Change of Inertia Tensor Due to a Severed Radial Boom for Spinning Spacecraft

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Spin-Stabilized Spacecraft

- Gyroscopic action
- Easy to tension long wire booms due to centrifugal force
- Magnetic and electric fields, plasma, etc.
- Thin boom structure can be a risk

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History of Radial Boom Breakage

- Fast Auroral SnapshoT (FAST)
  - 1996: Deployment failure
- Imager for Magnetopause-to-Aurora Global Exploration (IMAGE)
  - 2000: $^-$X antenna damage
  - 2001: $^+$Y antenna damage
  - 2004: $^+$Y antenna damage (again)
- ARTEMIS P1 (formerly THEMIS B)
  - 2010

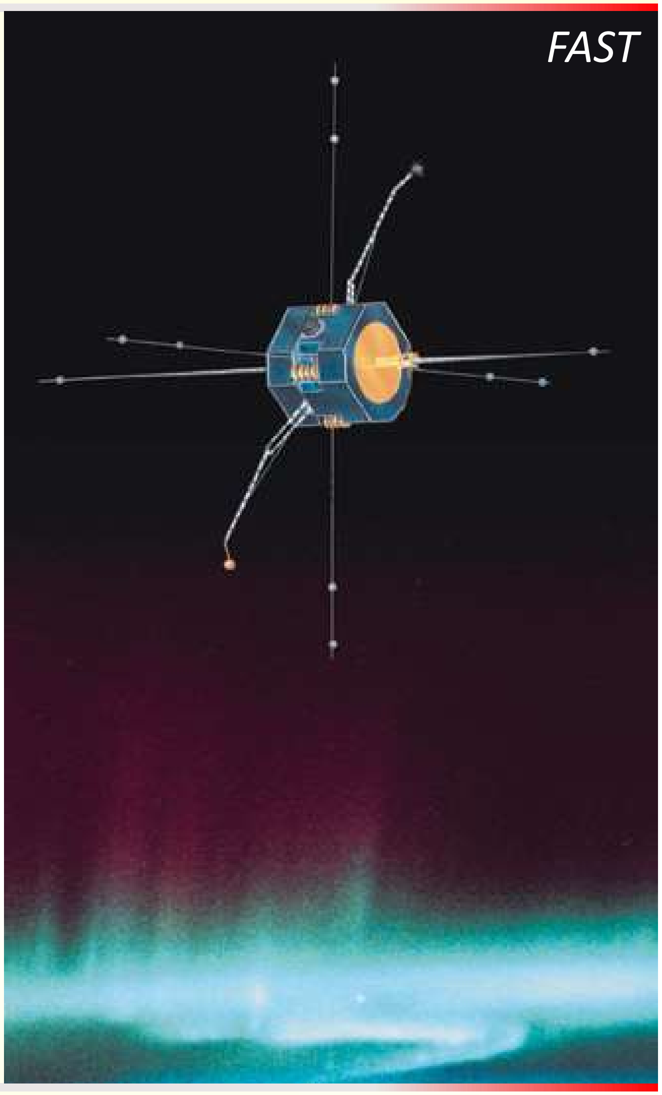

Image from https://www.nasa.gov/vision/universe/solarsystem/fast_10yr.html

Boom Contribution to Moment of Inertia

- Change in mass may be negligible ($100\ g$ lost versus $1{,}400\ kg$ spacecraft)
- Mass moment increases with square of distance

$$J = mr^2$$

- At a radius of $100\ m$, a $100\ g$ mass contributes $1{,}000\ kg \cdot m^2$ (typical total moment of inertia could be $5{,}000\ kg \cdot m^2$)

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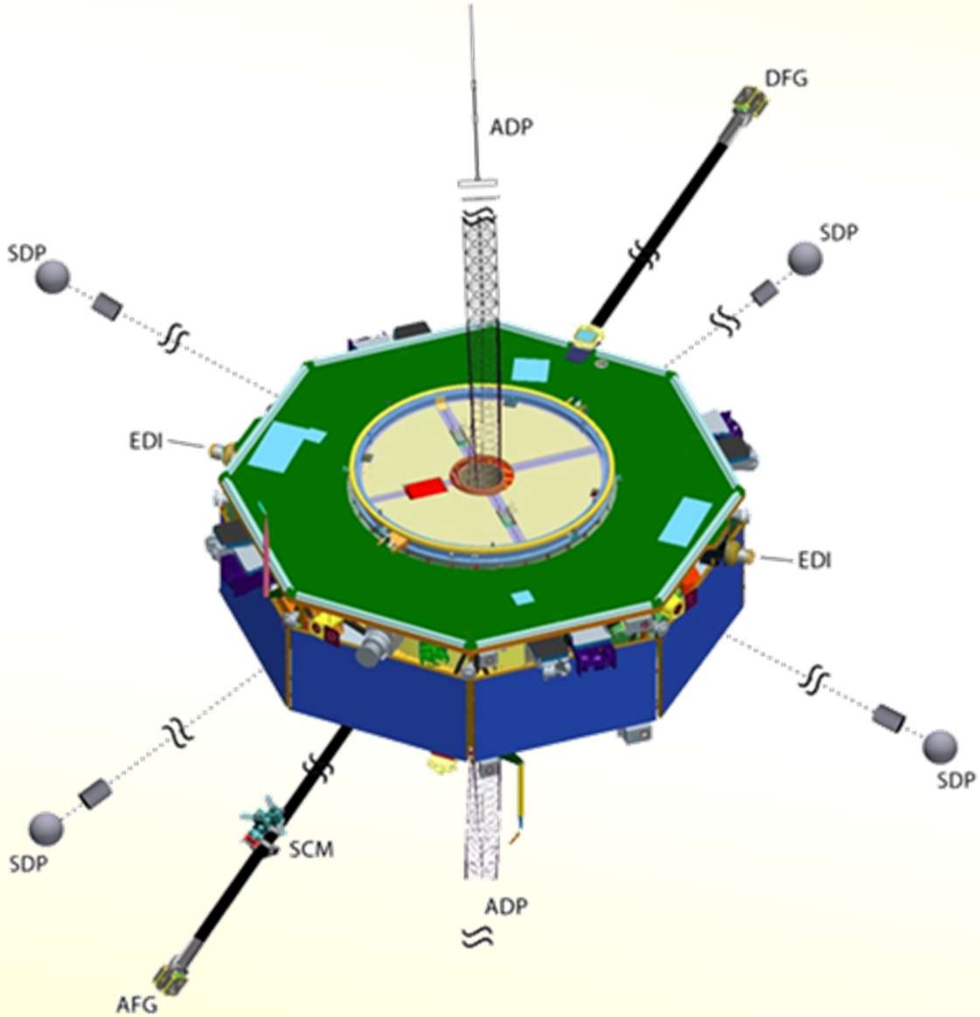
Motivation

1. Impact of radial boom anomaly to mass moment of inertia tensor is significant
2. While inertia tensor is not directly observable, direction of Major Principal Axis (MPA) <u>is</u> observable for some missions
3. Location of break along boom should be related to some change in MPA

Assumptions

1. Motion is steady-state, all vibrations damped
2. With no internal motion, inertia tensor is same as a rigid body
3. Torque-free motion
4. Given (1), (2), and (3): MPA, angular velocity $\vec{\omega}$, and angular momentum $\vec{L}$ all coincide

Magnetospheric MultiScale (MMS) Mission



- **Spin-plane Double Probe (SDP)**

- Axial Double Probe (ADP)
- Analog Flux Gate (AFG)
- Search Coil Magnetometer (SCM)
- Digital Flux Gate (DFG)
- Electron Drift Instrument (EDI)

The main coordinate system considered is the Observatory Coordinate System (OCS)

Figure used with permission of University of New Hampshire MMS-FIELDS team

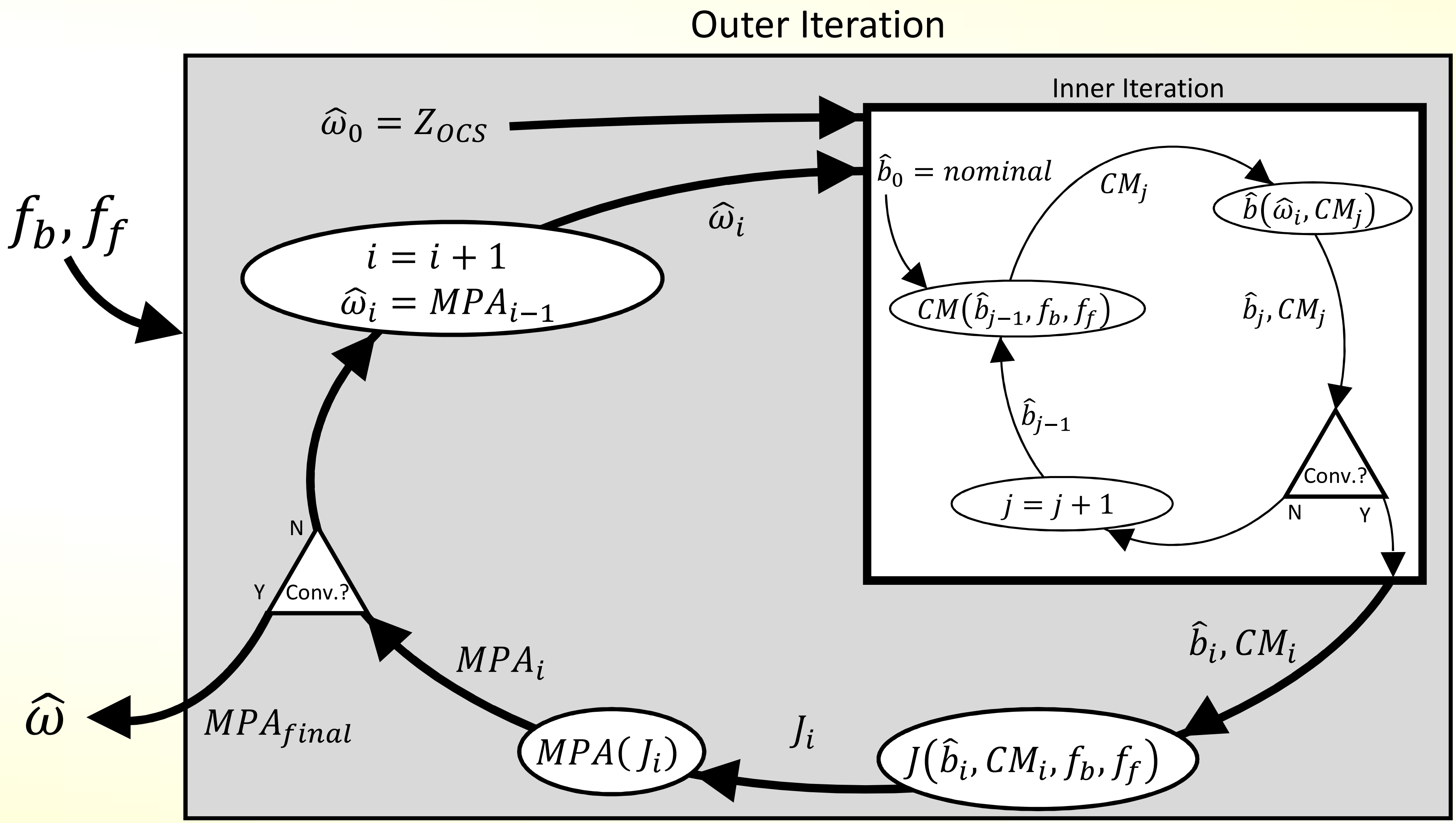
Existing MMS Attitude Ground System (AGS)

- Based on a software suite that has been used for many missions
- Center of Mass (CM) and inertia tensor models developed specifically for MMS
  - Use pre-launch determined values
  - Account for deployment status of booms
  - Assume nominal spin axis (OCS *Z*-axis)
- Inertia tensor calibration (fuel asymmetry)

Proposed Improvements

- Account for directions of booms at steady-state
  - Net torques and forces are zero
  - Radial to spin vector, not *Z*-axis
  - "Radial" = intersecting + perpendicular
- Account for mutual dependence of boom directions and MPA
- Account for fully or partially severed boom(s)

Big Picture of Improved Model



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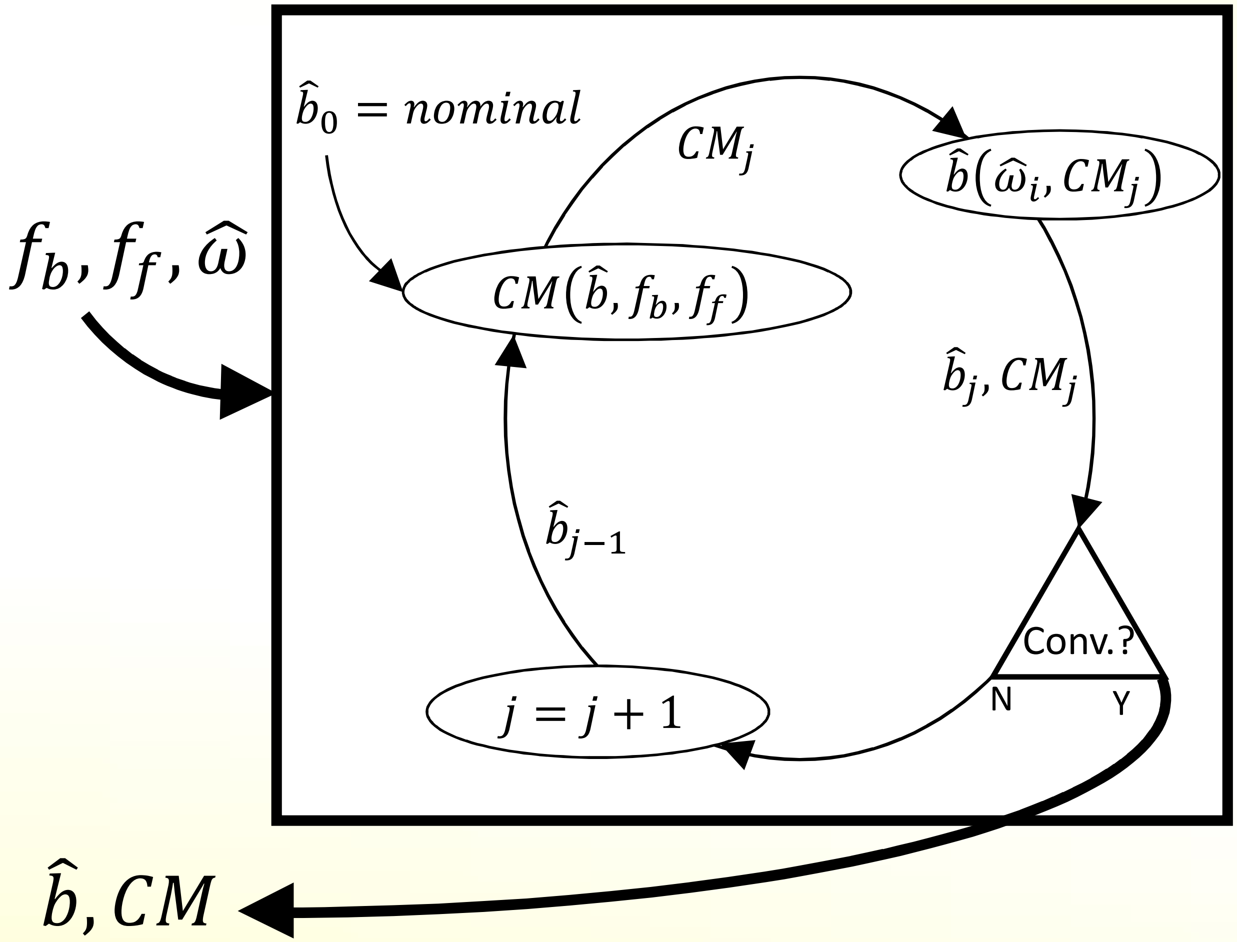
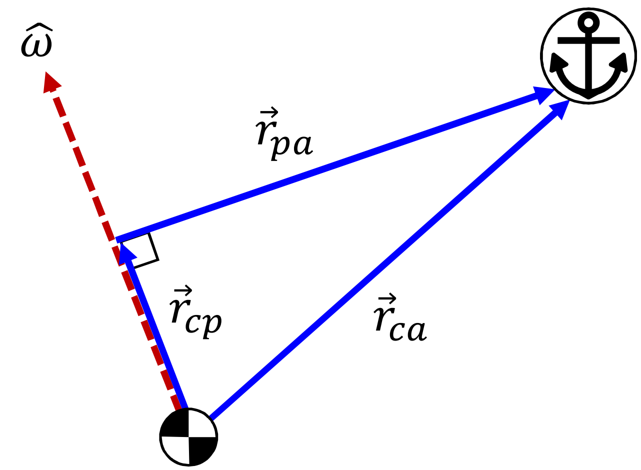
“Inner” Iteration



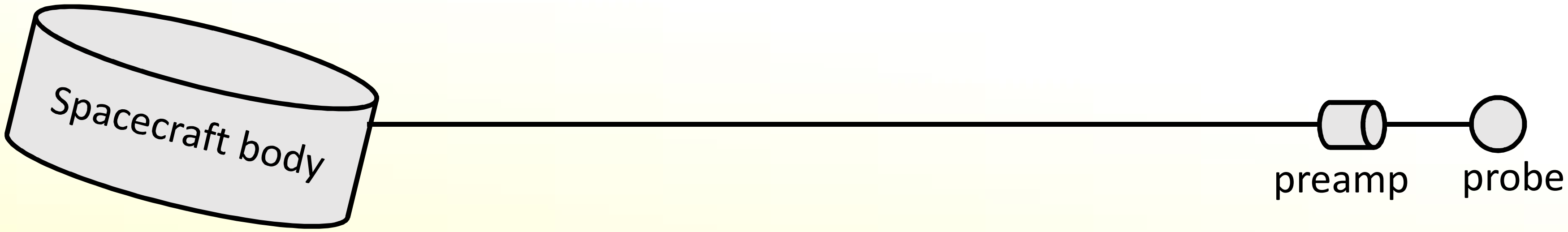
$$CM_{sys} = \frac{1}{M} \sum_{k=1,\ldots,N}^{N} m_k CM_k$$



$$\vec{r}_{cp} = (\vec{r}_{ca} \cdot \hat{\omega})\hat{\omega}$$
$$\vec{r}_{pa} = \vec{r}_{ca} - \vec{r}_{cp}$$
$$\hat{b} = \vec{r}_{pa} / \|\vec{r}_{pa}\|$$

Inertia Tensor (1 of 2)

- $\hat{b}$ and CM must be given (and f_b and f_f)
- "Build" inertia tensor of system from constituents that have known inertia tensors
  - Spacecraft body
  - Basic 3D solids (thin rod, sphere, cylinder)
- Parallel axis theorem translates inertia tensor to/from center of mass and arbitrary point



Inertia Tensor (2 of 2)

- Boom Direction Coordinate System (BDCS)

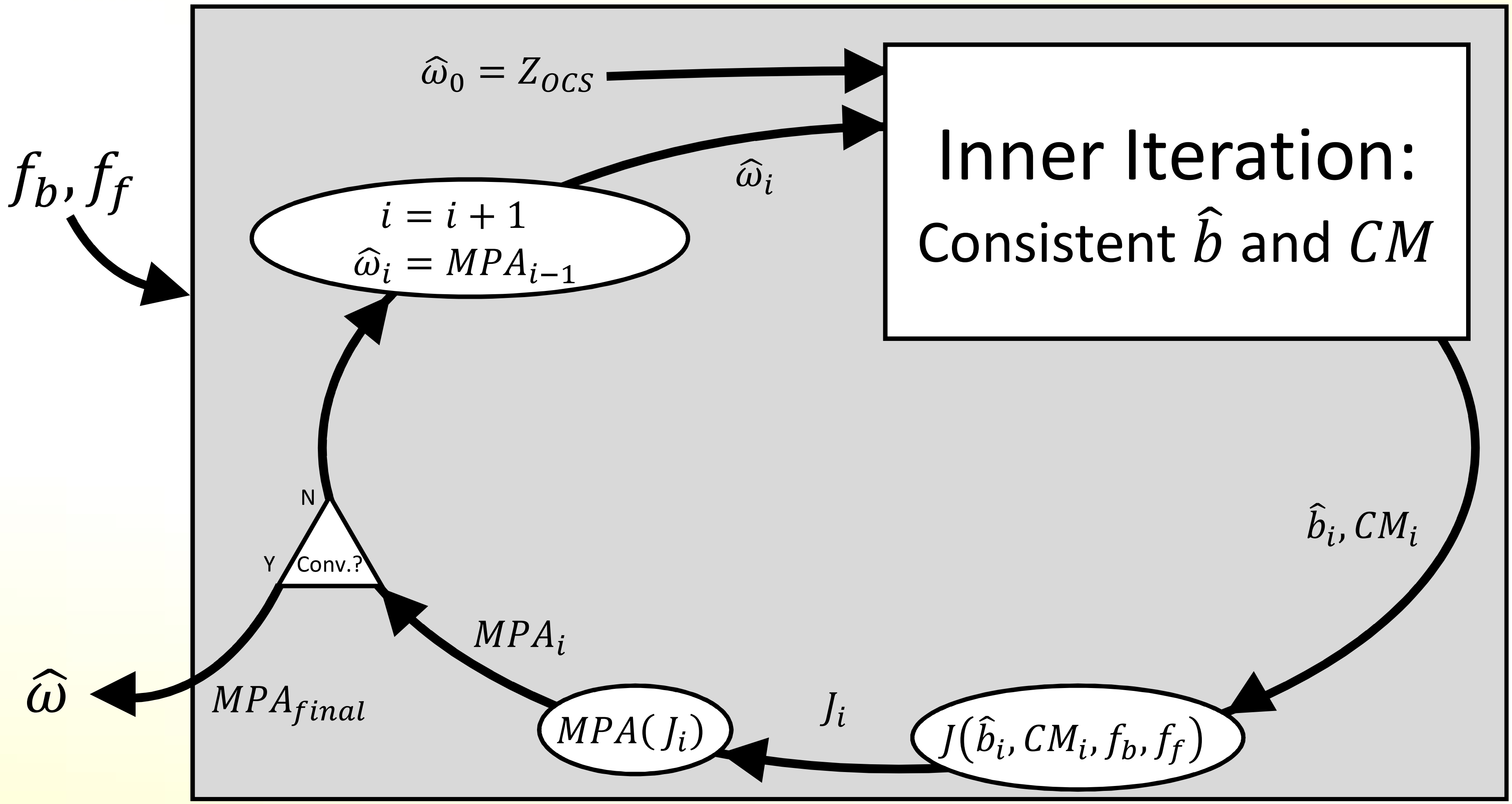
$$\begin{bmatrix} J_{axial} & 0 & 0 \\ 0 & J_{transverse} & 0 \\ 0 & 0 & J_{transverse} \end{bmatrix}$$

- Change tensor orientation via similarity transformation

$$A_{BDCS \leftarrow OCS} = f(\hat{b})$$
$$A = A^T_{BDCS \leftarrow OCS}$$
$$J_{OCS} = A\, J_{BDCS} A^T$$

- Overall process:
  1. Build each boom tensor (parallel axis theorem)
  2. Transform each boom tensor from $BDCS_{boom}$ to OCS
  3. Build total spacecraft tensor (parallel axis theorem)

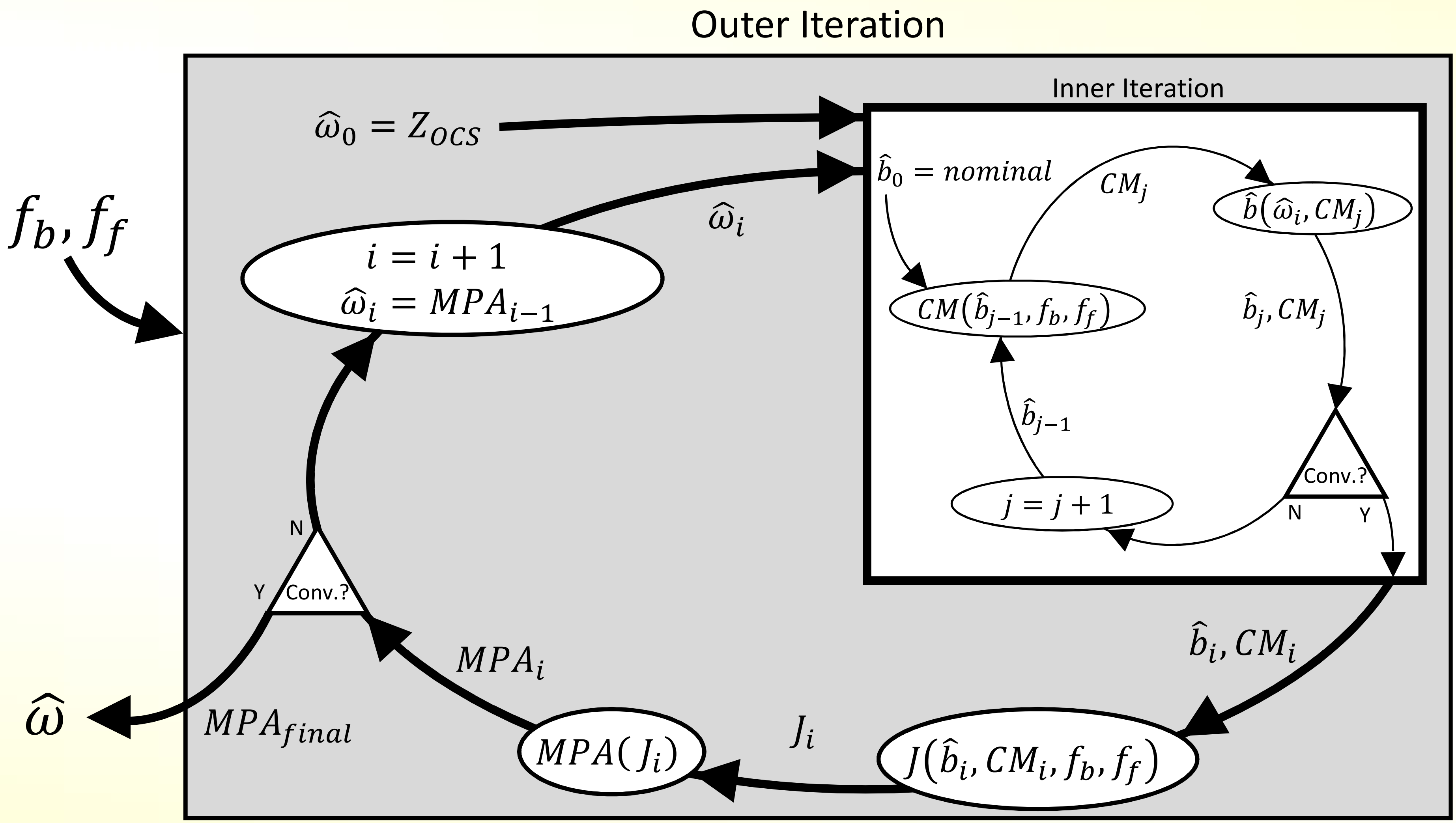
"Outer" Iteration (1 of 2)



"Outer" Iteration (2 of 2)

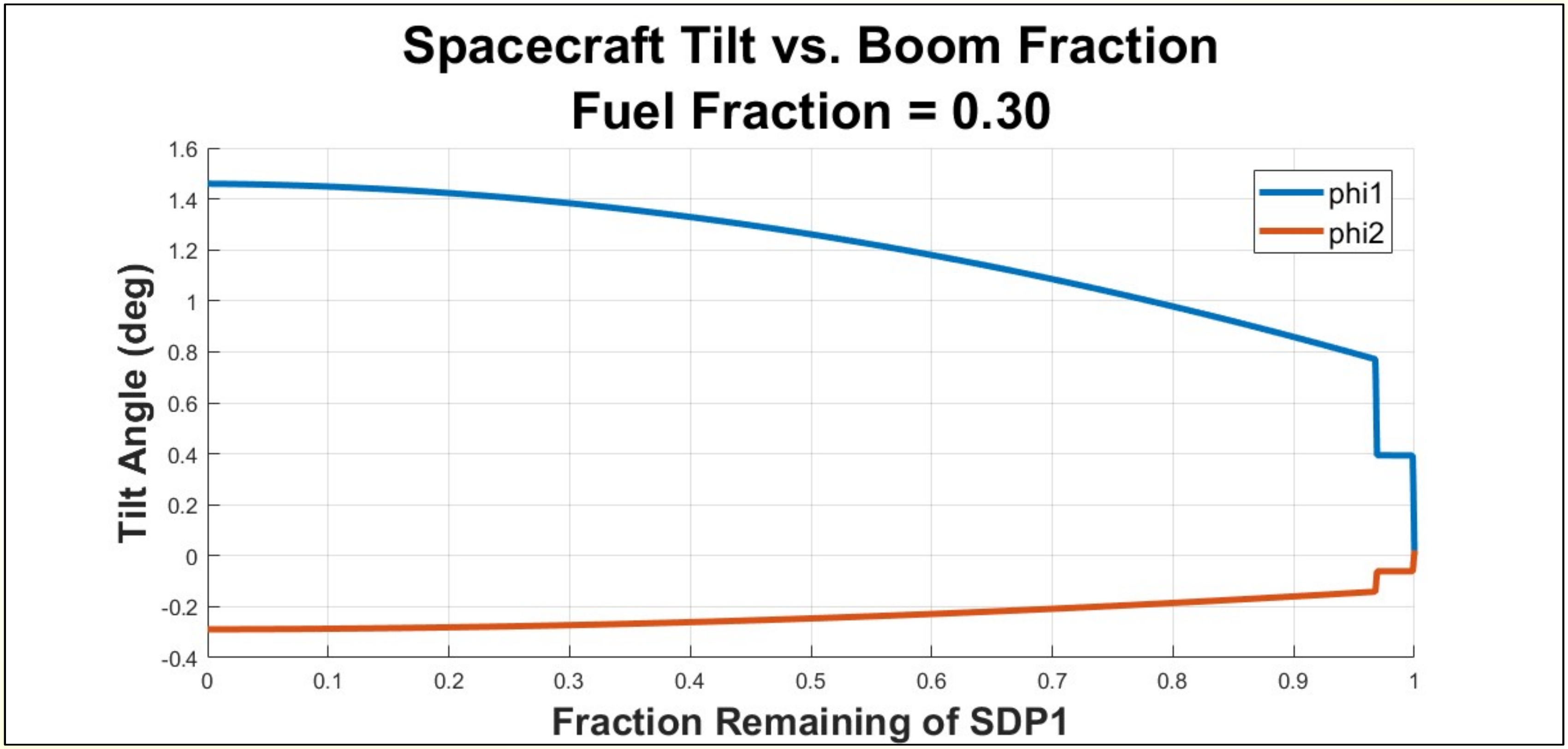
- Accelerated method converges in approximately ${}^{1}/_{10}$ to ${}^{1}/_{3}$ the number of iterations
- Based on assumption that error decreases roughly as a geometric progression ($\epsilon = ar^n$; $|r| < 1$)
- Related to Aitken's δ^2-process (a.k.a. Aitken extrapolation)

Big Picture of Improved Model (Review)

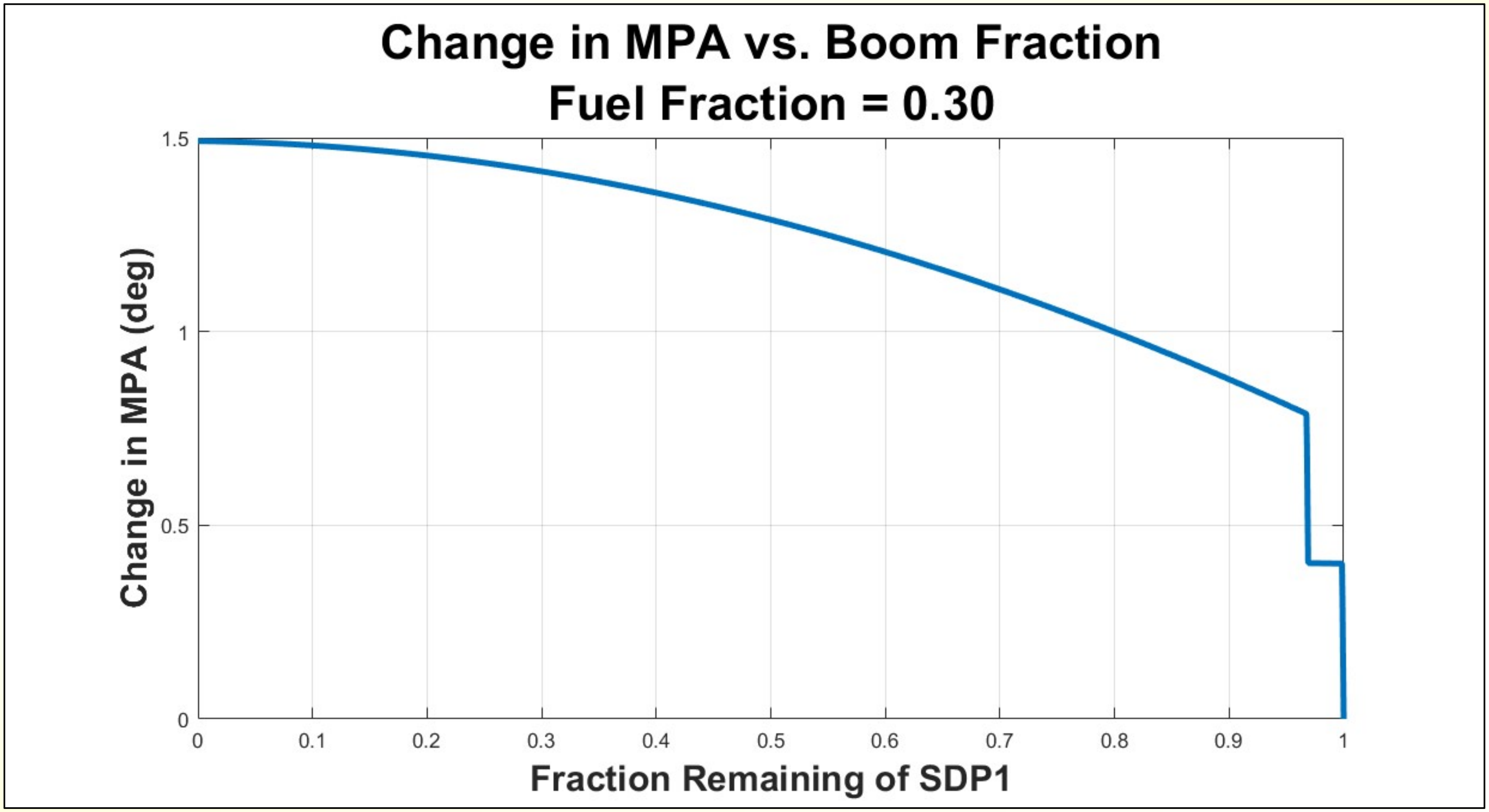


Results (1 of 5)

Suppose a coordinate system has its *X*-axis parallel to the nominal direction of the severed boom. Let φ_1 and φ_2 define the “tilt” of the MPA from the OCS *Z*-axis.



Results (2 of 5)



Results (3 of 5)

- Uncertainty in MPA is approx. $0.003°$ (3σ)
- Let Φ denote the change in MPA due to break
- Uncertainty in Φ is approx. $0.006°$ (3σ)
- Let X denote the break location, measured in meters from the attachment point of the boom
- Want to know uncertainty ΔX given uncertainty $\Delta\Phi$
- First order approximation:

$$\Delta X \approx \left|\frac{d\Phi}{dX}\right|^{-1} \Delta\Phi$$

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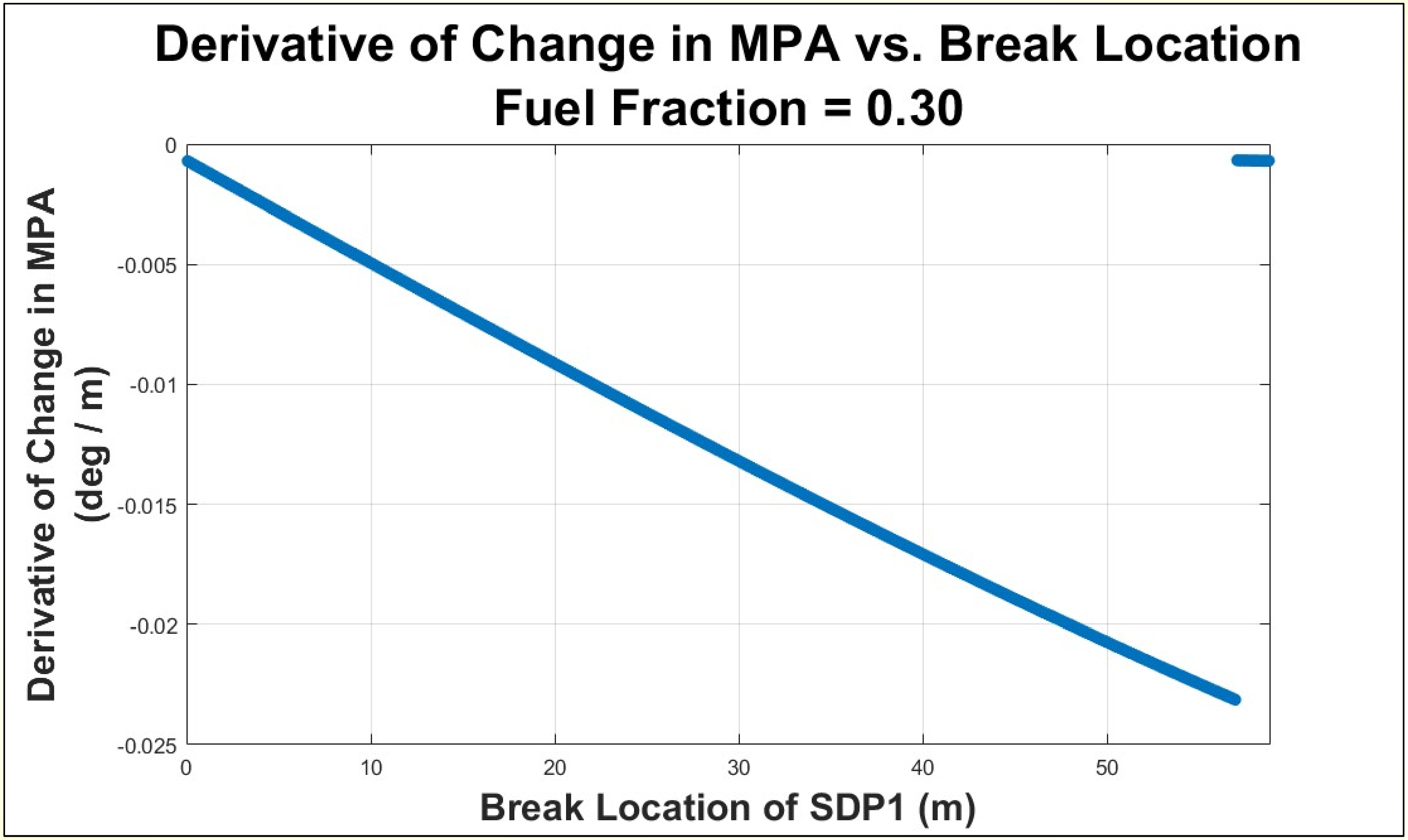
Results (4 of 5)



Results (5 of 5)

Approximate values for 3σ uncertainty in break location for various regions of the boom.

Break Region	3σ Uncertainty
Near Boom Attachment	6 m
Near Boom Midpoint	50 cm
Near Boom Tip	20-30 cm

MMS Application

- Predictive products:
  - Rigid body inertia tensor is used to calculate gravity gradient torque
- Definitive products:
  - Extended Kalman Filter (EKF) uses inertia tensor in propagation step
- Mass properties
  - CM and rigid body inertia tensor are reported for onboard use

Future Work

- Investigate whether choice of independent variable(s) (φ_1, φ_2, ΔMPA) affects accuracy of boom fraction mapping
- Implementation is already generalized for multiple breaks (f_b is a row vector)
  - May result in ambiguous solutions
  - Requires analysis
- Model boom deployment failure (requires little modification)
- Incorporate new model into inertia tensor calibration tool

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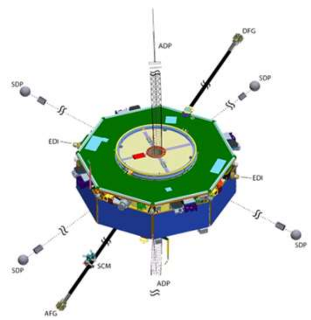
Summary

- The effects of a radial boom break were shown to be observable and quantifiable
- An improved model for CM and inertia tensor was developed for the MMS mission
- Based purely on attitude observations, location of boom break can be estimated to within a small uncertainty

Questions



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Backup Slide: Why MPA, $\vec{\omega}$, and $\vec{L}$ Coincide

1. $\vec{L}$ is fixed relative to space
2. MPA is fixed relative to the body
3. $\vec{\omega}$ nutates, tracing out "body cone" and "space cone"
4. Nutation of $\vec{\omega}$ induces internal motion
5. If all motion is damped, $\vec{\omega}$ is no longer nutating (angle between vectors is zero)

Good reading material: *Classical Mechanics* by Herbert Goldstein