SOLAR SAIL TRANSFERS FROM INVARIANT OBJECTS TO L_5 PERIODIC ORBITS

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The rising interest in a mission to the Sun-Earth L_5 point for heliophysics or Trojan asteroid search and the development of solar sails has opened the question of using solar radiation pressure for such a mission. Different solar sail trajectories to the Sun-Earth L_5 point are constructed. Different invariant objects in the neighbourhood of Earth are considered for departure: equilibrium points, families of periodic orbits and their associated invariant manifold. Using a multi-objective genetic algorithm, differential correction and the optimal control solver PSOPT the transfers are obtained. The approach followed results in fast solar sail transfers which can be used for the preliminary design of a mission to the Sun-Earth L_5 point.

INTRODUCTION

The equilateral libration points in the Sun-Earth system are very suitable for certain kinds of space missions—those being space weather and Trojan asteroids search. Since these points are stationary 60 degrees ahead and behind Earth, from them regions of the Sun inaccessible from Earth or the L_1 point can be observed. The satellite ACE at the L_1 point allows detecting geomagnetic storms about an hour before they hit Earth, but spacecraft on the equilateral points would allow earlier space weather predictions. Both equilateral points are suitable to study coronal mass ejections (CMEs). However, only the L_5 point is suitable to study corotation interaction regions (CIRs) as they pass by the L_5 point first, then Earth and then the L_4 point. Additionally, a spacecraft at the L_4 or L_5 points would obtain a side view from events like solar flares and CMEs which would help in developing a better understanding of these events as well as the magnetic reconnection that triggers them.[1] Due to the stable character of orbits around the equilateral points in the Sun-Earth system, bodies in orbit around these points are likely to have been there for a long time. The study of asteroids in orbits around these Lagrange points can therefore help in understanding the formation of the Solar System. Trojan asteroids have been found in orbits around the equilateral points in the Sun-Mars, Sun-Earth, Sun-Jupiter, Sun-Neptune and Saturn with some of its moons systems. At the Sun-Earth L_4 point, the asteroid 2010TK$_7$ was discovered in 2010 with NASAs WISE spacecraft.[2] The STEREO spacecraft visited both equilateral points in 2009 without spotting any Trojan and then in 2010, the only one known was discovered. This fact shows that there could still be other asteroids of small size or low albedo which have insofar not been discovered.

Due to the relevance of the L_5 point for space missions, previous work on transfers to L_5 periodic orbits and the triangular points neighbourhood have been done. Studies have shown the feasibility of

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transfers departing from 200 km altitude parking orbits around the Earth to specific periodic orbits around the L_5 point. The transfers considered require a ΔV of the order of 4 km/s, depending on the targeted periodic orbit and desired time of flight.[3,4] Solar sails provide an inexhaustible source of low-thrust that can decrease significantly if not completely the required propellant for a mission. Sood and Howell showed using invariant manifolds of Lyapunov orbits, differential correction and optimisation how the use of a solar sail for a mission departing from a 200 km altitude parking orbit around Earth and arriving at a periodic orbit around the L_5 point decreases the total ΔV for such a mission.[5] Alternatively, Farres, Heiligers and Miguel used Poincare sections and optimal control to compute solar sail transfers between the collinear points and the regions of practical stability around the equilateral libration points.[6]

In this study, a versatile approach to obtain solar sail transfers departing from invariant objects in the neighbourhood of the Earth to L_5 periodic orbits is presented. The invariant objects considered are equilibrium points, periodic orbits and their associated invariant manifolds. The previous studies target specific periodic orbits around the L_5 point or its associated region of practical stability. They consider the planar problem departing from Earth parking orbits or equilibrium points. Here, whole families of periodic orbits are considered for the initial and final conditions. Furthermore, the three-dimensional problem is also studied.

METHODOLOGY

The transfers are obtained using a genetic algorithm, differential correction and optimal control. The genetic algorithm finds almost feasible transfers. The differential correction transforms the initial guesses into feasible and sequentially lowers the time of flight. The solver PSOPT is then used to solve the optimal control problem by means of a pseudo-spectral method.

First, a multi-objective optimisation problem is defined such that a set of variables defines a guess for the transfer having as objectives the infeasibility ϵ_I of the guess and the time of flight (TOF). Such set of parameters vary depending on the case considered.

When the invariant objects are equilibrium points along the Sun-Earth line, their associated unstable manifold are integrated for five years. Then. given a family of periodic orbits around the L_5 point, the first variable d determines the size of the periodic orbit as the largest distance from the periodic orbit to its associated equilibrium point. The second variable τ determines the insertion point into the orbit as the propagated flow from some reference point for a time τT; where T is the periodic orbit period. A third variable α_f determines a constant attitude used for backwards integration from the periodic orbit for a five year period. Figure 1 depicts these variables. The initial guess is obtained by union of the unstable manifold of the equilibrium point and the backwards flow from the periodic orbit until the point of minimum euclidean norm in dimensionless phase space between both, which is then used as the infeasibility objective. Figure 4 shows the initial guess for a transfer from the L_1 point to L_5 natural periodic obits for $\beta = 0.02$, TOF= 699 days and $\epsilon_I = 0.079$.

For the natural collinear libration points, their associated unstable manifolds enter a complex region around Earth[6] which can cause issues with the described approach. Therefore for those cases the trajectory starts from the equilibrium points perturbed in the direction of the unstable manifold but with zero pitch angle. It can be shown that with such attitude and the lightness numbers considered, the trajectory escapes Earth in a short period of time. If the problem considered is three-dimensional, an extra variable is included for the attitude component out of the ecliptic plane for the backwards propagation from the periodic orbits in the neighbourhood of the L_5 point.

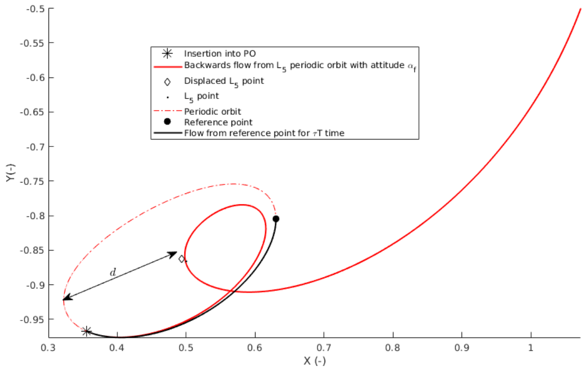

Figure 1: Genetic algorithm variables

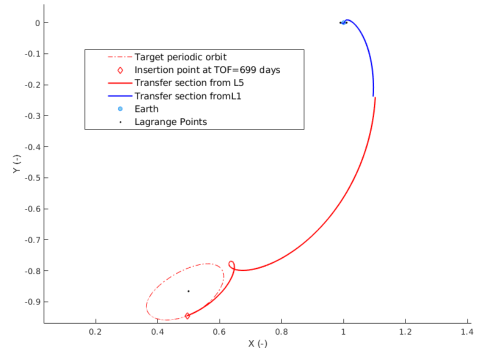

Figure 2: Initial guess example

When the departing invariant object is a periodic orbit in the neighbourhood of Earth, the trajectory starts along the unstable manifold associated with the departing periodic orbit at the departure point. The initial conditions are obtained by using variables analogous to the ones defining the target periodic orbit and the insertion point into it. The third case considered is when the solar sail is a secondary payload of a bigger mission. It is assumed that the bigger mission is on a transfer trajectory to a L_1 Halo orbit along it's stable manifold and that the solar sail will be deployed at some point along the transfer. Such case is of relevance given that a solar sail can be launched as a secondary payload.

The Pareto front obtained from the optimisation then gives several initial guesses that vary in feasibility and time of flight. Ideally the initial guess taken for the remaining part of the design is the guess which is sufficiently feasible and has the lowest time of flight. By sufficiently feasible it is meant that the differential correction can converge to a feasible solution from the initial guess. In theory, the initial guesses could be used directly with some optimiser, but since optimisation problems have several local minimums, an approach capable of decreasing the time of flight passing the local minimums is desirable. Therefore a differential corrector is proposed and used. Usually, a differential corrector is used to transform almost feasible transfers into feasible, but here a method to also minimise the time of flight is constructed. The initial guesses are divided into segments separated by nodes. With exception of the extreme nodes, every node can vary in phase space, solar sail attitude and integration time. As constraints, the initial node can be fixed or constrained to be in a periodic orbit belonging to certain family. The final point is also constrained to be in a periodic orbit within a family. The constraints to be within a periodic orbit are expressed with a periodicity condition which is similar to methods to compute periodic orbits by single shooting. The approach then uses the time of flight from the initial guess to compute a feasible trajectory and then by means of decreasing the imposed total time of flight, the flow, attitude and time of integration for each node are deferentially corrected until a new feasible trajectory is found. The final product is then a tool that modifies the periodic orbits considered in the transfer to sequentially find feasible transfers at lower time of flight. Since such method does not optimise, it could avoid getting stuck at local minimums. Furthermore, it allows for transfers between entire families, as opposed to methods that only consider one particular periodic orbit. Lastly, the optimal control problem is solved with PSOPT to obtain the optimal transfer from the transfer obtained by differential correction.

RESULTS

The described methodology was used for transfers between the collinear equilibrium points and periodic orbits in the neighbourhood of the L_5 point.

The initial guesses obtained with the genetic algorithm are sufficiently feasible to converge to feasible trajectories with the differential corrector, which is capable of decreasing the time of flight considerably. The optimal control solver PSOPT then converges to optimal solutions which are generally very close to the trajectories obtained with the differential corrector. When the transfers are from the natural or displaced L_1 point, PSOPT generally converges to slightly longer transfers, whereas when the transfers depart from the natural or displaced L_2 point, both PSOPT and the differential corrector converge to practically the same solution. Table 1 shows the time of flight for the optimised cases. From the table it is clear that targeting natural periodic orbits results in faster transfers. In terms of departure conditions, departing from the natural collinear points results in transfers faster than when departing from the displaced counterparts.

Table 1: TOF in days for transfers from the collinear equilibrium points to periodic orbits around the L_5 point

Lightness number	$\beta = 0.01$		$\beta = 0.02$		$\beta = 0.03$		$\beta = 0.04$		$\beta = 0.05$	
Method	DC	PSOPT	DC	PSOPT	DC	PSOPT	DC	PSOPT	DC	PSOPT
L_1-natural PO	943	962	612	613	486	486	434	435	402	418
L_1-solar sail PO	1094	1061	729	727	575	574	512	513	478	621
SL_1-natural PO	1094	1019	685	686	563	570	512	525	481	496
SL_1-solar sail PO	1194	1136	801	803	651	664	589	605	555	699
L_2-natural PO	846	846	599	598	481	480	429	428	396	396
L_2-solar sail PO	941	940	712	711	571	570	508	508	474	477
SL_2-natural PO	920	919	672	800	551	550	494	493	458	457
SL_2-solar sail PO	1015	1014	784	783	642	647	575	574	509	548

Since the differential corrector and PSOPT are built based on very different mathematics, it is expected to find some discrepancies in the optimised trajectories. Nevertheless, there is good agreement, which shows that the differential corrector finds trajectories very close to optimal. The cases where PSOPT converges to a transfer longer than the initial guess provided with the differential corrector are most likely caused by the close passage of the trajectory to Earth, which can cause PSOPT to struggle to find a solution. Furthermore, for departures from the natural or displaced L_1 point and $\beta = 0.01$, the initial guesses obtained include some revolution around Earth. This seems to cause some convergence difficulties for PSOPT, resulting in slightly more different transfers that the initial guess provided as compared with the remaining cases.

As an example for the transfers obtained, Figure 3 shows the trajectories obtained with the differential corrector for transfers between the L_1 point and L_5 natural periodic orbits. These transfers are the ones used as an initial guess for PSOPT; the optimal solutions obtained are almost indistinguishable from the initial guesses provided.

The method hereby presented successfully obtains fast optimal transfers between the collinear equilibrium points and periodic orbits around the L_5 point. The targeting of whole families of periodic orbits works well, allowing a more versatile design.

The complete manuscript will include cases departing from periodic orbits; both planar and three-dimensional. Furthermore, the case where a solar sail is a secondary payload of a bigger mission will also be studied.

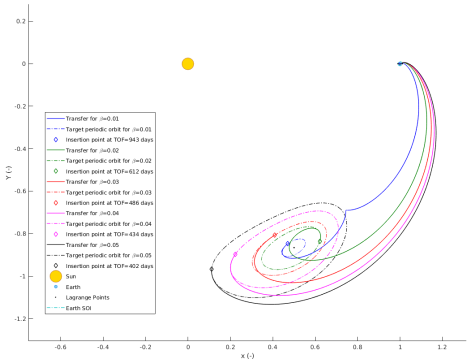

Figure 3: Transfers from the $L1$ point to L_5 natural periodic orbits.

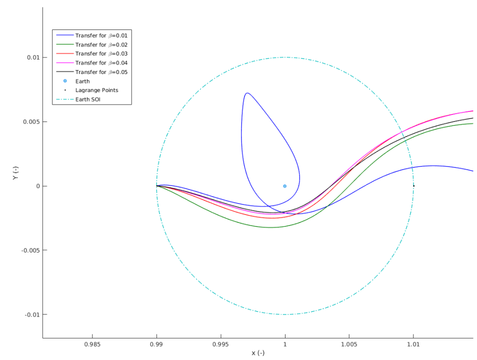

Figure 4: Close-up of the transfers in the neighbourhood of Earth.

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