



Safety and Mission Assurance (SMA)



Burden versus Capability Statistical Analysis for Structural Probabilistic Risk Assessments

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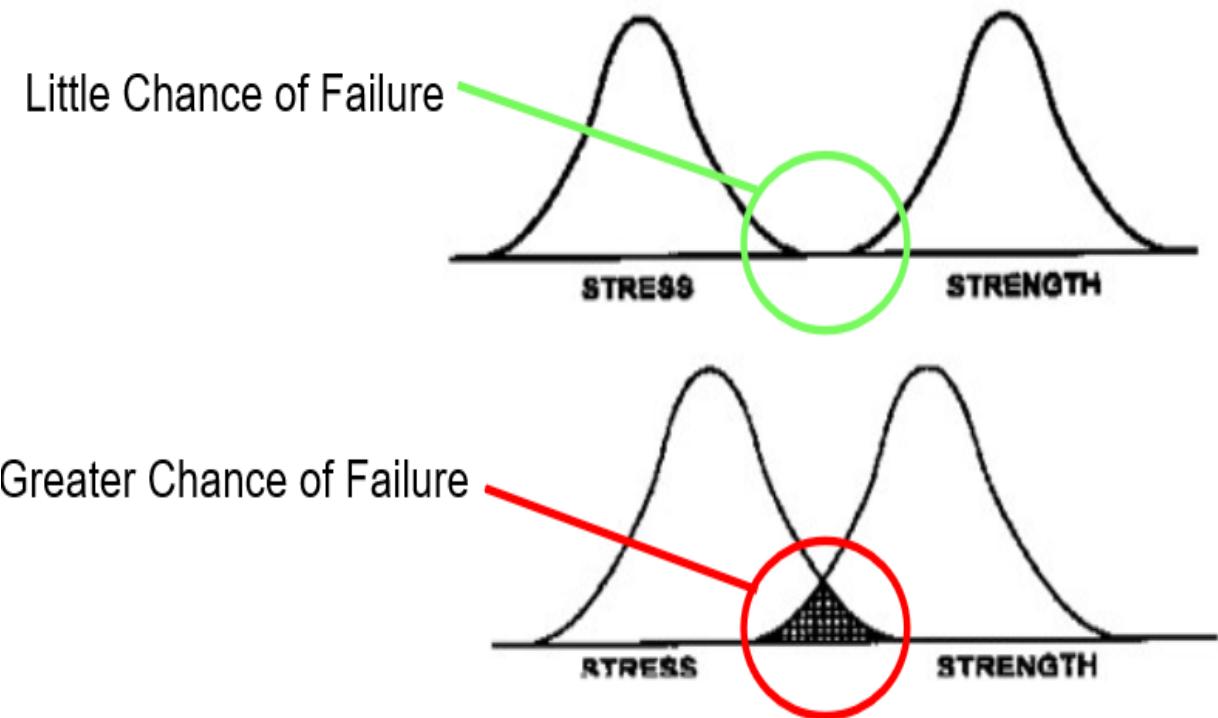


Background



- The purpose of the burden versus capability analysis is to analyze the ability of components to withstand the loads that they are subjected to
- When designing a launch vehicle, there is always a trade-off for the strength of the components versus the weight of the vehicle
- The vehicle needs to have some margin built in to the design, but this added margin should not add a significant amount of weight to the vehicle
- When the material properties and limits are known, estimated loads can be used to ensure that the vehicle will survive launch loads
- If the variation in the distributions can be quantified, the probability of failure can be estimated more accurately

- A burden versus capability analysis is the analysis of the strength of the component and the interference of the stresses placed on the component
- The overlap of the stress and strength distributions estimates the probability of failure



- The burden versus capability analysis relies on the ratio of the ultimate strength of the component to the stress of the component under design loads
- To simplify calculations, the realized factor of safety and max stress are used in place of the ultimate strength of the component and stress of the component under design loads

$$\frac{\text{Realized Factor of Safety}}{1} = \frac{\text{Ultimate Strength of the Component}}{\text{Stress of Component under Design Loads}}$$

- By definition, as long as the margin of safety is greater than zero, the design is meeting its safety factor requirements
- For example, if the design safety factor is 1.4 and the margin of safety is 0, the realized factor of safety will be 1.4

$$\text{Margin of Safety} = \frac{\text{Failure Load}}{\text{Design Load} \times \text{Design Safety Factor}} - 1$$

$$\text{Margin of Safety} = \frac{\text{Realized Factor of Safety}}{\text{Design Safety Factor}} - 1$$

Estimating the Stress Distribution

- Three parameters are used to create the lognormal distribution for the stress, or burden, estimated for the components:
 - CV_{stress} — The coefficient of variation assumed for the loads that the component is subjected to
 - Z_{Max} — The number of transformed normal standard deviations that is assumed between the loads that are used in the analysis (design loads) and the load mean
 - $Stress_{Max}$ — The stress that is expected for the component when applying the design loads

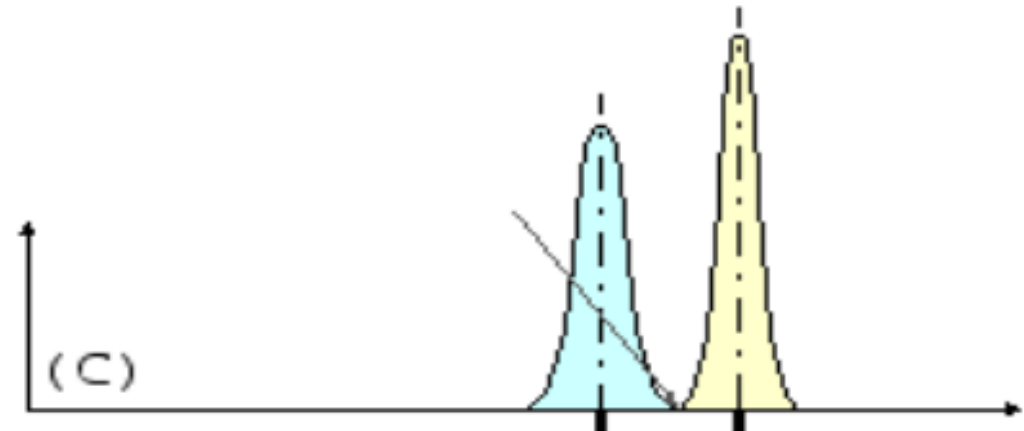
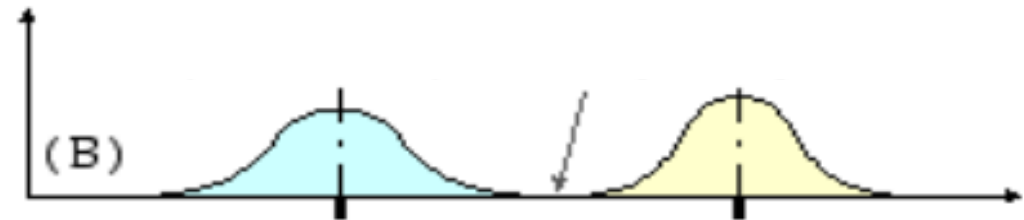
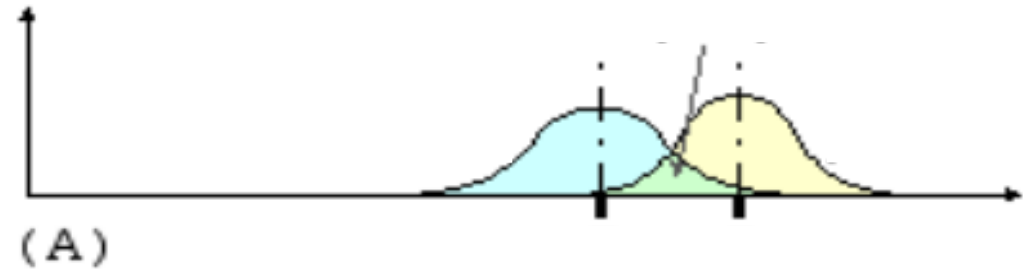
Estimating the Strength Distribution

- Similar parameters are used to create the lognormal distribution for the strength, or capability, estimated for the component:
 - $CV_{Strength(mean)}$ – The coefficient of variation assumed for the strength distribution, which is used to calculate the mean strength
 - $CV_{Strength(prob)}$ – The coefficient of variation assumed for the strength distribution, which is used to calculate the standard deviation of the strength
 - $K_{Strength}$ – The number of transformed normal standard deviations that is assumed between the mean material stress capability and the stress capability assumed in the stress analysis
 - Ultimate strength of the component – The predicted stress needed for the component to fail

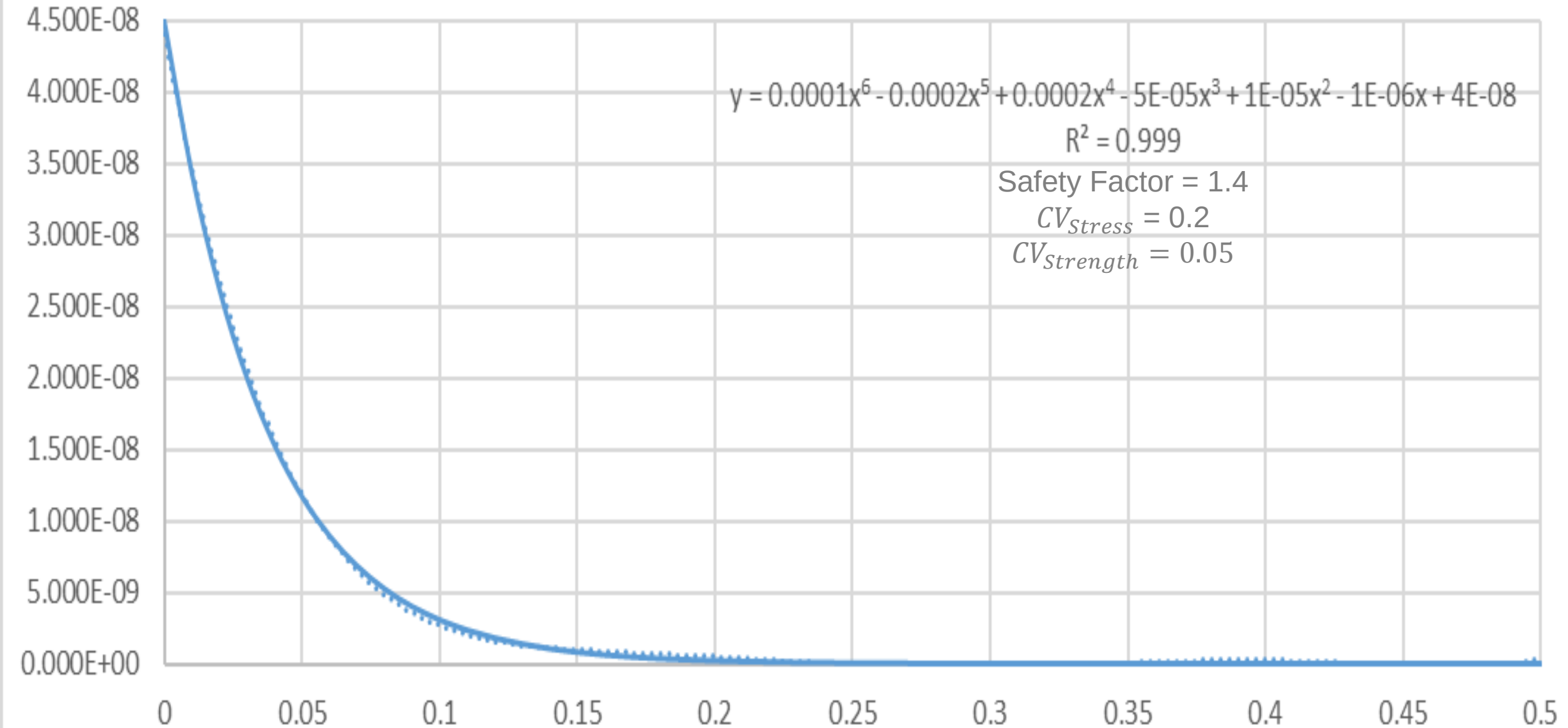
- Assume that both the design load (L) and the material strength (S) are random variables that have lognormal probability density functions (pdf) with parameters, μ_L, σ_L and μ_S, σ_S , respectively
- With these assumptions, $\text{Ln}(L) \approx N(\mu_L, \sigma_L)$ and $\text{Ln}(S) \approx N(\mu_S, \sigma_S)$
- Which leads to $S - L \approx N\left(\mu_L - \mu_S, \sqrt{\sigma_L^2 + \sigma_S^2}\right)$
- Failure occurs when the applied load exceeds the ultimate strength of the structural component, and the probability of failure of the component is calculated as $\text{Pr}(S-L < 0)$
- Using normal distribution theory, we can transform $S-L$ to a standard normal distribution $z \approx N(0, 1)$ by subtracting the means and dividing by the standard deviations

Examples

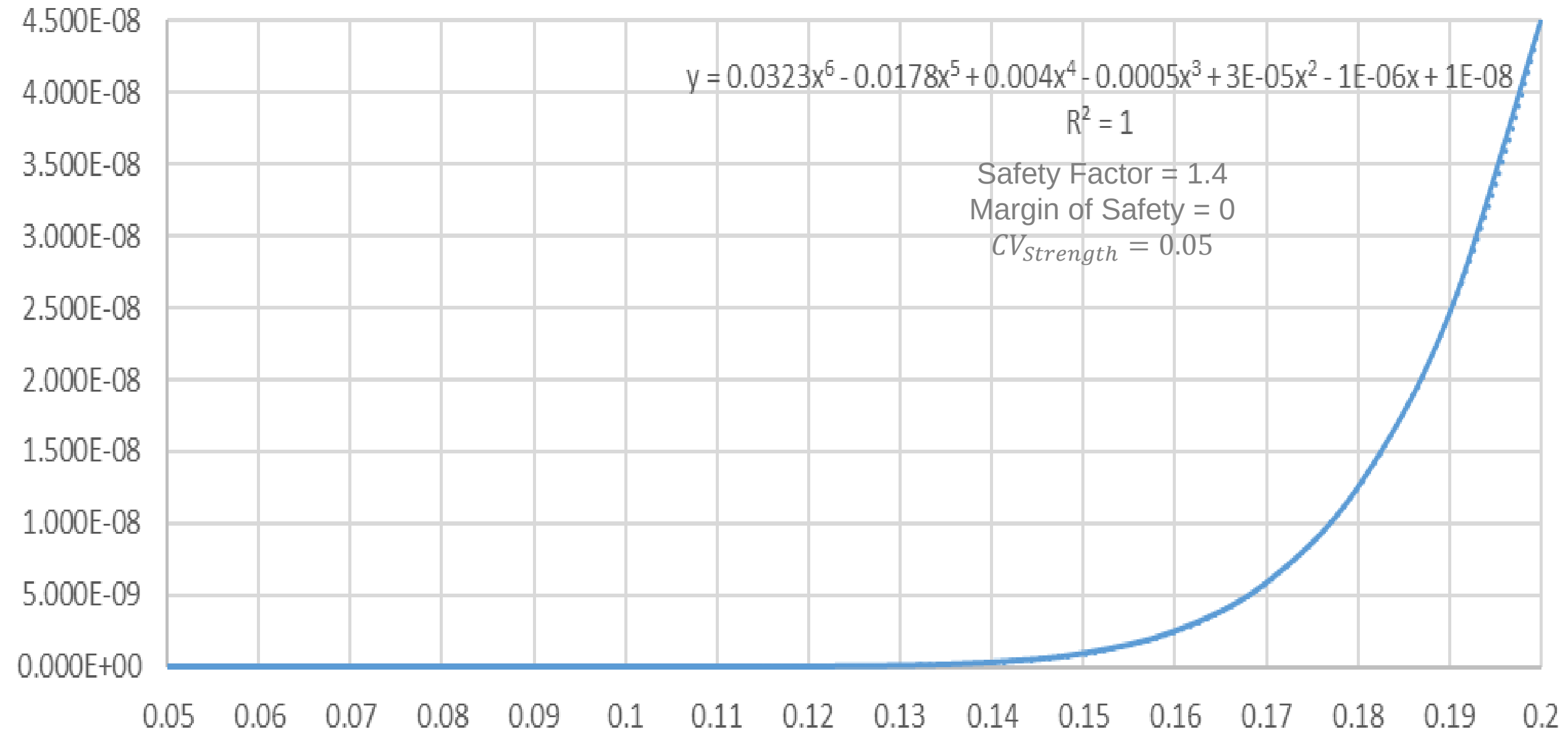
- As can be seen in the three graphs on this slide, the factor of safety and the variability within each of the distributions greatly influences the probability of failure of the component
- Having a high factor of safety and a low variability in the stress and strength distributions can help lower the probability of failure



Probability of Failure versus Margin of Safety



Probability of Failure versus CV-Stress



- Understanding the variability of stresses and strengths is useful in preventing overlap of the stress distribution onto the strength distribution in order to improve the reliability of a design
- Having less variability in the distributions, and having a higher factor of safety, are two ways to help improve the reliability of structural components



References



- <http://slideplayer.com/slide/6207332/>
- <http://www.writeopinions.com/safety-factor>
- http://www.mdp.eng.cam.ac.uk/web/library/enginfo/textbooks_dvd_only/DAN/SSS/safety/strenPops.gif