National Aeronautics and Space Administration

Orbital Maintenance for the Wide Field Infrared Survey Telescope: Effects of Solar Radiation Pressure and Navigation Accuracies on Stationkeeping

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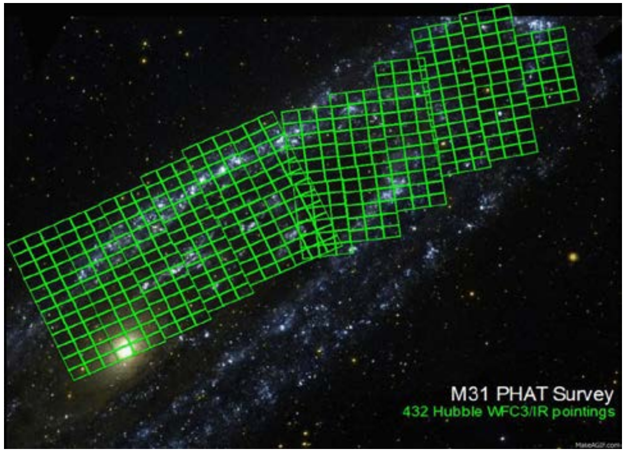
Outline

- Wide Field Infrared Survey Telescope (WFIRST)
- Force Models and Stationkeeping Strategy
- Modeling Solar Radiation Pressure (SRP)
- SRP Effect on Stationkeeping
- Navigation Errors Effect on Stationkeeping
- Conclusions and Future Work

Wide Field Infrared Survey Telescope

- Scheduled to launch in 2025 to an orbit about the Sun-Earth Libration Point 2 (SEL2)
- 2.4 meter primary mirror along with a Wide Field Instrument (WFI) will be used to scan up to 100x more sky than Hubble
- Coronograph Instrument (CGI) will be used to search for exoplanets

- Mission Objectives:
  - Explore exoplanets
  - Research into dark energy
  - Perform galactic and extragalactic surveys



Goal of the work

- WFIRST will be orbiting at Sun-Earth L2 around a Quasi-Halo orbit. To deal with the instability of the environment, and remain close to its nominal orbit, stationkeeping maneuvers will be performed every 21 days.

- Routine Momentum Unloads (MUs) will be performed to unload the stored momentum in the reaction wheels.

- The effect of Solar Radiation Pressure (SRP) on the stationkeeping Δv has been explored using different SRP models.

- The effect of Orbit Determination and Navigation errors, and maneuver execution errors on the stationkeeping Δv has also been explored.

Force Models

- **Circular Restricted Three Body Problem (RTBP)**

$$\ddot{\boldsymbol{R}} + 2\boldsymbol{\omega} \times \dot{\boldsymbol{R}} = \nabla\boldsymbol{\Omega} + \boldsymbol{a}_{\boldsymbol{srp}},$$

where $\boldsymbol{R} = (X, Y, Z)$ is the location of the satellite, $\Omega = \frac{1}{2}(X^2 + Y^2) + \frac{1-\mu}{r_{ps}} + \frac{\mu}{r_{pe}}$ is the gravitational potential, and $\boldsymbol{a}_{srp} = (a_X, a_Y, a_Z)$ is the SRP acceleration.

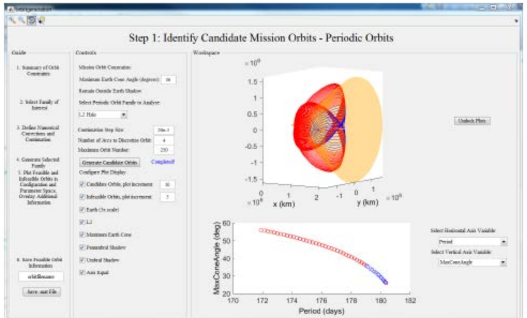
- **Point Mass Ephemeris Model**

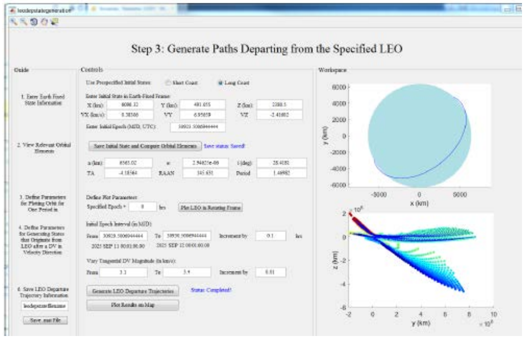
$$\ddot{\boldsymbol{R}}_{S,sc} = Gm_S \frac{\boldsymbol{R}_{S,sc}}{R_{S,sc}^3} + Gm_E \left(\frac{\boldsymbol{R}_{E,sc}}{R_{E,sc}^3} - \frac{\boldsymbol{R}_E}{R_E^3} \right) + Gm_M \left(\frac{\boldsymbol{R}_{M,sc}}{R_{M,sc}^3} - \frac{\boldsymbol{R}_M}{R_M^3} \right) + \boldsymbol{a}_{srp},$$

where $\boldsymbol{R} = (X, Y, Z)$ is the location of the satellite, $\boldsymbol{R_i} = (X_i, Y_i, Z_i)$ is the position of the Sun-Earth and Moon ($i = S, E, M$), and m_S, m_E, m_M their respective masses.

Adaptive Trajectory Design (ATD) module

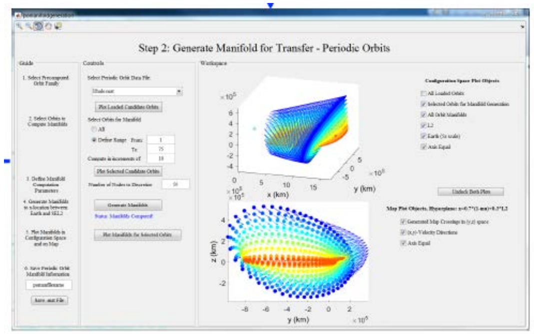
- The baseline trajectory for WFIRST has been computed with the ATD Module developed by Dr. Natasha Bosanac [1].

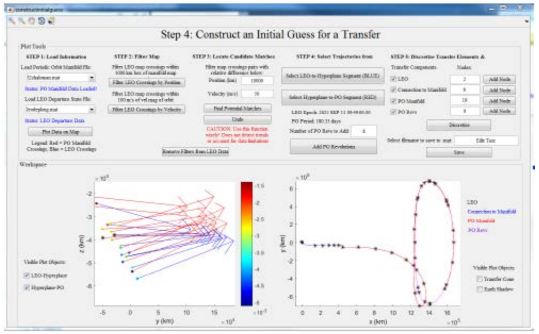


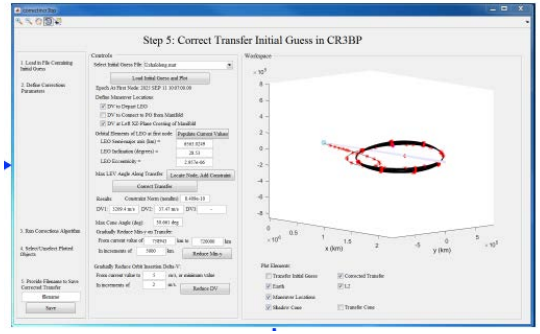


Main Steps:

1. Select candidate Halo orbit in CR3BP.
2. Find transfer from LEO to Halo in CR3BP.
3. Correct trajectory in CR3BP and Full Ephemeris.
4. Export to GMAT.



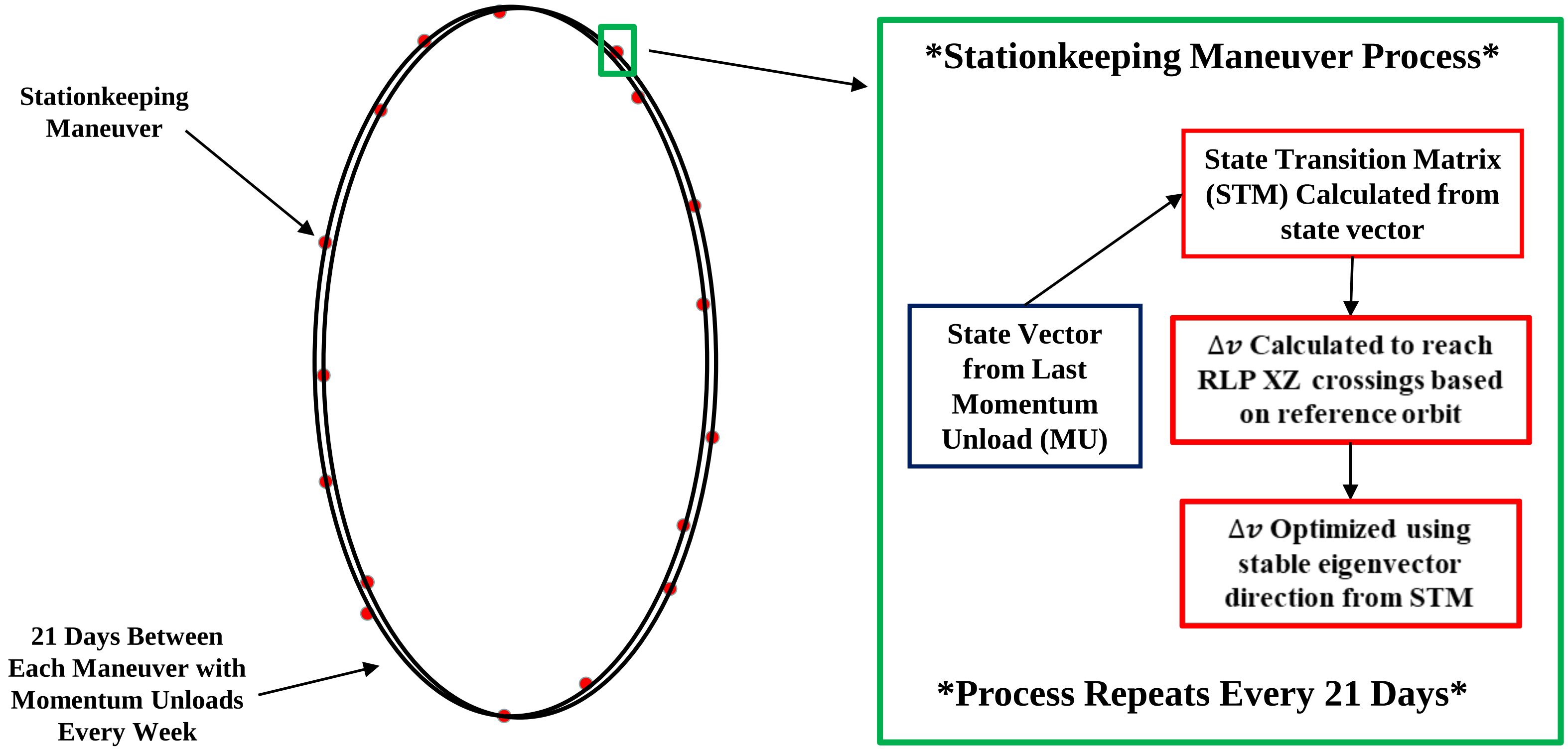




[1] N. Bosanac, C. M. Webster, K. Howell and D. C. Folta, *"Trajectory Design and Station-Keeping Analysis for the Wide Field Infrared Survey Telescope Mission",* in AAS/AIAA Astrodynamics Specialist Conference, 2017.

Stationkeeping Strategy

- We use information from the natural dynamics around a Halo orbit to determine the stationkeeping maneuver.



Stationkeeping Maneuver

21 Days Between Each Maneuver with Momentum Unloads Every Week

Stationkeeping Maneuver Process

State Vector from Last Momentum Unload (MU)

State Transition Matrix (STM) Calculated from state vector

Δv Calculated to reach RLP XZ crossings based on reference orbit

Δv Optimized using stable eigenvector direction from STM

Process Repeats Every 21 Days

Solar Radiation Pressure Models

- **Cannonball Model** (simple) the satellite's shape is approximated by a sphere:

$$\boldsymbol{a}_{srp} = -\frac{P_{srp} C_r A_{sat}}{m_{sat}} \boldsymbol{r}_s.$$

- **N-plate Model** (intermediate) the satellite's shape is approximated by a set of flat plates, each one with different reflectivity properties:

$$\boldsymbol{a}_{srp} = -\frac{P_{srp}}{m_{sat}} \sum_{i=1}^{N} \left(A_i \langle \boldsymbol{n}_i, \boldsymbol{r}_s \rangle \left[(1 - \rho_s^i) \boldsymbol{n}_i + 2 \left(\rho_s^i \langle \boldsymbol{n}_i, \boldsymbol{r}_s \rangle + \frac{\rho_d^i}{3} \right) \boldsymbol{r}_s \right] H(\theta_i) \right).$$

- **Finite Element Model** (high-fidelity) a CAD model is used to approximate the satellite's shape and ray-tracing techniques are used to approximate the SRP acceleration:

$$\boldsymbol{a}_{srp} = -\frac{P_{srp}}{m_{sat}} \int_{\partial\Omega} A \langle \boldsymbol{n}, \boldsymbol{r}_s \rangle \left[(1 - \rho_s) \boldsymbol{n} + 2 \left(\rho_s \langle \boldsymbol{n}, \boldsymbol{r}_s \rangle + \frac{\rho_d}{3} \right) \boldsymbol{r}_s \right] d\Omega.$$

[2] A. Farres, D. C. Folta and C. Webster, "Using Spherical Harmonics to Model Solar Radiation Pressure Accelerations,“ in *AAS/AIAA Astrodynamics Specialist Conference*, 2017.

Comparison between SRP models

- The main difference between the Cannonball and the N-plate model is that it does not account for the satellite's attitude.
- The 14-plate approximation for WFIRST shows good agreement with the Finite Element approximation.

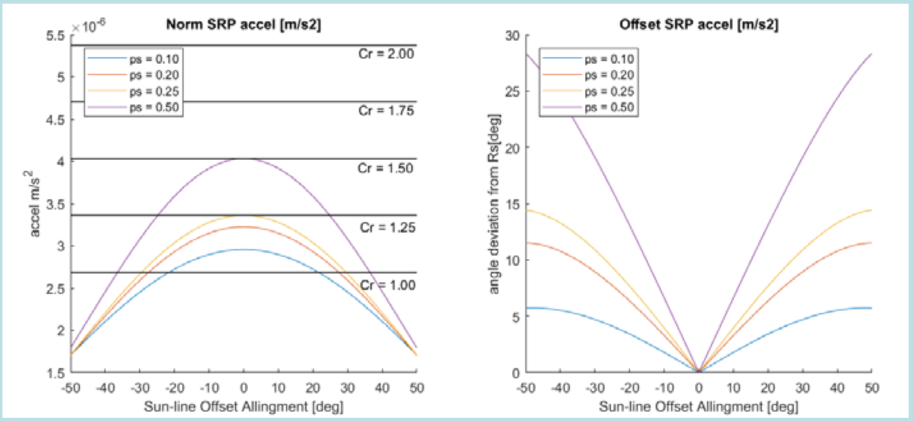


Fig 1: Cannonball vs 1-plate Model.

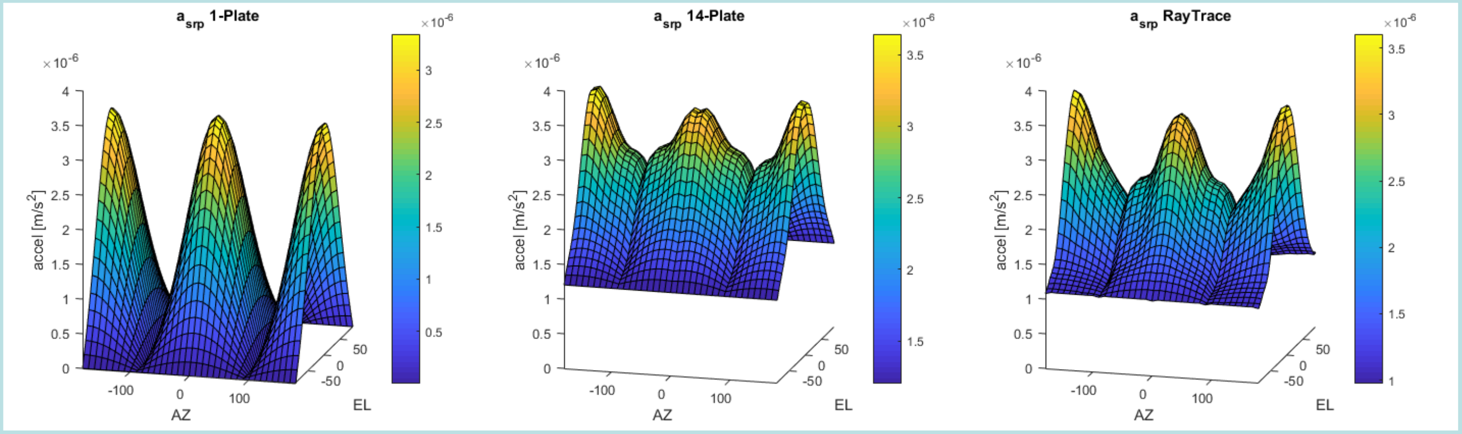


Fig 2: 1-plate vs 14-plate vs Finite Element

Effect of SRP on LPOs

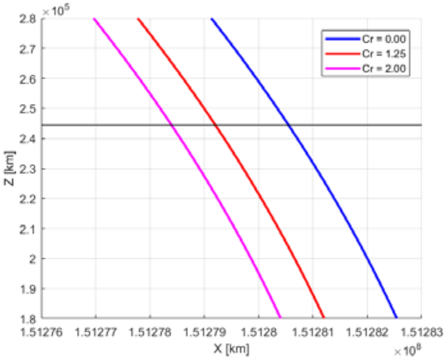
- The extra acceleration due to SRP essentially displaces the invariant objects toward the Sun.

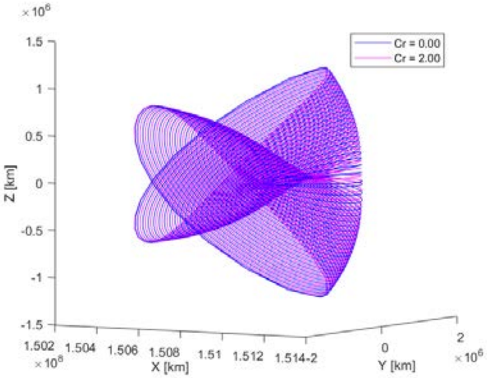
Table 1: Relationship between the location of L2 and Cr values.

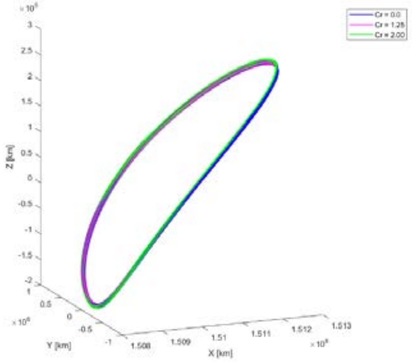
C_r	q_{srp}	L$_2$ location
$C_r^0 = 0.00$	0.0	151,105,099.17 km
$C_r^1 = 1.25$	5.7799×10^{-5}	151,104,145.49 km
$C_r^2 = 2.00$	9.2472×10^{-5}	151,103,573.97 km

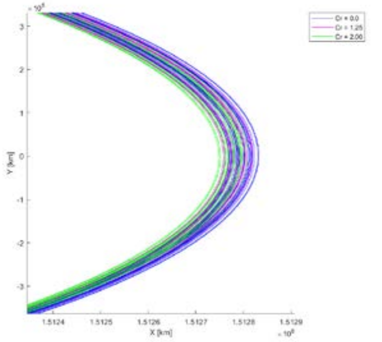
Table 2: Distance between L2 Halo orbits for different Cr values.

	$C_r^0 - C_r^1$	$C_r^0 - C_r^2$	$C_r^1 - C_r^2$
RTBP	1,300 km	2,000 km	800 km
Ephem	1,430 km	2,400 km	930 km









Effect of SRP on Stationkeeping

- Using the ATD module, two different reference orbits have been generated: one with $C_r = 0.0$ and another with $C_r = 2.0$.
- For each reference, orbit 5 simulations for stationkeeping over 5 years have been performed using three different C_r values ($C_r = 0.0, 1.25 \text{ and } 2.0$) and different MUs sizes (1.3 mm/s and 13.3 mm/s).
- Results show that following the reference orbit with the same C_r value helps lower the total Δv cost.
- Increasing the size of the MUs increases the Δv cost, and the accuracy in SRP models is less relevant.

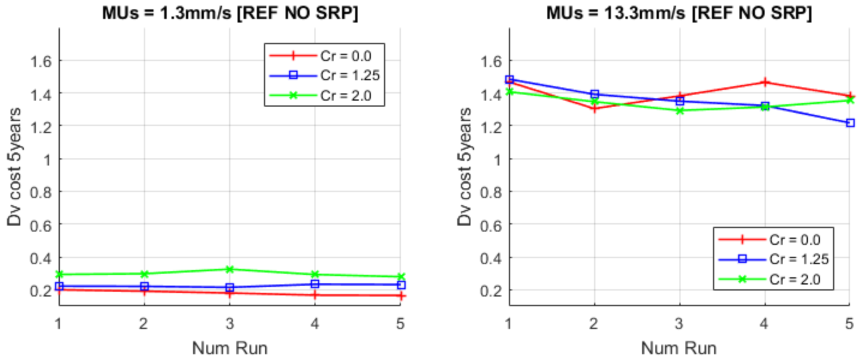


Fig 1. Total Δv cost for 5 years stationkeeping simulations using a No SRP reference trajectory for different Cr values.

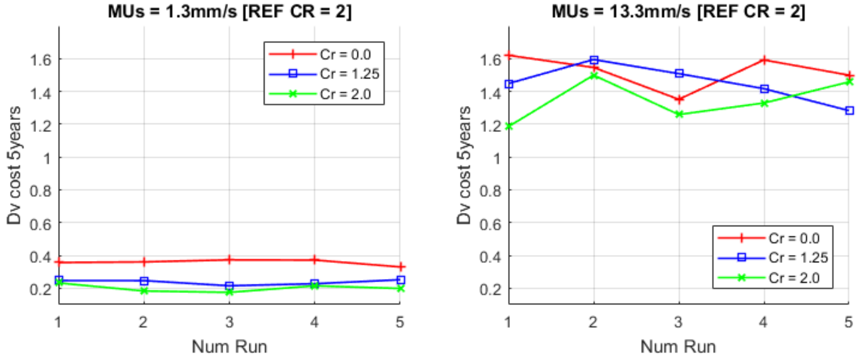


Fig 2. Total Δv cost for 5 years stationkeeping simulations using an SRP reference trajectory for different Cr values.

Effect of SRP on Stationkeeping

- Using a reference trajectory the same as in the previous examples ($C_r = 2.0$ with cannonball SRP).
- 5 simulations for stationkeeping over 5 years have been performed with different fixed offset angles and MUs sizes (1.3 mm/s and 13.3 mm/s).
- Results show that large offset angles result in larger total Δv cost.
- Increasing the size of the MUs increases the Δv cost, and the accuracy in SRP models is less relevant.
- Explorations with variable attitude will be done in the future.

Table 3: Total stationkeeping Δv cost with no MUs for a fixed plate offset.

Offset Angle	Total Δv
$\alpha = 0^o$	**0.1287 m/s**
$\alpha = 10^o$	**0.1355 m/s**
$\alpha = 20^o$	**0.1848 m/s**
$\alpha = 40^o$	**0.2754 m/s**

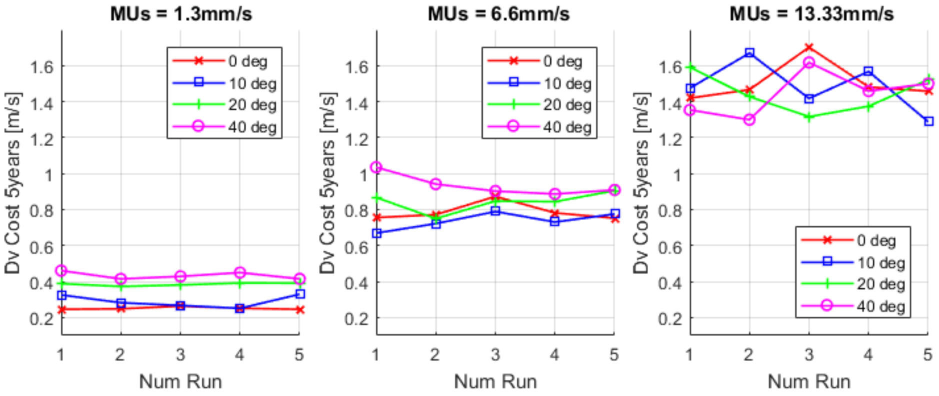
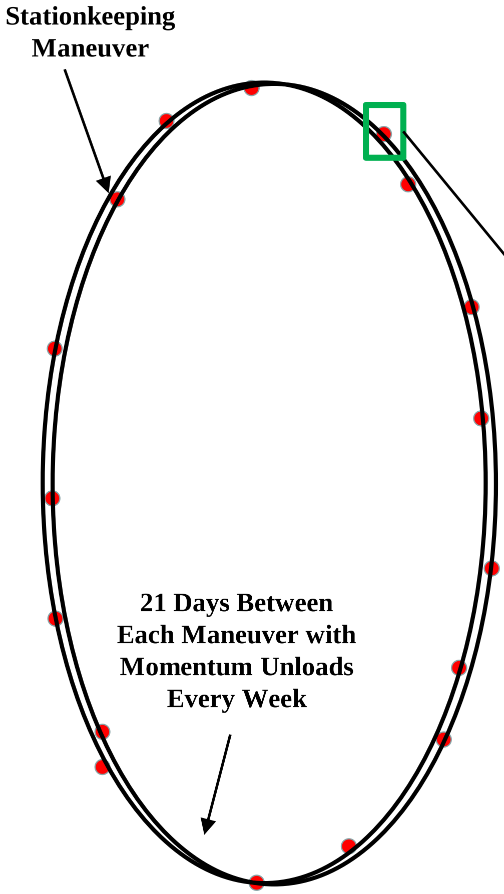
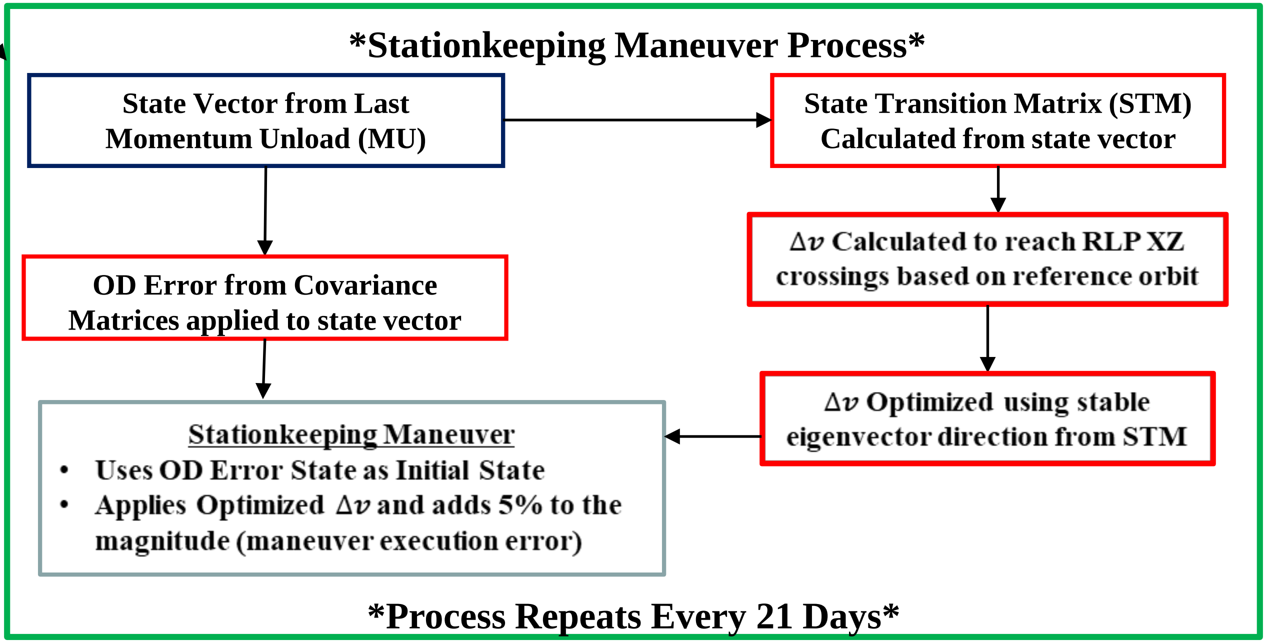


Fig 3. Total Δv cost for 5 years stationkeeping simulations using 1-plate model for SRP for different fixed offset angles.

Effects of Navigation Errors on Stationkeeping



- Currently NEN and DSN ground stations will be used for orbit determination.
- Having errors of 5 km on the steady-states and 5 cm/s on the velocity estimates.



Stationkeeping Maneuver Process

State Vector from Last Momentum Unload (MU)

State Transition Matrix (STM) Calculated from state vector

OD Error from Covariance Matrices applied to state vector

Δv Calculated to reach RLP XZ crossings based on reference orbit

Δv Optimized using stable eigenvector direction from STM

Stationkeeping Maneuver
- **Uses OD Error State as Initial State**
- **Applies Optimized Δv and adds 5% to the magnitude (maneuver execution error)**

Process Repeats Every 21 Days

Effects of Navigation Errors on Stationkeeping

- Four different cases have been analyzed: with no SRP ($C_r = 0.0$) and SRP ($C_r = 2.0$), each one taking different MUs sizes (1.3 mm/s and 13.3 mm/s).
- 10 simulations using the cannonball model for SRP have been performed including random OD errors and maneuver execution errors for 1 year of stationkeeping.

Table 4. Stationkeeping Δv with Orbit Determination Errors and Cannonball SRP Model.

Analysis Case	C_r Value Used in Analysis and Reference Orbit	Momentum Unload Residual Δv (mm/s)	Maximum Position OD Error (km)	Maximum Velocity OD Error (cm/s)	Average Total Stationkeeping Δv for 1 Year (m/s)
1	0	1.33	9.57	3.22	1.12
2	0	13.33	12.81	3.83	1.20
3	2	1.33	13.72	4.04	1.06
4	2	13.33	10.75	4.53	1.16

Effects of Navigation Errors on Stationkeeping

- 8 different cases have been analyzed using a 1-plate model for SRP: taking different fixed offset angles, each one taking different MUs sizes (1.3 mm/s and 13.3 mm/s).

Table 5. Stationkeeping Δv with Orbit Determination Errors and N-Plate Model.

Analysis Case	1-Plate Offset Angle (°)	Momentum Unload Residual Δv (mm/s)	Maximum Position OD Error (km)	Maximum Velocity OD Error (cm/s)	Average Total Stationkeeping Δv for 1 Year (m/s)
1	0	1.33	8.43	3.14	0.92
2	0	13.33	10.62	4.09	0.99
3	10	1.33	11.31	4.62	0.91
4	10	13.33	9.82	3.87	0.97
5	20	1.33	7.64	4.17	0.89
6	20	13.33	16.27	4.54	1.02
7	40	1.33	10.94	3.75	0.88
8	40	13.33	12.03	4.03	0.95

Conclusions and Future Work

- We have analyzed how SRP acceleration uncertainties, the size of MUs and OD errors affect WFIRST's total Δv for stationkeeping.
- Simulations without OD errors: agreement between the SRP model and reference orbit help lower the Δv cost. Moreover, large MUs increase the total Δv.
- Simulations with OD errors: the OD errors introduced are similar in size to the individual stationkeeping maneuvers. Better navigation errors (either using more tracking or Onboard OD) may reduce the total stationkeeping maneuver Δv.
- The total stationkeeping Δv increased significantly when OD errors were introduced vs. when just looking at SRP and MU sizes.
- In the future, using a variable attitude profile for WFIRST, the effects of SRP can fully be studied as WFIRST moves through its orbit with different orientations.

Questions ?

Thank you for your attention