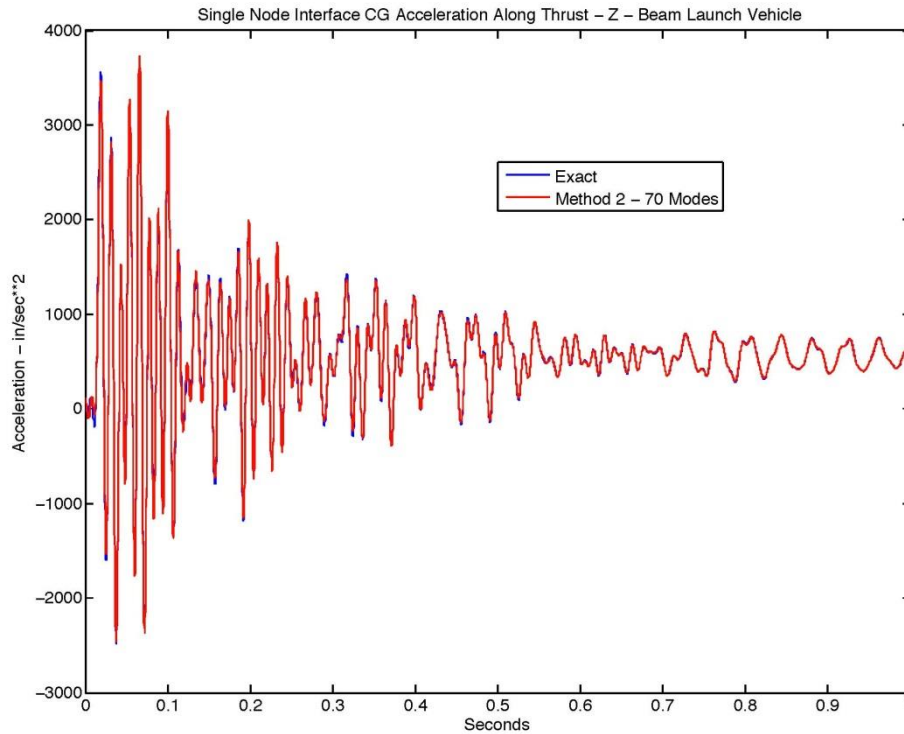


Figure 3-14. The Method 2 prediction of the response of the payload net CG acceleration is very accurate.

3.3 Generic Launch Vehicle Example

The most complex example used in this report is based on the same two QSAT and ISAT generic payload models introduced in section 3.2, but with a more realistic launch vehicle. The generic launch vehicle is also referred to as the Quartus launch vehicle (QLV).¹¹ The QLV is a generic launch vehicle model that is designed to be similar to a Delta II. The three primary parts of the QLV FEM are illustrated in Figure 3-15.

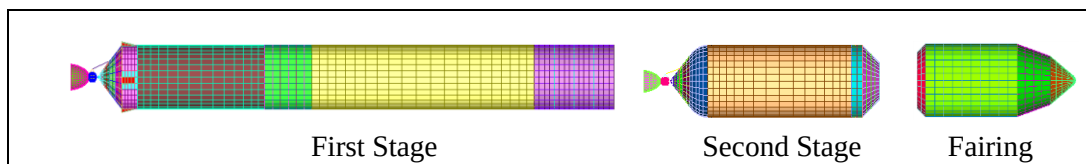


Figure 3-15. Generic launch vehicle finite element model.

The generic launch vehicle empty weights are 5,395 lbs for the first stage, 1,604 lbs for the second stage, and 1,379 lbs for the fairing. The example presented here is based on a liftoff configuration, which includes four hydroelastic slosh models of fluid in tanks. The fluid weights are 236,219 lbs for first-stage LOX, 95,990 lbs for first-stage RP, 27,228 lbs for second-stage LOX and 6,824 lbs for second-stage LH2. The total weight of the fully fueled vehicle, therefore, is 374,640 lbs, which is similar to the stick model.

¹¹ The generic model was provided by Chris Flanigan, Quartus Engineering, Inc., San Diego, California.

A total of eight slosh modes¹² were retained for each tank, resulting in 32 slosh modes for the vehicle below 2 Hz. The first bending modes are at 2.28 Hz with QSAT (Figure 3-16) and 2.36 Hz with ISAT. The first significant axial mode, which is a first-stage RP tank lower dome bulge mode, is at 5.40 Hz with both payloads (Figure 3-17). The major difference between this model and the stick is the modal complexity. While the stick has 32 modes with QSAT and 28 modes with ISAT below 50 Hz, the generic launch vehicle has 292 modes with QSAT and 287 modes with ISAT below 50 Hz. This results in a much more complex result and provides a significantly higher level of difficulty than the stick model.

V1

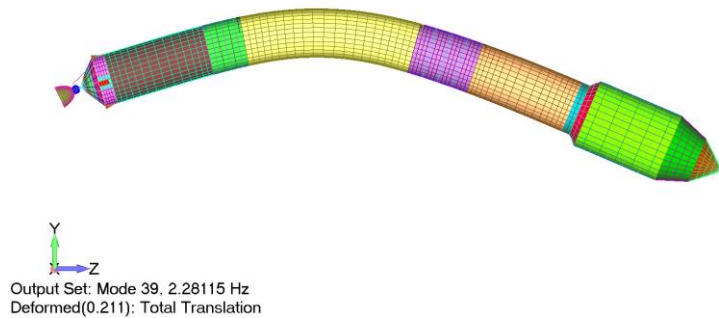


Figure 3-16. First bending mode of the QLV/QSAT system.

V1

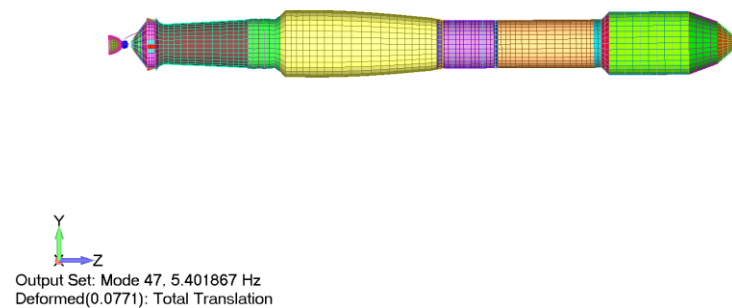


Figure 3-17. First axial mode of the QLV/QSAT system.

Two configurations of the QLV interface were modeled. The single-node interface configuration used a rigid element (RBE2) to connect a center node to the 32 nodes at the top of the payload adapter fitting (PAF) for attachment to a single-node payload model. The 32 node interface did not include the rigid element and was used to connect directly to 32 node payload interfaces. Results are presented for these two cases separately.

3.3.1 Single-Node Interface

In this case, the nodes around the base of the adapter rings for both payloads were rigidly connected to a central node that was then constrained to its counterpart in the launch vehicle. The nominal

¹² The eight slosh modes retained for each tank included the first three pairs of lateral modes and the first two axial modes.

HCB QSAT payload contains six physical DOF at the interface node and 74 fixed-interface modes ranging in frequency between 10.0 and 140.0 Hz.¹³ The modified HCB ISAT payload also has six interface DOF, and it has 43 fixed-interface modes between 20.0 and 175.0 Hz. The nominal launch system has 292 free-free modes with frequencies up to 50.0 Hz, while the modified system has 286 modes in the same frequency range. During simulation of the nominal system, 70% modal damping is applied to the residual vectors due to their high frequencies.

A step thrust of 489,600 lbs was applied at the top of the launch vehicle first-stage nozzle along the +Z-axis for 1.0 second, providing a rigid body acceleration of approximately 500.0 in/s². One-percent modal damping was assumed for both systems. Response of the nominal system was simulated in the time domain to generate the payload interface acceleration response required as input for the reanalysis methods. The modified system was also simulated in the time domain to get the correct response of the modified system interface for comparison with the predicted results. Figure 3-18 compares the acceleration of the interface node along the thrust direction for both payloads. The acceleration of the modified payload is significantly different from the nominal payload. Figure 3-19 illustrates the same response but in the frequency domain; below 20.0 Hz, the two payloads responses are effectively identical, indicating that the launch vehicle dominates the response in this frequency region. However, above 25.0 Hz the two payloads are dramatically different. This is especially the case at the upper end of the 0.0 to 50.0 Hz frequency range spanned by the nominal system modes.

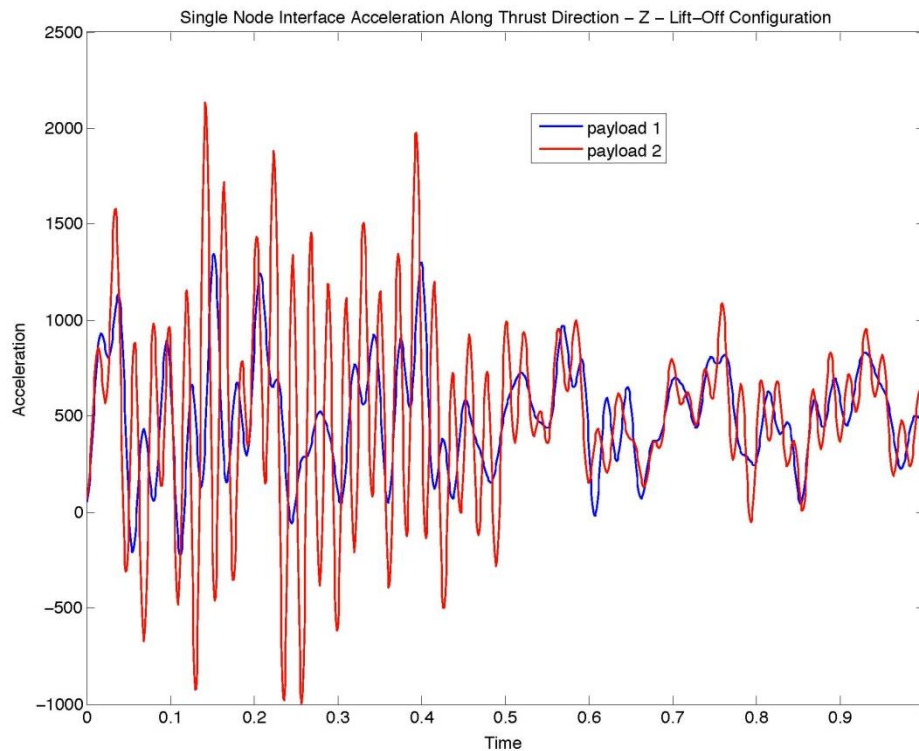


Figure 3-18. The correct payload interface accelerations (in/s²) along thrust (Z) direction are quite different in the time domain for the two payloads.

¹³ The modal truncation frequency for both QSAT and ISAT was 100 Hz, but six residual vectors were included, increasing the upper frequency range to 140 Hz for QSAT and 175 Hz for ISAT.

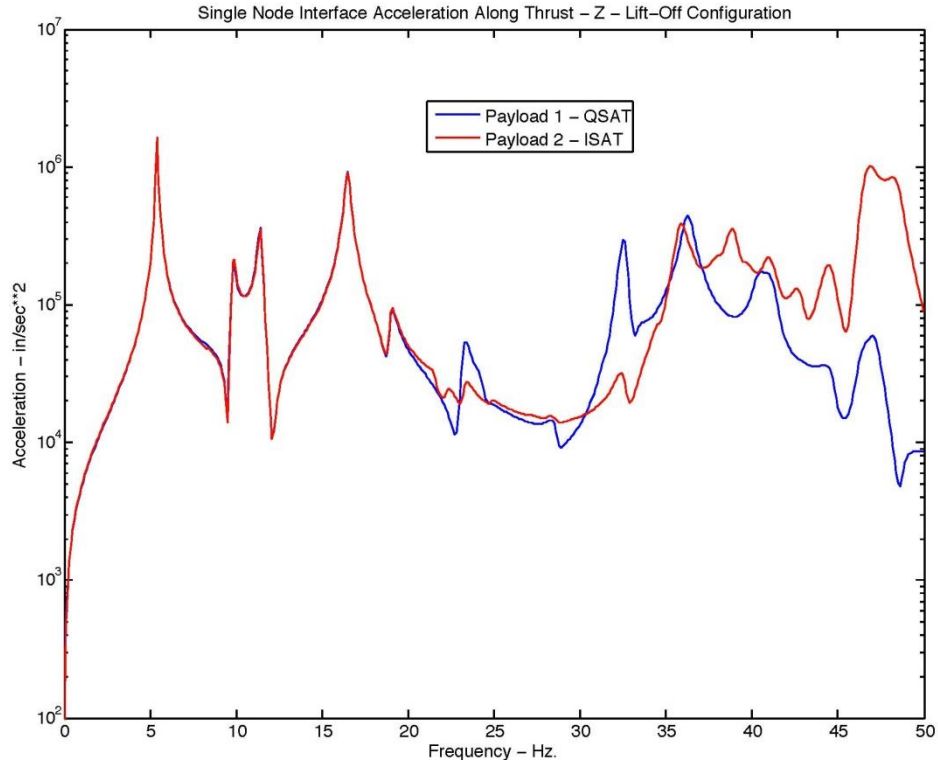


Figure 3-19. The correct payload interface accelerations along the thrust (Z) direction in the frequency domain are dominated by the launch vehicle at low frequency and therefore match, but they differ at frequencies above 20 Hz.

3.3.1.1 Reanalysis of Single-Node QLV Using Time-Domain JPL Method

The JPL method, described in section 2.1, was the first approach applied to the generic launch vehicle single-node interface reanalysis problem. In this method, there is no formulation of an augmented hybrid set of reanalysis differential equations to be integrated, so there is no need to reduce out massless DOF and no need to determine the number of reduced system modes to retain in the reanalysis simulation. At each frequency of interest, the frequency response of the Payload 2 interface is calculated directly from Eubsequently, the frequency response is transformed back to the time domain.

As mentioned, the JPL method requires that the payload fixed-interface modes are modally damped. In this analysis, it was assumed that each of the payloads had 1% modal damping. The nominal and modified launch systems were simulated using 1% system modal damping. Therefore, an inconsistent Payload 1 representation is being subtracted off in the JPL method reanalysis. It is believed that this inconsistency does not produce significant error for light damping.

Figure 3-20 illustrates a comparison between the reanalysis prediction and the exact solution for the interface node acceleration in the thrust direction. As shown, the predicted result is very inaccurate. Figure 3-21 illustrates the same data comparison but in the frequency domain. The Payload 2 response predicted by the JPL method is accurate in the 0.0 to 20.0 Hz frequency range but very inaccurate in the 20.0 to 50.0 Hz range. It is believed that the JPL method is especially susceptible to the large discrepancy between the two payloads in this region, particularly near the 50.0 Hz cutoff. It is likely that the nominal system modes do not do a very good job of representing the modified payload response in this frequency region. Figure 3-22 shows the interface

acceleration in the transverse X direction. This result is also very inaccurate and typical of the other interface node response directions.

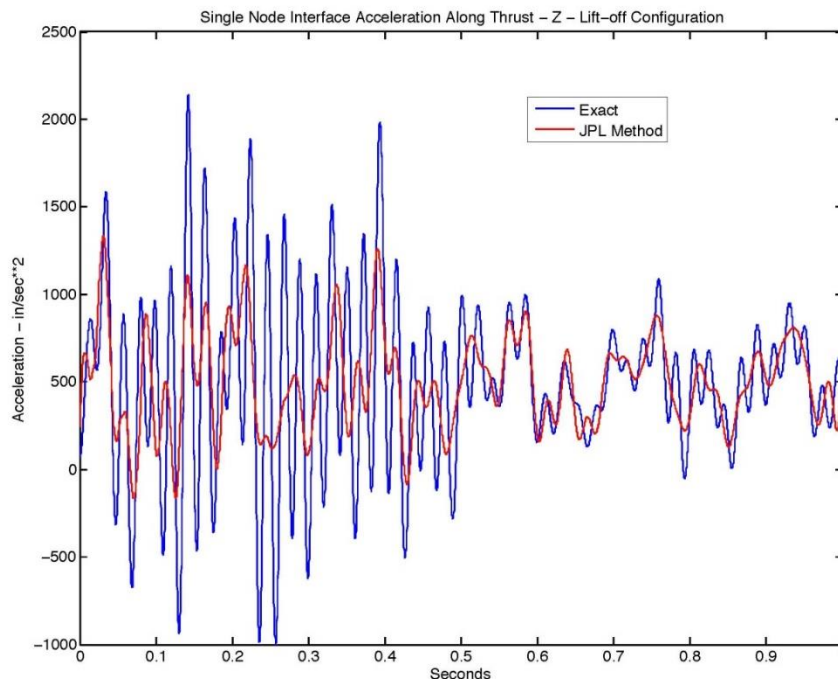


Figure 3-20. Comparison between the reanalysis prediction and the exact solution for Payload 2 interface acceleration along the thrust (Z) direction shows the reanalysis results to be very inaccurate.

In an effort to increase the modal richness of the nominal system and mitigate possible modal truncation issues, the six residual vectors were included in the simulation of the nominal system response and the subsequent reanalysis. It was found that the payload response predicted by the JPL approach was not affected by the inclusion of the residual vectors.

As a final check on the impact of modal truncation on the JPL reanalysis method, untruncated QSAT and ISAT system representations were used in the reanalysis. To do this, the QLV was reduced to an HCB representation with normal modes to 100 Hz and a residual vector associated with the applied load, and all the modes of the assembled system were calculated. Figure 3-23 compares the actual acceleration of the interface node along the thrust direction with the predicted result. Even in this case without any modal truncation, the JPL method generates poor results.

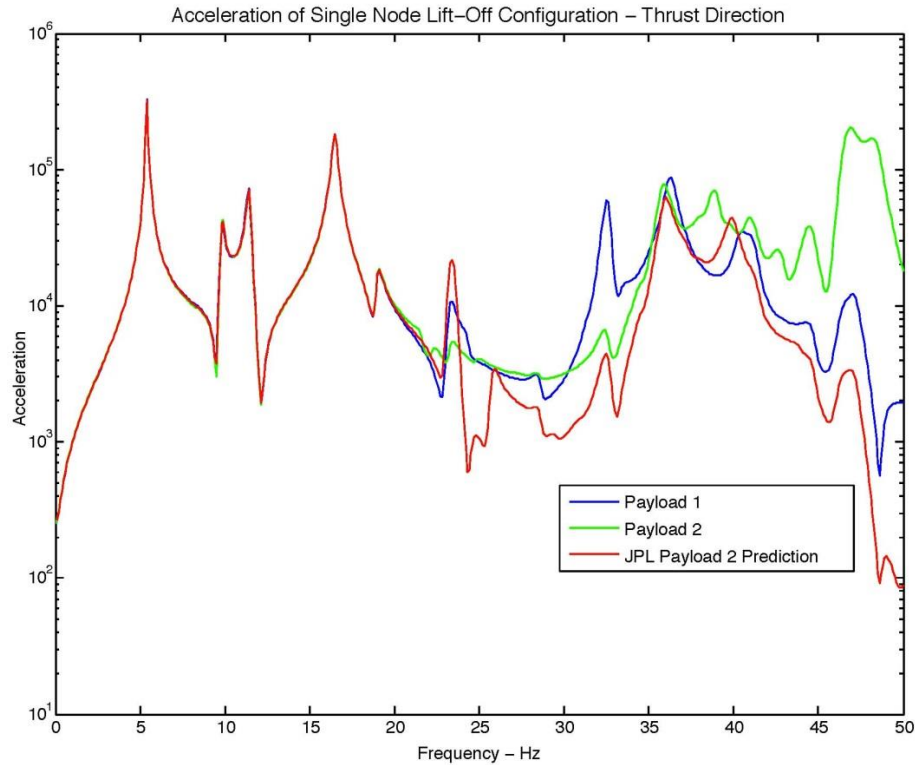


Figure 3-21. The frequency-domain result for Payload 2 interface acceleration along thrust (Z) direction predicted by the JPL method is accurate in the 0.0 to 20.0 Hz frequency range but very inaccurate in the 20.0 to 50.0 Hz range.

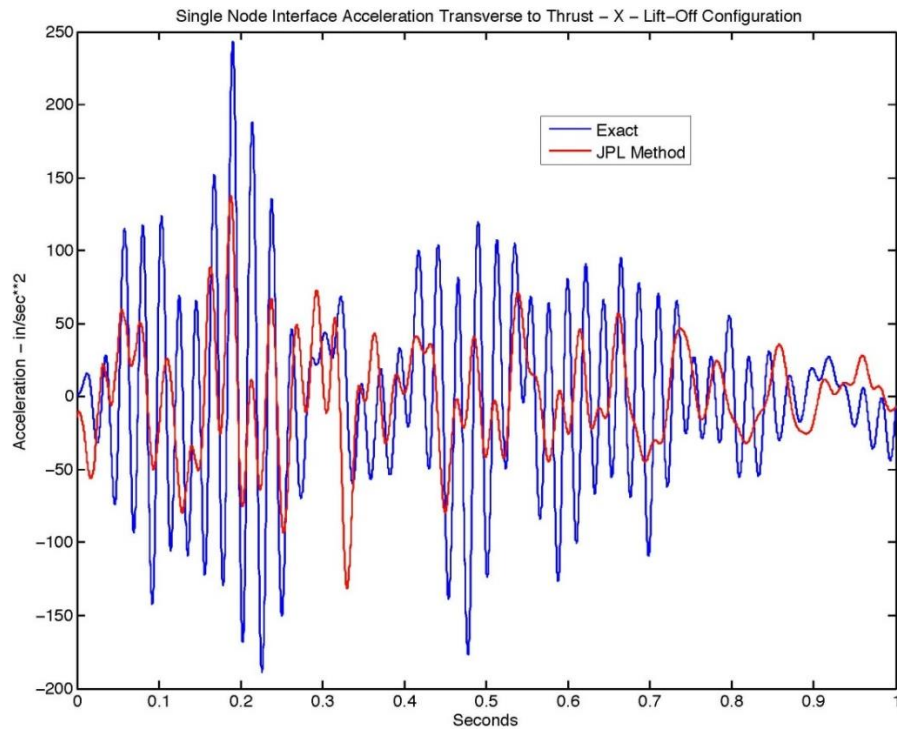


Figure 3-22. Reanalysis prediction for Payload 2 interface acceleration is also very inaccurate in the transverse (X) to thrust direction.

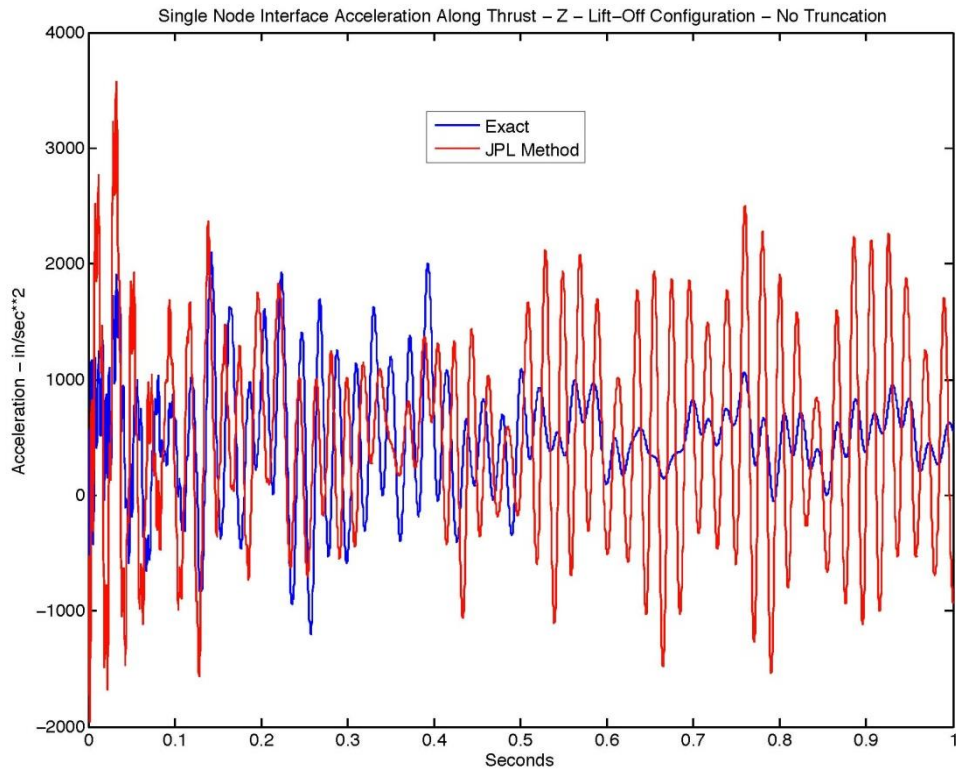


Figure 3-23. Comparison of actual Payload 2 interface acceleration along thrust (Z) direction with reanalysis predictions show that even in this case without any modal truncation, the JPL method generates poor results.

3.3.1.2 Reanalysis of Single-Node Generic Launch Vehicle Using Frequency-Domain JPL Method

Because of the inability to achieve convergence of the time-domain version of the JPL method when adding modes, the frequency-domain implementation was applied to the same problem. The frequency-domain code was written entirely independently of the time-domain code, though starting with the same equations.¹⁴ As demonstrated in section 3.1 and 3.2.1, the method works very well for the simpler cases, behaving exactly as expected. For the generic launch vehicle case, it was applied using a unit FRF input in the axial direction at the top of the first-stage engine gimbal, and responses were predicted at the payload interface. The resulting comparison in the axial direction with 1% modal damping applied at both the component and system level and with system modes truncated to 50 Hz is illustrated in Figure 3-24. As with the time-domain response, the predicted results to 50 Hz were very poor in the axial direction. The lateral direction is not plotted, but those results were worse.

¹⁴ The primary reason for developing two independent implementations of the JPL method was to check for possible errors in implementation. One implementation includes an FFT to convert time-domain signals to the frequency domain and then an IFFT to convert back, while the other includes only the frequency-domain calculation—Eq. (2-16).

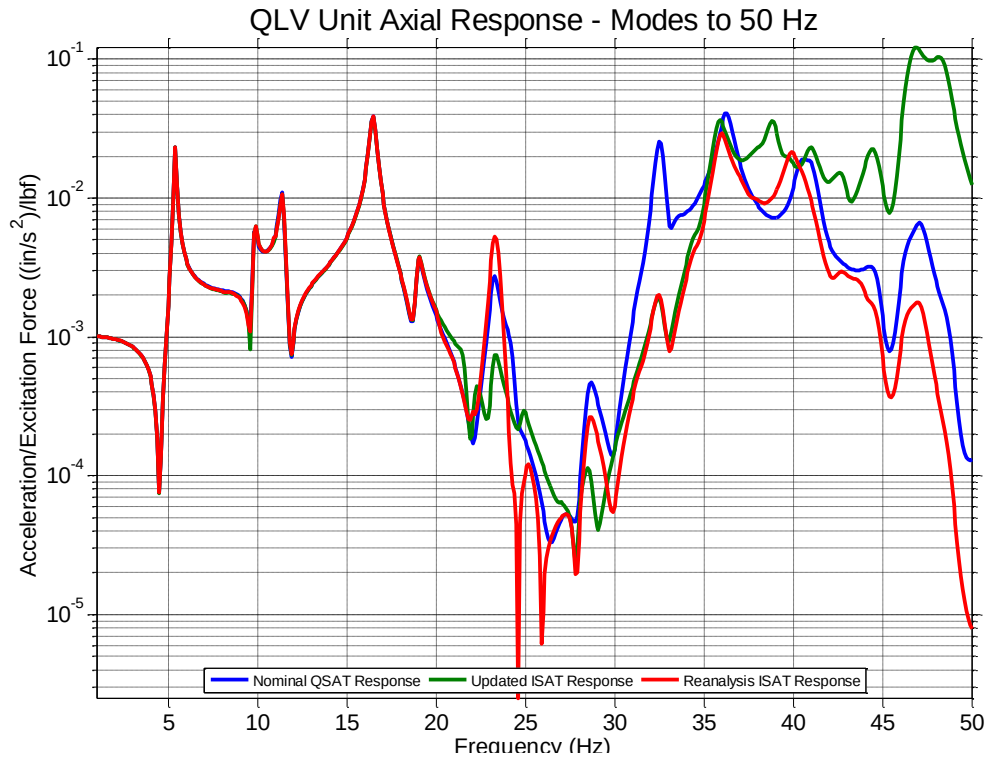


Figure 3-24. Comparison of the generic launch vehicle axial responses with 1% modal damping applied at both the component and system level and with system modes truncated to 50 Hz shows that the reanalysis result (in red) is a very poor match to the actual (in green).

The first step in attempting to improve these results was to include original system modes to 100 Hz rather than the nominal 50 Hz cutoff. These results are plotted in Figure 3-25 and appear to be unchanged, suggesting that adding additional modes is not improving convergence.

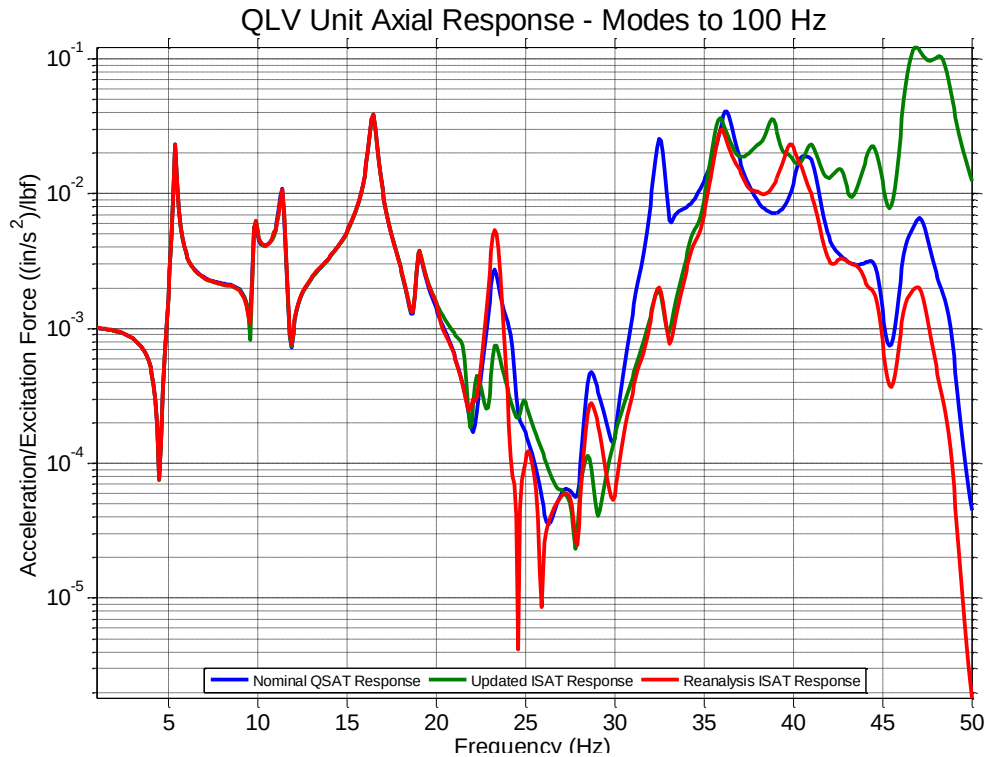


Figure 3-25. Generic launch vehicle axial responses for frequency-domain JPL reanalysis method (shown in red) are still a very poor approximation to actual responses (in green), even with modes to 100 Hz.

Next, we investigated the untruncated model calculated using all 823 modes of a system consisting of an HCB reduction of the launch vehicle. These results should be exact for all frequencies, so responses are plotted up to 100 Hz in Figure 3-26. The axial responses are much closer and are fairly accurate except for the 47–55 Hz range and a narrow frequency range around 62 Hz.

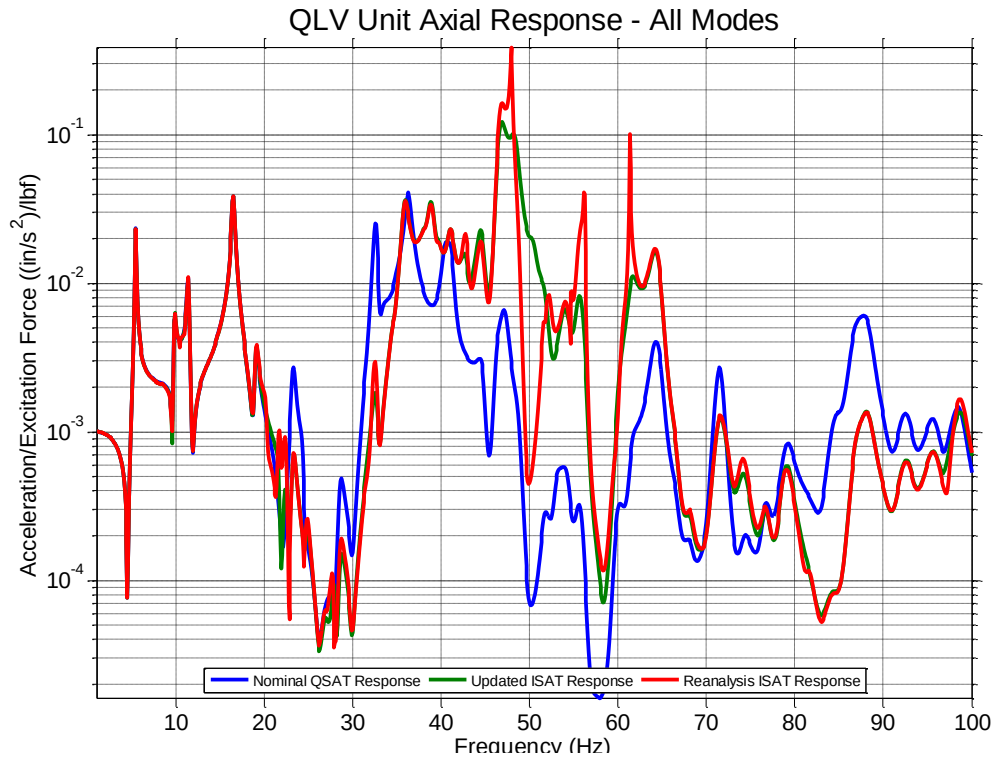


Figure 3-26. With no modal truncation, axial responses for the frequency-domain JPL method (shown in red) are fairly accurate compared to exact response (in green) except for the 47–55 Hz range and a narrow frequency range around 62 Hz.

The lateral (X) responses, however, are still very inaccurate, as illustrated in Figure 3-27. These results are consistent with the time-domain version of the JPL method, which showed that even when retaining all system modes, the JPL method did not converge on the correct results.

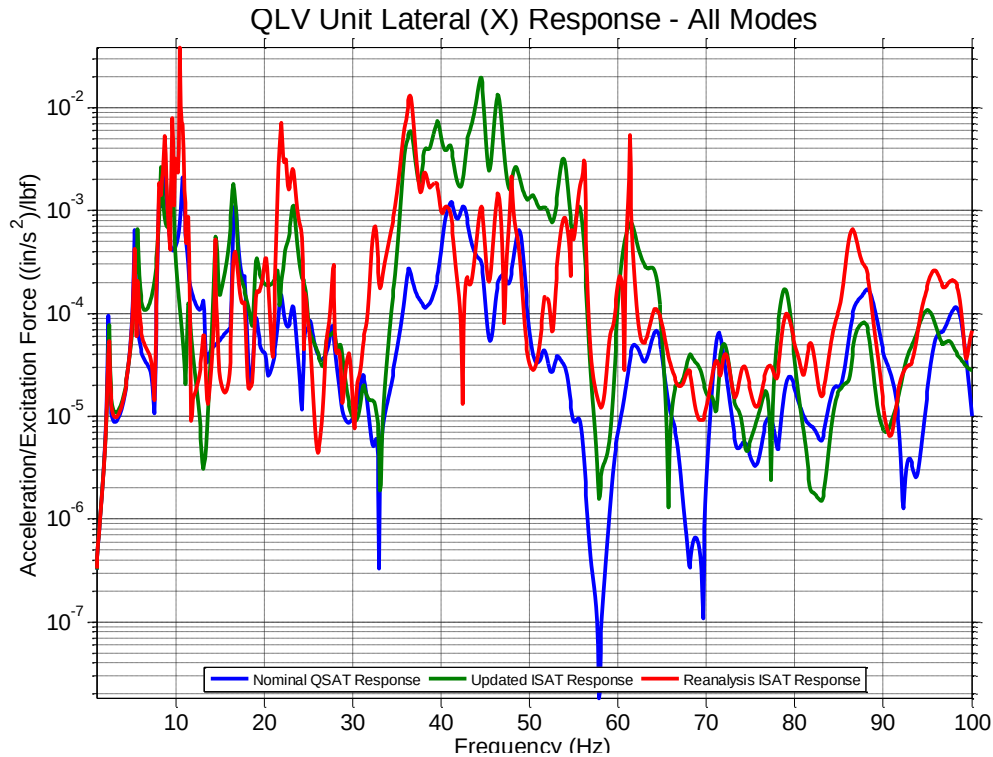


Figure 3-27. Generic launch vehicle lateral (X) responses for frequency-domain JPL method show that even when retaining all system modes, the JPL method did not converge on the correct results.

It was hypothesized that the inaccuracy of the method without any truncation might be due to either an inaccurate system modes calculation in Nastran, or possibly due to the transfer of single precision mode shape coefficients from Nastran to MATLAB. To eliminate this, the HCB model of the QLV was imported into MATLAB, and the component assembly and system eigenvalue solution were calculated in MATLAB. In addition, the assumption of 1% diagonal system damping was removed. Instead the 1% viscous damped component models were assembled, and a consistent coupled system damping matrix was calculated. Then both the FRF calculations and the reanalysis calculations were done with the coupled system damping matrix. The final results in the axial direction are illustrated in Figure 3-28. These results are much better, though still not exact. However, the lateral (X) responses illustrated in Figure 3-29 show that the method is still not very accurate.

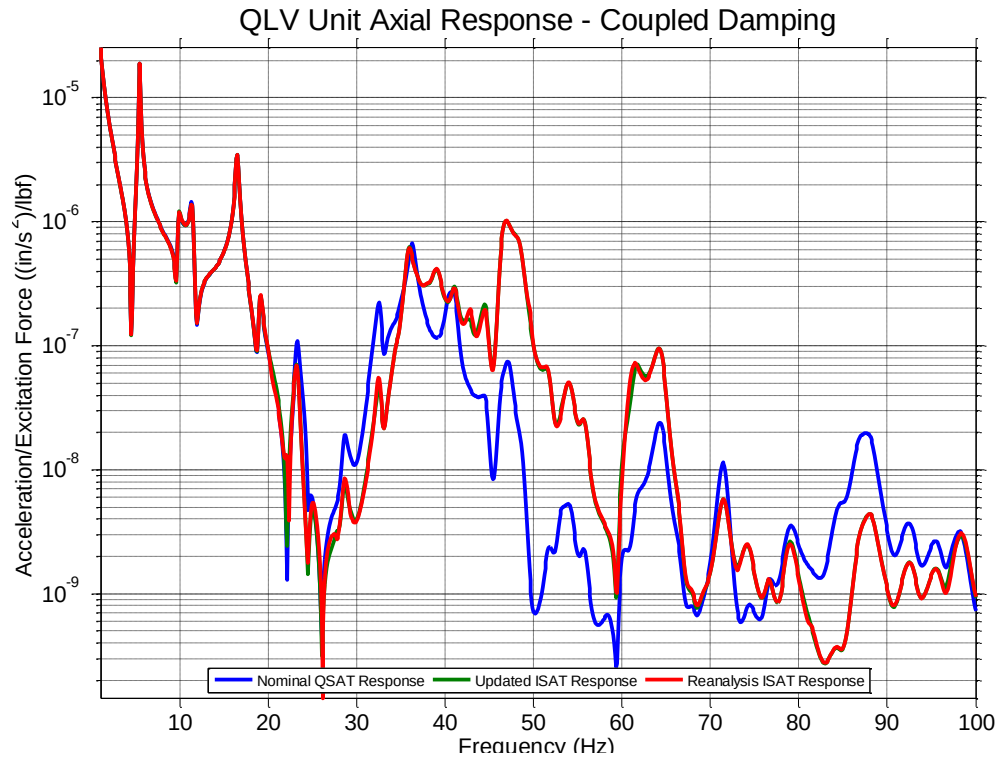


Figure 3-28. With coupled damping and no modal truncation, the generic launch vehicle axial response for the frequency-domain JPL method shows much better but not perfect match of reanalysis (in red) with exact (in green).

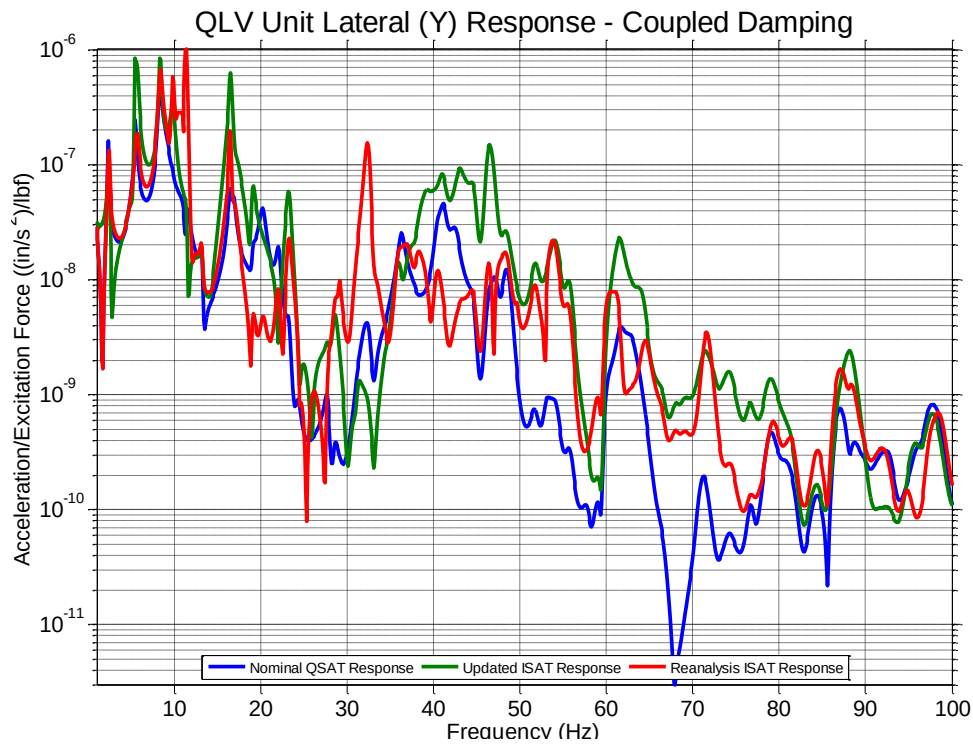


Figure 3-29. With coupled damping and no modal truncation, the generic launch vehicle lateral (Y) response for the frequency-domain JPL method (in red) is a very poor match to actual (in green).

The fact that the frequency-domain JPL implementation appears to be converging on the correct results as more and more assumptions are removed and the fact that it behaves exactly as expected for simpler cases suggest two things. The first is that this particular sample problem is extremely challenging, possibly due to some inherent numerical ill-conditioning, and that the JPL method, while accurate for simpler problems, is very numerically sensitive. This suggests that the method is particularly sensitive to small numerical errors in transfer functions, which is consistent with our prior experience using frequency-domain synthesis methods.

3.3.1.3 Reanalysis of Single-Node Generic Launch Vehicle Using Time-Domain Method 1

Next, Method 1, described in section 2.2, was applied to the single-node interface QLV reanalysis example. In this case, there are nine massless DOF that must be reduced from the reanalysis equations of motion using a singular value cutoff of 1.E-5. An eigenvalue solution was performed for the reduced mass and stiffness matrices in Eq. (2-24) to decouple the undamped system, and then 1% modal damping was added.¹⁵

Initially, 286 reduced system modes were used in the time-domain reanalysis response simulation because there are 286 modified system modes with frequencies less than 50.0 Hz. Figure 3-30 illustrates a comparison between the reanalysis prediction and the exact solution for the interface node acceleration in the thrust direction. As shown, the predicted result is quite accurate, though the peaks are slightly underpredicted. Figure 3-31 shows the predicted acceleration transverse to the thrust direction. The predicted response looks qualitatively accurate. The accuracy of the reanalysis predictions for the other interface node directions is comparable. Figure 3-32 illustrates the prediction of the first Payload 2 fixed-interface mode response. It is also accurate and is typical of the reanalysis predictions of the other Payload 2 fixed-interface modes. The response of the payload net CG acceleration gives an overall measure of the goodness of the reanalysis prediction. Figure 3-33 illustrates the Method 1 prediction of the Payload 2 net CG response. As can be seen, the prediction is quite accurate.

¹⁵ The time-domain method does not require that damping be defined at the component level unless system damping was calculated from component damping.

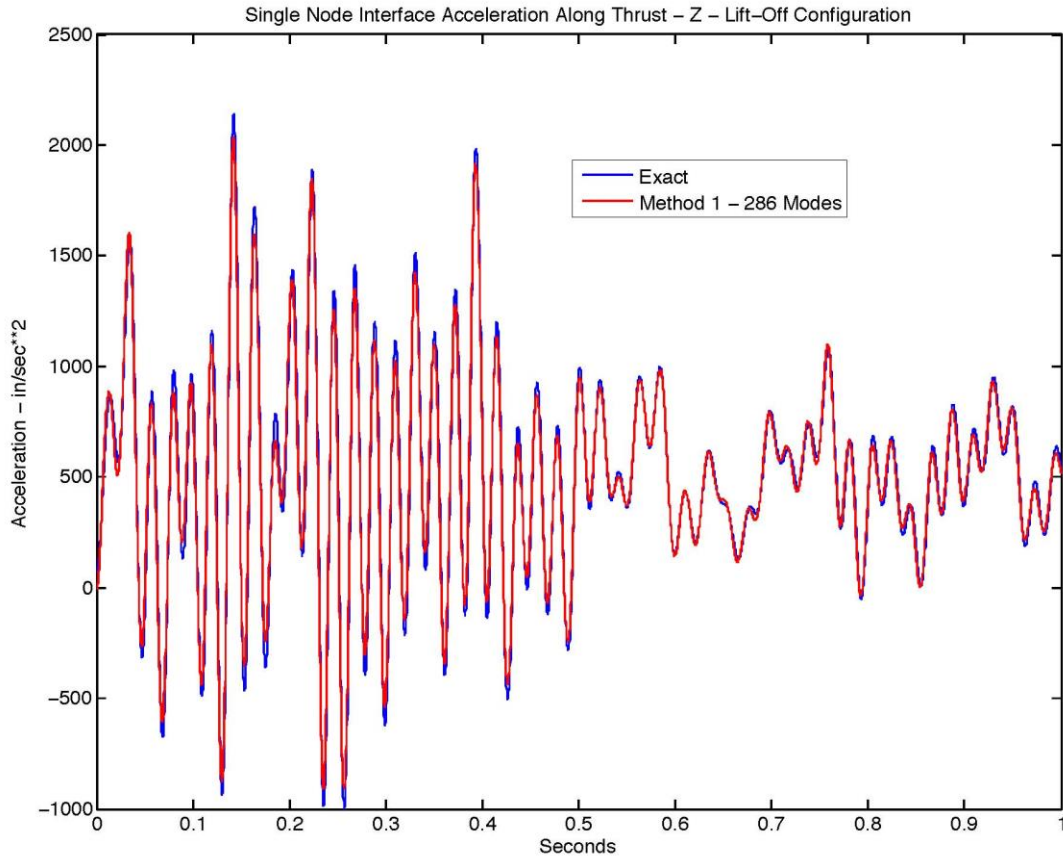


Figure 3-30. Payload 2 interface acceleration along thrust (Z) direction, 286 modes.

As a check of the proper application of Method 1 to the single-node interface example, a special case was considered in which reanalysis was performed to recover the response of Payload 1. The method essentially subtracts and then adds the same payload to the launch vehicle. The response of Payload 1 was originally simulated using 292 modes with frequencies below 50.0 Hz; therefore 292 reduced system modes were initially used in the reanalysis. The predicted acceleration of the interface along the thrust direction is almost exact. Errors in the response predictions for the other interface directions are relatively small.

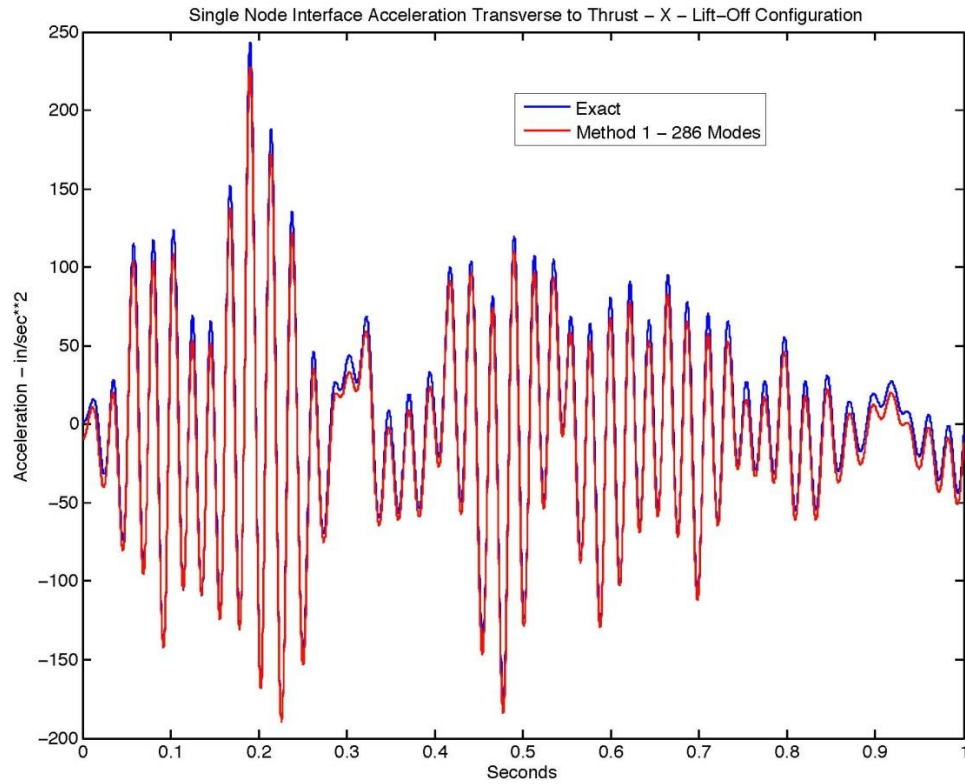


Figure 3-31. Payload 2 interface acceleration transverse to thrust (X) direction, 286 modes.

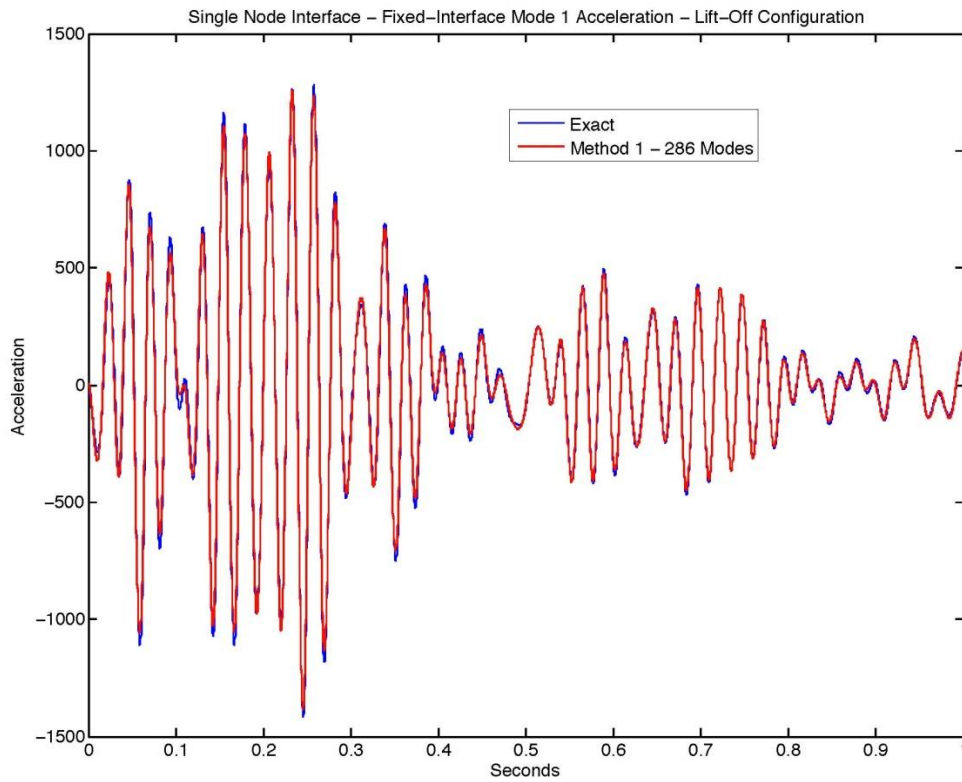


Figure 3-32. Payload 2 fixed-interface mode 1 acceleration, 286 modes.

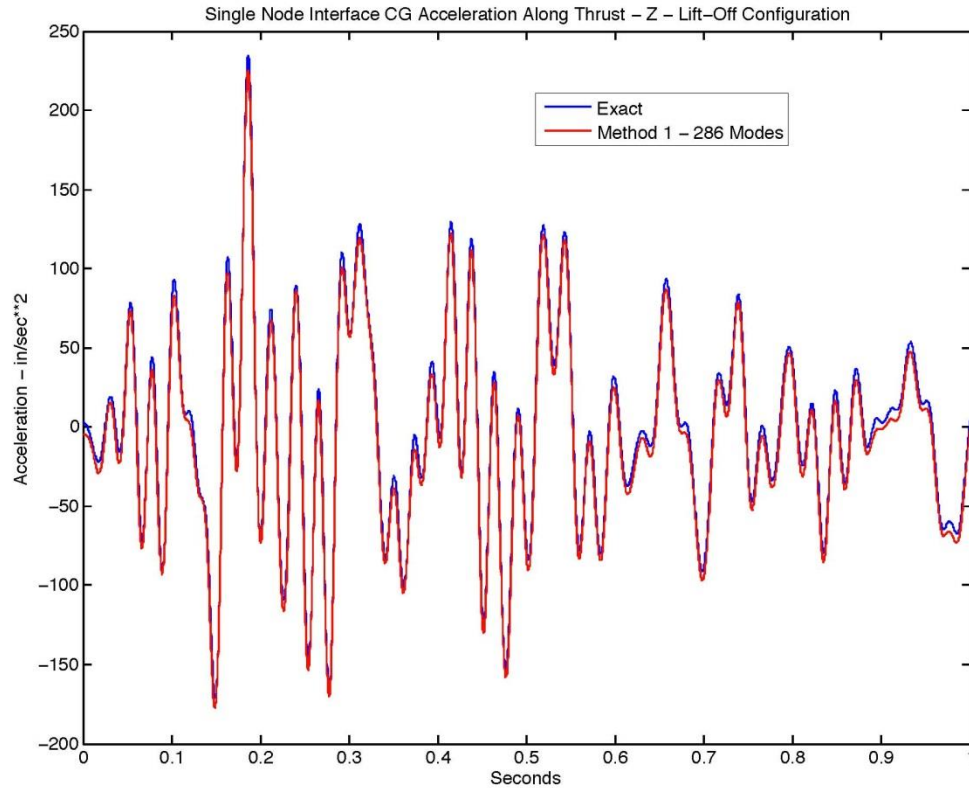


Figure 3-33. Payload 2 net CG acceleration along thrust (Z) direction, 286 modes.

As shown, the overall accuracy of Method 1 using 286 reduced system modes is very good, so modal truncation is not an issue for Method 1 in this example. However, the residual vectors were also included in the nominal system and the reanalysis to investigate the impact of modal truncation in this case. In the reanalysis process, twelve singular values were found to be less than the $1.E-5$ criterion. Therefore, the corresponding twelve massless DOF were reduced out of the equations of motion for this example. Figure 3-34 shows the prediction of the interface node thrust acceleration using Method 1 with residual vectors. The residual-vector-based prediction is an improvement over the response shown in Figure 3-30 computed without the residuals. All the other modified payload response predictions are very accurate.

As a final check on the impact of modal truncation on reanalysis Method 1, the untruncated QSAT and ISAT system representations were used in the reanalysis. In this case, 74 singular values violated the $1.E-5$ criterion, so the corresponding massless DOF were reduced from the equations of motion. The modified system contained 792 modes, so 792 modes were retained in the reanalysis. Figure 3-35 presents the predicted acceleration response of the payload net CG transverse to thrust (X). As expected, the result is very accurate. All other modified payload response predictions were equally accurate.

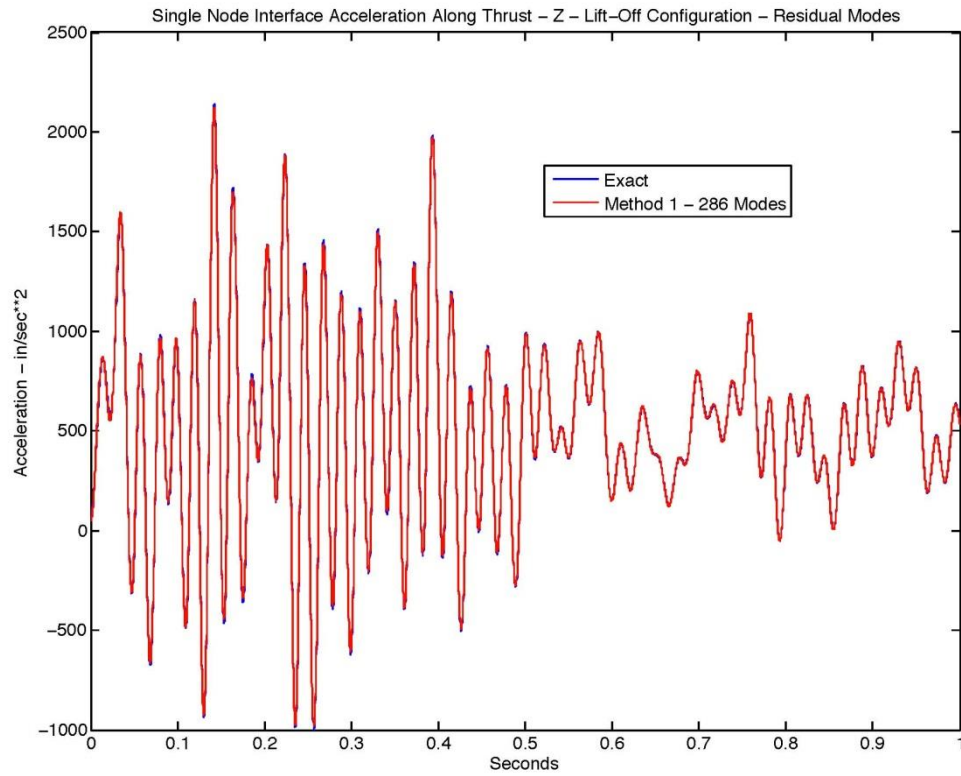


Figure 3-34. Payload 2 interface acceleration along thrust (Z) direction—286 modes plus residuals.

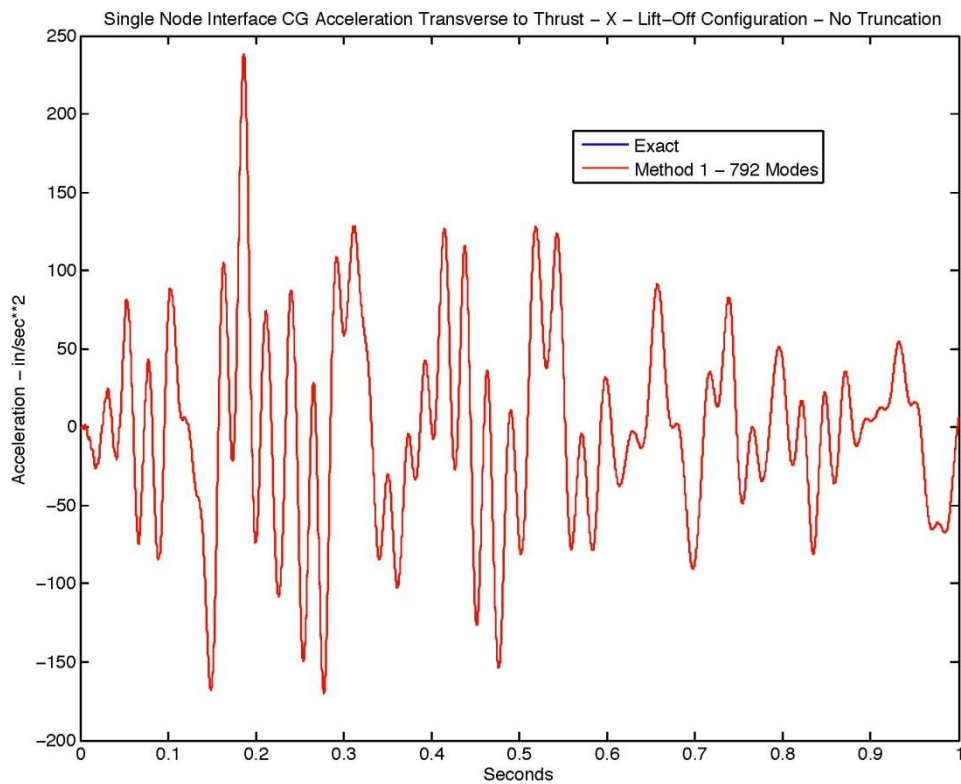


Figure 3-35. Payload 2 net CG acceleration transverse to thrust (X) direction, no truncation.

3.3.1.4 Reanalysis of Single-Node QLV Using Time-Domain Method 2

Next, Method 2, described in section 2.3, was applied to the single-node interface QLV reanalysis problem. As in the Method 1 application, there are nine massless DOF that must be reduced from the equations of motion. An eigenvalue solution was performed for the reduced mass and stiffness matrices in Eq. (2-29) to decouple the undamped system, and then 1% modal damping was added.

As with Method 1, 286 reduced system modes were initially used in the time-domain response simulation consistent with the number of modified system modes with frequencies less than 50.0 Hz. Figure 3-36 illustrates a comparison between the reanalysis prediction and the exact solution for the interface node acceleration in the thrust direction.

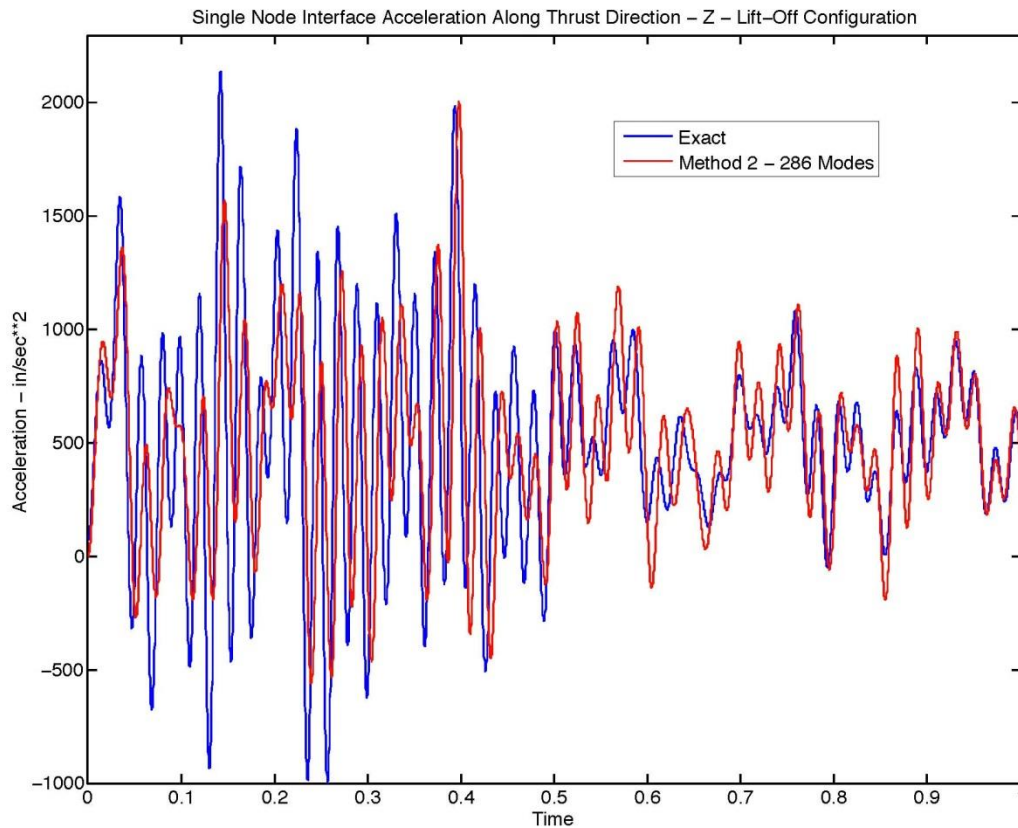


Figure 3-36. Payload 2 interface acceleration along thrust (Z) direction, 286 modes.

The agreement between prediction and the actual response is not very good. The reanalysis was repeated with the number of reduced system modes used in the simulation increased to 292. Figure 3-37 shows the actual interface response compared with the reanalysis prediction using 292 reduced system modes. The agreement is vastly improved, though peaks are slightly underpredicted. This result is unexpected, seeing as only 286 modified system modes below 50.0 Hz were used in the simulation of the actual Payload 2 response. However, since Method 2 is predicting the change in response rather than the modified response, it is clear that some of the higher-frequency modes are contributing to this change.

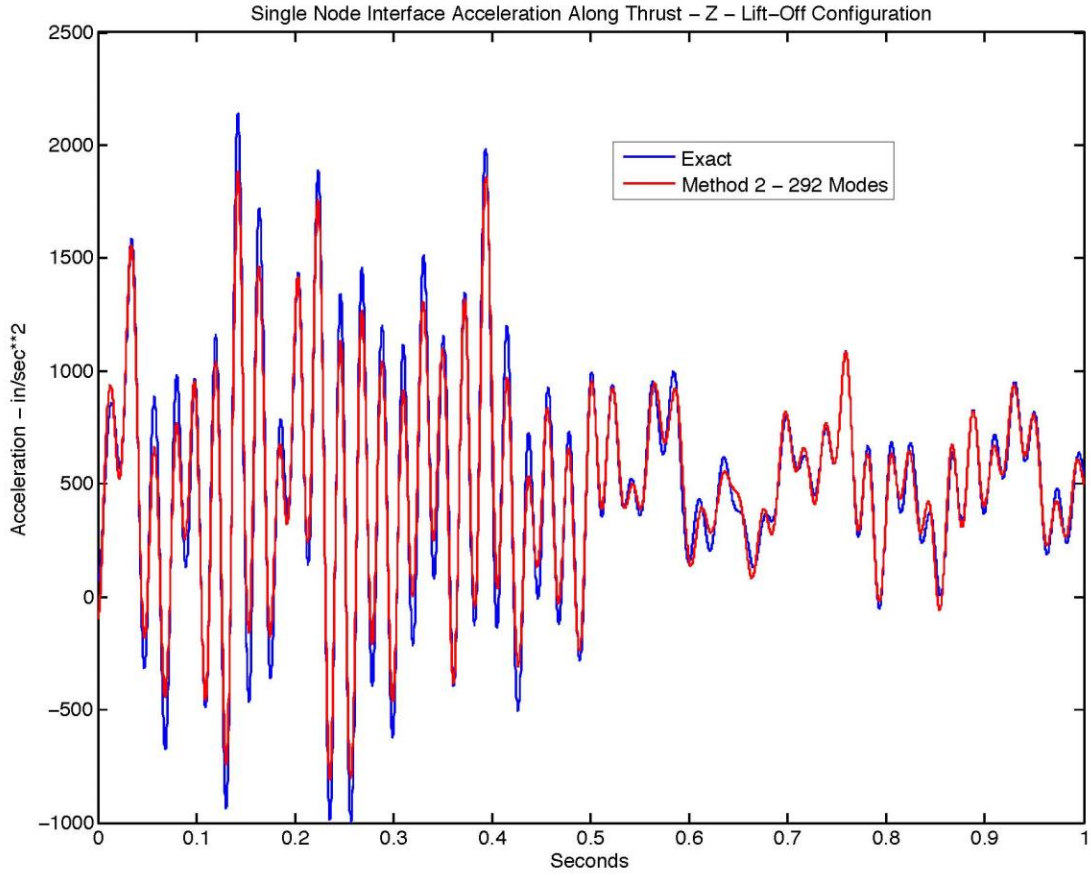


Figure 3-37. Payload 2 interface acceleration along thrust (Z) direction, 292 modes.

A convergence study was performed to determine the impact of including more reduced system modes, up to the maximum of 326, which is 292 nominal system modes, plus 43 Payload 2 fixed-interface modes, minus 9 zero singular values. An error metric was devised to give a quantitative measure of the agreement between the actual and predicted time-domain responses. The metric is given by the expression

$$e = 100 \frac{[(\ddot{u}_A - \ddot{u}_P)^T (\ddot{u}_A - \ddot{u}_P)]^{1/2}}{[\ddot{u}_A^T \ddot{u}_A]^{1/2}}$$

in which

- e = percent error in predicted time history
- $\ddot{u}_A$ = actual acceleration time-history vector
- $\ddot{u}_P$ = predicted acceleration time-history vector.

Figure 3-38 shows the percent error in the predicted acceleration of the interface node in the thrust direction as a function of the number of reduced system modes retained in the simulation. The error drops dramatically as the number of modes increases from 286 to 292. The error continues to drop slowly as the number of modes is increased beyond 292, and then decreases significantly as the number is increased from 315 to 320. The error then starts to increase as even more modes

are retained. This is due to the fact that very high-frequency modes retained in the simulation, up to 300 Hz, are excited by the sudden application of the thrust. The initial high-frequency transient dies out rapidly but leads to additional error. If reduced system modes beyond 70.0 Hz are heavily damped, the error can be reduced. If 20% damping is applied to reduced system modes 321–326, the percentage error is reduced to 0.83, as shown in Figure 3-38.

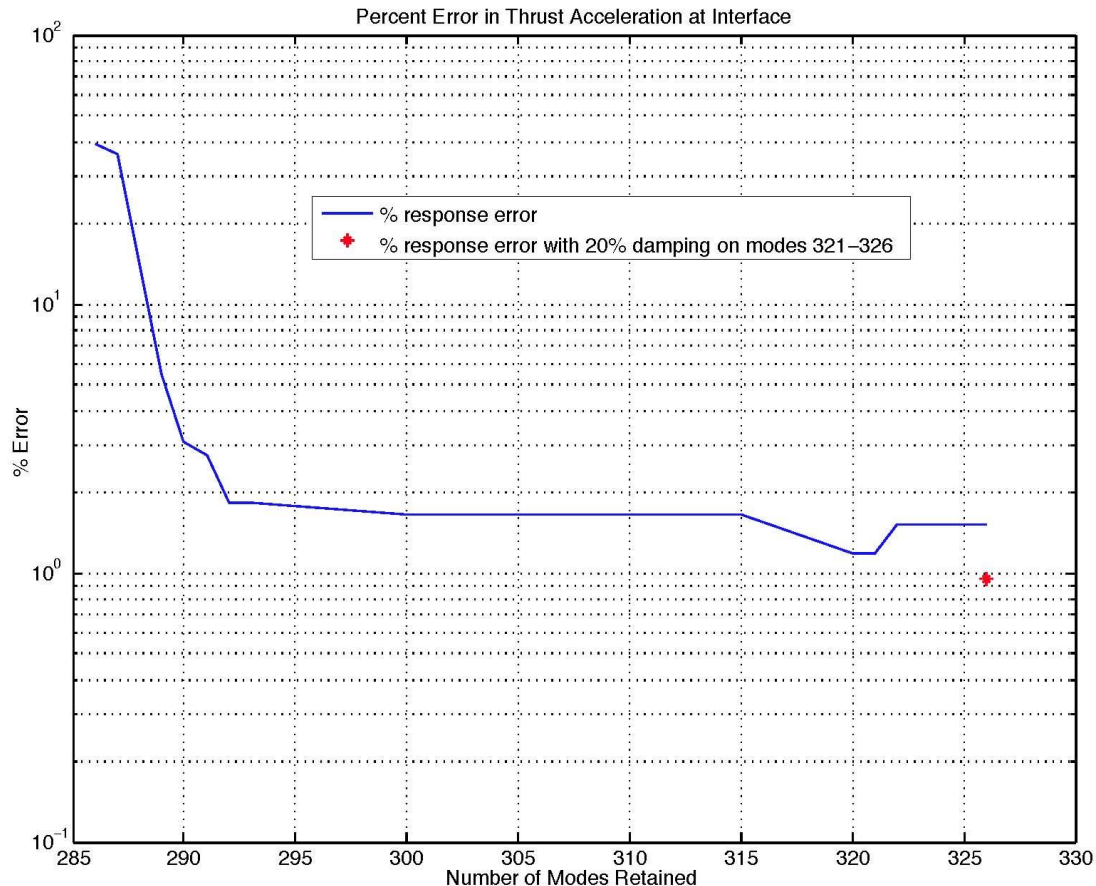


Figure 3-38. Error in predicted Payload 2 interface acceleration along thrust (Z) direction.

Figure 3-39 shows the predicted interface acceleration along the thrust direction when 326 reduced system modes are retained. The agreement with the actual response is very good. Note the small startup transient due to the high-frequency modes that dies out very quickly. Figure 3-40 illustrates the predicted interface acceleration in the X direction, transverse to the thrust. Once again, the agreement with the actual response is very good. Agreement between predicted and actual response for the other interface directions is comparable. Figure 3-41 shows the predicted response of Payload 2 fixed-interface mode 1. The agreement is also good and is typical of the other fixed-interface modes.

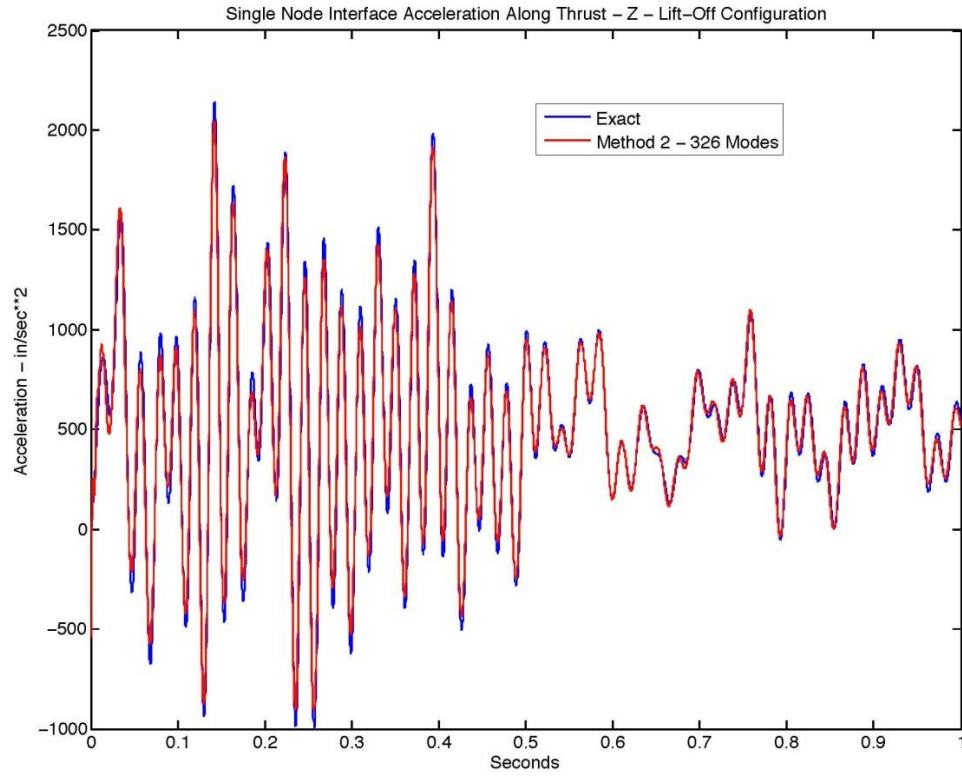


Figure 3-39. Payload 2 interface acceleration along thrust (Z) direction, 326 modes.

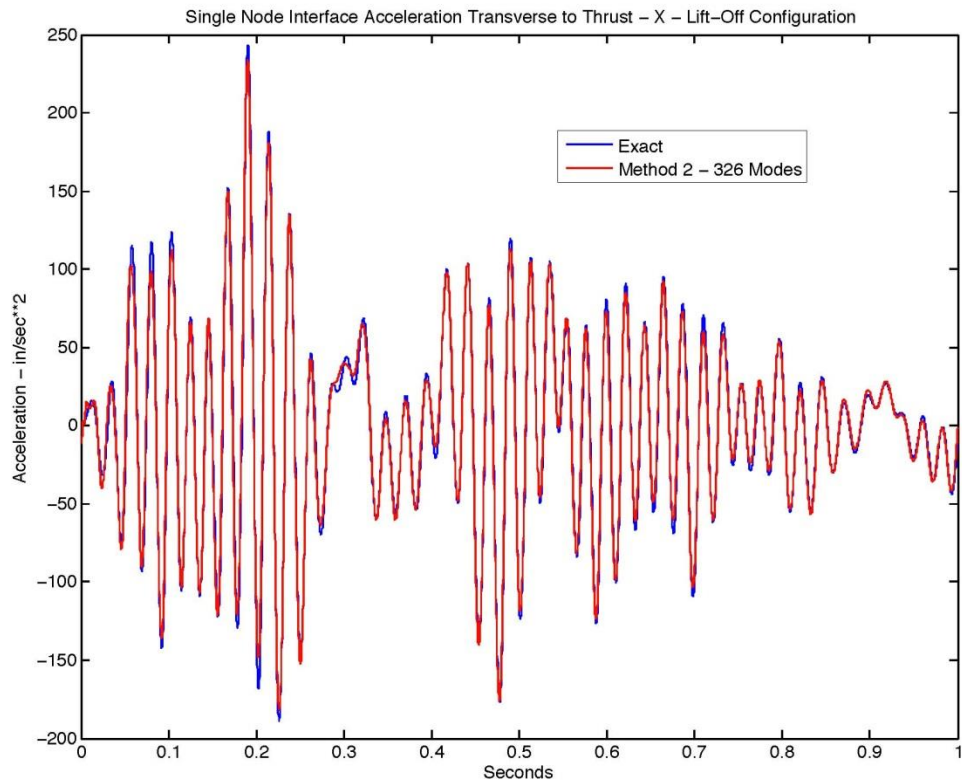


Figure 3-40. Payload 2 interface acceleration transverse to thrust (X) direction, 326 modes.

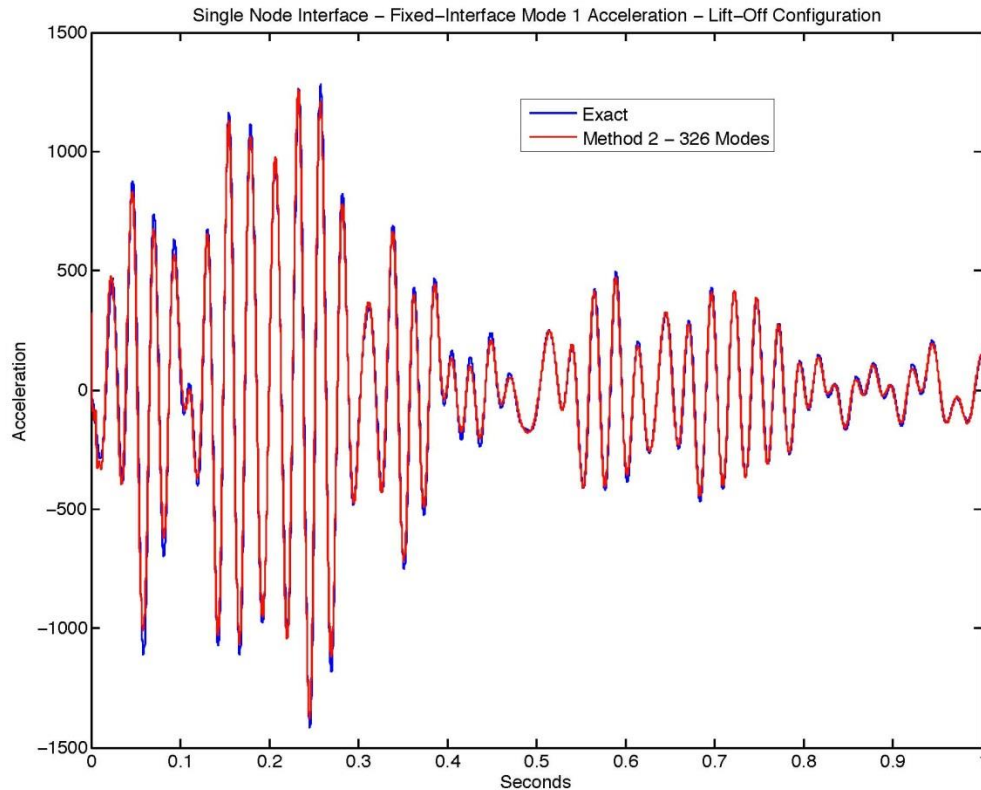


Figure 3-41. Payload 2 fixed-interface mode 1 acceleration, 326 modes.

An overall check of the accuracy of the reanalysis prediction is given by the recovery of the response of the Payload 2 net CG acceleration. Figure 3-42 illustrates the predicted response using 326 reduced system modes in the simulation. As with the interface response predictions, the agreement with the actual response is very good.

As a check of the proper application of Method 2 to the single-node interface example, reanalysis was performed to recover the response of Payload 1. The response of Payload 1 was originally simulated using 292 modes with frequencies below 50.0 Hz, therefore 292 reduced system modes were initially used in the reanalysis. Figure 3-43 shows poor agreement between the predicted and actual thrust-direction interface accelerations. As in the case of reanalysis of Payload 2, the predicted response for Payload 1 improves as more reduced system modes are retained in the reanalysis simulation, although the convergence is a bit slower. If the maximum number of reduced system modes are retained, 357, the predicted response of Payload 1 agrees quite well with the actual response, as shown in Figure 3-44.

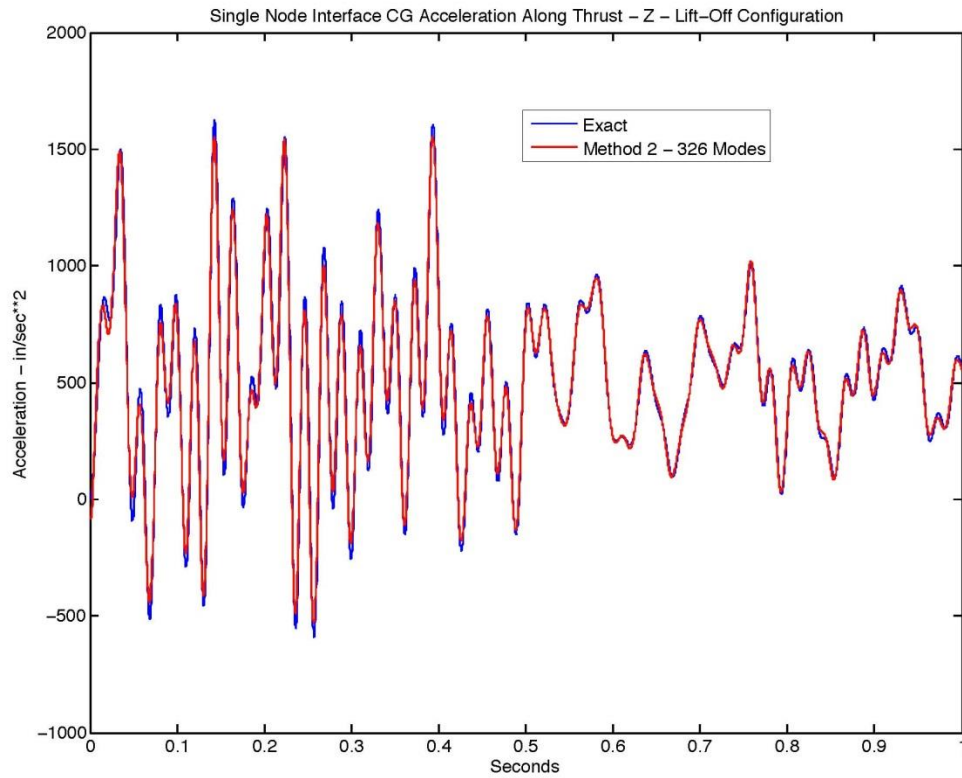


Figure 3-42. Payload 2 net CG acceleration along thrust (Z) direction, 326 modes.

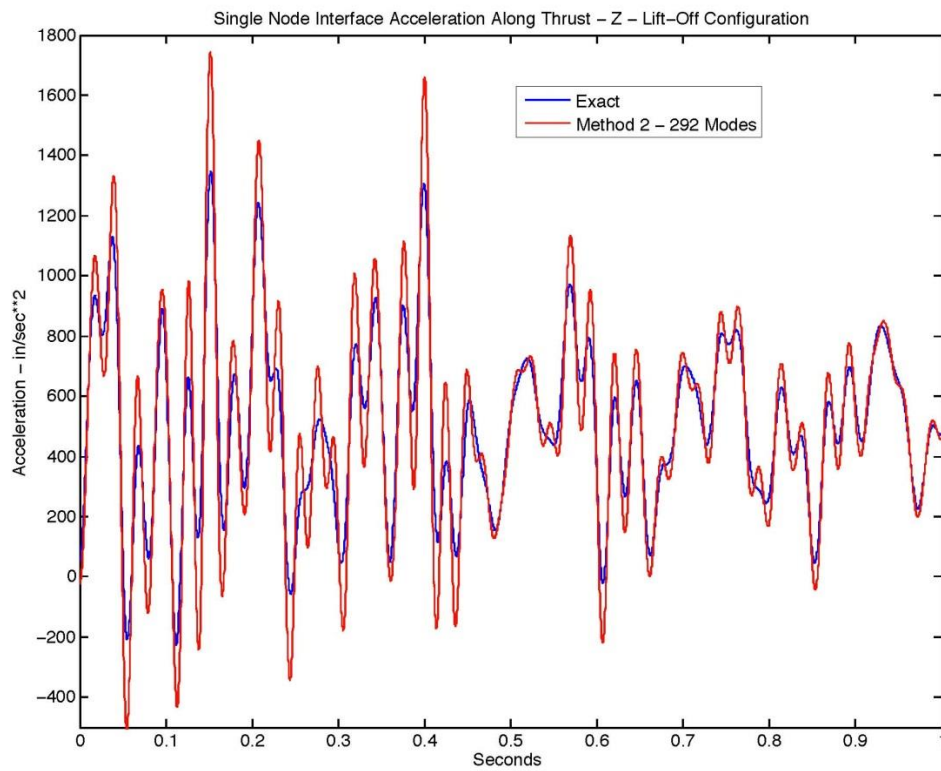


Figure 3-43. Payload 1 interface acceleration along thrust (Z) direction, 292 modes.

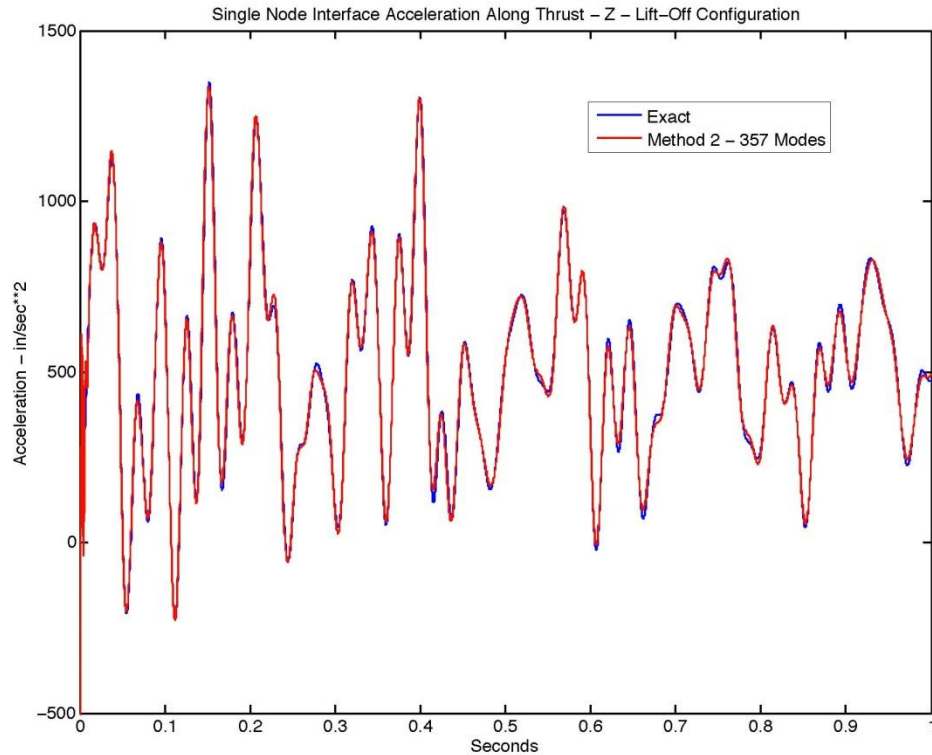


Figure 3-44. Payload 1 interface acceleration along thrust (Z) direction, 357 modes.

One of the early goals of this investigation was to examine the effect of a dynamic uncertainty factor (DUF) in the nominal payload response used in Method 2 reanalysis. The question was whether the DUF would properly propagate through the reanalysis into the elastic response of the new payload. To investigate this question, the rigid and elastic responses of the nominal payload were separately input to the Method 2 reanalysis code. During this investigation, it was found that the DUF did not affect the predicted rigid body response of the new payload, and it properly multiplied the elastic response of Payload 2 predicted by the reanalysis.

As suggested, modal truncation seems to be more of an issue in this example for reanalysis Method 2. Therefore, the method was applied again using the six residual vectors in the nominal system simulation and the subsequent reanalysis. Once again, twelve zero-mass DOF were reduced from the equations of motion during the reanalysis. Initially 286 reduced system modes were used in the reanalysis based on the number of modes in the modified system under 50.0 Hz. Figure 3-45 illustrates the predicted acceleration of the interface node along the thrust direction when the residual vectors are included. Comparing Figure 3-45 with Figure 3-37, it can be seen that predicted response is not affected by the inclusion of the residual vectors. As in the previous case without residual vectors, including all of the reanalysis reduced system modes dramatically improves the results, as shown in Figure 3-46. The other predicted modified payload responses have comparable accuracy. For this particular application, Method 2 does not benefit from the inclusion of the six residual vectors. However, modal truncation is obviously an issue.

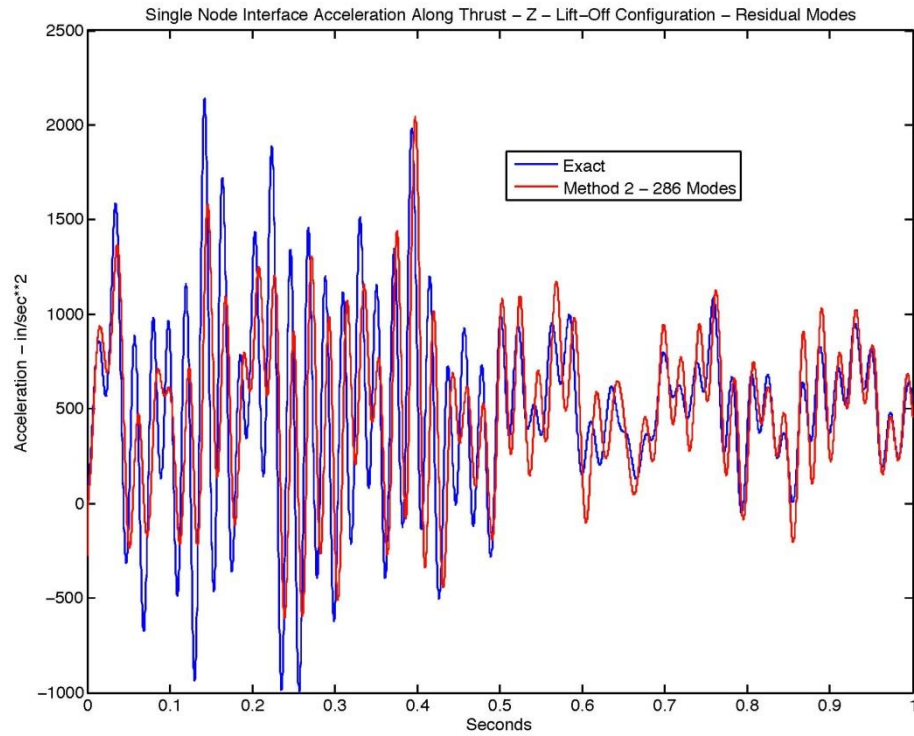


Figure 3-45. Payload 2 interface acceleration along thrust (Z) direction, 286 modes plus residuals.

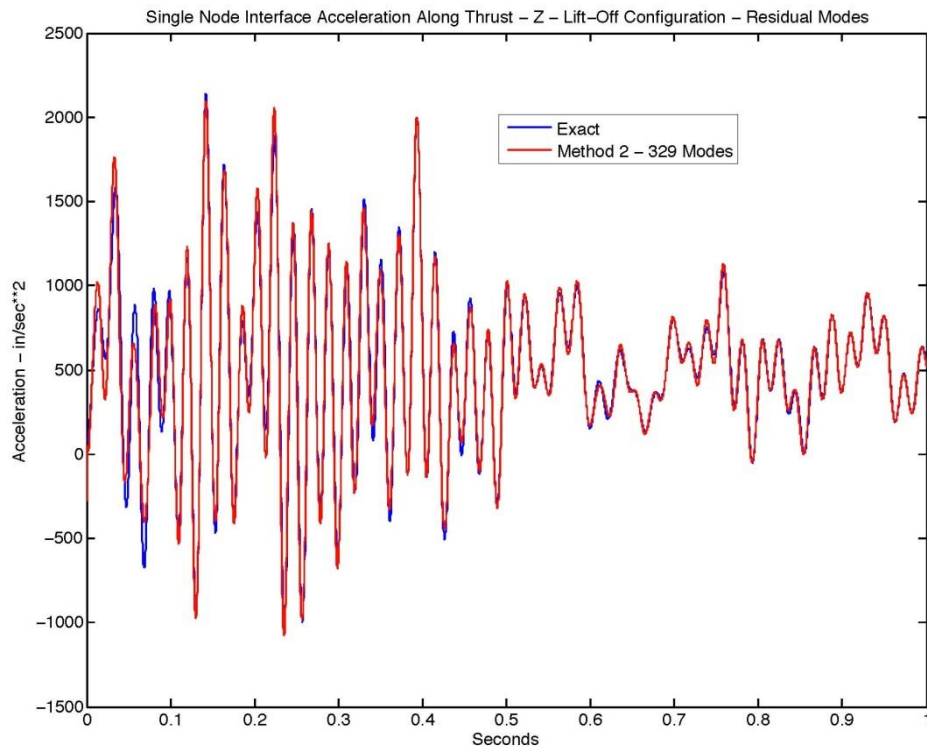


Figure 3-46. Payload 2 interface acceleration along thrust (Z) direction, 329 modes plus residuals.

As a final check on the impact of modal truncation on reanalysis Method 2, the untruncated QSAT and ISAT system representations were used in the reanalysis. As in the case with Method 1, 74 singular values violated the 1.E-5 value criterion, so the corresponding massless DOF were reduced from the equations of motion. The modified system contained 792 modes, so once again 792 modes were retained in the reanalysis. Figure 3-47 shows the predicted acceleration response of the payload net CG along the thrust direction. Once again, the result is very accurate, and all other modified payload response predictions were equally accurate. It is obvious that modal truncation can be a root cause of error in reanalysis Method 2.

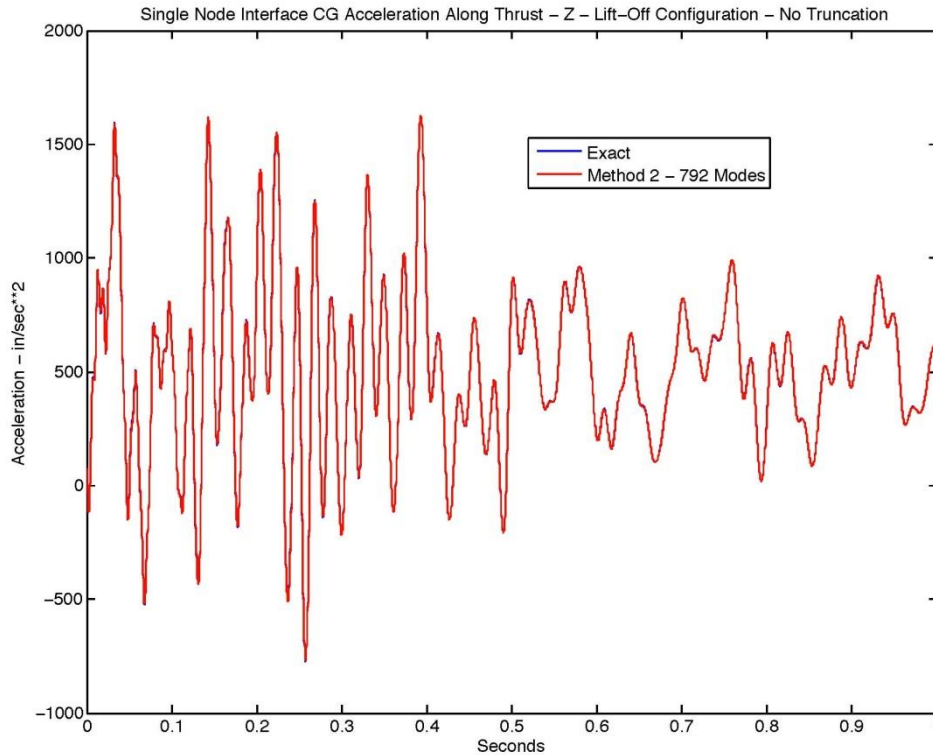


Figure 3-47. Payload 2 net CG acceleration along thrust (Z) direction, no truncation.

It should be noted that while both time-domain reanalysis methods were somewhat sensitive to modal truncation, both converged on the correct result when using a set of untruncated modes. Method 1 was accurate using the truncated modes, while Method 2 required the retention of a greater number of updated modes. However, both methods were considerably more robust with respect to modal truncation than the JPL method.

3.3.2 *Thirty-Two Node Interface*

For the QLV 32 node interface configuration, the nodes around the base of the adapter rings for both payloads were directly connected to the corresponding nodes on the launch vehicle with the rigid element removed. As in the single-node interface example, the nominal launch system has 292 free-free modes with frequencies up to 50.0 Hz, but now the modified system has 287 modes in the same frequency range. A total of 192 residual vectors corresponding to the interface DOF were computed for this example. During simulation of the nominal system, 70% modal damping was applied to the residual vectors due to their high frequencies.

A constant thrust of 489,600 lbs. was applied at the top of the launch vehicle first-stage nozzle along the +Z-axis for one second, providing a rigid body acceleration of approximately 500.0 in/s^2 . One-percent modal damping was assumed for the nominal system. Response of the nominal system was simulated in the time domain to generate the payload interface acceleration response required as input for the reanalysis methods. The modified system was also simulated in the time domain to get the correct response of the modified system interface for comparison with the predicted results.

3.3.2.1 Reanalysis of the 32 Node QLV Using the Time-Domain JPL Method

The time-domain JPL method, described in section 2.1, was applied to the 32 node interface QLV reanalysis problem. As mentioned previously, there is no formulation of an augmented hybrid set of reanalysis differential equations to be integrated; therefore there is no need to remove massless DOF and no need to determine the number of reduced system modes to retain in the reanalysis simulation.

Figure 3-48 illustrates a comparison between the reanalysis prediction and the exact solution for the acceleration at typical interface node 510101 in the thrust direction. As in the single-node interface example, the predicted result is very inaccurate. Figure 3-49 illustrates the same data comparison but in the frequency domain. As in the single-interface-node example, the Payload 2 response predicted by the JPL method is accurate in the 0.0 to 20.0 Hz frequency range but very inaccurate in the 20.0 to 50.0 Hz region. As mentioned, it is believed that the JPL method is especially susceptible to the large discrepancy in payload response at the high end of the frequency range. The inclusion of the residual vectors in the analysis does not improve the predicted results.

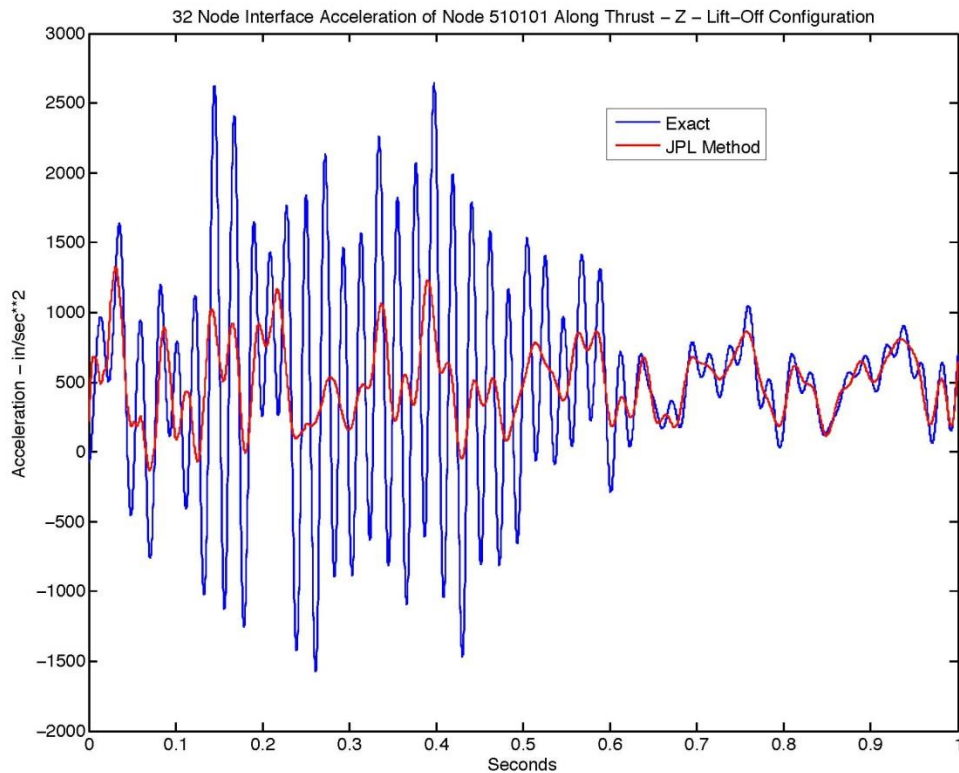


Figure 3-48. Payload 2 interface acceleration at node 510101 along thrust (Z) direction.

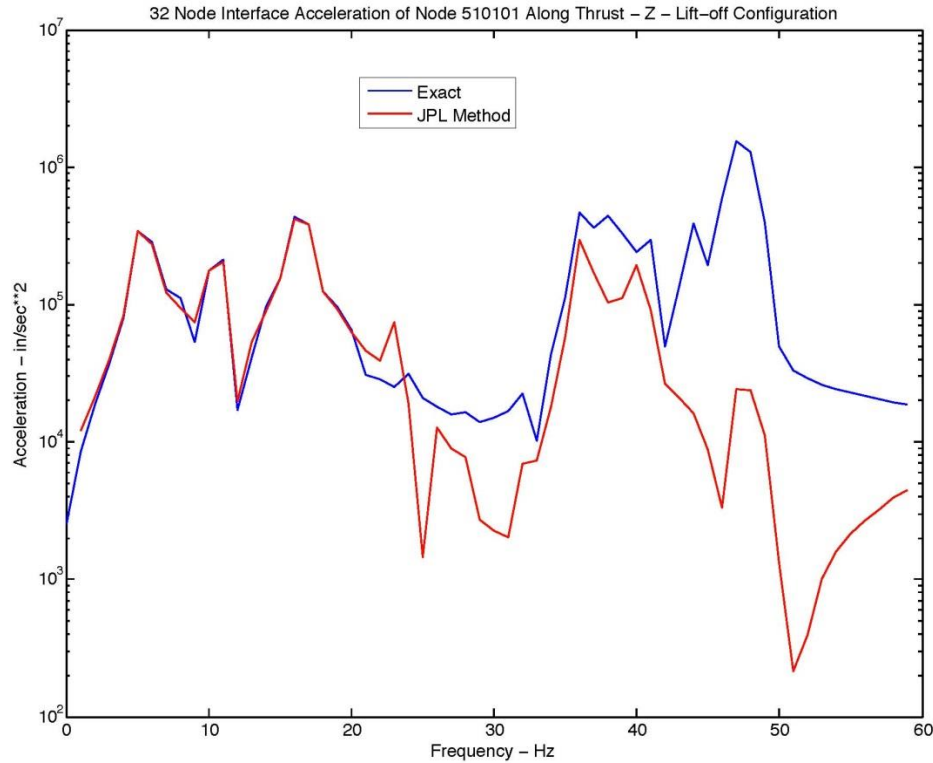


Figure 3-49. Payload 2 interface acceleration at node 510101 along thrust (Z) direction—frequency domain.

3.3.2.2 Reanalysis of 32 Node QLV Using Time-Domain Method 1

Method 1, described in section 2.2, was also applied to the 32 node interface QLV reanalysis problem. As in the single-node case, there were nine massless DOF that were reduced from the reanalysis equations of motion. An eigenvalue solution was performed for the reduced mass and stiffness matrices in Eq. (2-24) to decouple the undamped system, and then 1% modal damping was added.

Initially, 287 reduced system modes were used in the time-domain reanalysis response simulation, consistent with the fact that there are 287 modified system modes with frequencies less than 50.0 Hz in the 32 node interface example. Figure 3-50 illustrates a comparison between the reanalysis prediction and the exact solution for the acceleration of interface node 510101 in the thrust direction. The prediction is accurate over about the first 0.15 second but degrades after that.

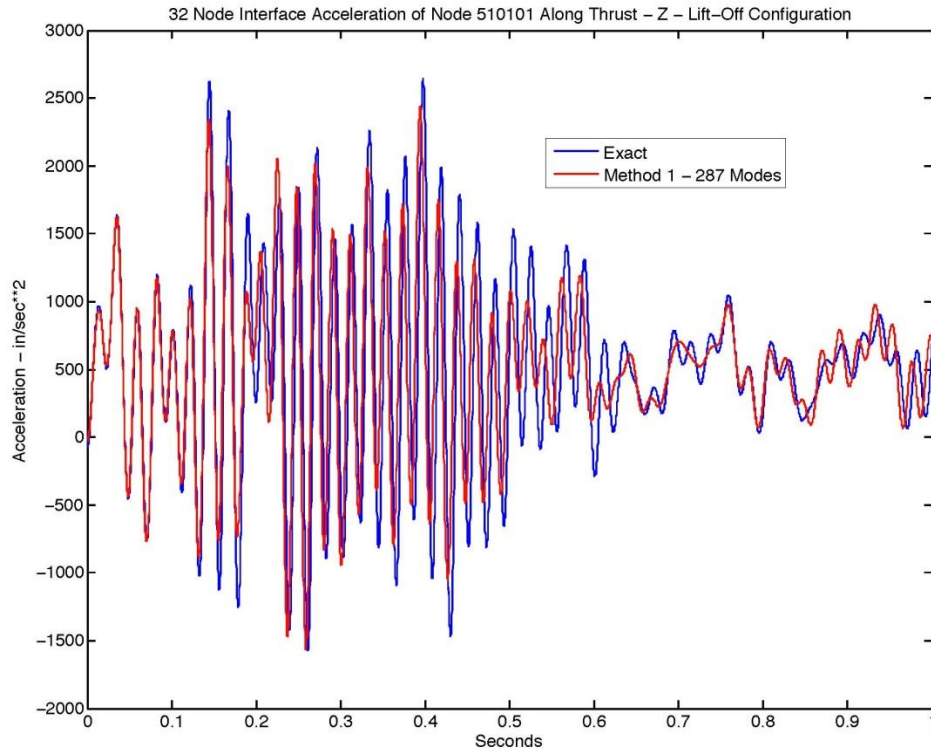


Figure 3-50. Payload 2 interface acceleration at node 510101 along thrust (Z) direction—287 modes.

Adding more reduced system modes to the reanalysis simulation does not improve the Payload 2 response prediction for the 32 node interface example. The overall accuracy of the Payload 2 response prediction can be gauged by recovering the payload net CG acceleration. Figure 3-51 shows the predicted Payload 2 net CG acceleration along the thrust direction. The prediction initially tracks the actual response but then also degrades.

A check of the proper application of the Method 1 procedure was also performed by applying the method to recover the response of Payload 1. The predicted results agree almost exactly with the actual Payload 1 response for 287 retained reduced system modes.

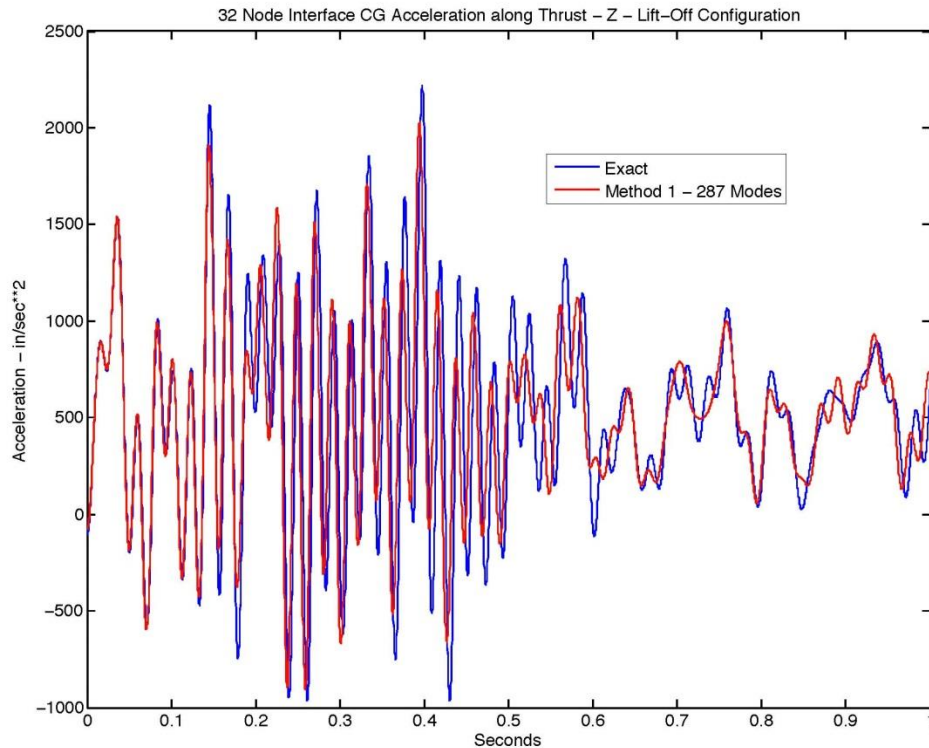


Figure 3-51. Payload 2 CG acceleration along thrust (Z) direction.

As in the single-node interface example, residual vectors were included in the nominal system simulation and the reanalysis in an attempt to mitigate modal truncation. In the single-node case, the six residual vectors had little impact on the predicted results. However, in the case of 32 interface nodes, there are 192 corresponding residual vectors, which provide a richer modal basis for reanalysis. Initially, 287 modes were used in the reanalysis simulation. Figure 3-52 illustrates the predicted acceleration of interface node 510101 along the thrust direction. The prediction is very accurate. When the residuals are not included, the Method 1 response predictions were very poor, as can be seen in Figure 3-50 and Figure 3-51. Figure 3-53 shows the predicted acceleration of the payload net CG along the thrust direction. Once again, it is very accurate. Response of a typical fixed-interface mode is presented in Figure 3-54. All other modified payload responses predicted by Method 1 are just as accurate. It is obvious that the inclusion of the residual vectors in the 32 node interface example has a profound effect on the predicted results. This means that modal truncation is an important issue for this example when using Method 1, though the modal truncation is handled very well by the inclusion of residual vectors at the interface DOF.

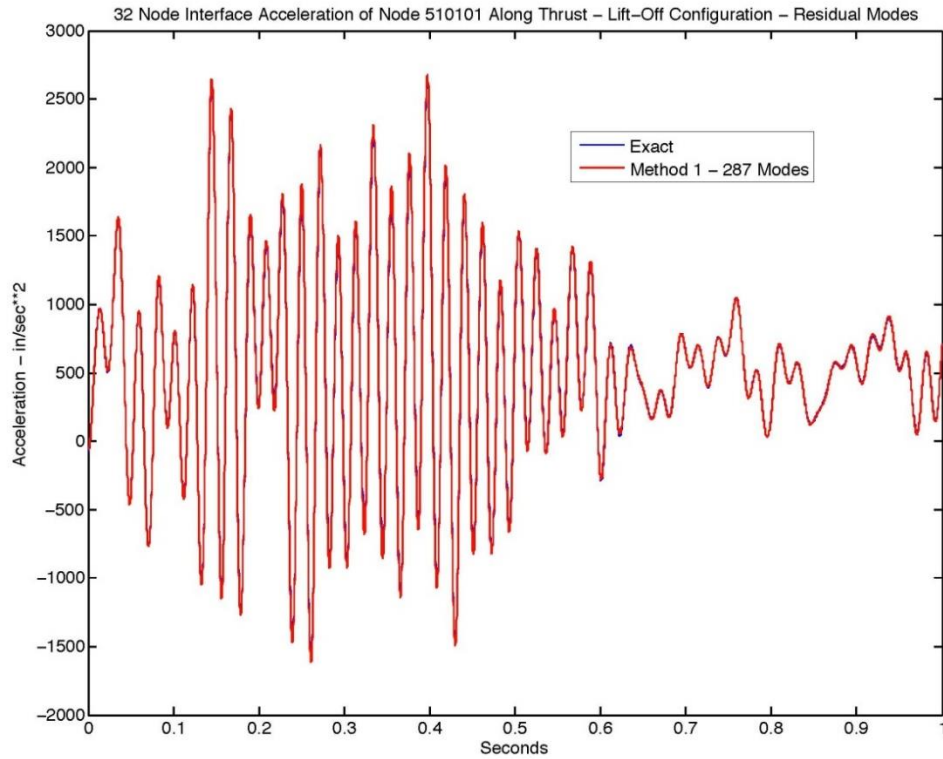


Figure 3-52. Payload 2 interface acceleration at node 510101 along thrust (Z) direction—287 modes with residuals.

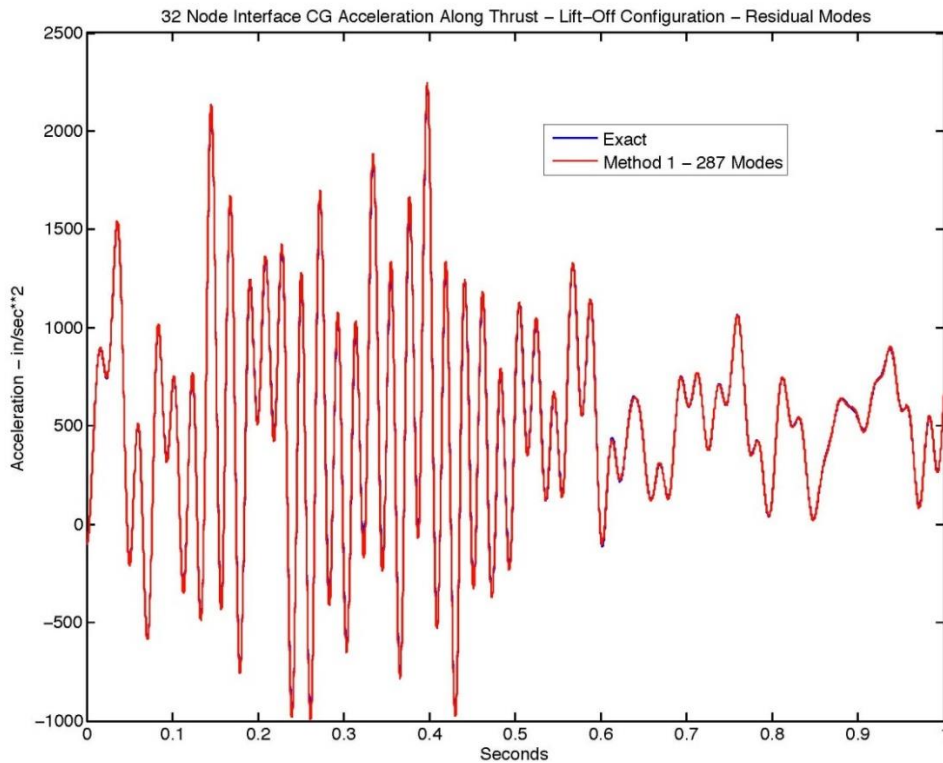


Figure 3-53. Payload 2 center of mass acceleration along thrust (Z) direction—287 modes with residuals.

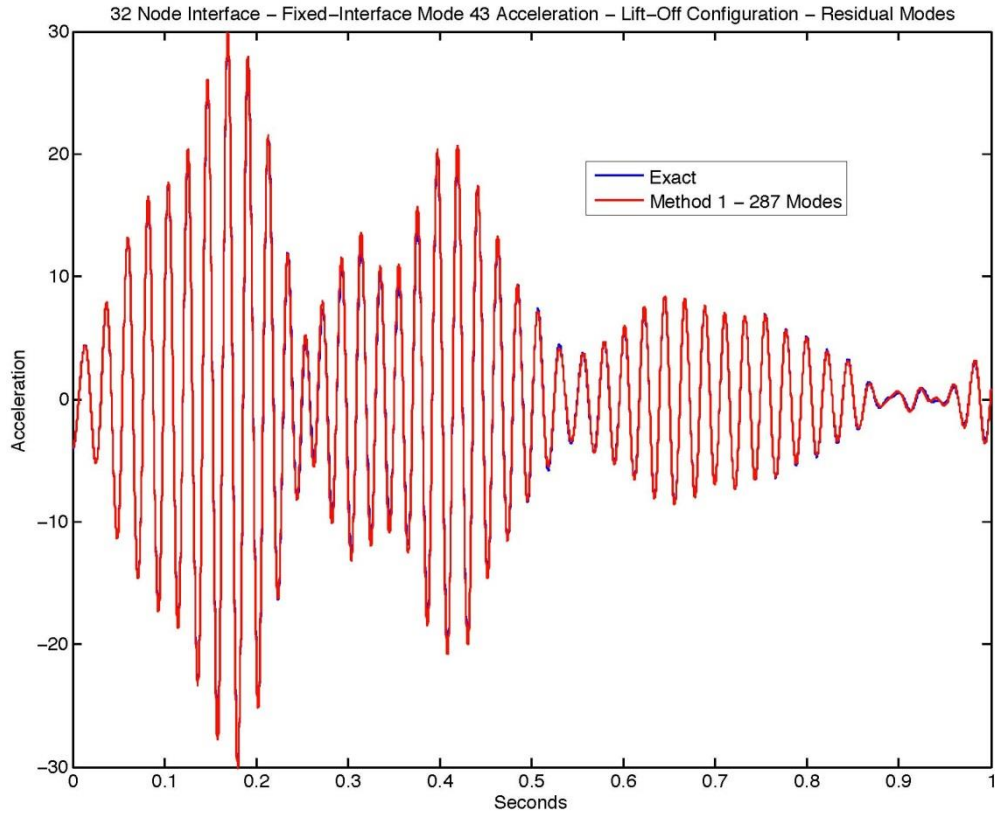


Figure 3-54. Payload 2 fixed-interface mode 43 acceleration—287 modes with residuals.

3.3.2.3 Reanalysis of 32 Node QLV Using Time-Domain Method 2

Method 2, described in section 2.3, was also applied to the 32 node QLV reanalysis problem. As in the single-node interface example, there were nine massless DOF, which were reduced out of the simulation using the singular value decomposition procedure. An eigenvalue solution was performed for the reduced mass and stiffness matrices in Eq. (2-29) to decouple the undamped system, and then 1% modal damping was added.

As in the Method 1 analysis, 287 reduced system modes were initially used in the time-domain response simulation. Figure 3-55 illustrates a comparison between the reanalysis prediction and the exact solution for typical interface node 510101 acceleration in the thrust direction. The result predicted using Method 2 is virtually identical to the prediction using Method 1, shown in Figure 3-50. The prediction is accurate over about the first 0.15 second but degrades soon after that.

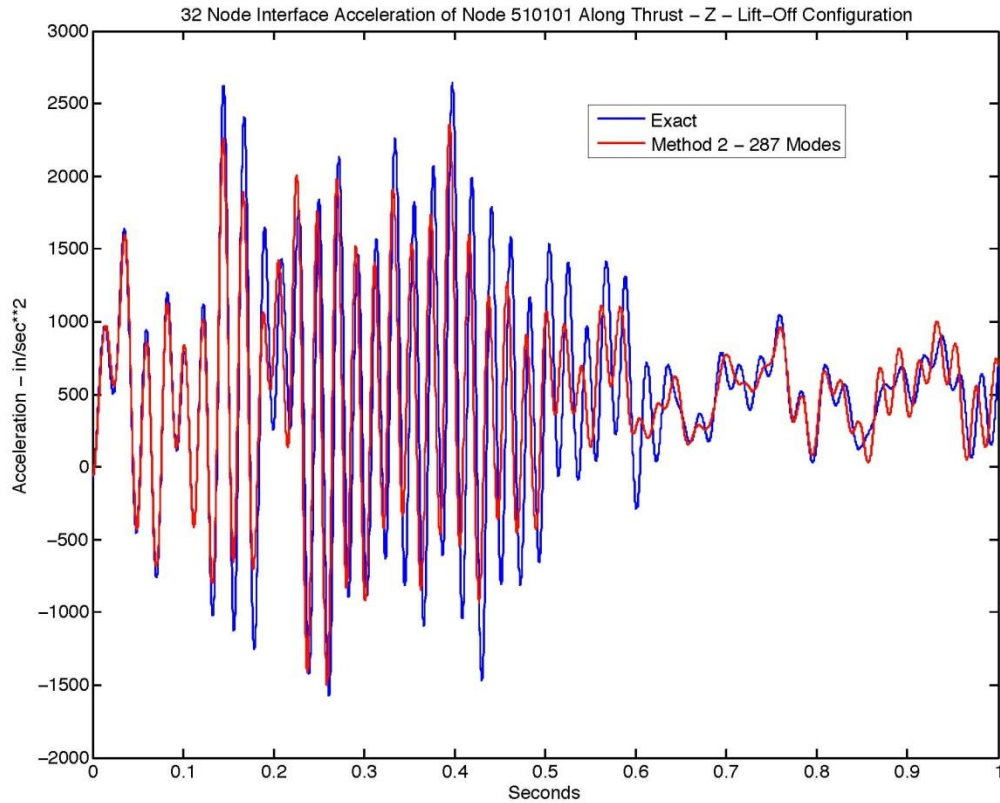


Figure 3-55. Payload 2 interface acceleration at node 510101 along thrust (Z) direction.

The corresponding transverse acceleration at node 510101 in the X direction is shown in Figure 3-56. Qualitatively, the predicted result looks inaccurate. The overall accuracy of the Payload 2 interface response prediction can be gauged by recovering the response of the net CG acceleration. Figure 3-57 shows the predicted Payload 2 net CG acceleration along the thrust direction. The predicted result is again almost identical to the result produced by Method 1. Figure 3-58 shows the same data but in the frequency domain. It is easily seen that the error in the prediction is due to disagreement in the 35.0 to 50.0 Hz frequency region. As in the single-node interface example, more reduced system modes were added to the reanalysis simulation; however, additional modes did not improve the predicted result for the 32 node interface example.

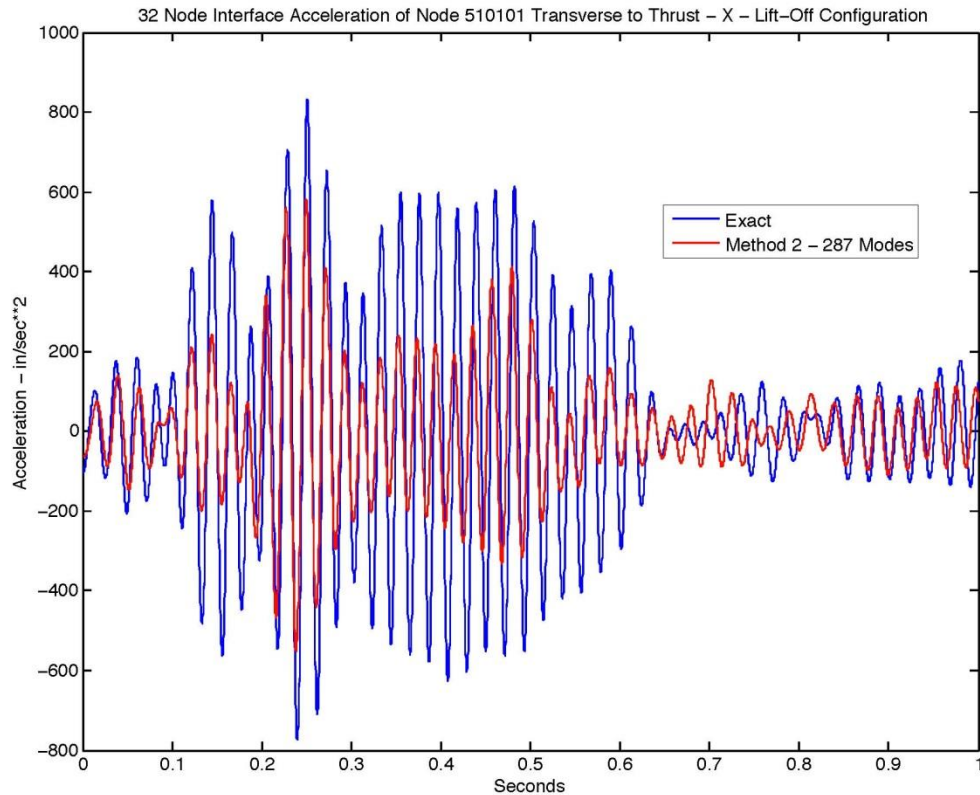


Figure 3-56. Payload 2 interface acceleration at node 510101 transverse to thrust (X) direction.

Proper application of the Method 2 procedure was also checked by attempting to recover the response of Payload 1. Figure 3-59 illustrates the acceleration of interface node 510101 for Payload 1 along the thrust direction using 287 reduced system modes. The prediction tracks the actual response, but it is not very accurate. A subsequent series of reanalysis simulations was performed including additional reduced system modes. The predicted response converged very slowly to the correct result by the time that 355 modes were included. Figure 3-60 illustrates the thrust direction acceleration of interface node 510101 using the maximum number of modes, 357. The Method 2 reanalysis now correctly predicts the response of Payload 1.

Overall, Method 2 did not perform as well on the 32 node example as it did on the single-node example. The large interface DOF set combined with the large difference between the payloads at the upper end of the frequency range make the 32 node example very challenging. To further investigate the impact of modal truncation on the Method 2 predictions, the reanalysis was repeated including the 192 residual vectors. Initially, 287 reduced system modes were retained in the reanalysis simulation. Figure 3-61 shows the predicted acceleration at interface node 510101 along the thrust direction. The response predicted by Method 2 is very accurate. Comparing this to the result with no residuals, shown in Figure 3-55, it is clear that including the residual vectors has a dramatic impact. Figure 3-62 illustrates a response transverse to thrust direction at node 510101. Once again, the prediction is very accurate. The overall accuracy of the modified payload predictions is given by the accuracy of the response predicted for the payload net CG. The acceleration of the payload net CG along the thrust direction is shown in Figure 3-63. The prediction is very accurate, and the predicted results for the other five directions at the center of

mass are just as accurate. As with Method 1, modal truncation is an important issue when applying Method 2 to the 32 node interface example.

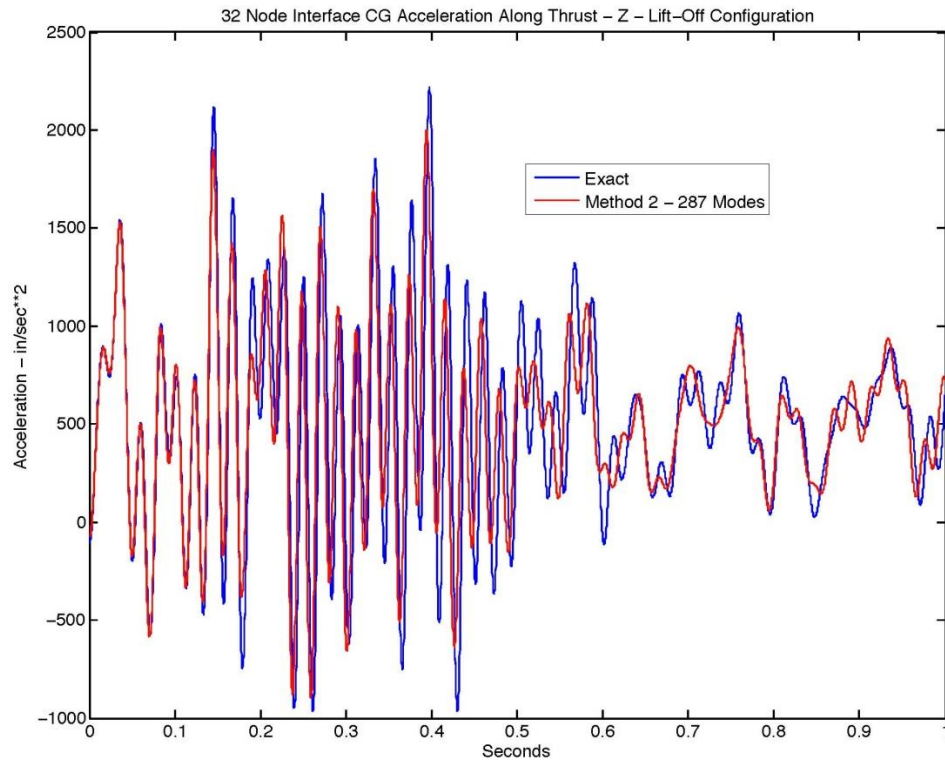


Figure 3-57. Payload 2 net CG acceleration along thrust (Z) direction.

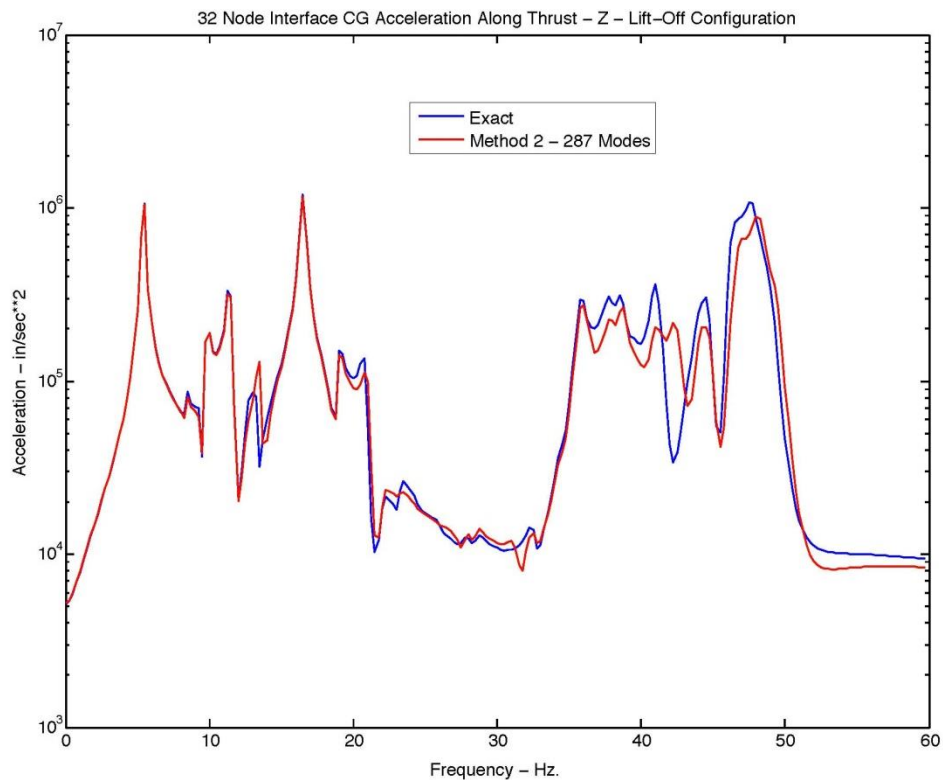


Figure 3-58. Payload 2 net CG acceleration along thrust (Z) direction—frequency domain.

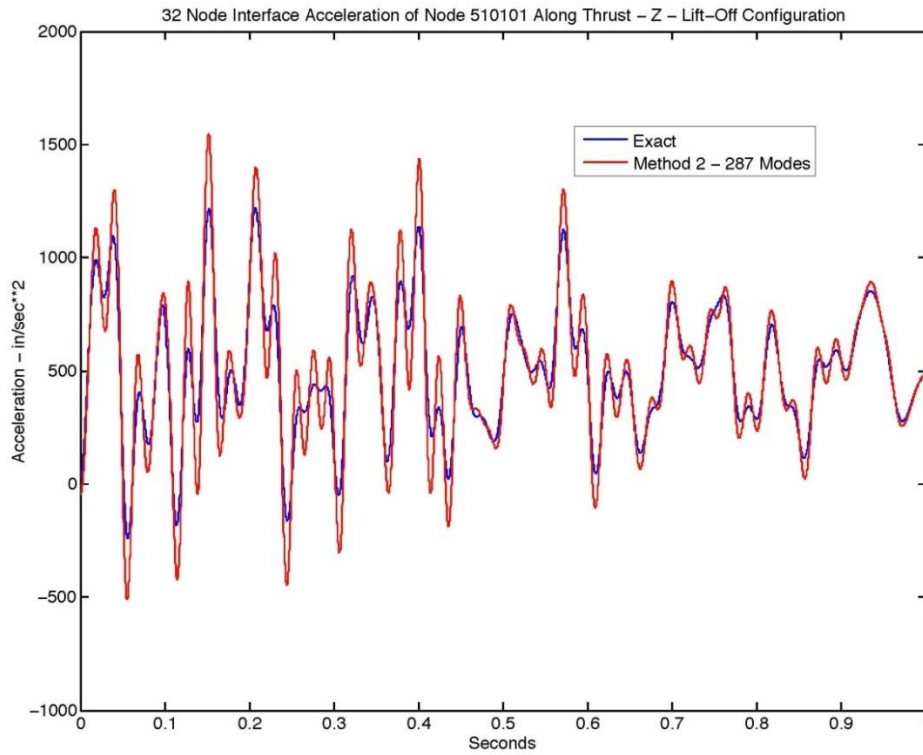


Figure 3-59. Payload 1 interface acceleration at node 510101 along thrust (Z) direction, 287 modes.

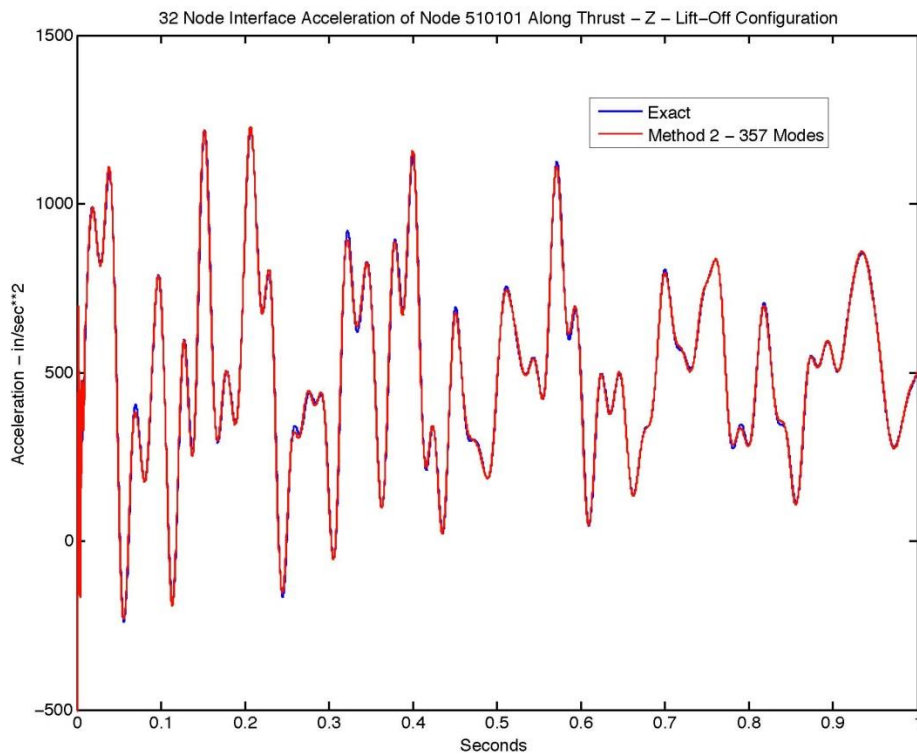


Figure 3-60. Payload 1 interface acceleration at node 510101 along thrust (Z) direction, 357 modes.

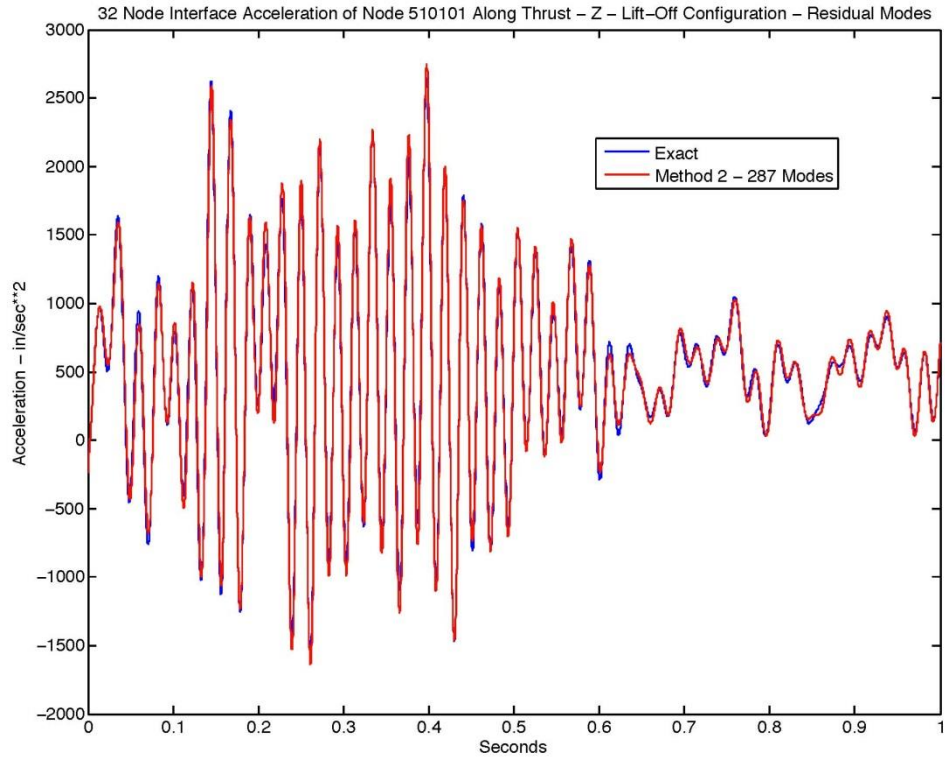


Figure 3-61. Payload 2 interface acceleration at node 510101 along thrust (Z) direction—287 modes with residuals.

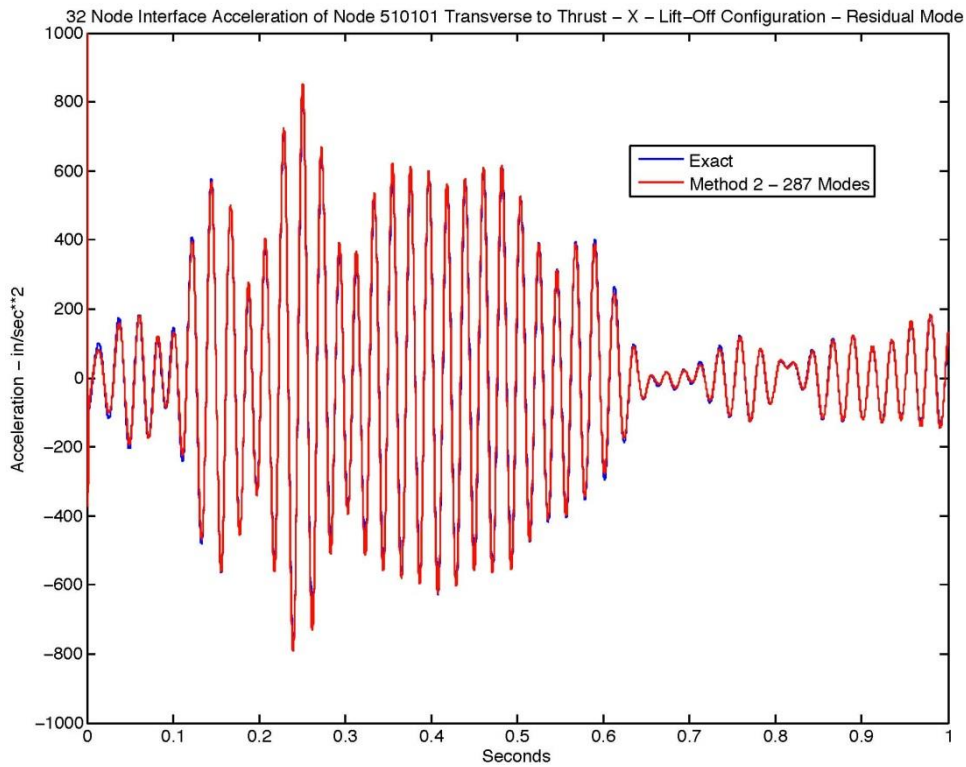


Figure 3-62. Payload 2 interface acceleration at node 510101 transverse to thrust (X) direction—287 modes with residuals.

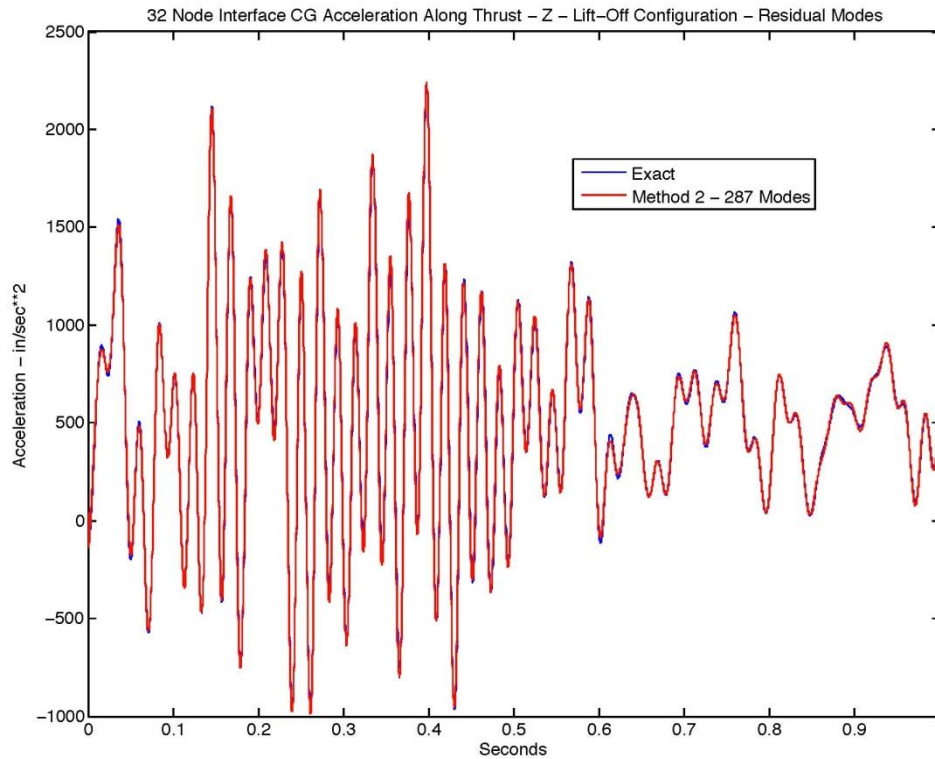


Figure 3-63. Payload 2 center of mass acceleration along thrust (Z) direction—287 modes with residuals.

4.0 Conclusion

Three different time-domain CLA reanalysis methods were investigated. The first of these was a method implemented at JPL, referred to as substitution analysis, as described in Trubert and Peretti [5]. This is a frequency-domain method, which can be used in the time domain by first transforming the time histories to the frequency domain and then back to the time domain after applying the substitution equations. The second two methods are described in Blueloch and Flanigan [6] and the *Reanalysis User's Guide* [7]. These methods are implemented entirely in the time domain without any requirement to transform to the frequency domain. All three methods are based on system modal properties of a preliminary CLA, along with HCB models of the original and updated payloads.

Three sample problems were used. The first was a simple three-mass model, which demonstrated that the JPL method is exact when all modes are retained but suffers from some degree of error when a truncated set of system modes is used. The second example was somewhat more complex, consisting of realistic spacecraft models coupled with a simplified stick model of a launch vehicle. This method likewise demonstrated that the JPL method is exact when all modes are retained, but it also demonstrated that both the true time-domain methods are considerably more robust than the JPL method when using a truncated set of modes.

The final sample problem was the most complex, consisting of the same spacecraft models, but coupled with a realistic launch vehicle model in a liftoff configuration. This proved to be particularly challenging. In this case, the JPL method gave very poor results. Though the results improved when using an untruncated version of the problem, the results were still far from exact.

Both time-domain methods were also somewhat inaccurate when using only modes up to the truncation frequency of 50 Hz, though they were considerably more accurate than the JPL method. Time-domain Method 1 was accurate when residual vectors were included in the set of initial modes, and Method 2 was accurate when including all modes of the updated system rather than truncating those modes at 50 Hz.

Since the focus of the NESC study is the NTRC approach, it is worth comparing the characteristics of the methods presented here to NTRC. The primary difference is that NTRC is an “exact” method, in the sense that it uses “exact” FRFs to represent the payload and the launch vehicle and, as long as sufficient care is taken to avoid numerical issues, it recovers “exact” system response. All the methods presented in this report are approximate in the sense that they attempt to represent the response of a coupled system using a set of truncated modes from a different coupled system. The accuracy of the methods is a function of how well a linear combination of the truncated modes of the nominal coupled system can represent the coupled modes of the modified system. This is a classic case of structural modification, when a modal representation of one system is used to approximate the dynamic response of another system.

While an exact method would presumably be preferred over an approximate method, there are reasons to consider the approximate methods:

1. In our experience, while methods based on combining FRFs are theoretically exact, they can also be very sensitive to relatively minor errors. This is certainly true of the JPL substitution analysis method. While this method is exact when using exact FRFs (no modal truncation), the errors can be very large when the system-level response is based on a set of truncated modes, and in the generic launch vehicle example the errors were unacceptable even without modal truncation. We have experienced similar behavior in other applications, where an FRF synthesis method that is producing outstanding results can produce very poor results due to a very small change. It is likely that the NTRC method will prove to be significantly more robust than the JPL method because it avoids replacing one payload with another and avoids the pole-zero cancellations that result from that substitution. However, based on our experience with FRF synthesis methods, we recommend that the sensitivity to small model changes be examined.
2. The reason that the three methods presented here use system modes from a previous CLA is that it was assumed that this data would be readily available. Motion of the payload HCB DOF is often provided for the purposes of additional internal data recovery in any case, so the only additional required data are the CLA system frequencies and mode shape coefficients at the HCB DOF. In practice it has been more difficult to get this data than anticipated, but it should be much easier to provide than the full interface impedance of the uncoupled launch vehicle for every flight condition.
3. The reason that the three methods presented here start from a CLA with an existing payload is twofold. The first is that it is presumed that such a CLA already exists, while data for an unloaded interface would need to be prepared specially. The second and more important reason is that when using a modal method, it is important that the modes of the initial system represent the modified system as closely as possible. Using modes with an unloaded interface would not do a good job in this regard. When a model of the unloaded launch vehicle is used instead, it becomes especially critical that the transfer functions at the interface accurately represent the interface flexibility, which is accomplished either by using an HCB model of the launch vehicle or by using residual vectors.

A conclusion worth emphasizing is that FRF-based methods (of which NTRC is an example) can be very sensitive to numerical problems. For the two simple examples presented here, the JPL method was exact, as expected, when modal truncation was eliminated, though not as accurate as the time-domain methods with truncated modes. However, for the third example an acceptable answer was never found, even when all sources of modal truncation were eliminated, implying that other numerical issues were causing the unacceptable results. The time-domain methods more closely approximate the standard CLA process and have proven to be considerably more robust.

5.0 References

- [1] Craig, R.R., and M.C.C. Bampton. “Coupling of Substructures for Dynamic Analysis.” *AIAA Journal* 6, No. 7 (1968): 1313–1319.
- [2] Jetmundsen, B., R.L. Bielawa, and W.G. Flannelly. “Generalized Frequency Domain Substructure Synthesis.” *Journal of the American Helicopter Society* 33 (1988): 55–64.
- [3] Gordis, J.H., R.L. Bielawa, and W.G. Flannelly. “A General Theory for Frequency Domain Structural Synthesis.” *Journal of Sound and Vibration* 150 (1989): 139–158.
- [4] Gordis, J.H. “Structural Synthesis in the Frequency Domain: A General Formulation.” *Shock and Vibration* 1, No. 5 (1994): 461–471.
- [5] Trubert, M., and L. Peretti. “A Cost-Effective Modes Analysis for Shuttle Payloads using a Combination of Frequency Domain and Time Domain Approaches.” 26th Structures, Structural Dynamics, and Materials Conference. 1985. 391–403.
- [6] Blelloch, P.A., and C.F. Flanigan. “A Time Domain Approach for Spacecraft Reanalysis.” 33rd Structures, Structural Dynamics, and Materials Conference. 1992. 2295–2306.
- [7] SDRC, Inc. *Reanalysis User’s Guide*, Version 1. 1993.

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