

Conjunction Assessment Risk Analysis

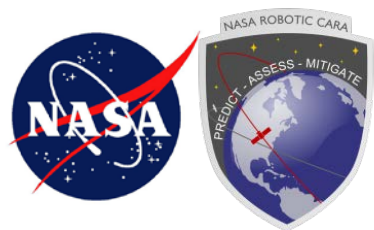


CA Risk Assessment Approaches

M.D. Hejduk

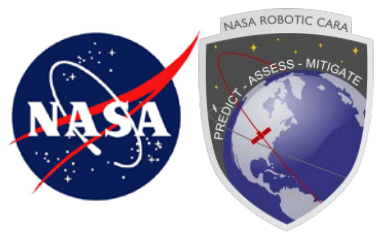
D.E. Snow

November 2018



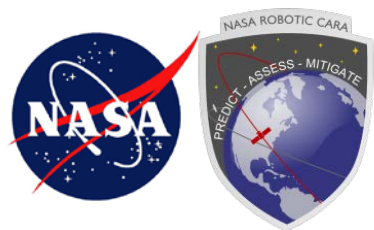
Agenda

- **Basics of hypothesis testing**
- **Hypothesis testing as applied to CA**
- **“Dilution region” and implications for CA**
- **Individual risk assessment approach categories**
 - Probabilistic
 - Current practice, miss distance confidence intervals, Wald test
 - Plausibilistic Type I
 - CNES covariance scaling, CARA P_c uncertainty
 - Plausibilistic Type II
 - Alfano/Balch dilution region remediation
 - Possibilistic
 - Balch overlapping ellipsoids



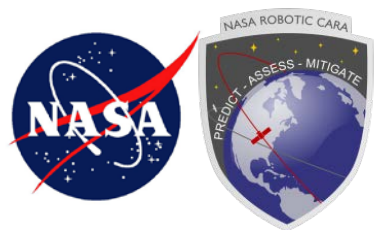
Assumptions

- **Approaches consider the risk assessment only at decision time**
 - Certain methods evaluable only at decision time (e.g., Wald ratio test)
 - “Wait for more data” option meets natural end at decision time
 - Even if no more obs data forthcoming, waiting can decrease prediction and result in better space weather predictions
 - Need to put evaluation stop to this option



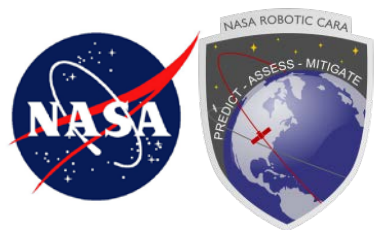
Hypothesis Testing Background: Social Science Example (1 of 2)

- **Question: do violent cartoons result in violent play among toddlers?**
- **Experiment:**
 - Divide toddlers into two groups
 - Show one violent cartoons; show the other peaceful cartoons
 - Move each group to separate play area and observe play
 - Rate each toddler on a scale of 1 to 10 for violence of play
 - Compute the mean “violence scale” value for each group
- **Null hypothesis: cartoon violence has no effect on violence of play**
- **Research hypothesis: cartoon violence increases violence of play**
- **Statistical test**
 - Presume the violence scores for each group are normally distributed
 - Calculate the t-test statistic for the two means
 - Determine the one-sided p-value for the calculated test statistic



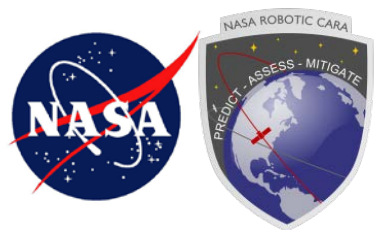
Hypothesis Testing Background: Social Science Example (2 of 2)

- **In this example, p-value indicates the likelihood that the difference between the two means would arise if both subgroups are taken from the same parent distribution**
 - If randomly constructing two subgroups from same distribution, occasionally the means (and variances) of the subgroups will differ substantially
 - Question is the likelihood that a given difference would arise in this way
- **Usually, if p-value less than some small value (set by the researcher but often in 5% to 1% range), natural occurrence of the observed difference is considered highly unlikely, and the null hypothesis is rejected**
 - Here, one would reject the null hypothesis (that cartoon violence has no effect on play violence) and embrace the research hypotheses (that there is an effect)
 - Violence mean would need to be greater than non-violence mean
 - Premise of the one-sided test applied earlier



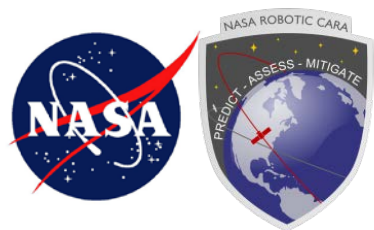
CA Risk Assessment: An Estimation or Statistics Problem?

- **Some argue is simply an estimation problem (J. Frisbee)**
 - Mean states and covariances arise from an estimation process
 - P_c just a calculation from these products
 - P_c simply compared to a threshold, and appropriate actions follow
- **However, as soon as estimated results compared to a threshold, the situation becomes a statistics problem**
 - “Critical region” defined (here P_c area above threshold)
 - “Confidence interval” sought (“What is the true likelihood that the P_c is above the threshold?”)
 - Ancillary circumstances can prevent direct comparison (e.g., poor OD)
 - Evokes similarity to hypothesis test—if no comparison possible, then one remains with the null hypothesis (not to maneuver)
 - If it looks, smells, and quacks like a statistics problem . . .
- **Advantages to be gained if full formalisms of statistics problems used**



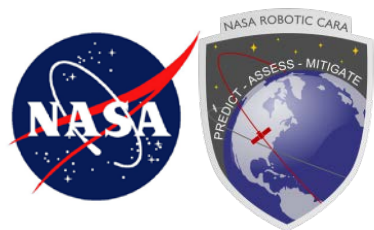
Setting the Null Hypothesis

- **There appears to be no single mandated method for framing the null hypothesis**
 - Most guidance recommends the “typical” or “ordinary” outcome to be the null hypothesis
 - Should be where “presumption” lies, meaning the burden of proof falls on the alternative hypothesis
- **Fundamental question**
 - Do the presented data provide evidence to justify a decision to maneuver?
- **Null hypothesis**
 - The miss distance is greater than the HBR
- **Alternative hypothesis**
 - The miss distance is less than the HBR
- ***Similar flexibility available for Type I / Type II error assignment***
 - *Nuisance state typically assigned to Type I error (false alarm)*
 - *Process oversight assigned to Type II error (missed detection)*



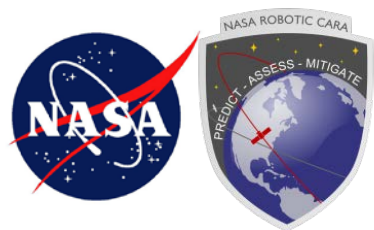
Setting the Null Hypothesis: Other CA Publications

- **Early publication by Coppola introduced the following definition of collision:**
 - “The miss distance is less than or equal to the HBR”
- **R. Carpenter, in Wald test and ASA paper (which included M. Hejduk as coauthor), used this as null hypothesis**
 - Remediation thus set as the default position
 - Null hypothesis rejected if $P_c <$ small threshold
- **Issue: places presumption with the unusual action**
 - In the absence of compelling evidence to maneuver, CARA presumes that not maneuvering is the safer choice
 - Way of considering inherent risks of maneuvering and other intangibles
 - Could be construed to require a maneuver for cases in which poor data quality indicates questionable actionability



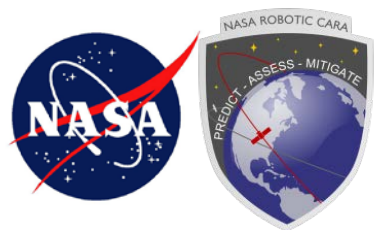
Hypothesis Testing: Application to CA (1 of 2)

- **In CARA, we implicitly embrace the following null hypothesis:**
 - “The miss distance (MD) between the two objects is greater than or equal to the combined HBR”
 - Some practitioners ascribe importance to the placement of the equals sign
 - Since very rare for MD, as a continuous variable, to equal HBR exactly, placement of equals sign not significant here
- **We reject this null hypothesis and recommend a maneuver if all of the following are true:**
 - The P_c is greater than the established threshold
 - The primary and secondary ODs (state and covariance) are well determined
 - The space weather situation is sufficiently stable
- **If any of the above conditions are not met, we cannot justify rejecting the null hypothesis**
 - Burden of proof is thus borne by the decision to maneuver; refraining from a maneuver is the default



Hypothesis Testing: Application to CA (2 of 2)

- **Mean (estimated) miss distance $<$ HBR**
 - Some practitioners (Alfano, Carpenter) appear to counsel a remediation action in such cases, regardless of the P_c value
 - With this approach, null Hypothesis would thus have two conditions
 - One statistical – the probability of the miss distance being less than the HBR
 - One deterministic – the value of the miss distance estimate itself
 - Can result in maneuvers for very low P_c events
 - If P_c is low, produces little, if any, reduction in statistical risk
 - May result in a large reduction of political risk: “Why didn’t you maneuver when a collision (*i.e.* miss distance $<$ HBR) was predicted?”
 - Occurs very rarely in operations
 - From CARA database, roughly 0.005% of events have miss distance less than 20m
 - Thus makes little difference whether or not included
- **If used in risk assessment process, would be essentially for political purposes only**



The “Dilution Region”

- **Two ways in which a P_c can be low**

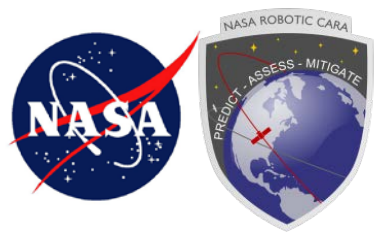
- Positions of both objects can be very well determined (smallish covariances), so the set of MDs assumes a bounded and relatively small set of values
 - One can then decisively conclude that the likelihood that the MD < HBR is low
- Positions of both objects can be very poorly determined (largish covariances), so the set of MDs could assume a very large and broad set of values
 - Given this broad set of possibilities, the portion of those MDs expected to fall within the HBR is small, so the P_c is low

- **Dilution region is the region for which the latter is true**

- Determined by considering the ratio of the miss distance to covariance size
- When this ratio greater than $\sqrt{2}$, in dilution region

- **Typical event development**

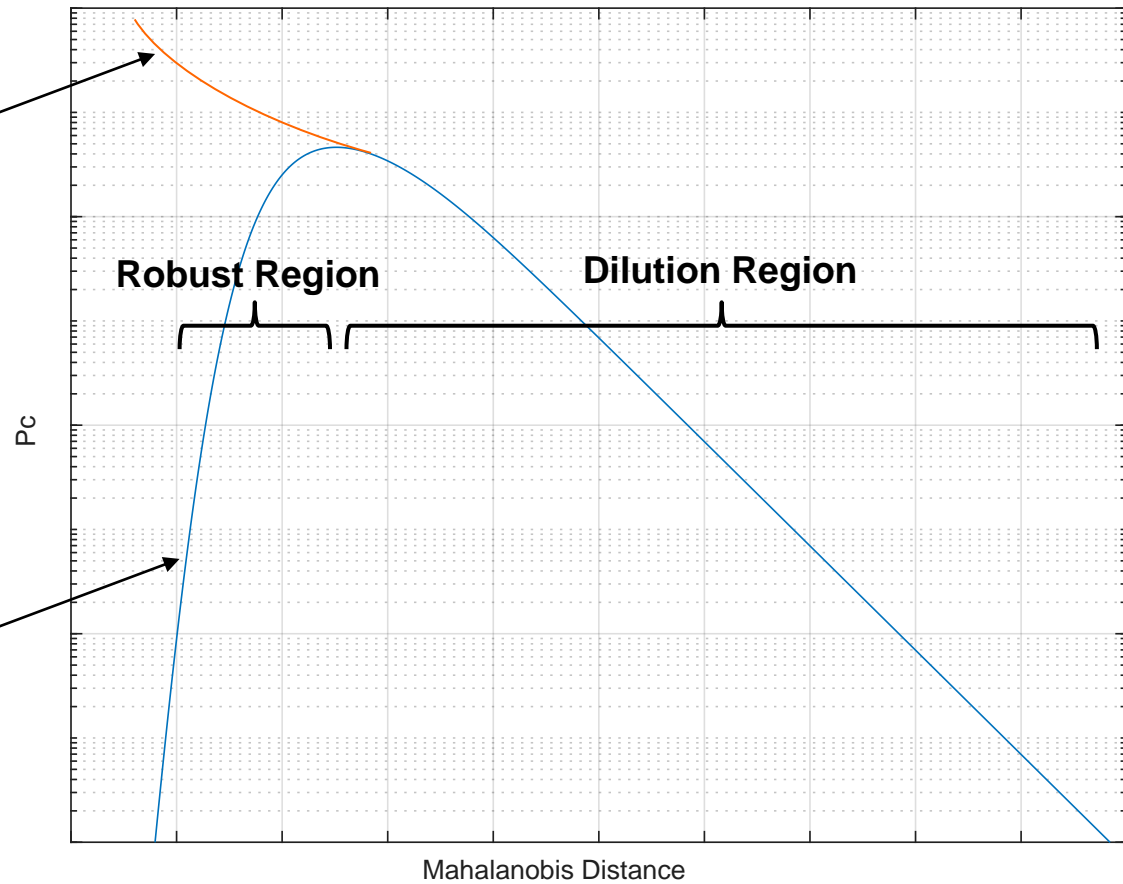
- In dilution region when first discovered (large covariances due to long prop)
- Moves to “robust” region as prop times decrease and covariances shrink

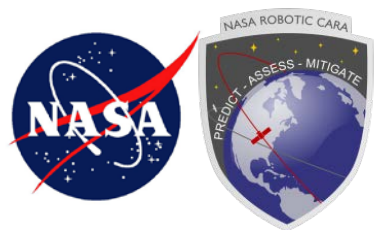


Dilution and Robust Region Graph

**Collision event:
Pc trending
towards unity**

**Miss event:
Pc trending
towards zero**





Risk Assessment by (Robust or Dilution) Region

- **Robust region**

- If P_c is high, can conclude that situation is risky
- If P_c is low, can conclude that situation is safe

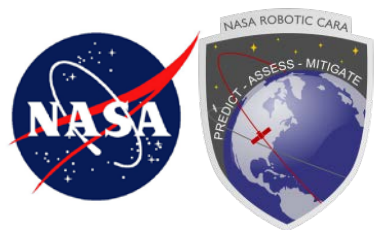
- **Dilution region**

- If P_c is high, can conclude that situation is risky
- If P_c is low, cannot actually draw a conclusion
 - In particular, CANNOT conclude that the situation is safe, only that we know too little about the situation to determine definitively whether risky or safe

- **Appropriate action for low P_c in dilution region determined by chosen form of null hypothesis**

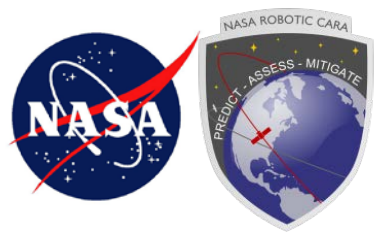
- Inadequate grounds to reject null hypothesis
- If null hypothesis is not to maneuver, then no maneuver warranted
- If null hypothesis is to maneuver, then maneuver warranted
 - Some argue this is the appropriate safer course
 - Of course, selection of maneuver impaired by these same problems

- **CARA practice is not to maneuver in such cases**

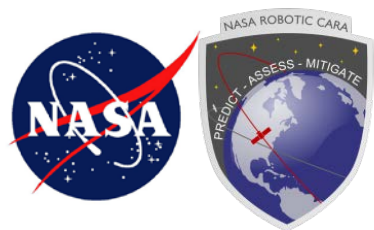


Dilution Region and the Space Fence

- **Concern expressed regarding large number of expected dilution region conjunctions with Space-Fence-only objects**
- **If SF covariances realistic (even if large), problem self-correcting**
 - If in dilution region and P_c high enough to exceed threshold, then should remediate
 - Diluted event still clears threshold, so no question about how to proceed
 - If in dilution region and P_c below threshold, then not enough information available to justify an act of remediation
 - Presumes CARA-recommended formulation of null hypothesis; alternative is to maneuver incessantly and perhaps never even launch
 - Risk no different from situation before Space Fence was deployed—potentially serious encounters with (then untracked) debris occurred regularly, but without notice
 - Use of CARA formulation of null hypothesis makes this straightforward
- **If SF covariances unrealistically small, then situation is problematic**
 - Will require covariance irrealism remediation efforts

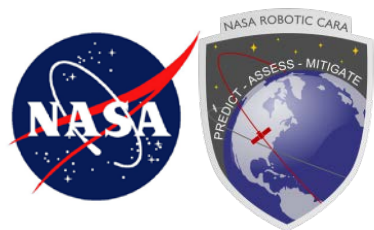


INDIVIDUAL RISK ASSESSMENT APPROACHES



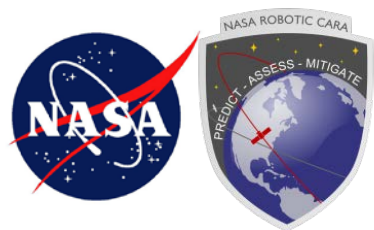
Different Risk Assessment Paradigms: General Considerations

Method	Fundamental Question	Null Hypothesis	Distance from Current Event Data
Probabilistic Single Pc Evaluation MD Confidence Int. Wald Test	Do the data justify a decision to maneuver?	The conjunction miss distance is greater than the HBR	Derives entirely from current CDM's state estimates and covariances
Plausibilistic Type I CNES Pc Sensitivity CARA Pc Uncertainty	Given the data and covariance realism assumptions, does the Pc range of values justify a decision to maneuver?	The conjunction miss distance is greater than the HBR	Tied to current states; uses historically-informed modulation of covariances
Plausibilistic Type II Alfano Max Pc Balch "Blue Fuzzy"	Given the data and assumptions on possible values of the covariance, does the maximum plausible value of risk justify a decision not to maneuver?	The conjunction miss distance is less than the HBR	Tied to current states; retains only aspect ratio of covariance, or no covariance data at all
Possibilistic Balch "Red Fuzzy" Overlapping Ellipses	Do the data support the possibility of a conjunction?	N/A	Tied to current states and covariances, but asks entirely different question



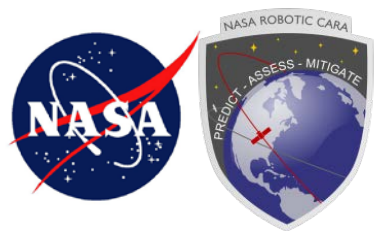
PROBABILISTIC APPROACH

Method	Fundamental Question	Null Hypothesis	Distance from Current Event Data
Probabilistic Single Pc Evaluation MD Confidence Int. Wald Test	Do the data justify a decision to maneuver?	The conjunction miss distance is greater than the HBR	Derives entirely from current CDM's state estimates and covariances



Probabilistic Approach #1: Current Pc Practice (1 of 2)

- **Point estimate of Pc calculated and compared to a threshold value**
 - If exceeds threshold, event is considered serious and requires remediation
 - Additional considerations applied when Pc near the actual threshold, from either side
- **If Pc exactly on threshold, strength of test is 50/50**
 - Because right at boundary of critical region
- **Further into critical region, strength of test increases**
 - But no defined function to determine precisely by how much
- **Threshold presumably chosen because significantly larger than background collision risk**
 - Reasonable method of choosing threshold, if in fact this is the methodology that was followed



Probabilistic Approach #1: Current Pc Practice (2 of 2)

- **Advantages**

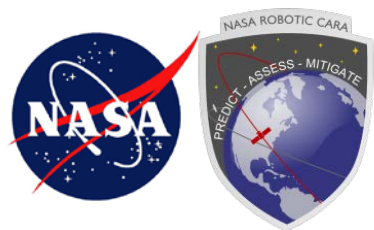
- Established practice
- Easy to explain and administer

- **Disadvantages**

- As typically practiced, no formal recognition of hypothesis test underpinnings of method
- No significance value or confidence intervals associated with procedure
- No formal recognition of or accommodation for dilution region
 - However, with proper framing of null hypothesis, probably operationally acceptable
- No accommodation for covariance or HBR uncertainties

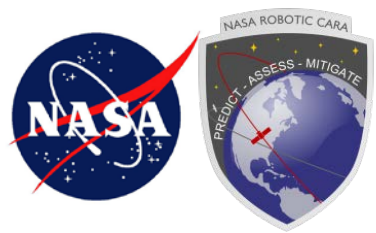
- **Acceptable for risk assessment?**

- Yes



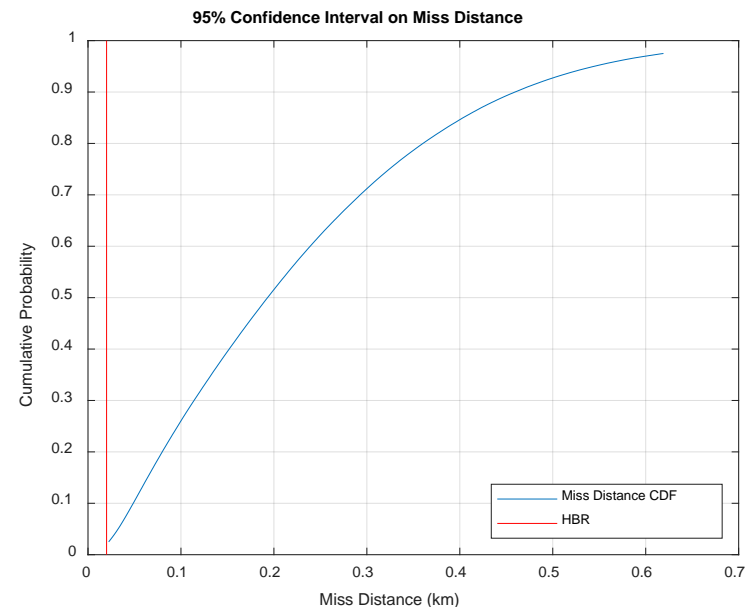
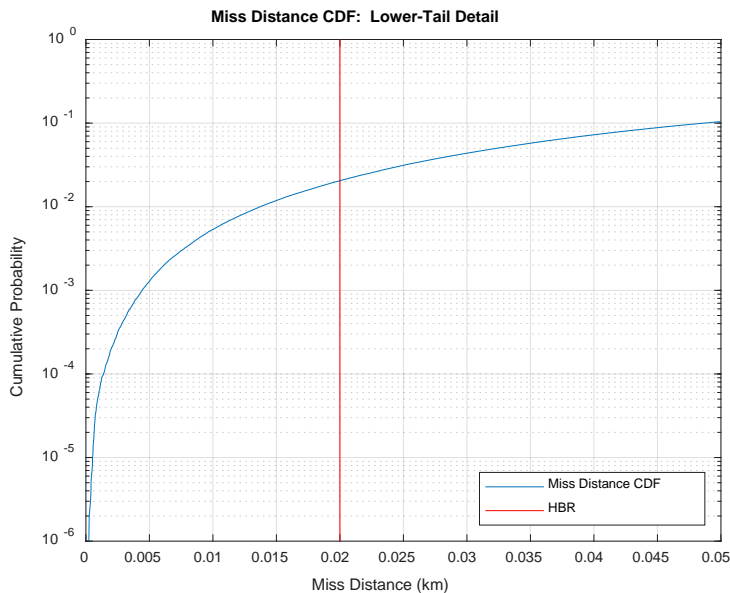
Probabilistic Approach #2: Confidence Interval on MD (1 of 4)

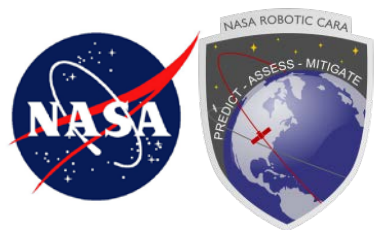
- **Proposed by R. Carpenter in ASA paper**
- **Instead of P_c value, produce a CDF of the possible miss distances**
- **Define a one-tail confidence value as a threshold**
- **If the miss distance confidence value for the HBR is greater than the threshold, then event is high risk, and mitigation required**
- **Additionally, if $MD < HBR$, then maneuver**
 - MD is maximum likelihood estimate of the miss; presume that this means a collision is likely
- **One-tailed confidence interval operates as test statistic**
- **Two-tailed confidence interval gives visual clue to whether dilution region present**
 - Broad confidence interval means that MD not well determined, and therefore dilution region in play
 - Narrow confidence interval indicates a well-determined situation



Probabilistic Approach #2: Confidence Interval on MD (2 of 4)

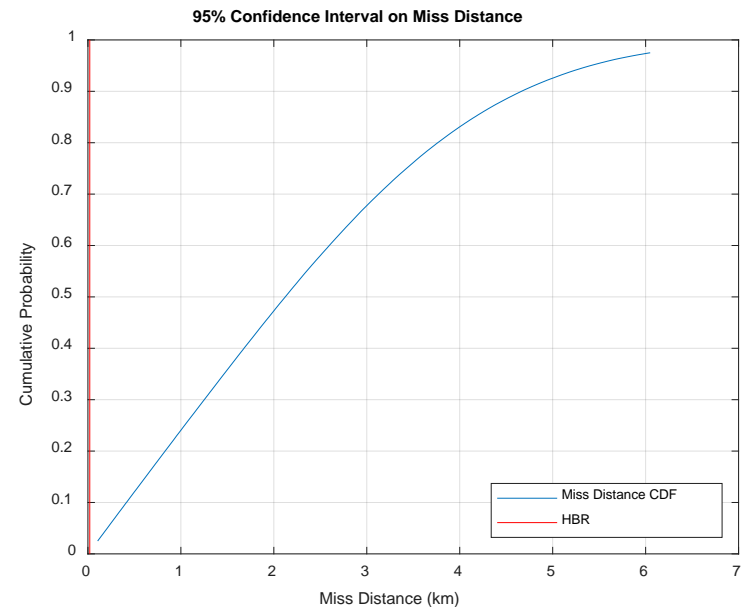
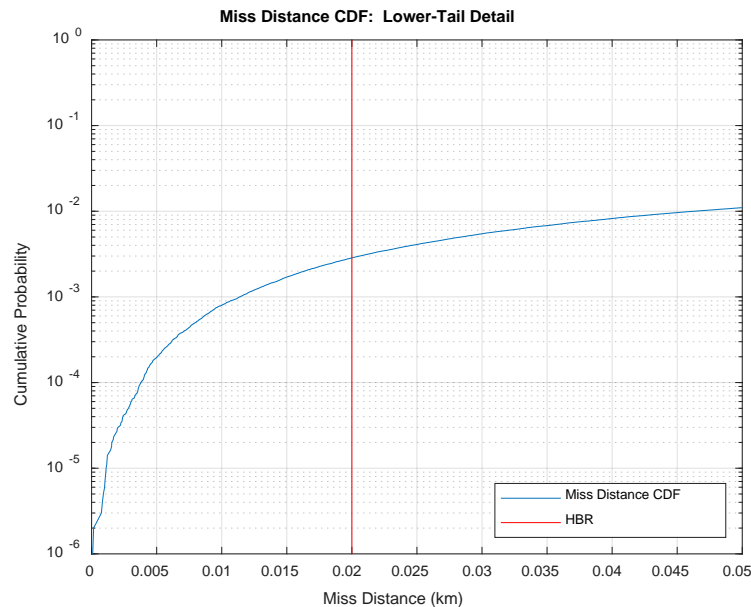
- **Example event has P_c 2.06E-02, Mahalanobis distance (M) of 0.85**
- **One-sided test (left graph) reduces to P_c calculation**
 - Curve crosses HBR line at 2E-02
- **Two-sided test (right graph) shows 95% span of MD distribution**
 - Fairly narrow, in line with $M < \sqrt{2}$

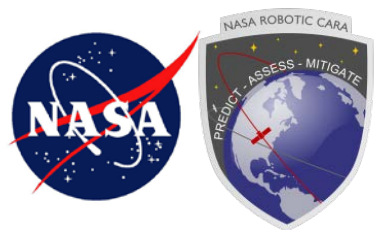




Probabilistic Approach #2: Confidence Interval on MD (3 of 4)

- **Example event has P_c 2.77E-03, Mahalanobis distance (M) of 4.64**
- **One-sided test (left graph) reduces to P_c calculation**
 - Curve crosses HBR line at 2.7E-03
- **Two-sided test (right graph) shows 95% span of MD distribution**
 - Fairly broad, as expected with $M > \sqrt{2}$





Probabilistic Approach #2: Confidence Interval on MD (4 of 4)

- **Advantages**

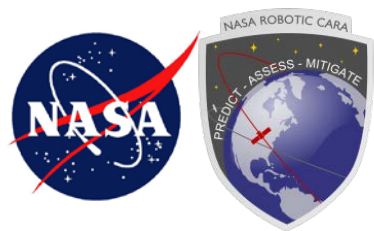
- Easy to understand—speaks in terms of miss distance
- Power of test made evident by presentation and results
 - Tight vs broad confidence interval
 - Need to use only one-tailed interval for test evaluation; two-tailed for assessment of confidence interval
- Can immediately see the change in risk posture for a modified HBR

- **Disadvantages**

- Would have to retool thinking around miss distance confidence
 - But not unrelated to P_c
- Does not define clear dilution region boundary
 - Can do this more directly by Mahalanobis distance calculation
- Does not consider covariance uncertainty

- **Acceptable for risk assessment?**

- Yes—in some ways more straightforward than P_c -based approaches



Probabilistic Approach #3: WSPR Test: Basic Explanation

- **WSPR: Wald Sequential Probability Ratio test**

- Developed by R. Carpenter for the MMS program

- **Framing question**

- “How much worse is present conjunction from what is presented by the ‘usual’ interaction of these two objects over time?”

- **The Ratio itself**

- Ratio of the odds of a conjunction for this particular encounter to the odds of a conjunction between these two objects in general

- Restated: how much worse is this particular encounter from the collision risk we would generally expect between these two objects

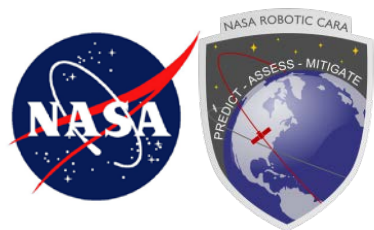
- **Nomenclature**

- $P_{c|o}$: the background collision risk between the two objects outside of this event

- P_{fa} : the false alarm rate one is willing to tolerate

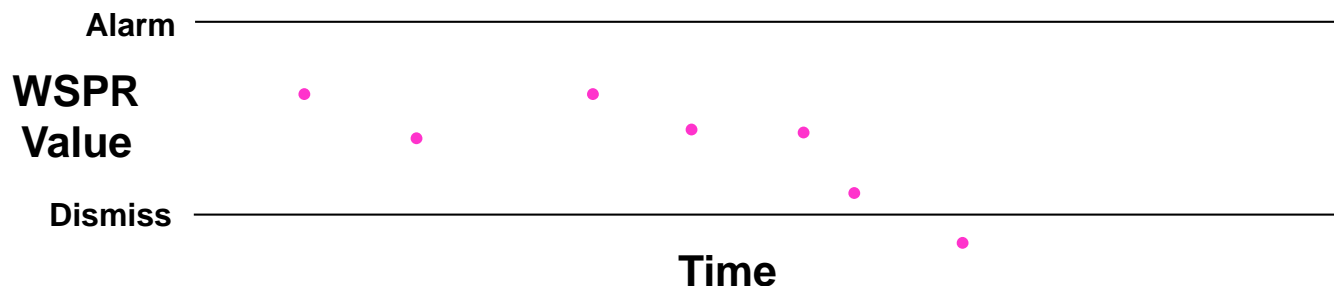
- P_{md} : the missed detection rate one is willing to accept

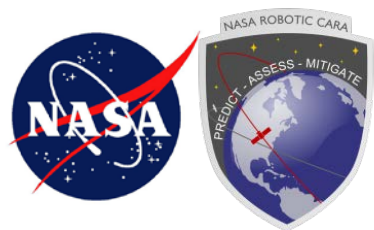
- MD here means the failure to call for a maneuver when one should have, not an actual collision arising



Probabilistic Approach #3: WSPR Test: Dynamics

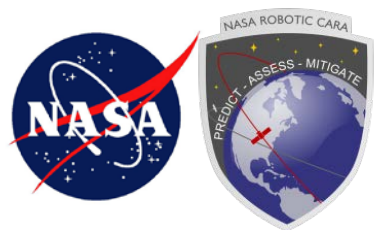
- **Specified P_{fa} and P_{md} (along with $P_{c|o}$) establish the following:**
 - Alarm condition: WSPR above this value, should maneuver
 - Dismiss condition: WSPR below this value, should dismiss event
 - “Wait” condition: WSPR between these values, should wait for more data
- **When event first identified, process begins with first ratio calculated**
- **With each successive update, determine whether situation meets alarm, dismiss, or wait condition; and act accordingly**





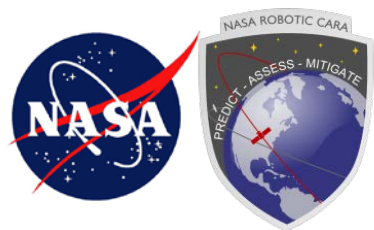
Probabilistic Approach #3: WSPR Test Issue #1: JSpOC Data Mismatch

- **Bayesian nature of WSPR requires full accumulation of OD data from the initial event discovery through remediation decision**
 - Presumes a sequential estimator (which is probably used for MMS system)
- **However, JSpOC OD approach is moving-window batch updates**
 - As event develops, in most cases observational data are left behind as moving-window moves forward in time
- **Repair possibilities not very appealing**
 - Freeze beginning of fit-span as event develops
 - Will produce distorted OD and poor covariance realism—excessive length of fit span will result in inappropriate covariance size
 - Create running sum of information matrices as event develops (approach used in CARA implementation of WSPR)
 - Considers same datum multiple times, thus also producing undersized covariance
 - This and related difficulties documented in MMS team's AIAA paper on approach
- **WSPR in current implementation not appropriate for CA with JSpOC data; cannot therefore be recommended**



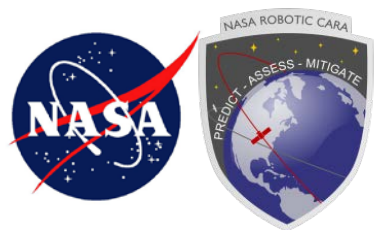
Probabilistic Approach #3: WSPR Test Issue #2: Calculation of $P_{c|o}$

- **For MMS program, $P_{c|o}$ calculated by “on demand” Monte Carlo runs**
 - For the two satellites in the current conjunction
 - From the time of last constellation configuration until current conjunction TCA
- **Because of intra-constellation recontact potential, $P_{c|o}$ calculated this way will have finite value**
- **For conjunctions between primary and any random catalogued object, situation more problematic**
 - $P_{c|o}$ calculated this way may never be non-zero
- **No obvious way to choose appropriate time period over which to evaluate $P_{c|o}$**
 - Extremely long precision propagations not tenable; would need some sort of analytic method
- **$P_{c|o}$ for a primary versus any secondary might be a tolerable substitute, but this is not clear**
 - Because foundational parameter for this method, clear and convincing methodology needed



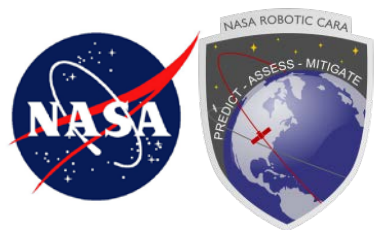
Probabilistic Approach #3: WSPR Test Issue #3: RA Dynamics Mismatch

- **CA *modus operandi* centers on the maneuver commit point**
 - Gather as much data as possible—do not render a decision until necessary
 - At the maneuver commit point, take the decision; for it must be taken then
- **WSPR suggests a different approach**
 - As soon as an alarm/dismiss condition met, then decision can be taken
 - However, CA history frequently shows substantial changes in event risk with additional data, so why would these not be considered?
 - If neither condition met by maneuver commit point, then WSPR offers no real recommendation
 - One could presume that in such a case a maneuver is warranted, “to be safe”; but WSPR did not actually assist the decision when between alarm/dismiss conditions
- **Therefore, utility of WSPR strongest if either alarm or dismissal condition happens to be met at the maneuver commit point**



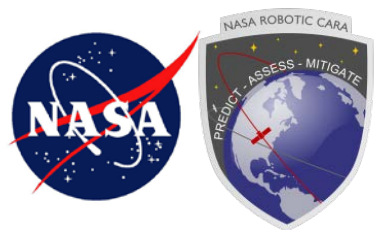
PLAUSIBILISTIC APPROACH I

Method	Fundamental Question	Null Hypothesis	Distance from Current Event Data
Plausibilistic Type I CNES Pc Sensitivity CARA Pc Uncertainty	Given the data and covariance realism assumptions, does the Pc range of values justify a decision to maneuver?	The conjunction miss distance is greater than the HBR	Tied to current states; uses historically-informed modulation of covariances



Plausibilistic Approach I #1: Covariance Modulation (1 of 3)

- **Regular method of P_c calculation used**
- **However, P_c evaluated for a range of scaled covariances for the primary and secondary satellites**
 - Covariance for each satellite multiplied by a range of scale factors
 - Typical range might be from 0.5 to 3
 - All combinations of these scale factors used for primary and secondary; if n scale factors, $n \times n$ number of P_c values
 - Values determined from notional understanding of potential covariance errors or analysis of previous CDM covariances
 - P_c values represented graphically in 2-D “Heat Map”
 - X and Y are primary and secondary covariance scale factors
 - Intensity is P_c value
- **Presently used by CNES**
 - Remediation threshold adjusted for this technique (made more permissive)
 - Any plausible P_c that exceeds this threshold prompts remediation consideration



Plausibilistic Approach I #1: Covariance Modulation (2 of 3)

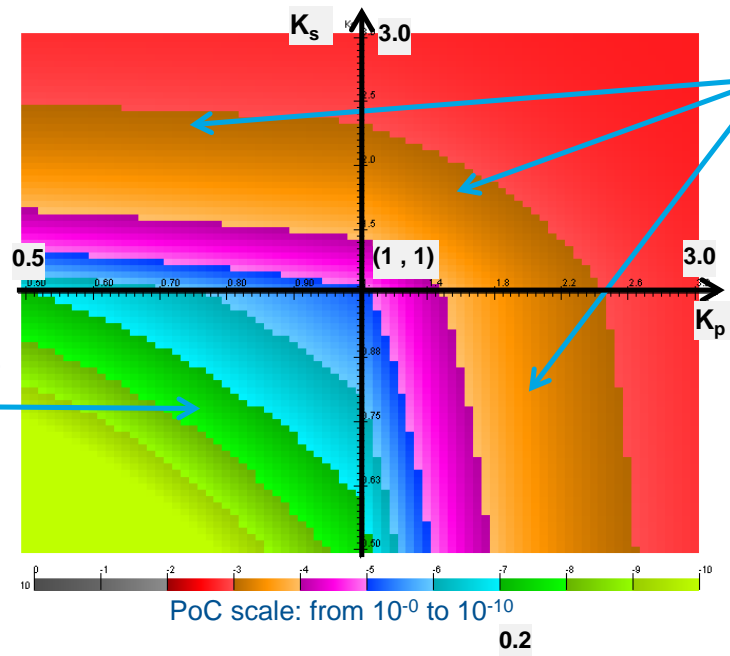
CDM Analysis:

Covariance at the TCA: PoC* for a PoC analysis

Covariance sensitivity analysis on PoC(Kp, Ks)

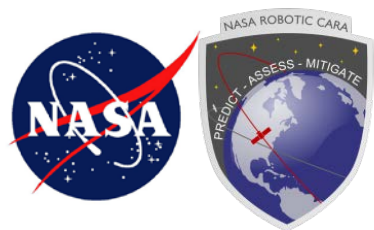
Example of display: Kp in [0.5 ; 3.] and Ks in [0.5 ; 3.]

If Primary's and Secondary's covariance are pessimistic the risk is over-estimated



If Primary's or Secondary's covariance is optimistic the risk is under-estimated





Plausibilistic Approach I #1: Covariance Modulation (3 of 3)

- **Advantages**

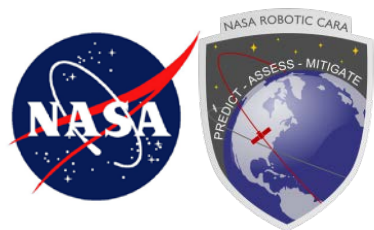
- Construct allows consideration of covariance uncertainty, which certainly exists
- Naturally addresses the “low miss, low P_c ” phenomenon by characterizing the effect of plausible small changes in the covariance

- **Disadvantages**

- Range of scale factors often not well determined
 - Difficult to establish without access to CSpOC covariance realism evaluation products
- Does not consider likelihood of any particular combination of scaling factors actually occurring
 - With such scaling, a certain P_c value is shown to be plausible; but is it likely?
- Such P_c values are therefore actually not event probabilities of collision
 - Actual probability is the product of the plausible P_c and the likelihood of the covariance scale factors used actually inhering for the current conjunction

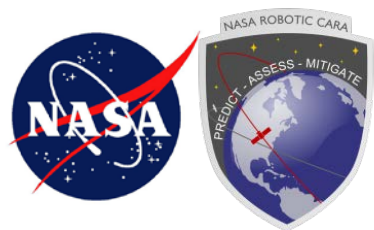
- **Acceptable for risk assessment?**

- Yes, so long as threshold(s) set appropriately



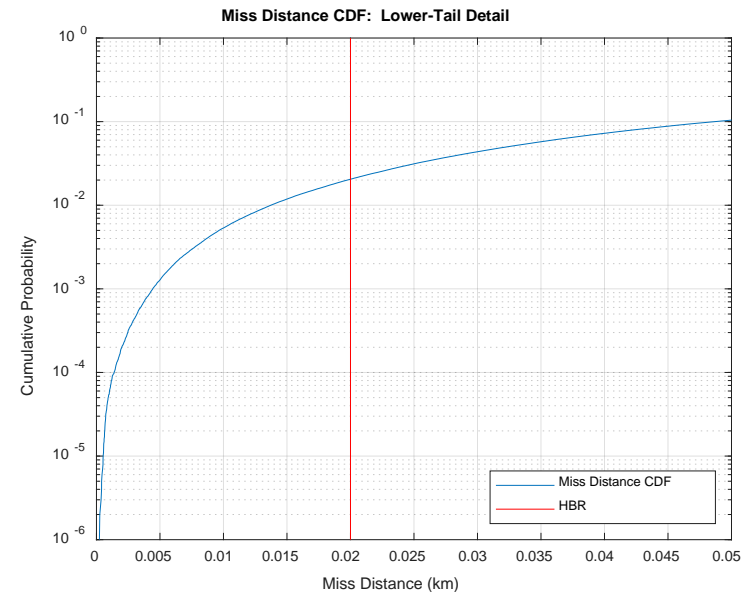
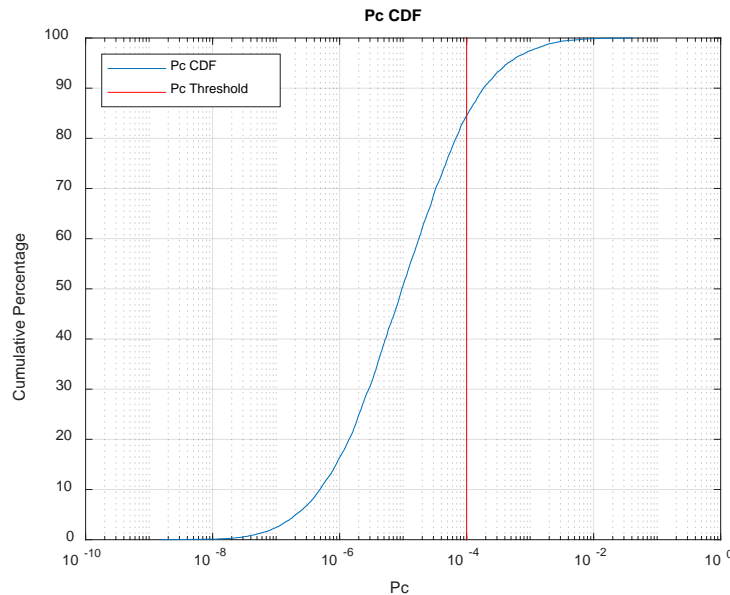
Plausibilistic Approach I #2: Pc Uncertainty (1 of 3)

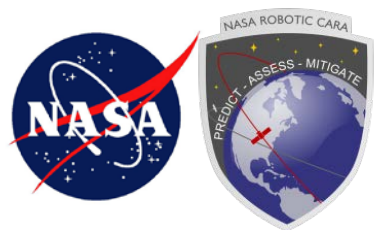
- **Considers uncertainties in the covariances (based on covariance realism examination methods)**
 - More elaborate methods to generate statistical distributions of uncertainties
 - Scale-factor percentile-matching
 - Bayesian seeding for construction of statistical distribution of realistic covariances
- **Can consider HBR uncertainties also**
- **Monte Carlo trials drawing from variations in primary/secondary covariances and in HBR produces PDF/CDF of Pc values**
 - CDF of Pc values allows a threshold and significance level to be specified
 - *E.g.*, if at least x% of probability density over threshold, then remediate
 - Equivalent to: if at least x% of a chance that the Pc is actually greater than the threshold, then remediate
 - Presumes a null hypothesis of miss distance being greater than HBR



Plausibilistic Approach I #2: Pc Uncertainty (2 of 3)

- CDF shows range of Pc values with uncertainties considered
- Portion of probability density above threshold (1E-04 here) dictates whether to pursue remediation
- Can also render results as miss distance CDF plot





Plausibilistic Approach I #2: Pc Uncertainty (3 of 3)

- **Advantages**

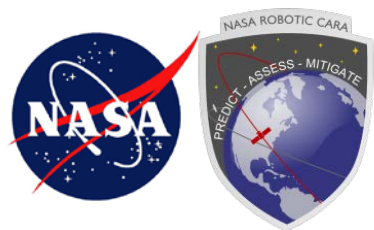
- Provides both critical region and significance level, thus going beyond current methodology
- Allows decision-maker to determine sensitivity to particular threshold used
- Naturally handles “low-miss low Pc” situations by actually determining ease with which Pc can become high

- **Disadvantages**

- Requires examining an entire CDF rather than a single value
 - Although can of course be reduced to single value if significance level imposed
 - Can also frame in terms of miss distance confidence interval / test
- Requires covariance realism data to be present for the calculation

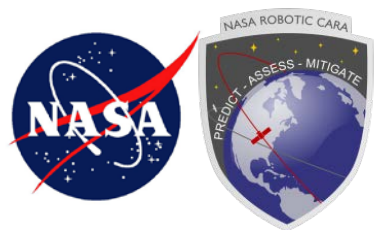
- **Acceptable for risk assessment?**

- Yes, so long as threshold set appropriately



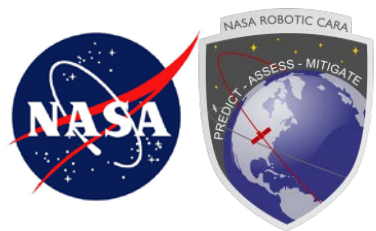
PLAUSIBILISTIC APPROACH II

Method	Fundamental Question	Null Hypothesis	Distance from Current Event Data
Plausibilistic Type II Alfano Max Pc Balch “Blue Fuzzy”	Given the data and assumptions on possible values of the covariance, does the maximum plausible value of risk justify a decision not to maneuver?	The conjunction miss distance is less than the HBR	Tied to current states; retains only aspect ratio of covariance, or no covariance data at all



Plausibilistic Approach II: Maximum Collision Probability (1 of 3)

- **Technique originally constructed by Alfano (2005) to address issue of Probability Dilution; developed more rigorously by Balch (2016)**
 - If in non-dilution region, use joint covariance as submitted
 - If in dilution region
 - Freeze miss distance
 - Contract covariance incrementally, maintaining aspect ratio
 - Find maximum P_c value (peak of Mahalanobis distance curve) and use this value
 - If this value above threshold, pursue maneuver to remediate it
- **Essentially moves along P_c curve by varying covariance for given miss distance until maximum P_c found for covariance smaller than or equal to given value**
 - Paradigm is that more data would produce a smaller covariance, moving to the max value of P_c
 - However, paradigm breaks down as even more data would make the covariance even smaller – driving P_c to zero
- **Similar to CNES approach but with scaling factors of 0 to 1**



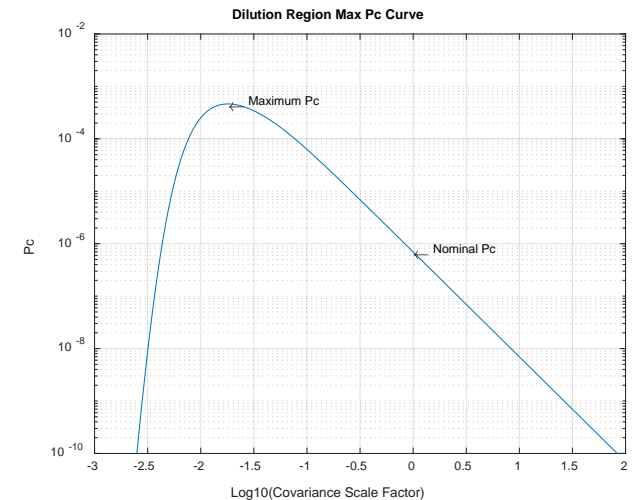
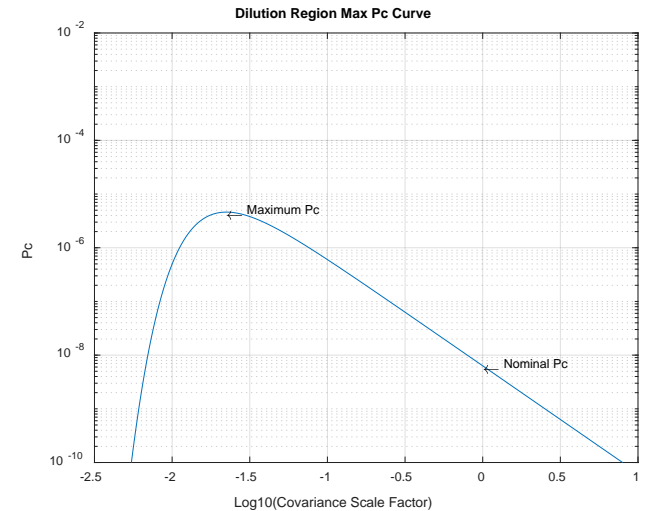
Plausibilistic Approach II: Maximum Collision Probability (2 of 3)

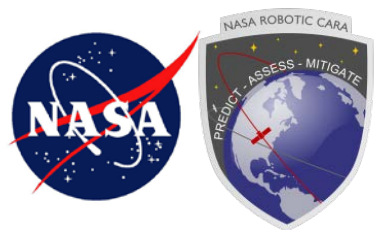
- **Top graph example event**

- Nominal P_c of $6.3E-09$; maximum P_c (peak of curve) $4.6E-06$
- “True” P_c (which could have been obtained with more adequate data) could be as high as $4.6E-06$
- Below threshold of $1E-04$, so can conclude that this dilution region event not dangerous

- **Bottom graph example event**

- Nominal P_c of $6.9E-07$; maximum P_c (peak of curve) $4.6E-04$
- Here, maximum P_c above threshold, so situation remains inconclusive





Plausibilistic Approach II: Maximum Collision Probability (3 of 3)

- **Advantages**

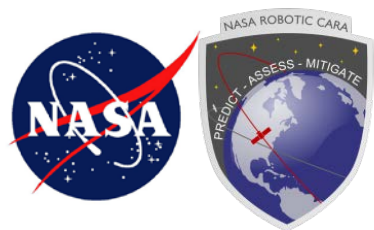
- Determines the worst case that could be expected with a covariance of a given aspect ratio
- Further relaxation of constraints can produce maximum P_c for any covariance

- **Disadvantages**

- As with scale-factor covariance modulation, no articulation of probability that covariance that produces maximum P_c would actually arise
- As such, not an event probability of collision
- With null hypothesis of not maneuvering, construct cannot inform risk assessment situation

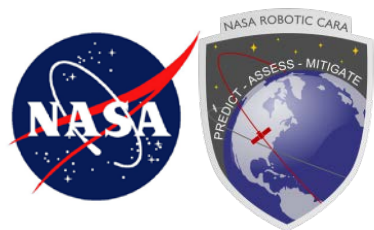
- **Acceptable for risk assessment?**

- Yes, although unnecessarily conservative



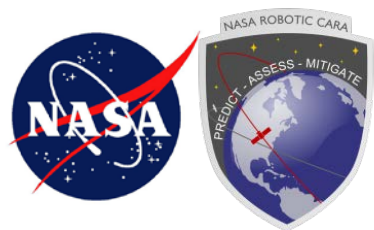
POSSIBILISTIC APPROACH

Method	Fundamental Question	Null Hypothesis	Distance from Current Event Data
Possibilistic Balch “Red Fuzzy” Overlapping Ellipses	Do the data support the possibility of a conjunction?	N/A	Tied to current states and covariances, but asks entirely different question



Possibilistic Approach: Red Fuzzy / Ellipse Overlap (1 of 3)

- **Advocated by Balch (2016, 2017)**
- **Formulated to address problem of low P_c in dilution region**
 - P_c may be low, but no guarantee that one is safe
- **For a given sigma level, determined by risk threshold:**
 - Do the joint covariance and HBR impinge on each other? (“Red Fuzzy”, 2016)
 - Do the primary and secondary covariances impinge on each other? (2017)
- **If they do, then a maneuver should be pursued to increase MD to the point that they no longer overlap**
- ***N.B.: Balch states that these approaches “...may not be a usable metric for satellite collision risk, but rather may serve as a blueprint for getting to one.”***



Possibilistic Approach: Red Fuzzy / Ellipse Overlap (2 of 3)

- **Advantages**

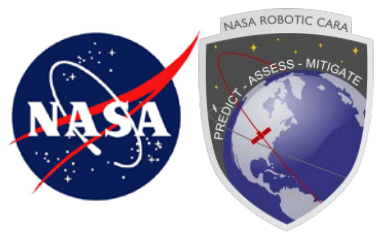
- Addresses potential lack of safety in dilution region
- Simple to explain and straightforward to implement

- **Disadvantages**

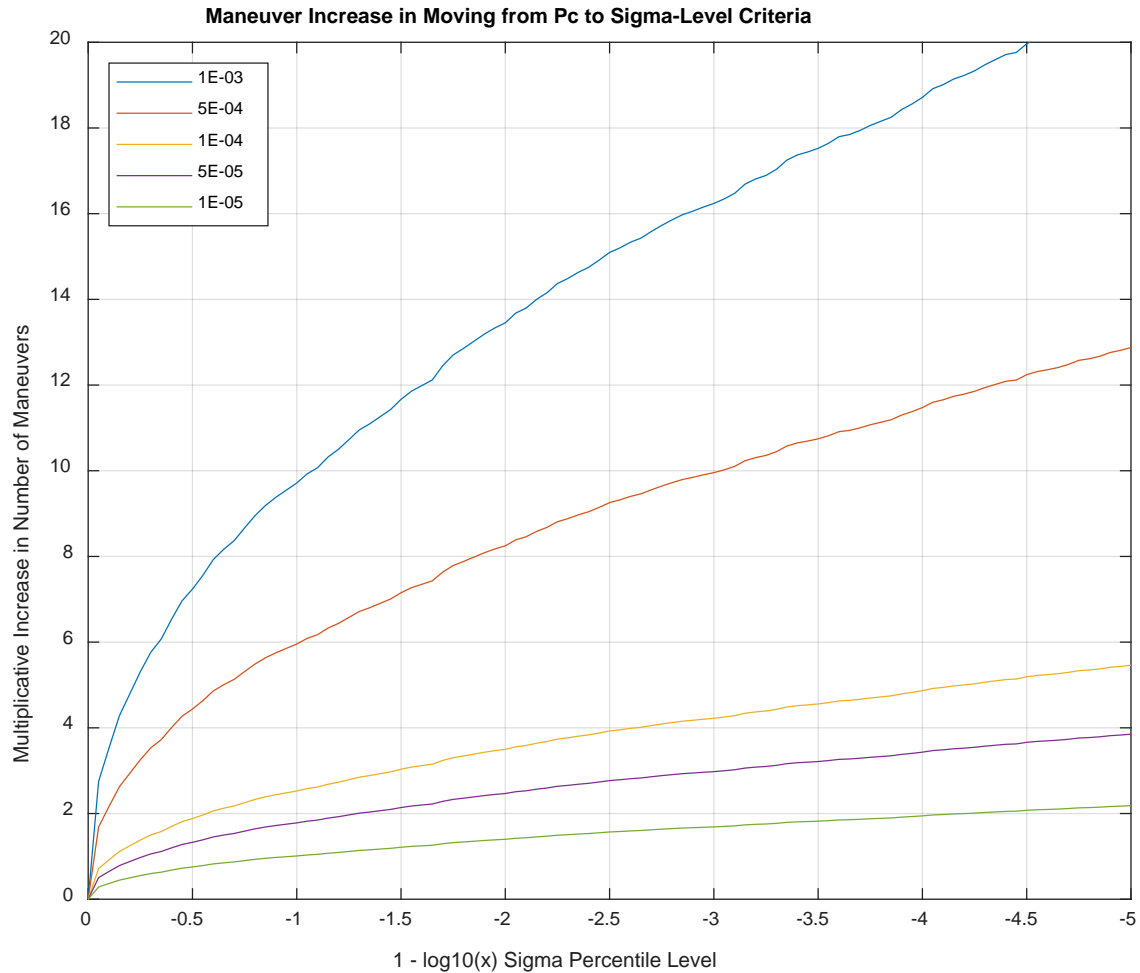
- Actually a response to a different question
- Extremely conservative
 - Probabilistic/Possibilistic maneuver rate comparison on next slide
- Not the way risk is typically handled
 - There was a possibility you could have been killed driving to work today to attend this meeting, but did you therefore stay home?

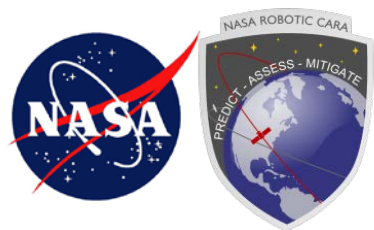
- **Acceptable for risk assessment?**

- Yes, although very much excessively conservative



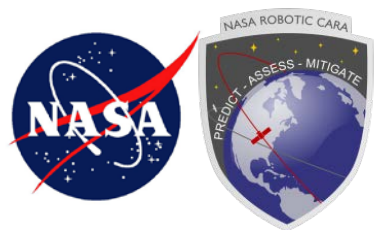
Possibilistic Approach: Red Fuzzy / Ellipse Overlap (3 of 3)





Different Risk Assessment Paradigms: Reprise

Method	Fundamental Question	Null Hypothesis	Distance from Current Event Data
Probabilistic Single Pc Evaluation MD Confidence Int. Wald Test	Do the data justify a decision to maneuver?	The conjunction miss distance is greater than the HBR	Derives entirely from current CDM's state estimates and covariances
Plausibilistic Type I CNES Pc Sensitivity CARA Pc Uncertainty	Given the data and covariance realism assumptions, does the Pc range of values justify a decision to maneuver?	The conjunction miss distance is greater than the HBR	Tied to current states; uses historically-informed modulation of covariances
Plausibilistic Type II Alfano Max Pc Balch "Blue Fuzzy"	Given the data and assumptions on possible values of the covariance, does the maximum plausible value of risk justify a decision not to maneuver?	The conjunction miss distance is less than the HBR	Tied to current states; retains only aspect ratio of covariance, or no covariance data at all
Possibilistic Balch "Red Fuzzy" Overlapping Ellipses	Do the data support the possibility of a conjunction?	N/A	Tied to current states and covariances, but asks entirely different question



Summary

- **Current single-Pc approach works well**
 - When subtended by appropriate hypothesis-testing framework
- **Reframing construct in terms of miss distance distribution would confer certain advantages**
 - Reduces to same import as Pc, but conveys some additional information
- **Methods to incorporate known uncertainties in covariances and HBR should be pursued**
 - Can also be framed in terms of miss distance distribution
- **Plausibilistic II and Possibilistic constructs, while adequate from a safety perspective, are not recommended**
 - Very much overly conservative
 - Could create pressure to relax thresholds or downplay CA importance