Simulation of Stochastic Mud-Crack Damage Formation in an Environmental Barrier Coating

Noel Nemeth[1], Subodh Mital[2],

Pappu Murthy[1], Brett Bednarcyk[1], Evan Pineda[1],

Dongming Zhu[1], Haydn Wadley[3], Steven Arnold[1],

Multiscale & Multiphysics Branch, NASA Glenn Research Center

United States Advanced Ceramics Association (USACA) 42nd Annual Conference on Composites, Materials, and Structures

January 22-25, 2018, Radisson Resort at the Port,

Cocoa Beach, Florida

[1]NASA Glenn Research Center Cleveland Ohio; [2]University of Toledo, Toledo, Ohio; [3]University of Virginia, Charlottesville, Virginia

GRC
**Glenn Research Center at Lewis Field**

Abstract:

The integrated Finite Element Analysis–Micromechanics Analysis Code/Ceramics Analysis and Reliability Evaluation of Structures (FEAMAC/CARES) program was used to simulate the formation of mudflat-cracks from thermomechanical loading on a multi-layered Environmental Barrier Coating (EBC) system deposited on a ceramic substrate. FEAMAC/CARES combines MAC/GMC multiscale composite micromechanics code with CARES/Life probabilistic multiaxial failure criteria code and Abaqus finite element analysis. In this work, step function elastic modulus reduction of randomly damaged finite elements was used to represent discrete cracking events. The use of many small-sized low-aspect-ratio finite elements enabled the depiction of crack boundaries and formation of mudflat patterned damage. Demonstrated examples include finite element models of button–sized disk–shaped 3-D specimen, and a 2-D model of through-the-thickness cross-section. All models were subjected to a progressive cool down from 1300° C to room temperature. Mudflat crack damage in the coating system resulted from the buildup of residual tensile stresses between the individual material constituents from thermal expansion mismatch. A 2-parameter Weibull distribution characterized the coating layer stochastic strength response and the effect of the Weibull modulus on the formation of damage was studied here.

Surface cracking allows environmental penetration, accelerating material degradation. Simulating the process / physics of crack formation is necessary for development of an EBC design & life prediction methodology

Mud-flat cracking examples:

Rare earth silicate EBC after heat flux testing
Figure courtesy of Dr. Dongming Zhu

1

NEP51-8 6.0kV 12.0mm x30 SE(L) 2/16/2017   1.00mm
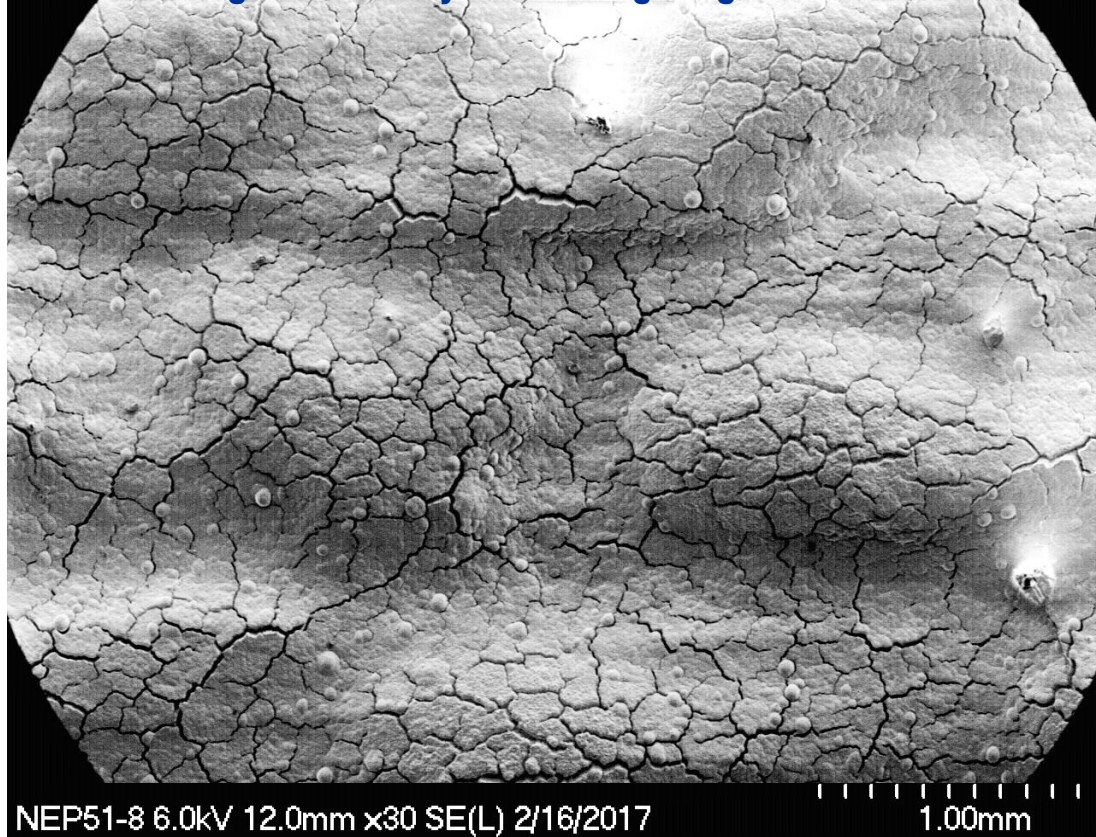
2
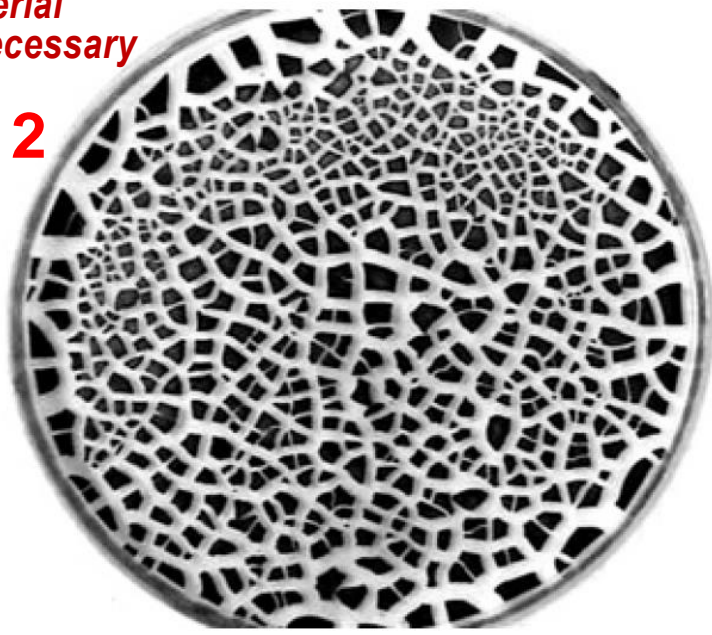
Desiccated layer of synthetic clay suspension on circular petri dish (10 cm in diameter)[a]

3

X-MAS Cookie
Courtesy of Dr. Roy Sullivan

(a) Sadhukhan, et al.: "Desiccation Cracks on Different Substrates: Simulation by a Spring Network Model." J. of Physics: Condensed Matter, Vol. 19, 10pp, 2007.

Objective

1. Demonstrate a generalized life prediction tool for EBC subcomponent subjected to thermomechanical loading that captures some of the salient features of EBC *mechanical* failure modes

2. Demonstrate with this tool that a 2-parmeter Weibull distribution describing brittle material failure strength can stimulate spontaneous formation of mud-flat cracking in a multilayered coating system on a rigid substrate when a thermal cool-down load is applied

Outline

1. FEAMAC/CARES code / methodology

2. Applying FEAMAC/CARES to simulate stochastic damage initiation and progression in an environmental barrier coating (EBC) from thermal cool-down from an initial processing temperature

   - Demonstrate methodology and contrast prediction of damage pattern formation for three different values of Weibull modulus (the Weibull "scatter" parameter or "shape" parameter m) in an **Ytterbium Monosilicate** multi-layered coating system on a silicon carbide substrate

   - 2-D finite element model of EBC coating cross-section
   - 3-D finite element model of 1cm diameter EBC coated disk

FEAMAC/CARES:

Stochastic-strength-based Life Prediction & Component Design of Composites

Combines codes:

- *MAC/GMC composite micromechanics analysis*
- *CARES/Life ceramics reliability analysis*
- *Abaqus finite element analysis*

❖ FEAMAC/CARES Capability:

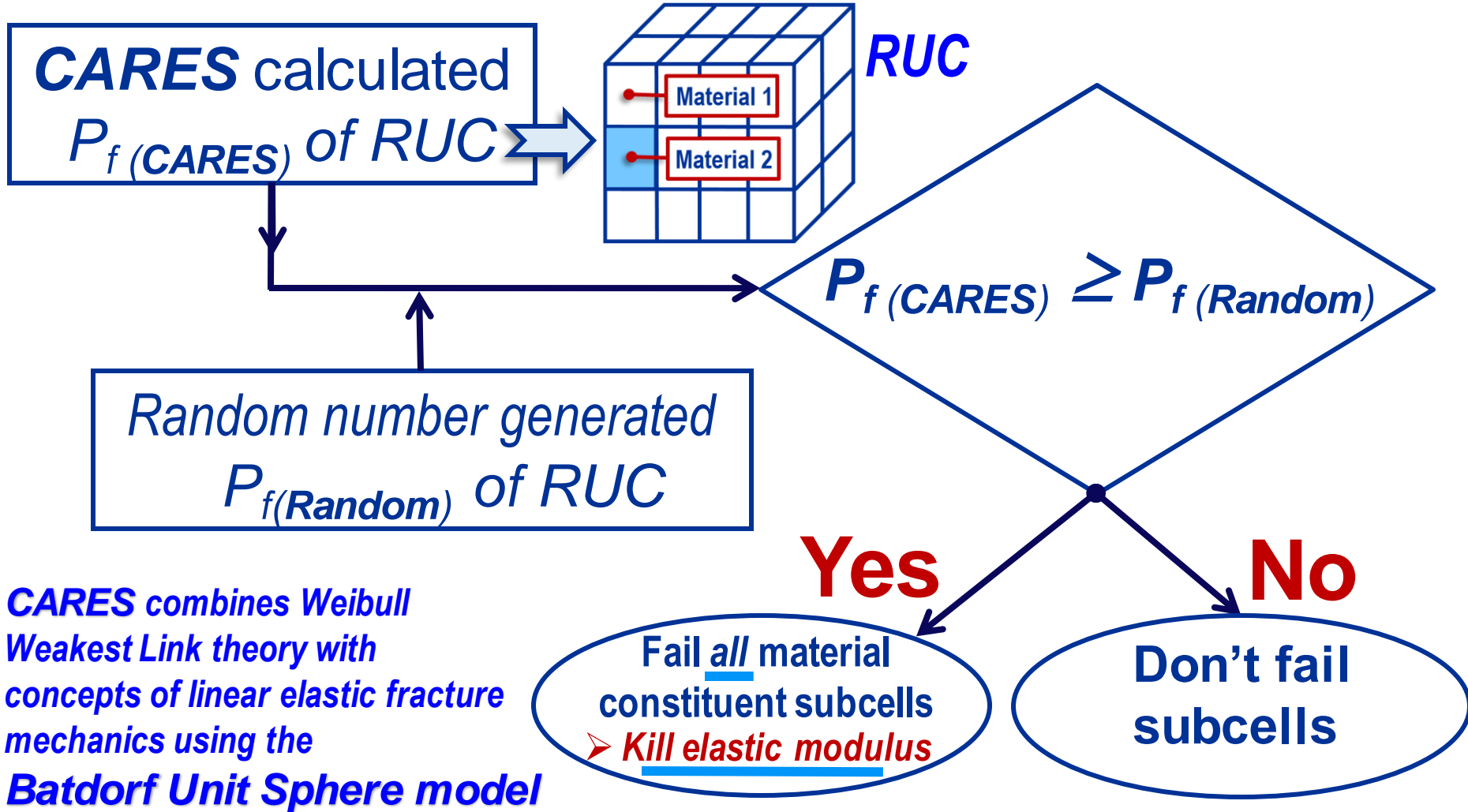- **Individual constituent and component level probability of failure tracked (for failure initiation)**

- **Progressive damage capability/simulation**
  - ➢ **Subcells elastic modulus reduced (killed) at random failure thresholds**

FEAMAC/CARES

$\left\{\begin{array}{c}\textbf{Abaqus UMAT}\\ \textit{"User Material"}\end{array}\right\}$

Structural-Scale FEA

Element/Integration Point

Micromechanics Analysis

(MAC/GMC)

RUC

(CARES)
Subroutine

Reliability analysis at the repeating unit cell (RUC) level

Fiber Interface Matrix

GRC
Glenn Research Center at Lewis Field

NASA

Stochastic Progressive Damage Criterion

Calculate failure probability, P_f , <u>for each material constituent of the RUC</u> associated with an element integration point

CARES calculated $P_{f\,(CARES)}$ of RUC

RUC

Material 1

Material 2

$P_{f\,(CARES)} \geq P_{f\,(Random)}$

Random number generated $P_{f(Random)}$ of RUC

CARES combines Weibull Weakest Link theory with concepts of linear elastic fracture mechanics using the **Batdorf Unit Sphere model**

Yes

No

Fail *all* material constituent subcells
➤ *Kill elastic modulus*

Don't fail subcells

Fracture-mechanics-based *multiaxial* failure criteria to predict probability of failure / damage of a material constituent over time

Unit Sphere
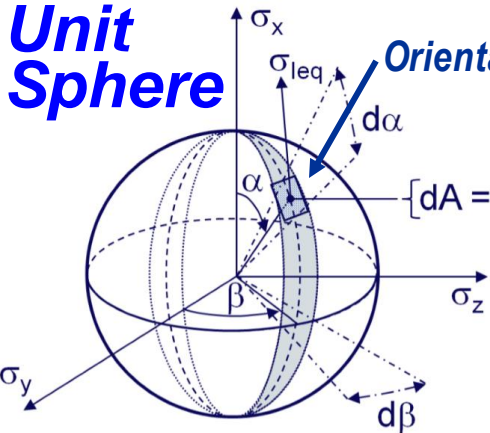
Orientation normal to a crack plane

$$dA = \{(\sin \alpha)\, d\beta\}\, d\alpha$$

- crack shape
- mixed-mode fracture criterion

Unit Sphere Probability Density Distribution For Orientation Of Critical Flaws
(Isotropic Material)

Uniaxial tension

Equibiaxial tension

Shear: shear-sensitive flaws

Uniaxial compression: Compression criterion

σ_Y

Most probable angle (normal to flaw plane)

$\sigma_Y = \sigma_Z$

τ_{YZ}

$-\sigma_X$
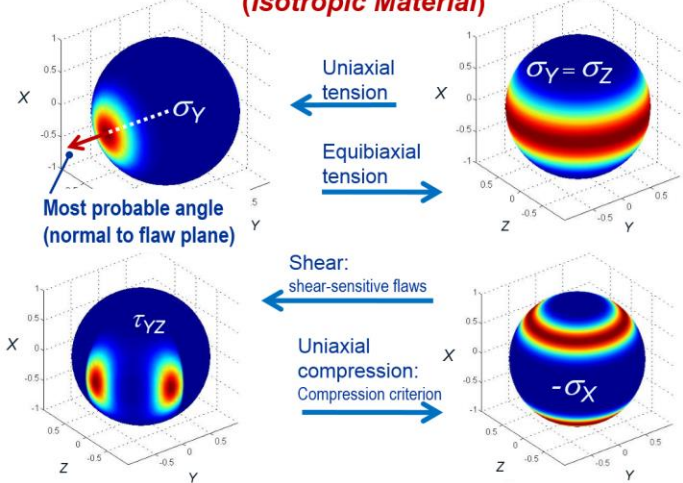
Anisotropic material capability also

Cellular Automaton:
Failure probability thresholds of elements adjacent to failed elements adjusted to promote a biased damage direction

Failed element
Adjacent element
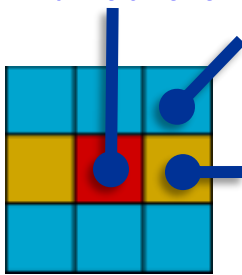
Adjacent 2 elements with highest

$P_{f\,(CARES)}$ **has**

$P_{f\,(Random)}$ **adjusted**

0° Composite ply for 25x25 FEA mesh of shell elements

Random element failure:
➤ simulates stochastic toughening

Loading and fiber Direction
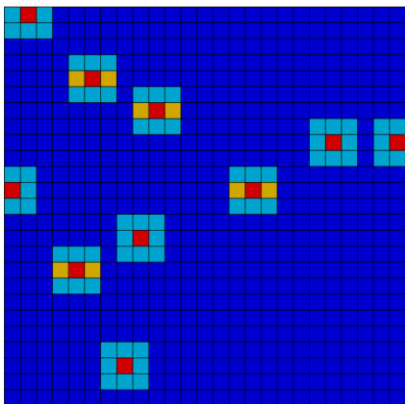
Cellular automaton:
➤ "crack-like" growth patterns
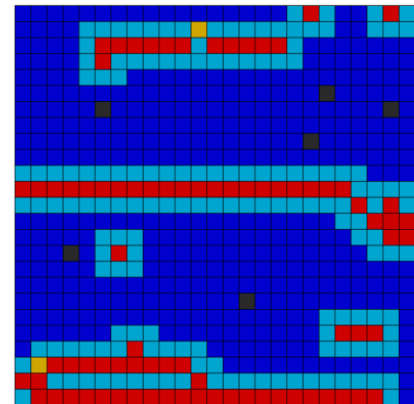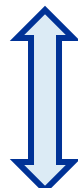
Environmental Barrier Coating Failure/Life Modeling

Environmental barrier coatings (EBCs) display complex failure modes that evolve with time and fluctuating load
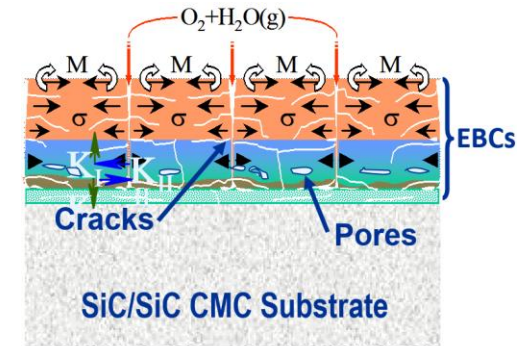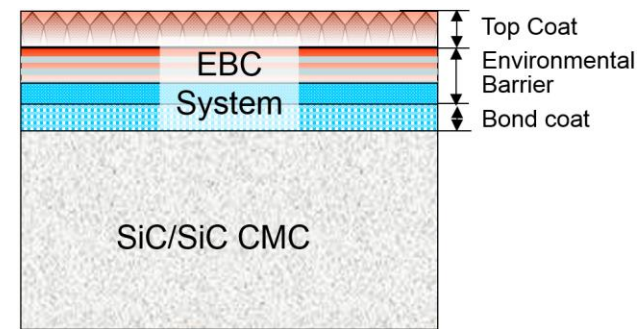

APPROACH:

Use FEAMAC/CAREs to simulate stochastic damage initiation and progression in an environmental barrier coating (EBC) from thermal cool-down from an initial processing temperature

Generalized EBC Failure Mechanisms

APPLIED TO:

A Ytterbium monosilicate (YBSM) EBC on a silicon carbide substrate undergoing thermal cool down from an initial processing or annealing temperature of 1300° C to 23° C

DESIRED RESULTS & BENEFITS:

❑ **Predict onset of coating microcracking; crack propagation, delamination, and spallation**

❑ **Demonstrate ability to reproduce or simulate formation of mud-flat cracking and create a "parameter space" with which these failure mechanisms can be explored and controlled**

❑ **Demonstrate a physically based model that more accurately reproduces progressive damage failure modes under generalized transient loading conditions**
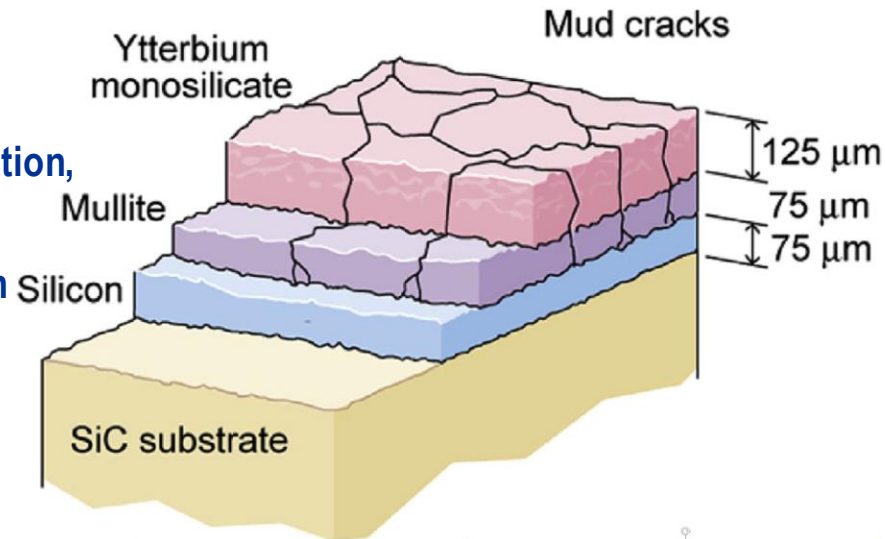

Figure adapted from Richards et al.: Fracture Mechanisms of Ytterbium Monosilicate Environmental barrier coatings during cyclic thermal exposure" Acta Materialia, 103, pp. 448-460, 2016.

Environmental Barrier Coating Failure/Life Modeling
Our approach…

❑ **Each coating layer is modeled as a discrete material component in FE model**

❑ **each coating layer is described by a separate MAC/GMC input file.**

❑ **A very simple MAC/GMC repeating unit cell (RUC) of a material element:**

> ➤ **RUC consists of only *a single (monolithic) material subcell* representing one of the constituent materials of the EBC material system or substrate.**

❑ **Damage (defined as failure) for an element or element integration point is an abrupt (99%) step function stiffness reduction**

❑ **As a consequence; the finite element model uses many small cubic-shaped elements because a failed element is representative of a crack or discontinuity**

❑ **Irregular top surface (surface roughness) is not initially modeled here with finite elements. Instead a low Weibull modulus top material layer can be optionally substituted to imitate the wide scatter in fracture strength the top surface would induce**

2-D EBC Cross-Section FE model (2mm X 1mm)

27,600 S4R reduced integration shell elements

Edges are free:

No constraint or periodic boundary conditions;
Bottom of substrate: fixed in y direction

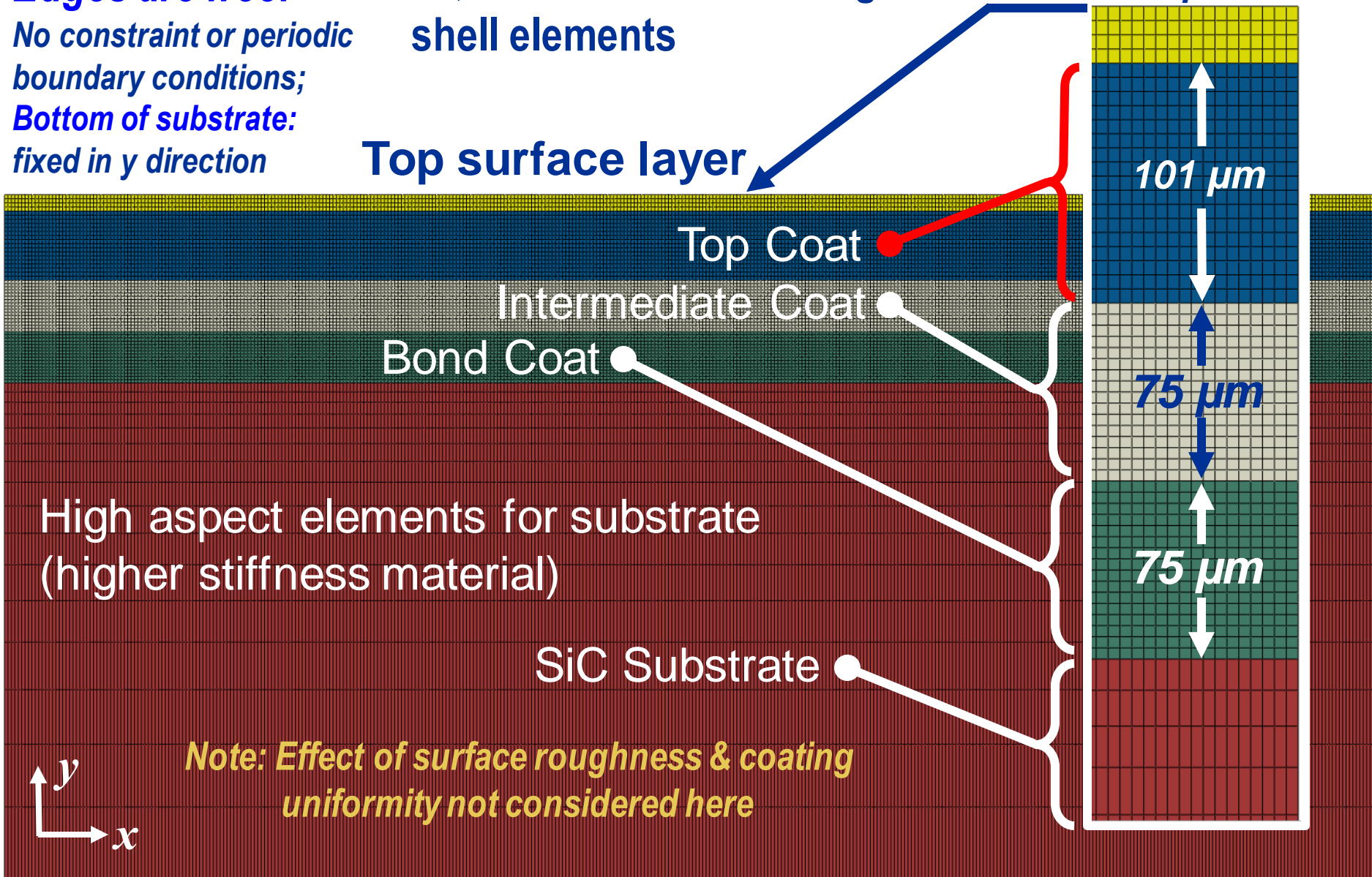**Top surface layer**

24 µm

Top Coat

Intermediate Coat

Bond Coat

101 µm

75 µm

High aspect elements for substrate
(higher stiffness material)

75 µm

SiC Substrate

Note: Effect of surface roughness & coating uniformity not considered here

y

x

Properties Table

(Tensile strength/Weibull parameters adopted from Abdul-Aziz et al.[2], None listed in Richards et al[1])

Report	Material	E (Gpa)	Poisson, v	Therm. Expan., α, m/m/$^oC \cdot 10^{-6}$	Weibull modulus, m	Weibull Scale Parameter, σ_o, MPa·mm$^{3/m}$
YBSM (Richards et al. 2016)[1] Using 50% reduced properties listed in reference	Top Coat Suface: Ytterbium monosilicate	86	0.27	7.5	2.5 ; 5. ; 10.	14 (assumed)
	Top Coat: Ytterbium monosilicate	86	0.27	7.5	2.5 ; 5. ; 10.	28 (assumed)
	Intermediate Coat: Mullite	110	0.28	5.3	2.5 ; 5. ; 10.	28 (assumed)
	Bond Coat: Silicon	82	0.223	4.1	2.5 ; 5. ; 10.	40 (assumed)
	Substrate: SiC (Monolithic)	430	0.14	4.6	2.5 ; 5. ; 10.	321 (assumed)

[1] Richards et al.: Fracture Mechanisms of Ytterbium Monosilicate Environmental barrier coatings during cyclic thermal exposure" Acta Materialia, 103, pp. 448-460, 2016..

[2] Abdul-Aziz, et al.: "Durability Modeling of Environmental Barrier Coating (EBC) Using Finite Element Based Progressive Failure Analysis" J. of Ceramics, pp. 1-10, 2014
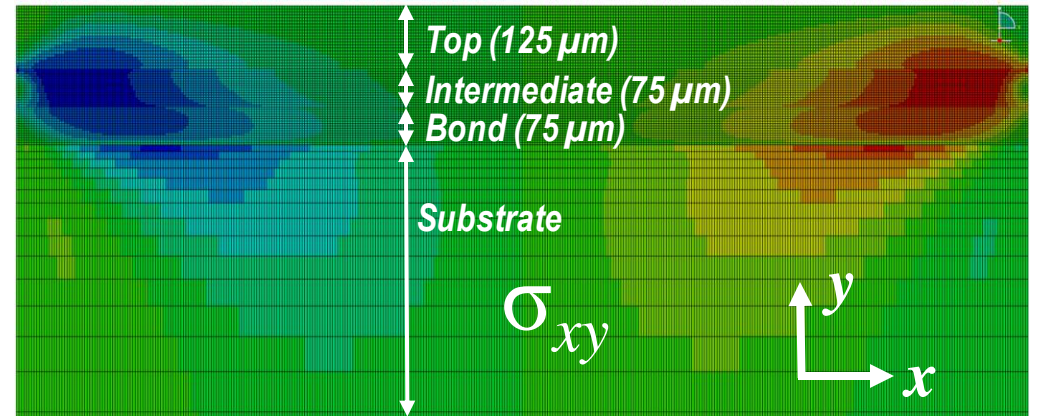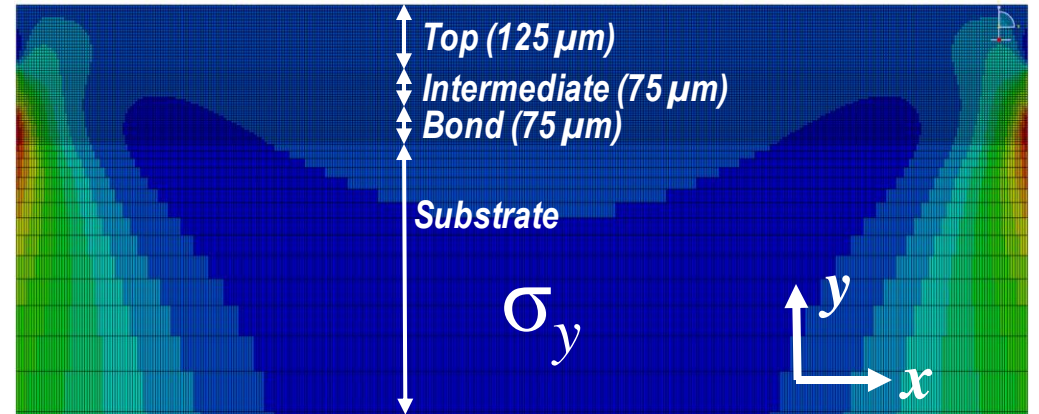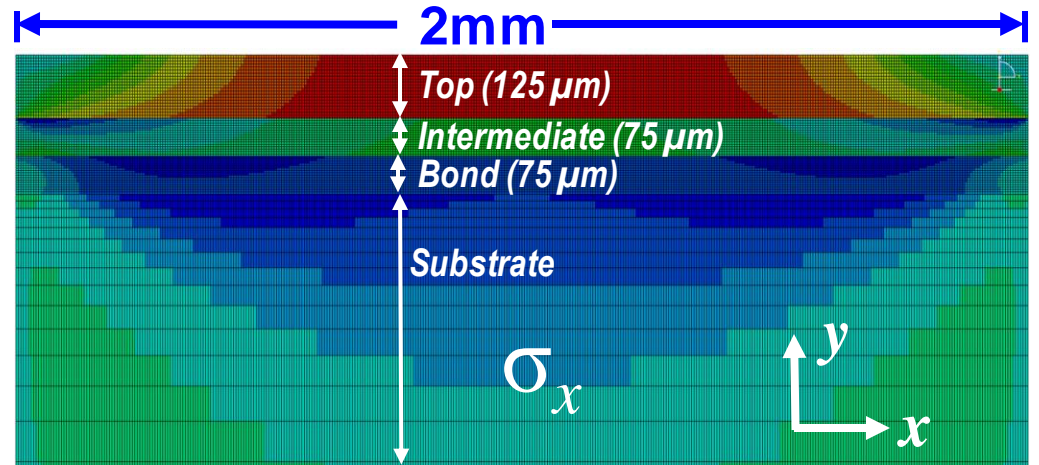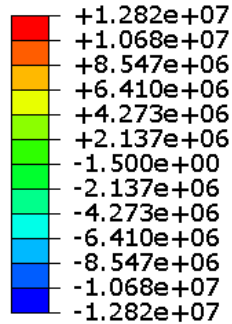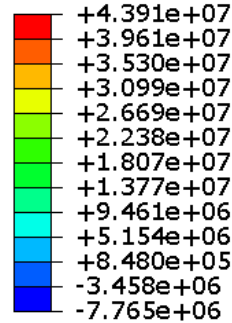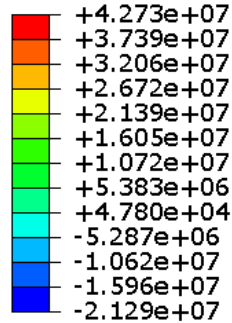
Ytterbium Monosilicate (YBSM)

Stresses (Pa) in undamaged EBC layers :
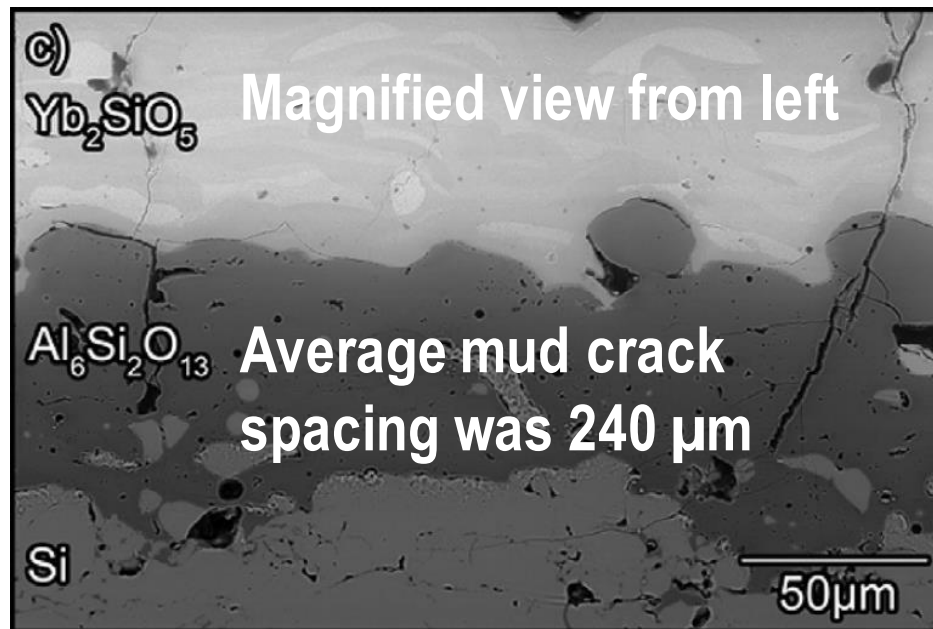
200° C cool-down from annealing Temp.

Left & right edges unconstrained
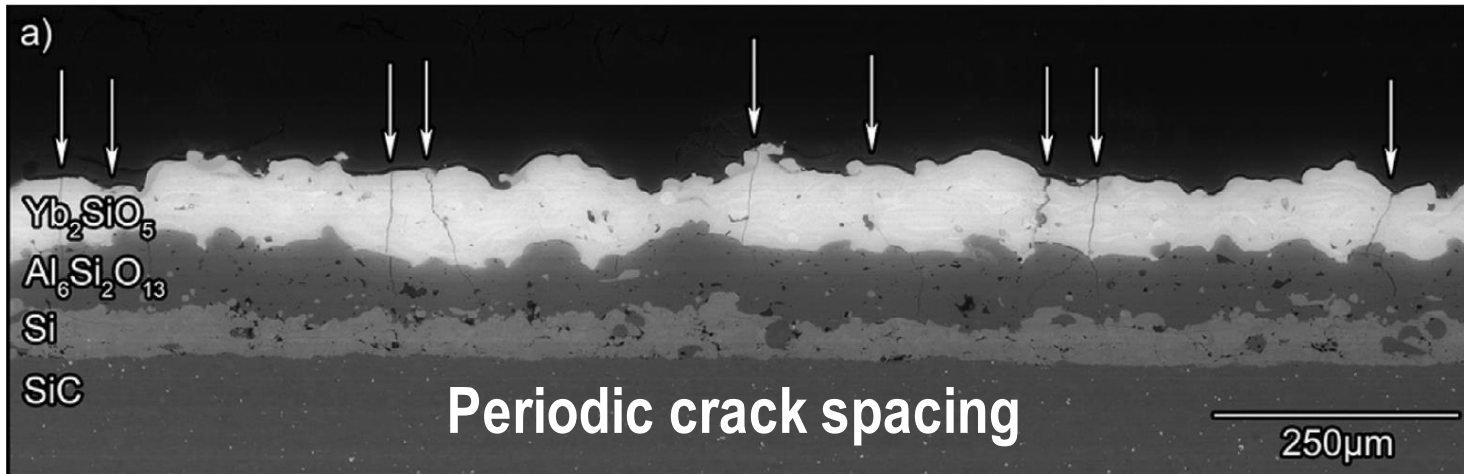
Bottom of substrate fixed in y direction

2mm

Top (125 μm)
Intermediate (75 μm)
Bond (75 μm)
Substrate

+4.273e+07
+3.739e+07
+3.206e+07
+2.672e+07
+2.139e+07
+1.605e+07
+1.072e+07
+5.383e+06
+4.780e+04
-5.287e+06
-1.062e+07
-1.596e+07
-2.129e+07

σ_x

Top (125 μm)
Intermediate (75 μm)
Bond (75 μm)
Substrate

+4.391e+07
+3.961e+07
+3.530e+07
+3.099e+07
+2.669e+07
+2.238e+07
+1.807e+07
+1.377e+07
+9.461e+06
+5.154e+06
+8.480e+05
-3.458e+06
-7.765e+06

σ_y

Top (125 μm)
Intermediate (75 μm)
Bond (75 μm)
Substrate

+1.282e+07
+1.068e+07
+8.547e+06
+6.410e+06
+4.273e+06
+2.137e+06
-1.500e+00
-2.137e+06
-4.273e+06
-6.410e+06
-8.547e+06
-1.068e+07
-1.282e+07

σ_{xy}

EBC coating after 1300 C annealing of as processed coating
Figure 6 from Richards et al.

a) Periodic crack spacing

Yb$_2$SiO$_5$
Al$_6$Si$_2$O$_{13}$
Si
SiC

250μm

Channel crack

Fig. 1 from Richards et al.

b) Yb$_2$SiO$_5$
← mud crack →
Al$_6$Si$_2$O$_{13}$
Si
SiC

Magnified view

100μm

c) Yb$_2$SiO$_5$
Magnified view from left

Al$_6$Si$_2$O$_{13}$
Average mud crack spacing was 240 μm

Si

50μm
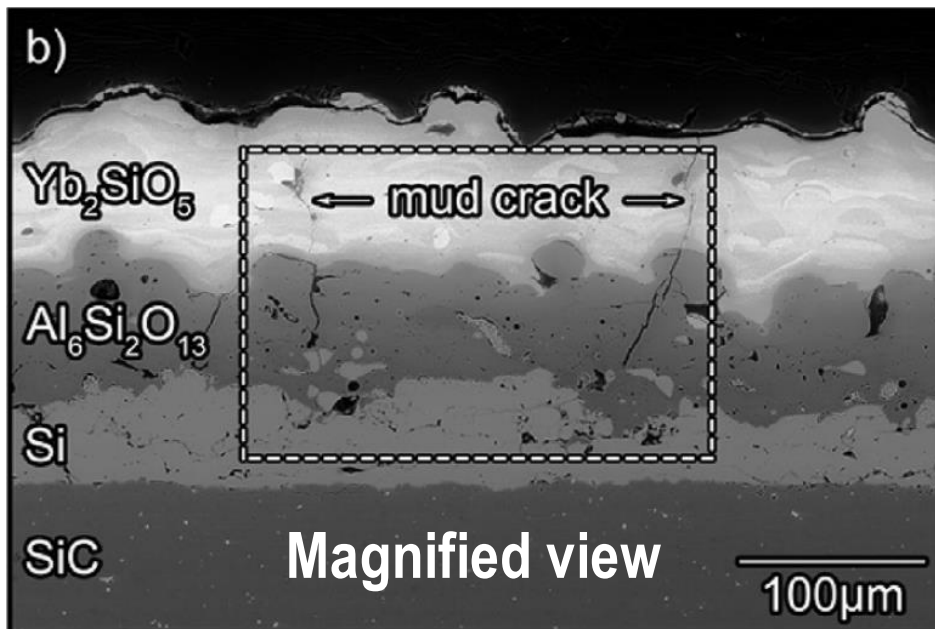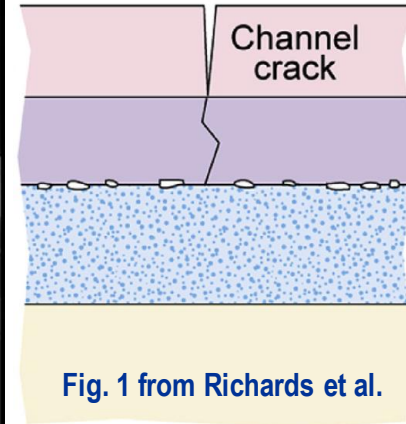
Ytterbium Monosilicate: 2D cross-sectional model
Damage at 23° C increment 810 *Complete substrate not shown*

Top coat:
low strength layer
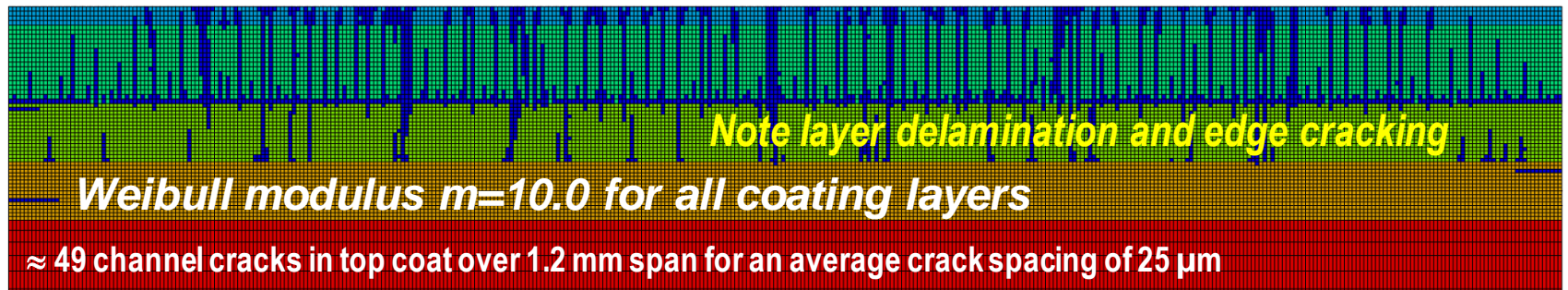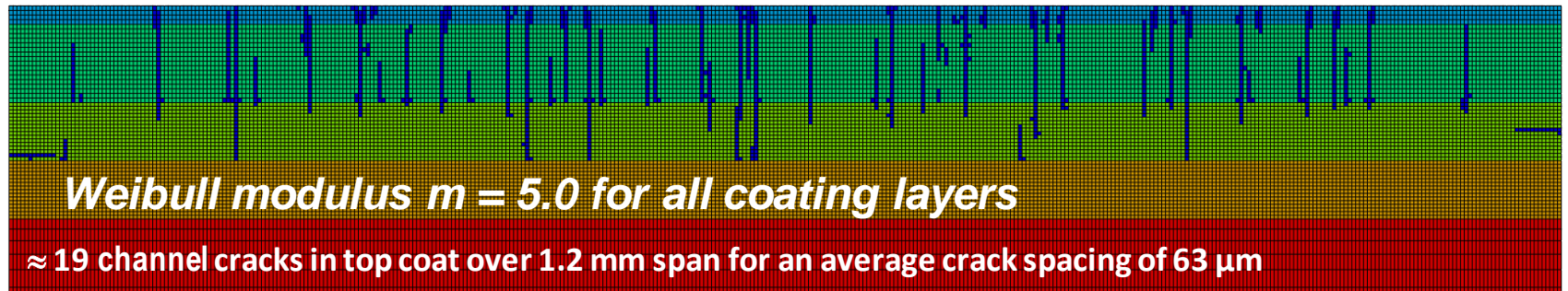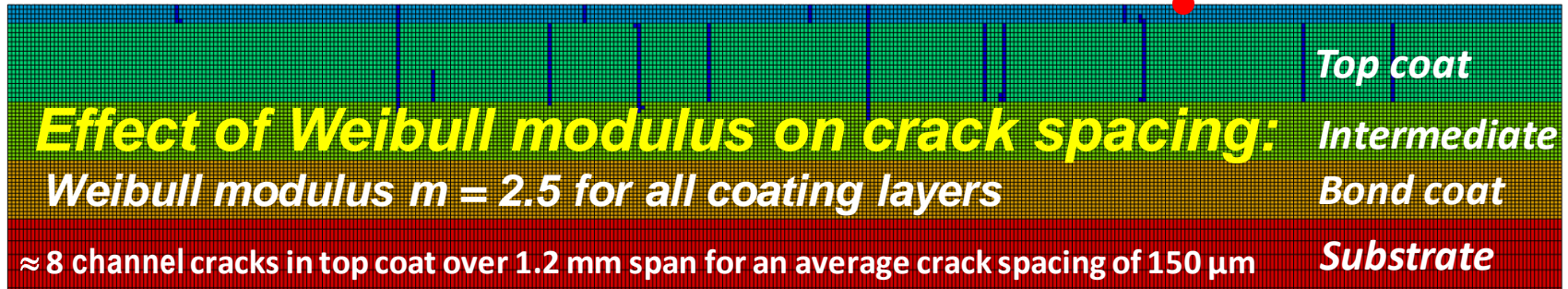
Effect of Weibull modulus on crack spacing:

Top coat

Weibull modulus m = 2.5 for all coating layers

Intermediate

Bond coat

≈ 8 channel cracks in top coat over 1.2 mm span for an average crack spacing of 150 μm

Substrate

Weibull modulus m = 5.0 for all coating layers

≈ 19 channel cracks in top coat over 1.2 mm span for an average crack spacing of 63 μm

Note layer delamination and edge cracking

Weibull modulus m=10.0 for all coating layers

≈ 49 channel cracks in top coat over 1.2 mm span for an average crack spacing of 25 μm

But we don't know the temperature when microcracking initiated !

810 load increment steps (ramp load):
- First 10 early cool-down steps from 1300° C to 1200° C
- 800 increment steps from 1200° C to 23° C

3-D FE model of EBC (top coat (blue and light green), intermediate coat (green), bond coat (orange), on a rigid SiC substrate (red))

10mm dia. *X* 1mm thick disk model with about 280,000 solid C3D8R reduced integration elements
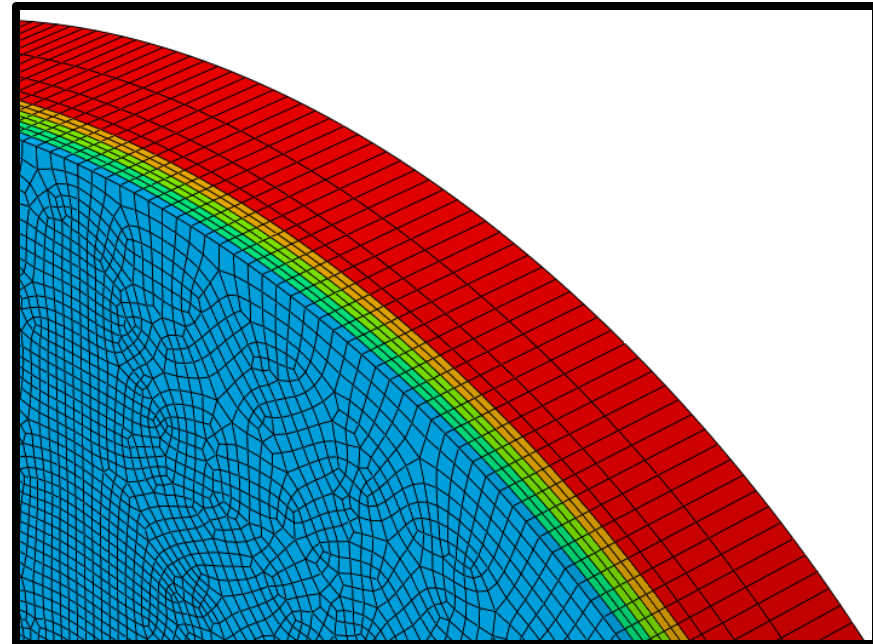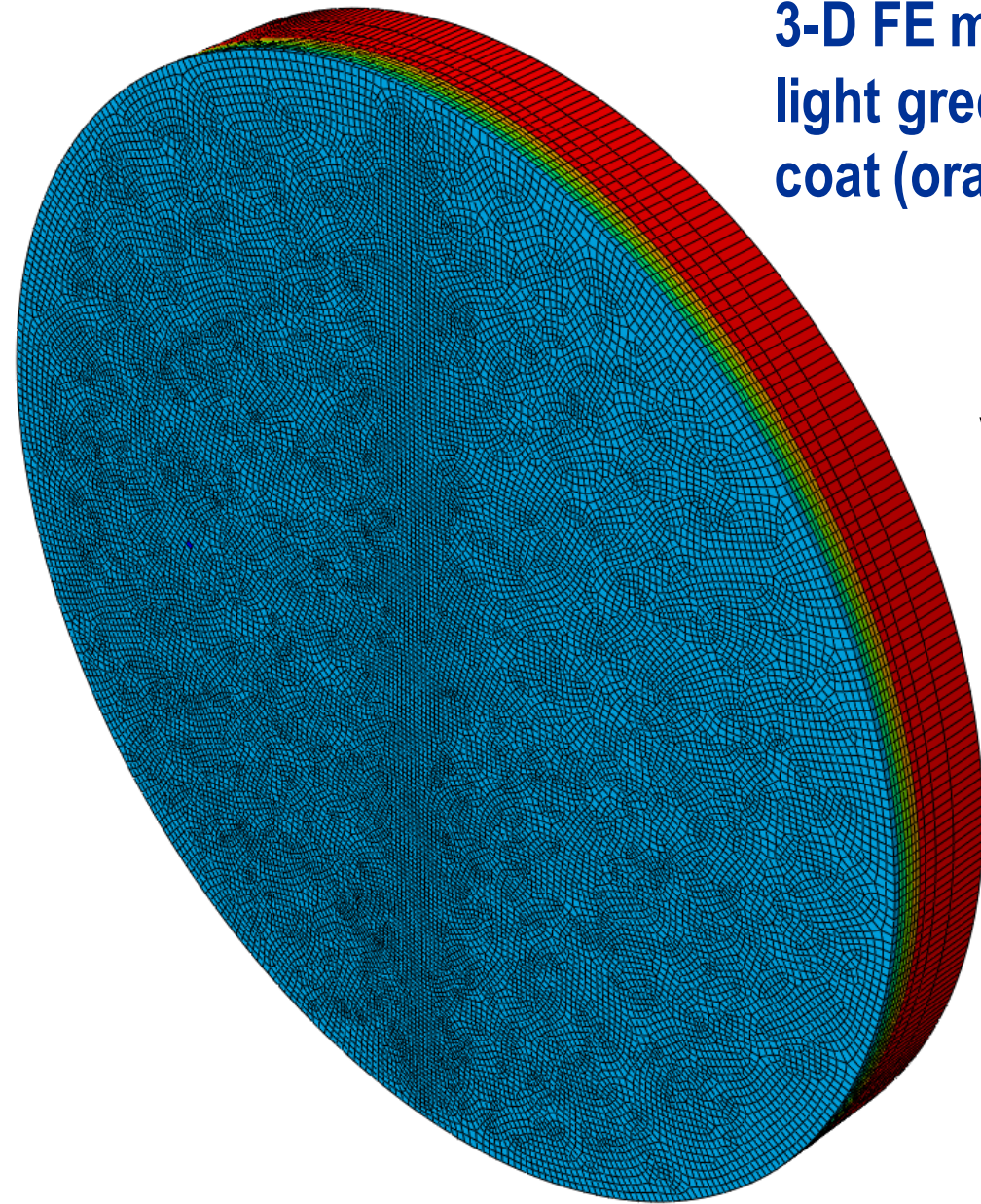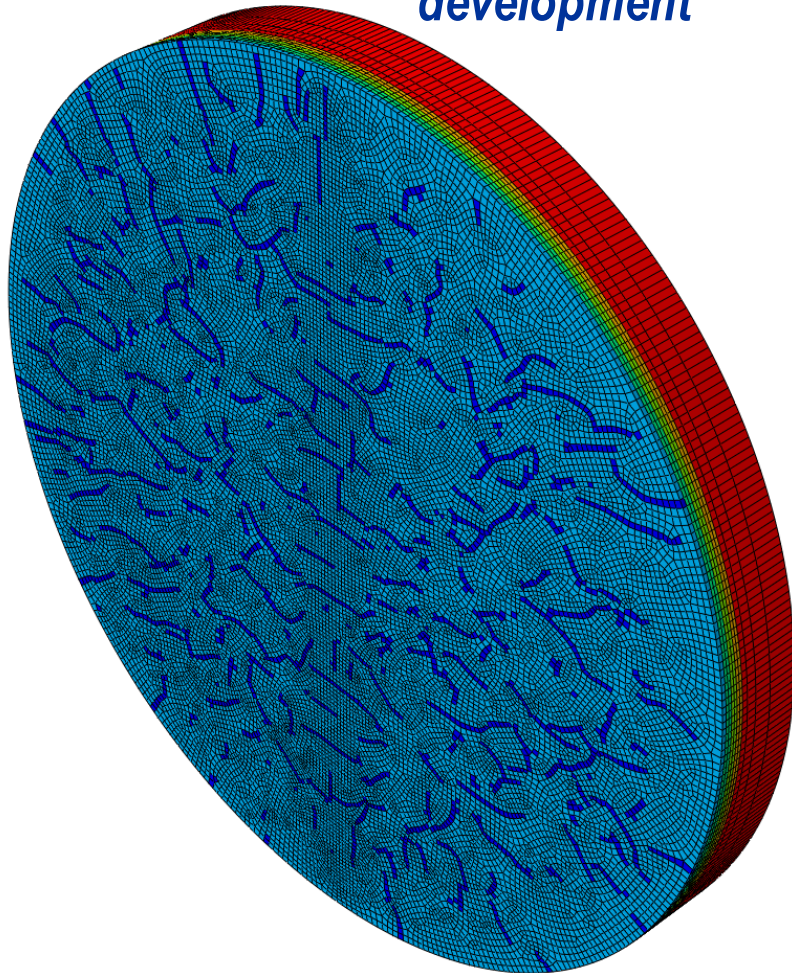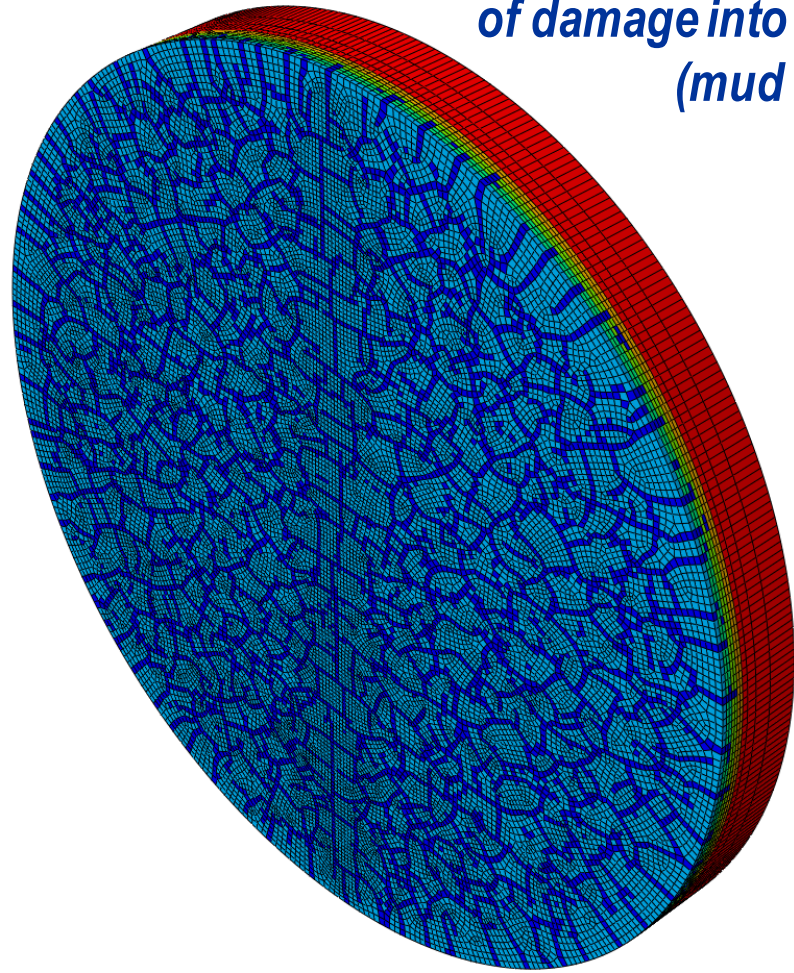
Damage Pattern In 3-D EBC Stochastic Progressive Damage Simulation
Spontaneous development of mud flats fragment into progressively smaller sizes when cooling from 1300º C to room temperature (fractal fracture pattern)

Early damage development

Advanced development of damage into cells (mud flats)

Weibull modulus m = 5

EBC coating cool down from 1300° C annealing/processing temp.
Simulation shows qualitative resemblance to mudflat cracking experiment

FEAMAC/CARES simulation of YBMS EBC

**Real life:
Synthetic clay in petri dish**

*Shown in black
& white*
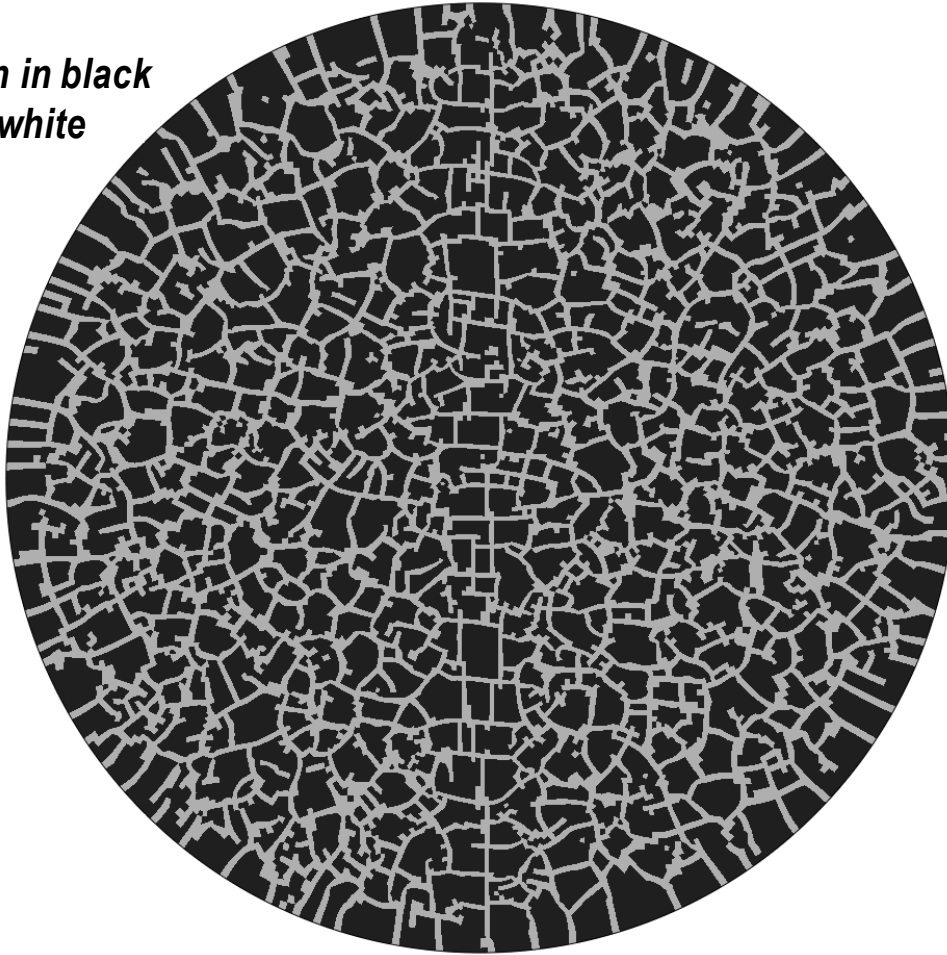
1 cm dia. EBC multilayer coating on SiC substrate
Weibull modulus m = 5.0
758.6° C (increment 160)
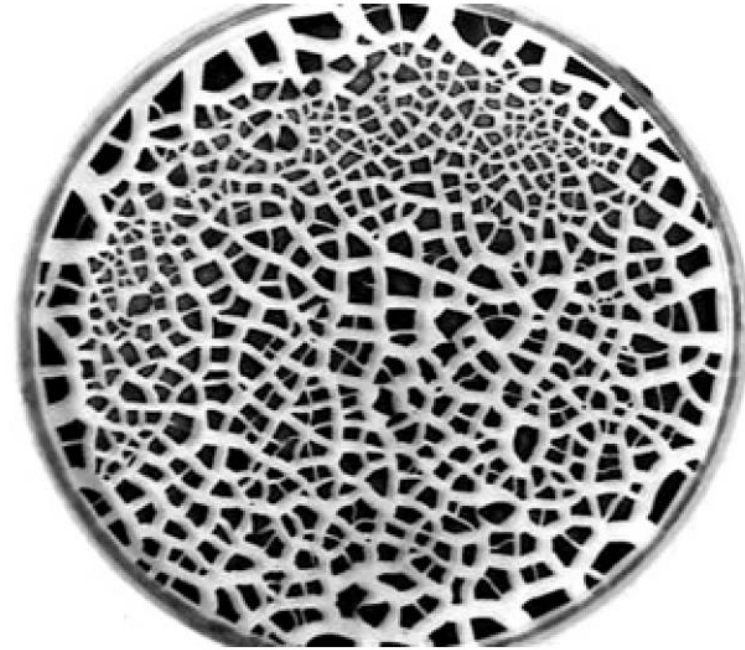
Desiccated layer of synthetic clay
suspension on circular petri dish
(10 cm in diameter)[a]

Sadhukhan, et al.: "Desiccation Cracks on
Different Substrates: Simulation by a Spring
Network Model." J. of Physics: Condensed
Matter, Vol. 19, 10pp, 2007.

Channel cracks on Periphery :

Damage on top surface for EBC coating Cool down from 1300° C to 23° C

Weibull modulus *m* = 2.5
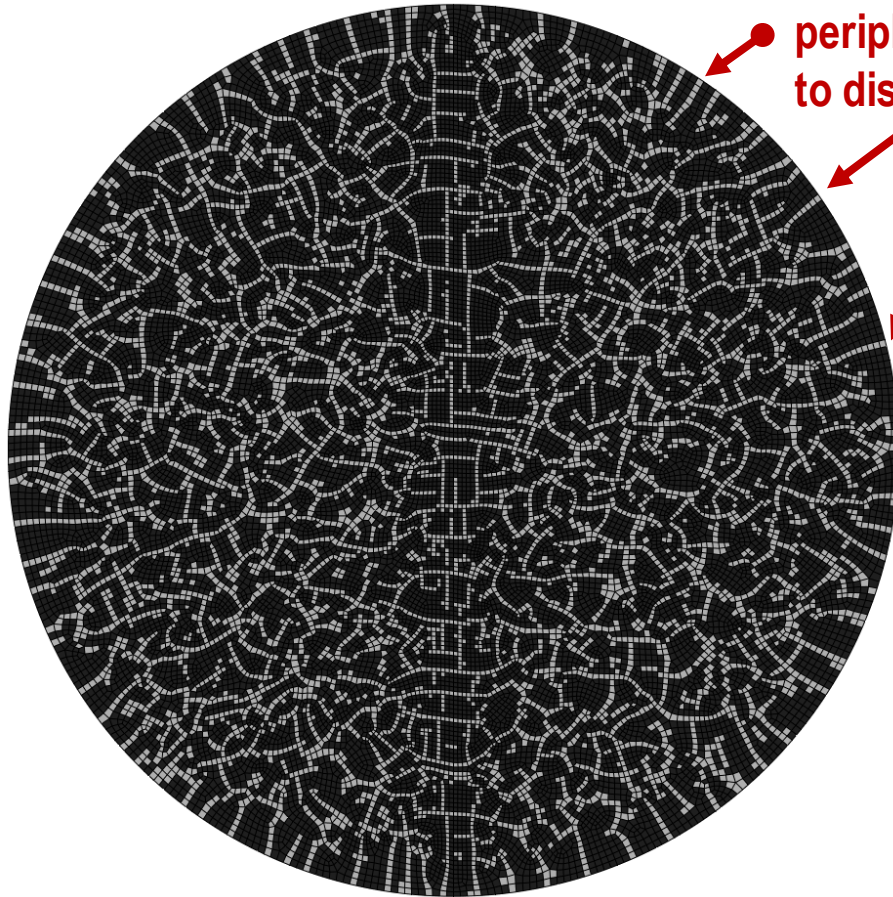
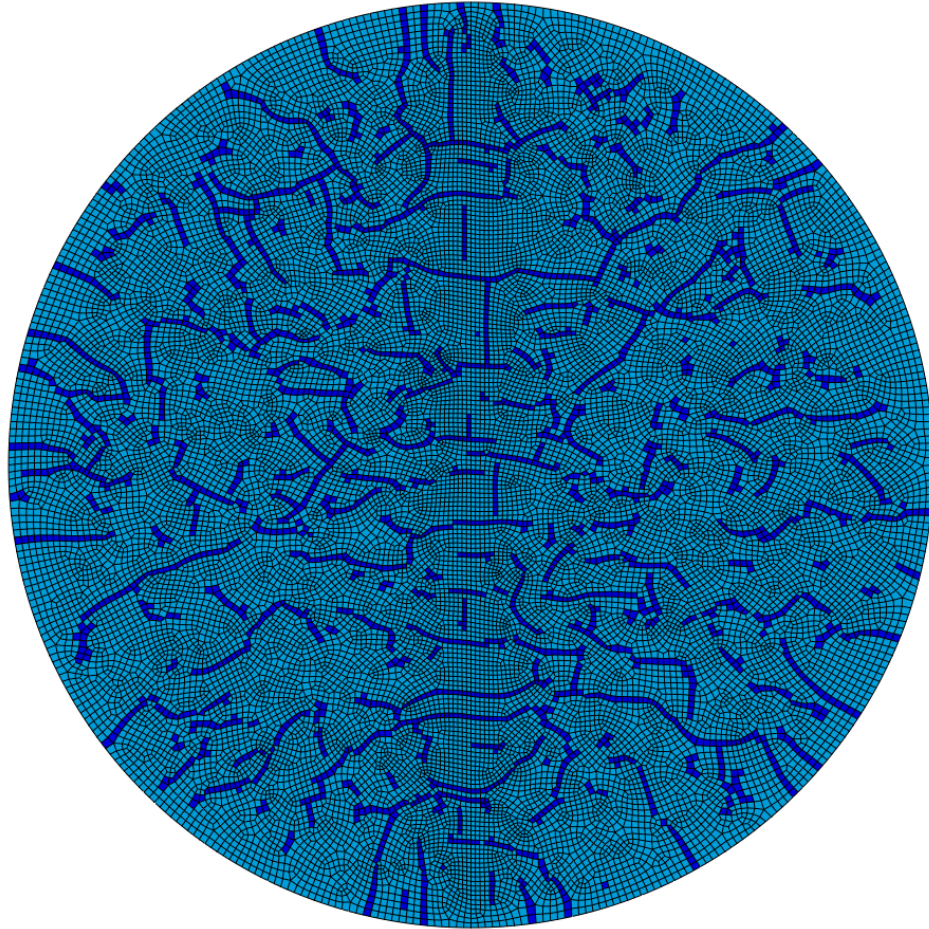**Channel cracks on periphery are normal to disk edge**

***m* = 5.0**
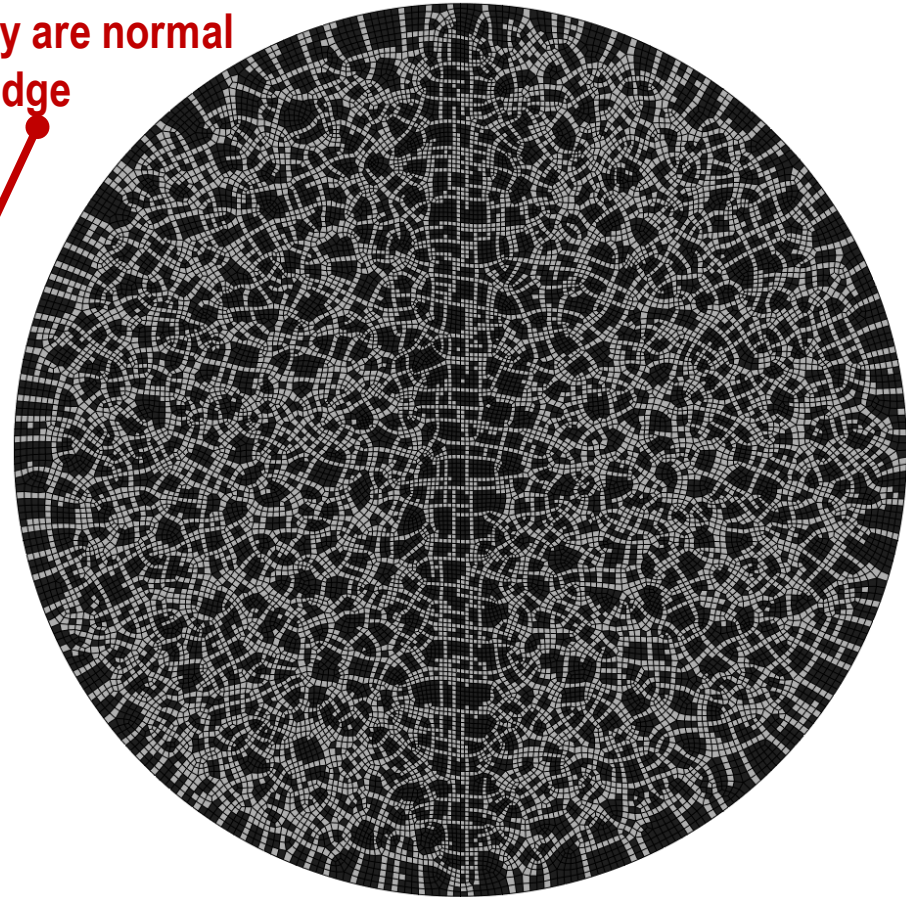
≈ 77 Channel cracks about periphery
for 408 *μm* average crack spacing

≈ 102 Channel cracks about periphery
for 308 *μm* average crack spacing

Damage of top surface for EBC coating:
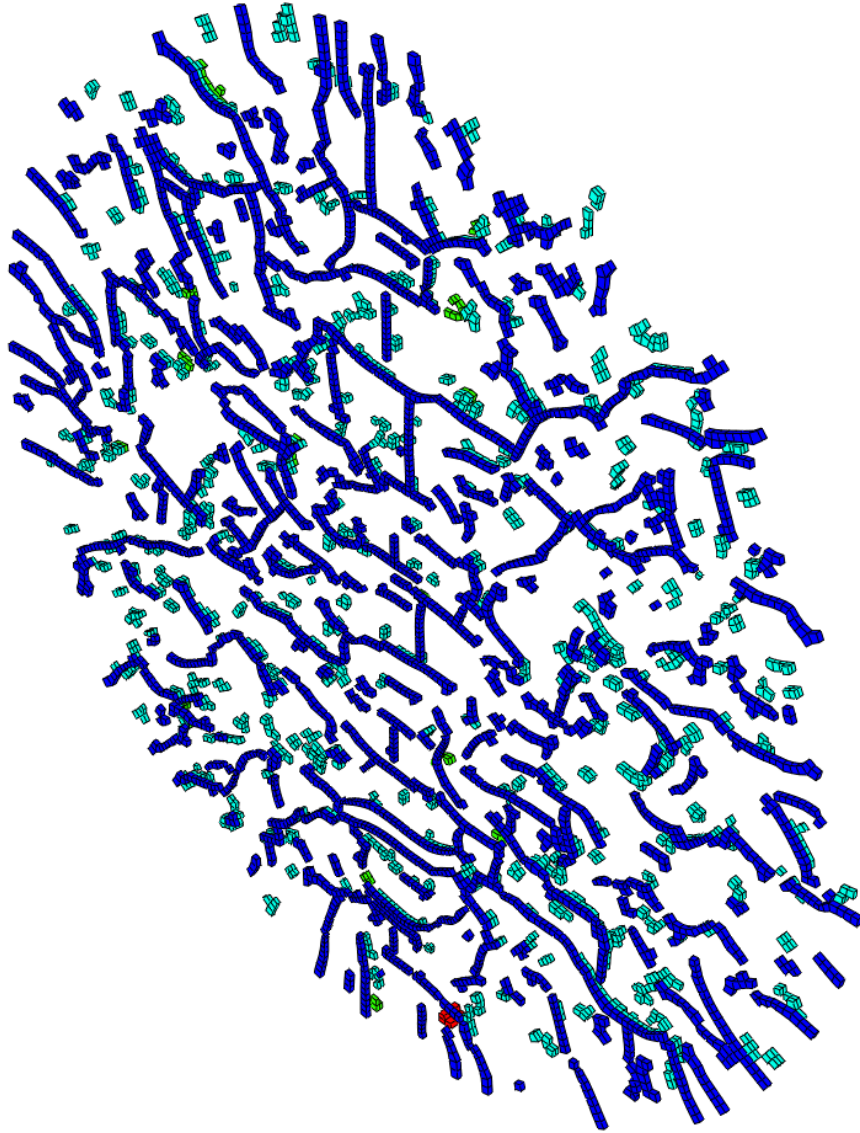## *Weibull modulus m = 2.5*
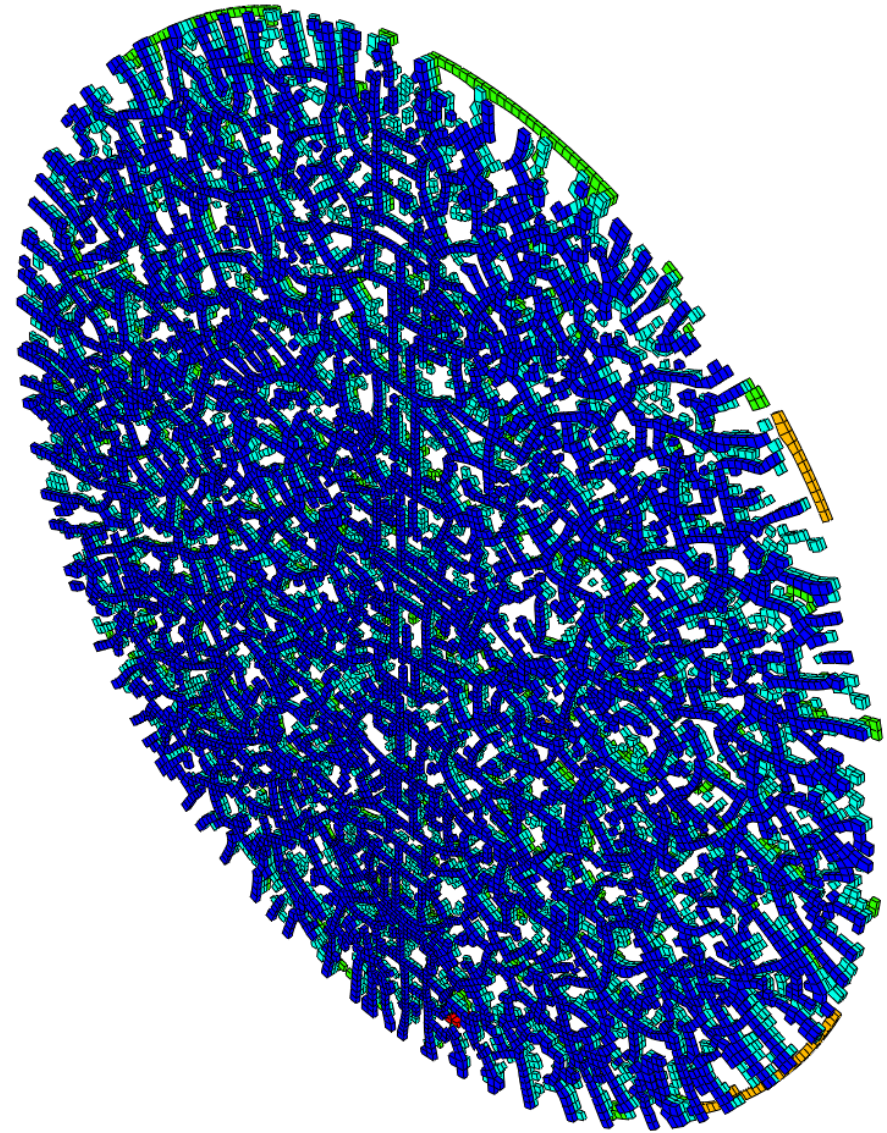

905.8° deg. C (110 steps)

23° deg. C (410 steps)

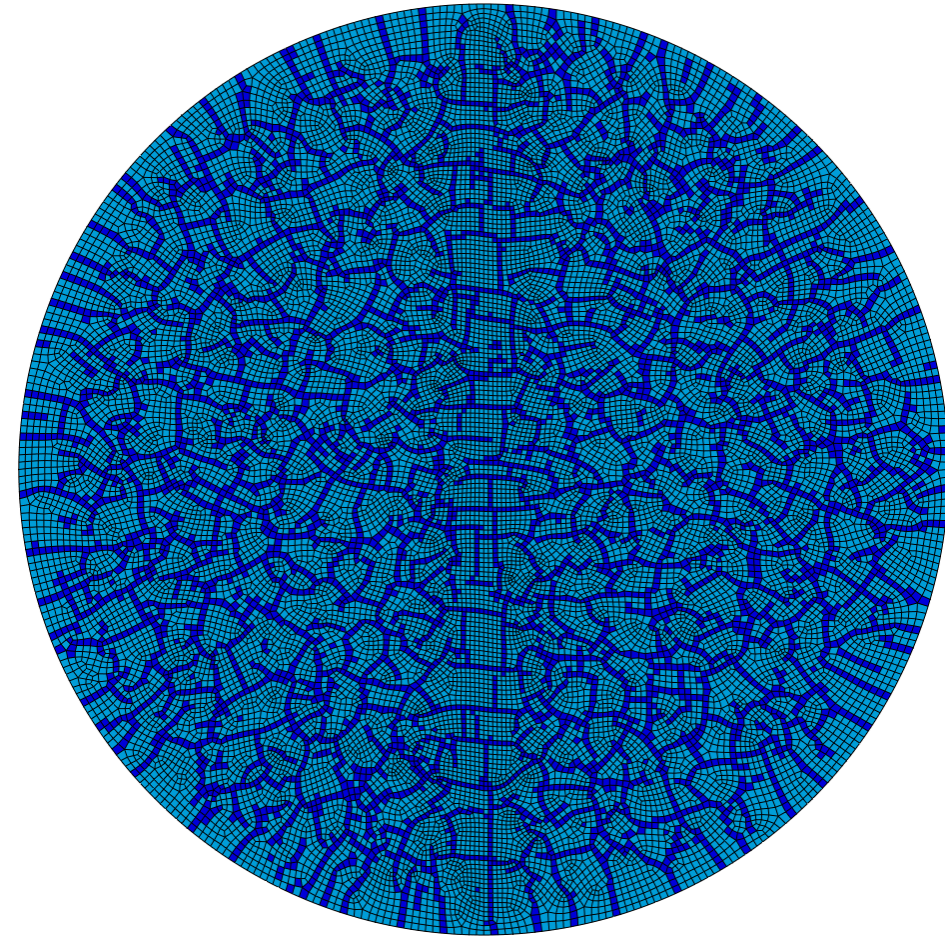# Damage through the coating layers:
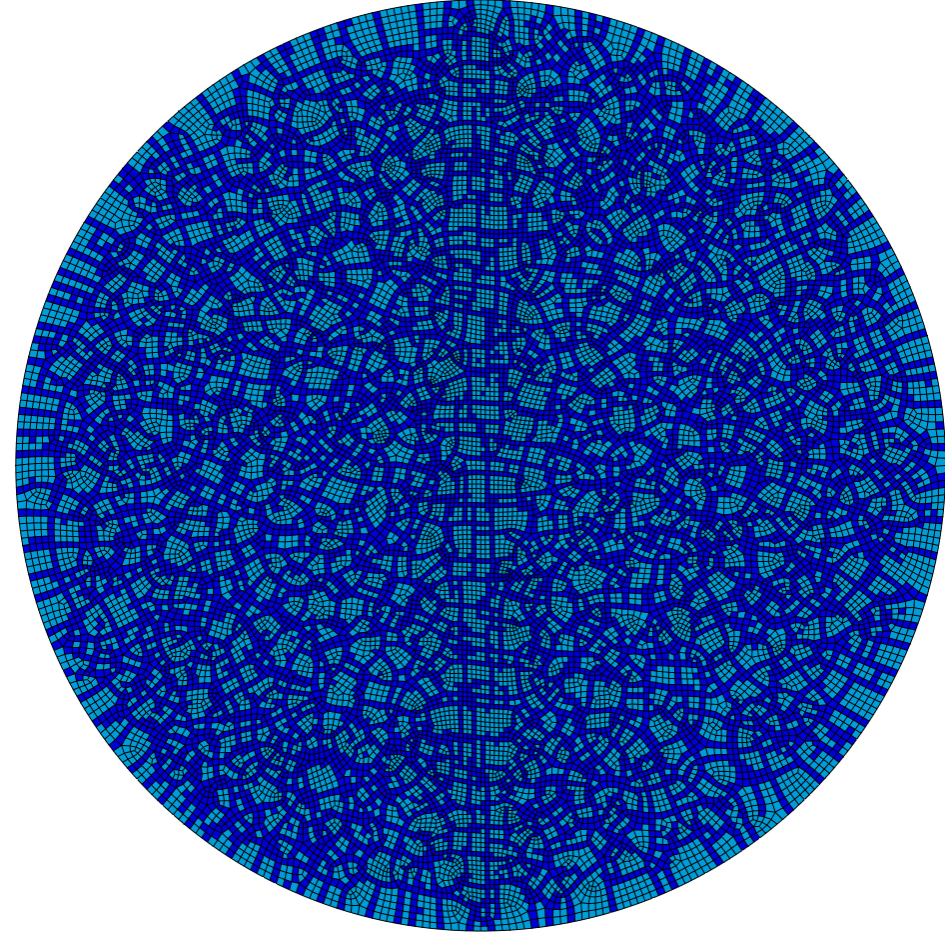## *Weibull modulus m = 2.5*

905.8º deg. C (110 steps)

23º deg. C (410 steps)

Damage of top surface for EBC coating:
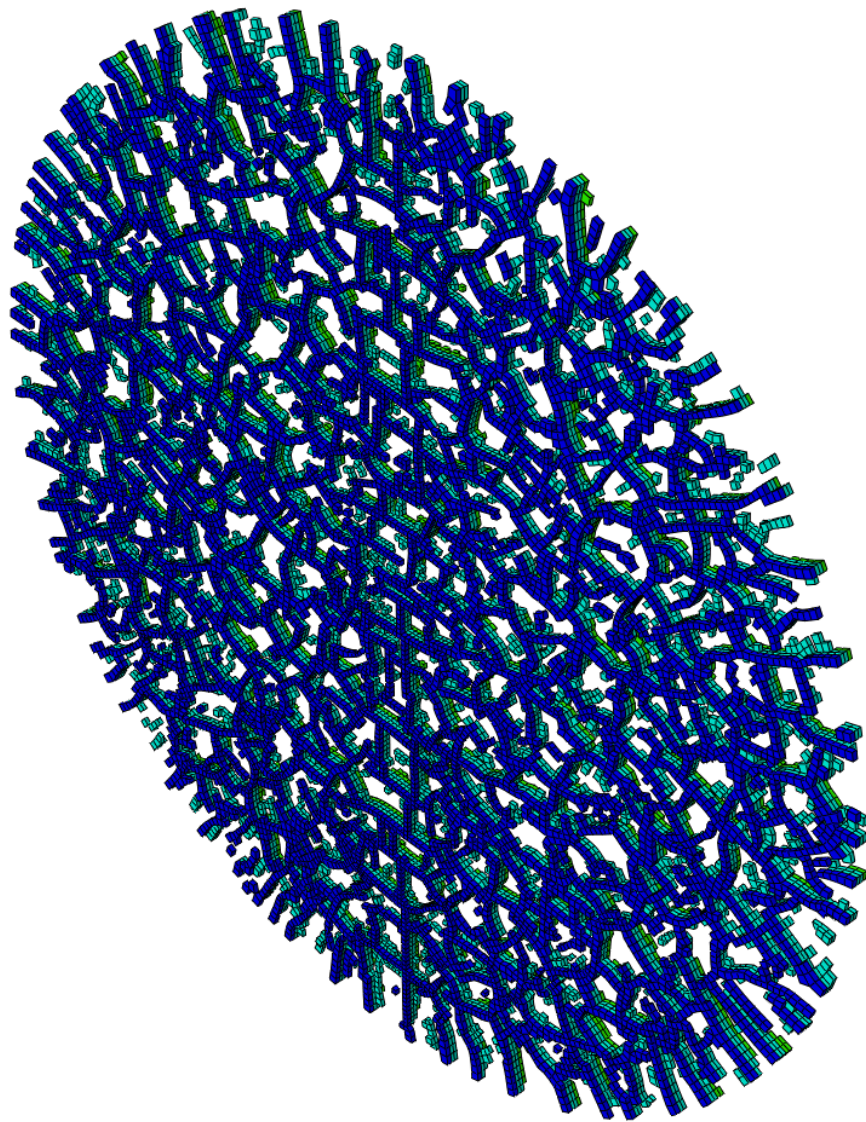## *Weibull modulus m* = 5.0
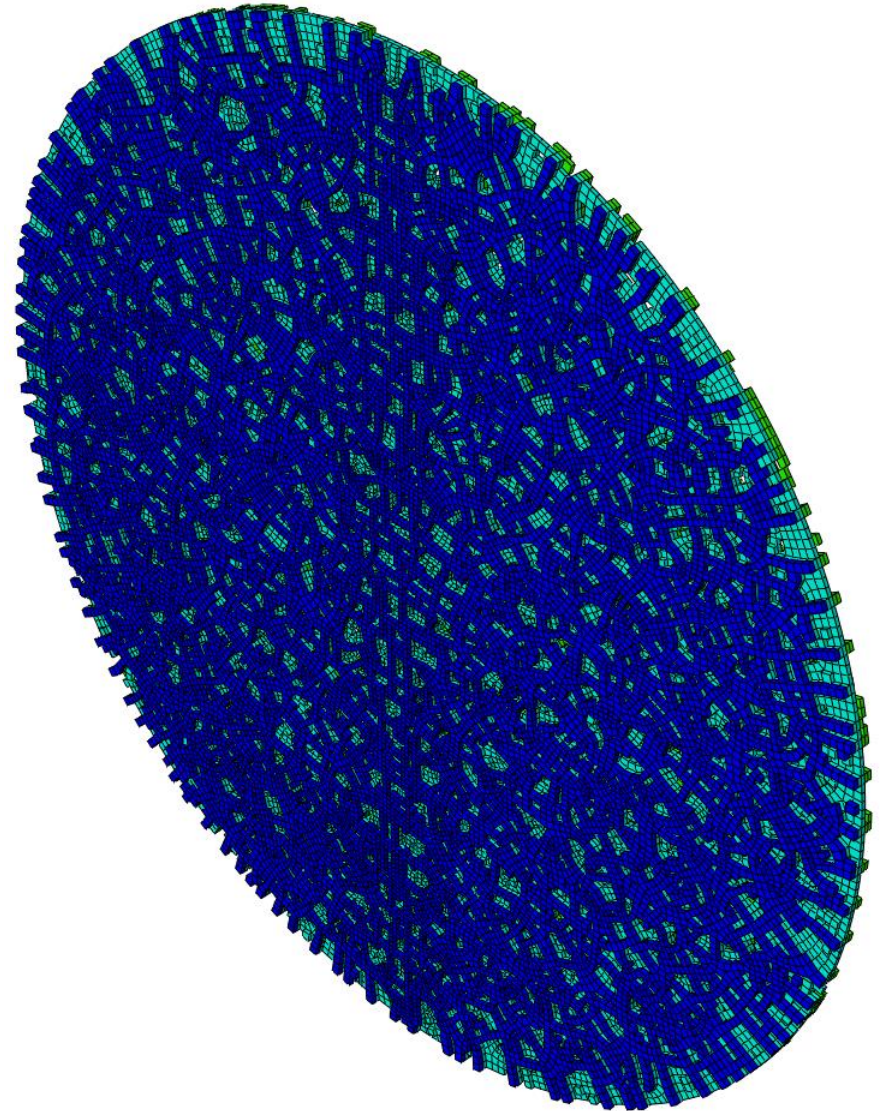

905.8° deg. C (110 steps)

23° deg. C (410 steps)

Damage through the coating layers:
Weibull modulus m = 5.0
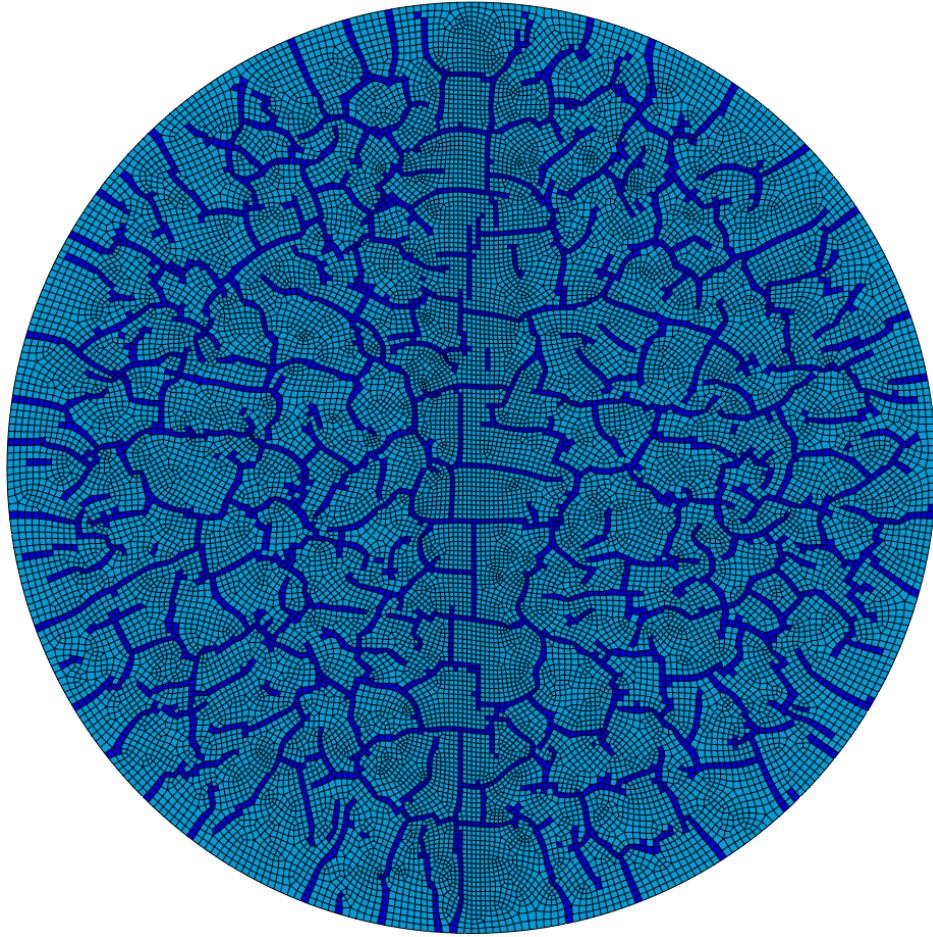

905.8º deg. C (110 steps)　　　　**23º deg. C (410 steps)**
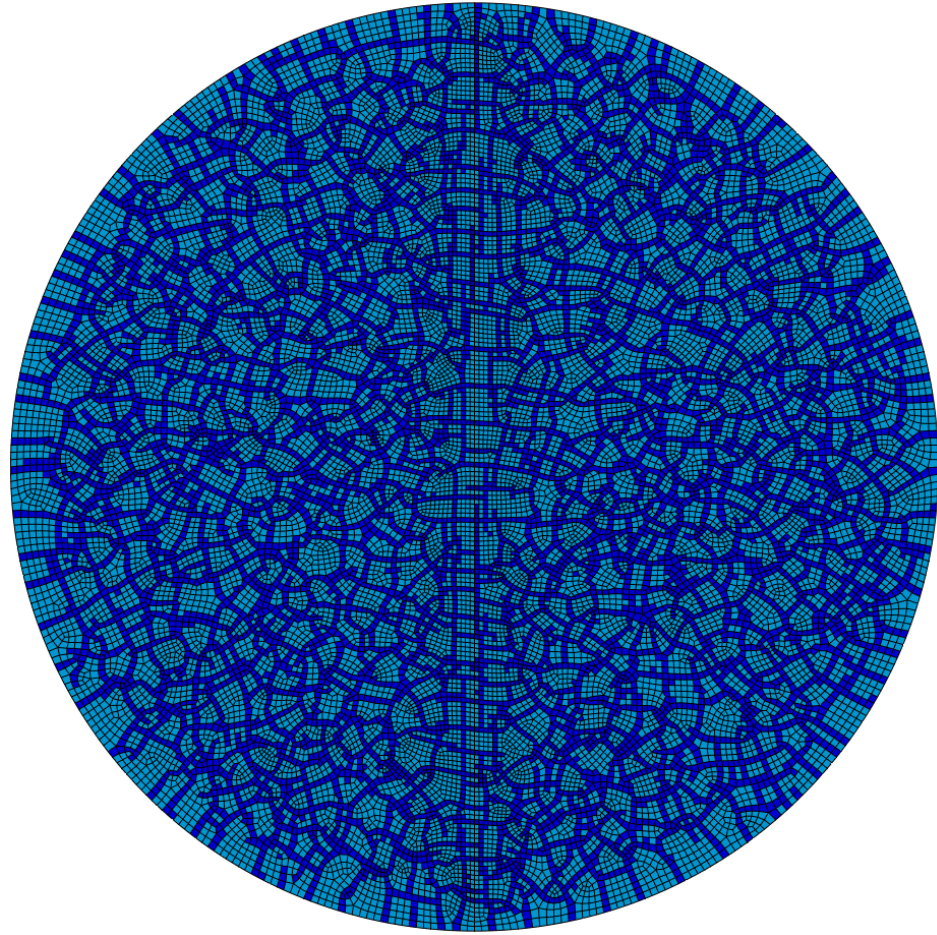
Damage of top surface for EBC coating:
Weibull modulus m = 10.0

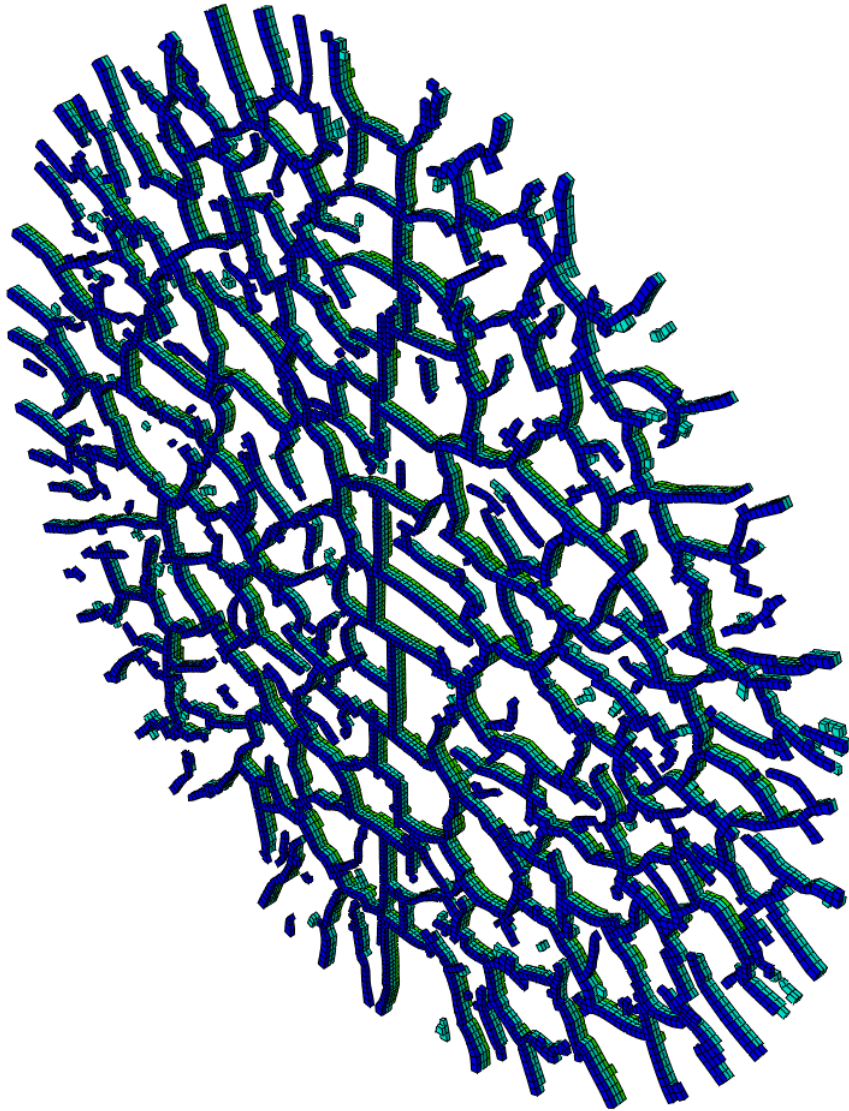**1150.0° deg. C (110 steps)**   **850° deg. C (410 steps)**
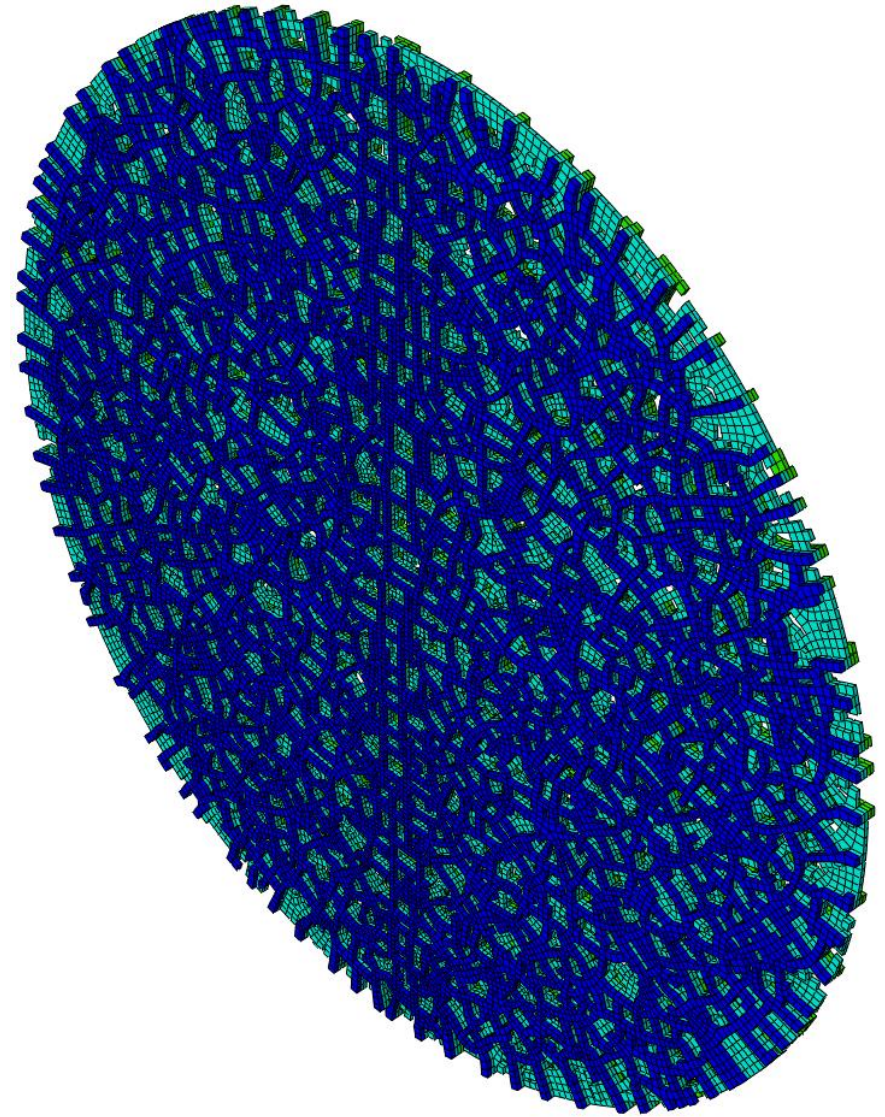
Damage through the coating layers:
Weibull modulus m = 10.0

1150.0° deg. C (110 steps)   850° deg. C (410 steps)

Conclusions

❖ **Progressive damage simulation of EBC coated structure incorporating probabilistic material strength model demonstrated with the FEAMAC/CARES code**

Problem investigated:

- **Damage development resulting from build-up of residual stresses from thermal cool-down (from processing)**
  - ➢ **3-D finite element model of disk-shaped specimen**
  - ➢ **2-D model of material cross-section detailing individual coating layers**

Shown was:

- **Crack boundaries could be reasonably mimicked with dense mesh of low-aspect-ratio 2-D and 3-D elements and crack growth could be promoted with a cellular automation methodology**

Observed was:

- **Periodic cracking arose from 2-parameter Weibull distribution describing coating layer stochastic strength response**
- **(2-D model) periodic channel crack formation observed**
- **(3-D model) periodic formation of mud cracks on the EBC**
  - ➢ **Mud flat cells became progressively smaller or sub-divided as loading progressed: consistent with fractal-like behavior**
- **Effect of value of Weibull modulus shown to affect crack spacing density with higher Weibull modulus correlating with higher density**
- **Channel crack spacing could be analyzed on outside edge of 3-D FE model of disk-shaped specimen**
- **Could not calibrate model parameters since the temperature where microcracking initiated is not known – *Need to know when microcracking starts when performing an experiment***

Future work:

- **reduce fracture path mesh dependency, Incorporation of evolving properties (e.g., thermal conductivity) as a function of damage accumulation, investigating effect of interfacial surface roughness between material layers, creep and environmental effects, TGO layer**

Weird bug…

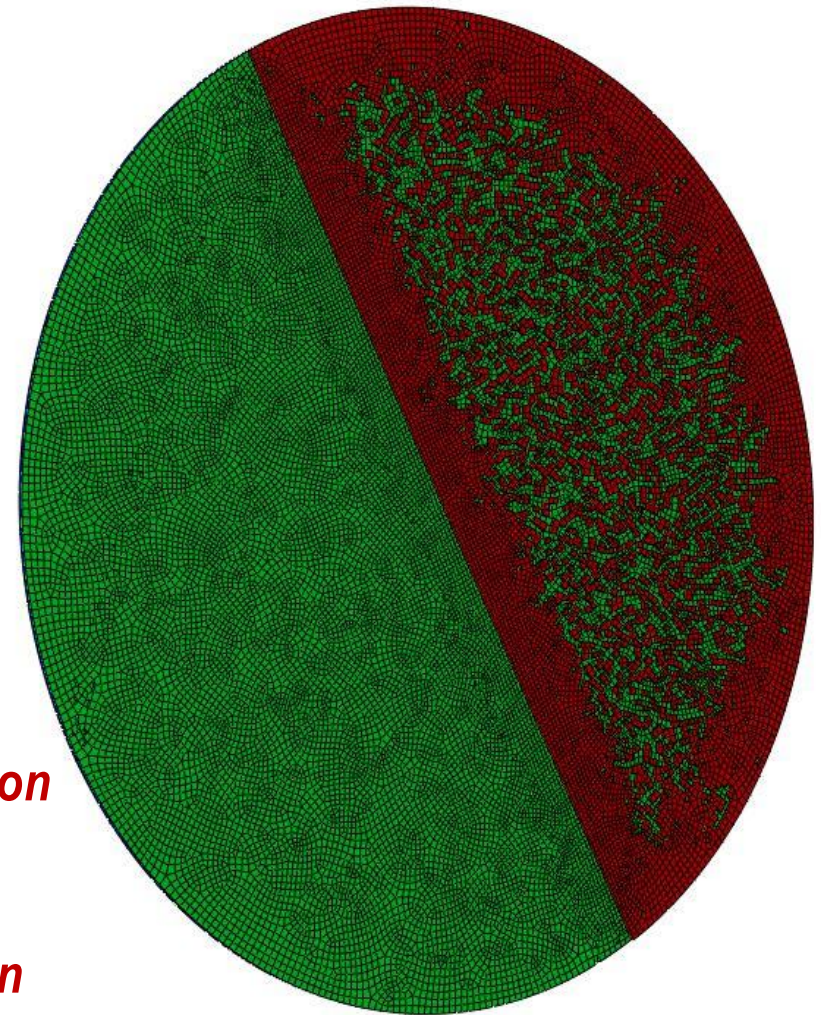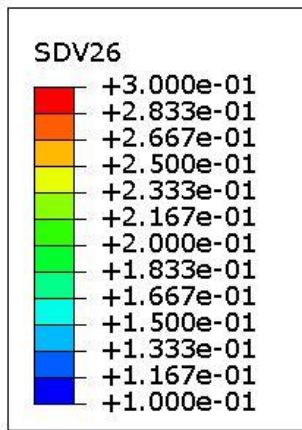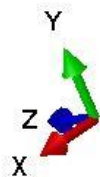**Attempting to put a pressure load after thermal cool down so as to demonstrate a service loading condition after characterizing initial damage state of material from initial coating deposition**

…yet to be tracked and fixed…

SDV26
+3.000e-01
+2.833e-01
+2.667e-01
+2.500e-01
+2.333e-01
+2.167e-01
+2.000e-01
+1.833e-01
+1.667e-01
+1.500e-01
+1.333e-01
+1.167e-01
+1.000e-01

Always occurs on 4th incremental load step after initlal cool down

ODB: EBC-3D-Base-File-a-R-6 40 step pressure bad result.odb   Abaqus/

Step: Pressure
Increment   4: Step Time =   0.2000
Primary Var: SDV26
Deformed Var: U   Deformation Scale Factor: +2.119e+02

Y
Z
X

GRC
Glenn Research Center at Lewis Field

NASA

Extra Material

Demonstrate Life Prediction Tool For EBC/CMC Subcomponent Subjected To Thermomechanical Loading

PROBLEM:

Environmental barrier coatings (EBCs) on Ceramic Matrix Composites (CMC) display complex failure modes that evolve with time, fluctuating load, and environmental exposure

APPROACH:

Use the newly developed FEAMAC/CARES code (which combines GRC codes {Composite Micromechanics Code (*MAC/GMC*) & Ceramics Analysis and Reliability Evaluation of Structures (*CARES/Life*)} with finite element analysis to simulate the stochastic damage evolution of EBC material system under generalized and transient thermomechanical loading over time and cyclic loading

INITIAL RESULTS:

- Established a probabilistic methodology to first damage event and subsequent evolution under multiaxial thermomechanical loading
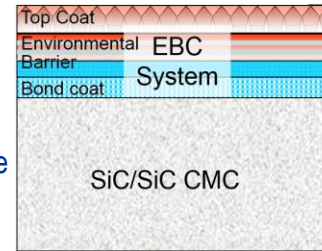- Demonstrated the spontaneous formation of mud cracks on an EBC subjected to thermal loads.

SIGNIFICANCE:

- Provides capability to optimize/design EBC *mechanical* performance based on a 2-parameter Weibull distribution of the strength and failure probability of individual brittle material coating layers
- Reproduce and understand EBC failure modes such as mud flat cracking and delamination which lays the foundation for future enhancements aimed at modeling effect of oxidizing species penetration within mud-cracks over time and the effect of thermally grown oxide (TGO) layer

FOLLOW-ON TOPICS:

- Incorporation of Creep and Environmental effects
- Develop algorithm to reduce fracture path mesh dependency (various means – simple, and more sophisticated)
- Incorporation of evolving properties (e.g., thermal conductivity) as a function of damage accumulation
- Incorporation of interfacial surface roughness between material layers thus inducing stress concentrations and fracture sites

Top Coat
Environmental Barrier — EBC System
Bond coat
SiC/SiC CMC

*Stochastic Progressive Damage Simulation Successfully Predicts Mud Flat Damage Pattern In EBCs*

Spontaneous development of mud flats fragment into progressively smaller sizes when cooling from 1300º C to room temperature (fractal fracture pattern)
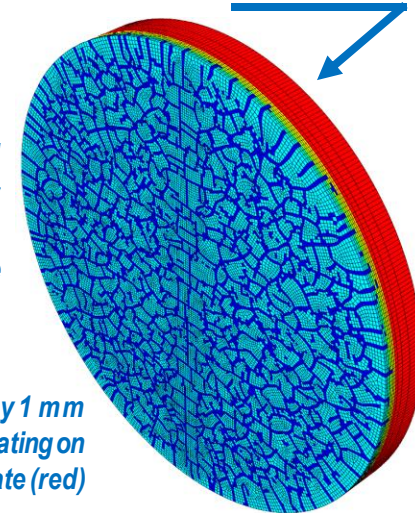
10 mm dia. by 1 mm multilayer coating on SiC substrate (red)

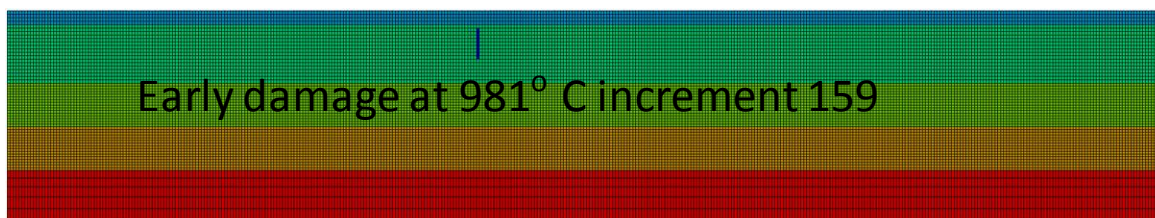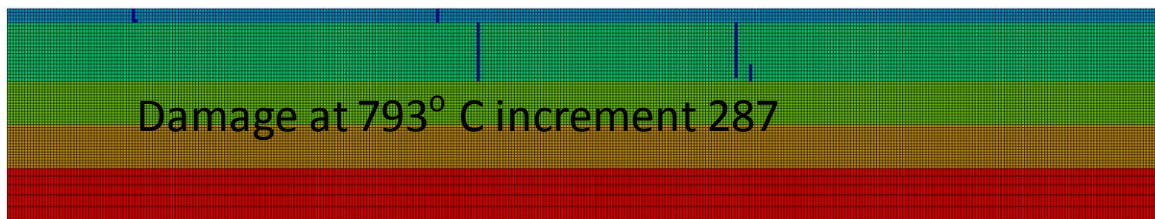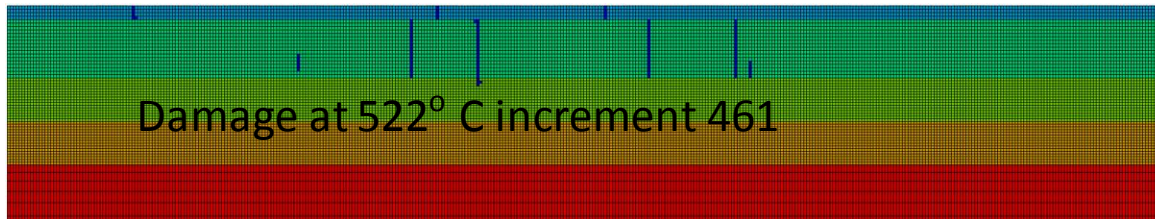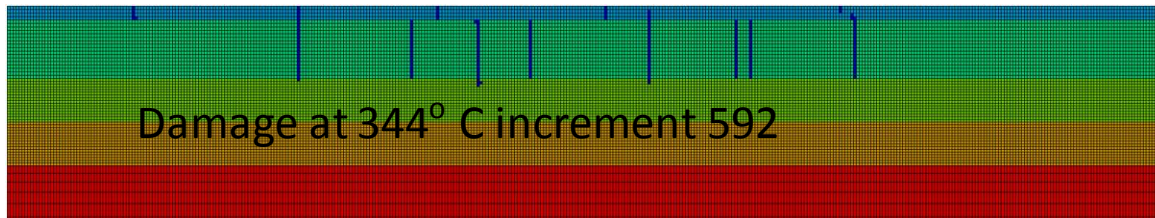*Compare to rare earth silicate EBC after heat flux testing showing mud flat damage*
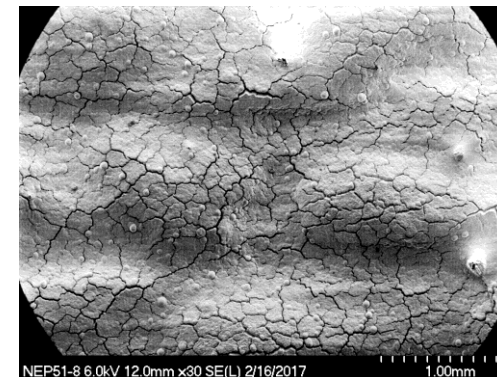
NEP51-8 6.0kV 12.0mm x30 SE(L) 2/16/2017   1.00mm

POC: Steven Arnold/ (LMS), Noel Nemeth/ (LMS)

Ytterbium Monosilicate: 2D cross-sectional model
Weibull modulus m=2.5 for all coating layers

Top coat:
low strength layer

Top coat
Intermediate coat
Bond coat
Substrate

Damage at 23° C increment 810

Damage at 344° C increment 592

Damage at 522° C increment 461

Damage at 793° C increment 287

Early damage at 981° C increment 159

2 mm X 1 mm specimen
Complete substrate not shown

810 load increment steps (ramp load):

- First 10 early cool-down steps from 1300° C to 1200° C
- 800 increment steps from 1200° C to 23° C

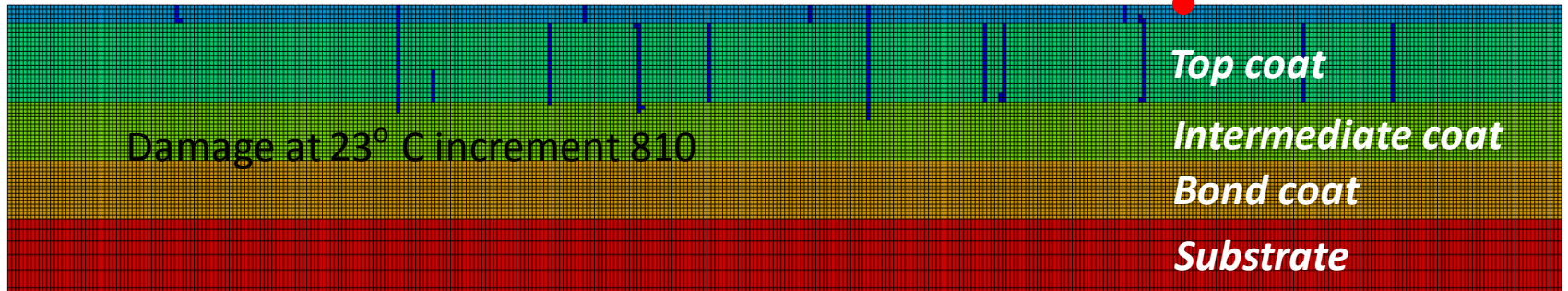**≈ 8 channel cracks in top coat over 1.2 mm span for an average crack spacing of 150 μm**

Ytterbium Monosilicate: 2D cross-sectional model
Weibull modulus m=5.0 for all coating layers

*Top coat:
low strength layer*

Top coat
Intermediate coat
Bond coat
Substrate

Damage at 23° C increment 810

Damage at 258° C increment 564

Damage at 464° C increment 500

Damage at 884° C increment 215

Early damage at 1044° C increment 106

2 mm X 1 mm specimen
Complete substrate not shown

810 load increment steps (ramp load):

- First 10 early cool-down steps from 1300° C to 1200° C
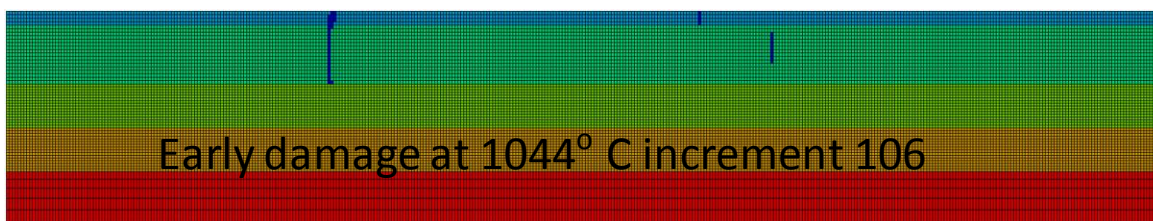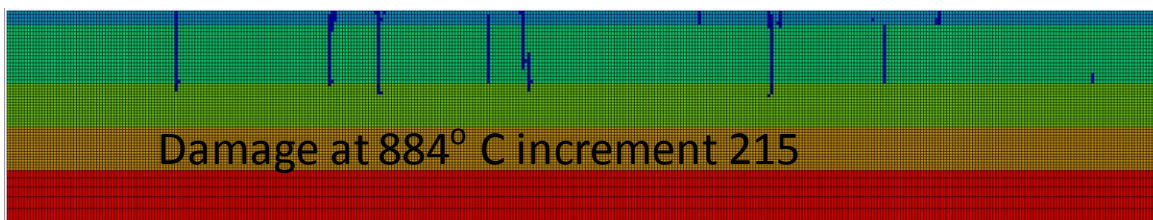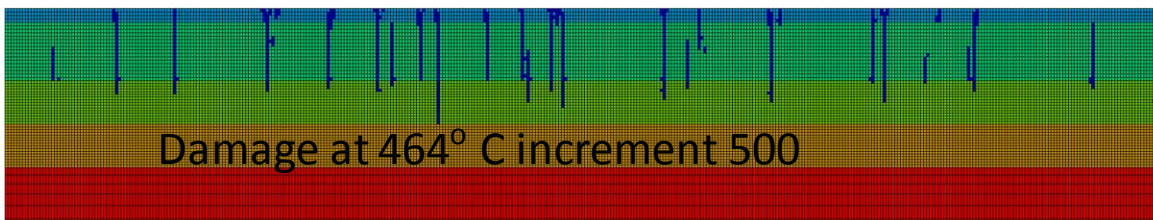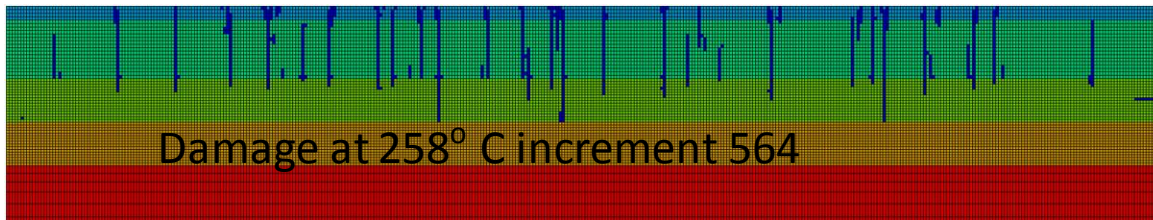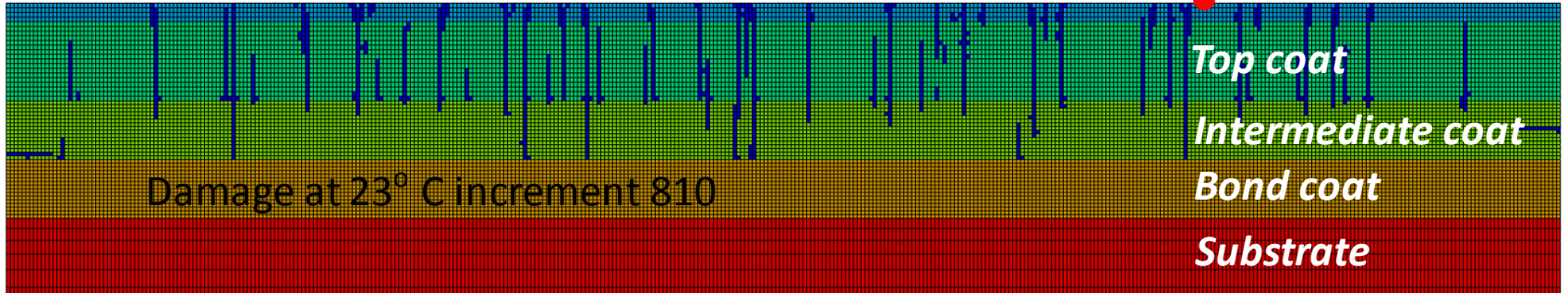- 800 increment steps from 1200° C to 23° C

≈ 19 channel cracks in top coat over 1.2 mm span for an average crack spacing of 63 µm

Ytterbium Monosilicate: 2D cross-sectional model
Weibull modulus m=10.0 for all coating layers

Top coat:
low strength layer

Top coat
Intermediate coat
Bond coat
Substrate

Damage at 23° C increment 810

Damage at 436° C increment 519

Damage at 654° C increment 371

Damage at 1037° C increment 111

Early damage at 1137° C increment 43
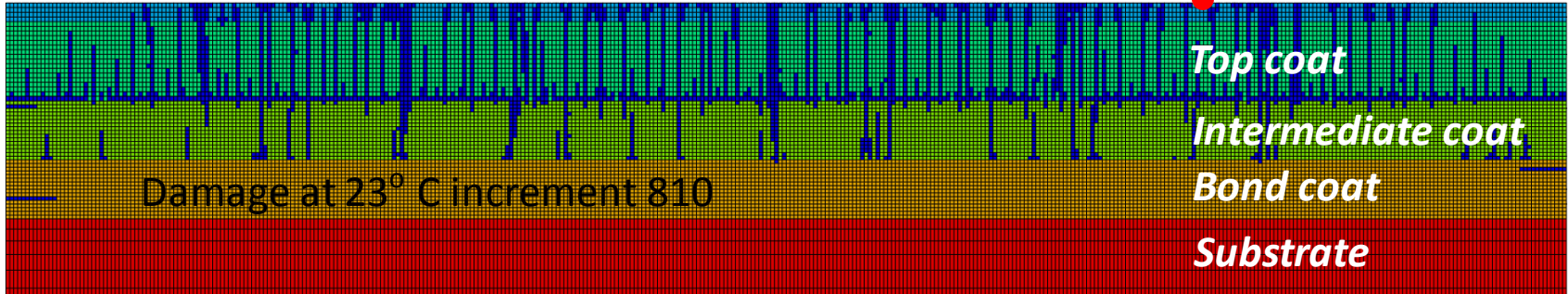
2 mm X 1 mm specimen
Complete substrate not shown

810 load increment steps (ramp load):

- First 10 early cool-down steps from 1300° C to 1200° C
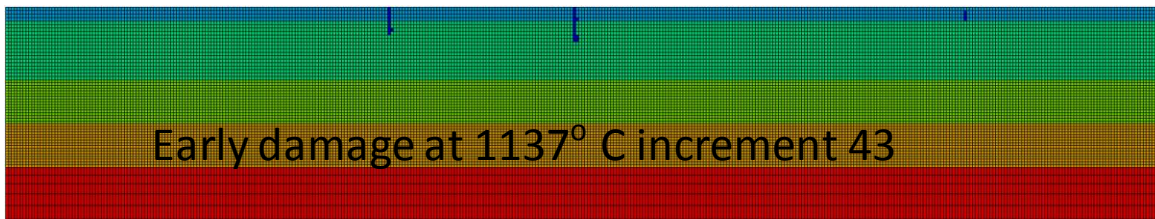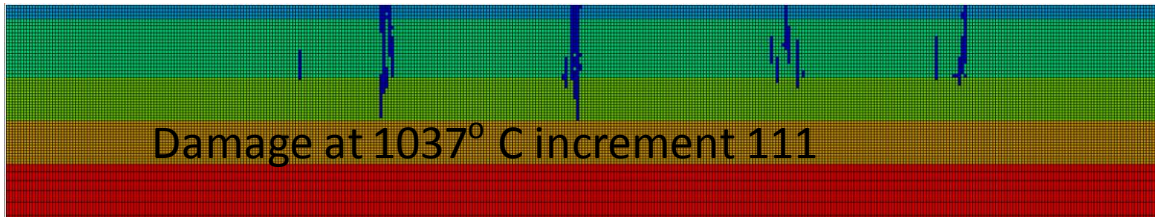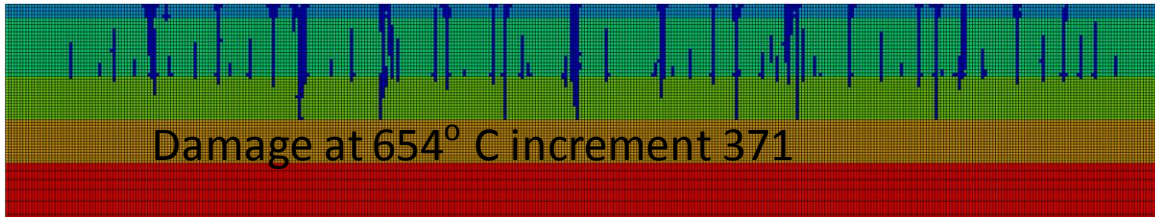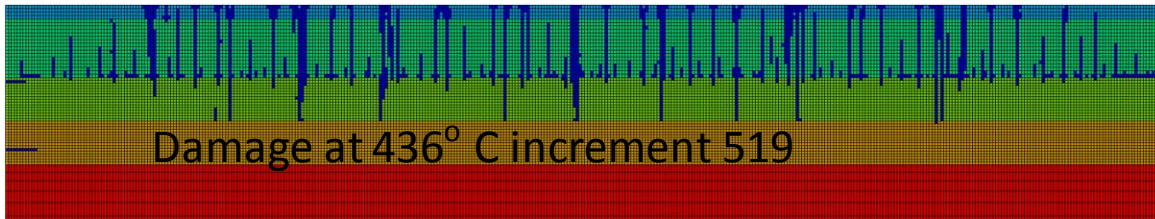- 800 increment steps from 1200° C to 23° C

$\approx$ **49 channel cracks in top coat over 1.2 mm span for an average crack spacing of 25 μm**

Properties Table (Tensile strength/Weibull parameters adopted from Abdul-Aziz, None listed in Richards)

Report	Material	E (Gpa)	Poisson, v	Therm. Expan., α, m/m/°C · 10^{-6}	Weibull modulus, m	Weibull Scale Parameter, σ_o , MPa·mm$^{3/m}$
YBSM (Richards et al. 2016)[1] Using 50% reduced properties listed in reference	Top Coat Suface: Ytterbium monosilicate	86	0.27	7.5	5	14 (assumed)
	Top Coat: Ytterbium monosilicate	86	0.27	7.5	5	28 (assumed)
	Intermediate Coat: Mullite	110	0.28	5.3	5	28 (assumed)
	Bond Coat: Silicon	82	0.223	4.1	5	40 (assumed)
	Substrate: SiC (Monolithic)	430	0.14	4.6	5	321 (assumed)
BSAS (Abdul-Aziz et al. 2014)[2]	Top Coat Surface: BSAS	32	0.19	5.6	5	14
	Top Coat: BSAS	32	0.19	5.6	5	28
	Intermediate Coat: BSAS+Mullite	37.4	0.179	5.7	5	28
	Bond Coat: Silicon	97	0.21	4.5	5	40
	Substrate: SiC (CMC)	285	0.17	2.71	5	321

[1] Richards et al.: Fracture Mechanisms of Ytterbium Monosilicate Environmental barrier coatings during cyclic thermal exposure" Acta Materialia, 103, pp. 448-460, 2016..

[2] Abdul-Aziz, et al.: "Durability Modeling of Environmental Barrier Coating (EBC) Using Finite Element Based Progressive Failure Analysis" J. of Ceramics, pp. 1-10, 2014

2-D EBC Cross-Section FE model (2mm X 1mm)

25,600 S4 Shell elements

9.375 μm

Edges are free:
No constraint or periodic boundary conditions

Top surface layer

65.625 μm

Top Coat

Intermediate Coat

75 μm

Bond Coat

High aspect elements for substrate (higher stiffness material)

75 μm

SiC Substrate

Note: Effect of surface roughness & coating uniformity not considered here

y

x
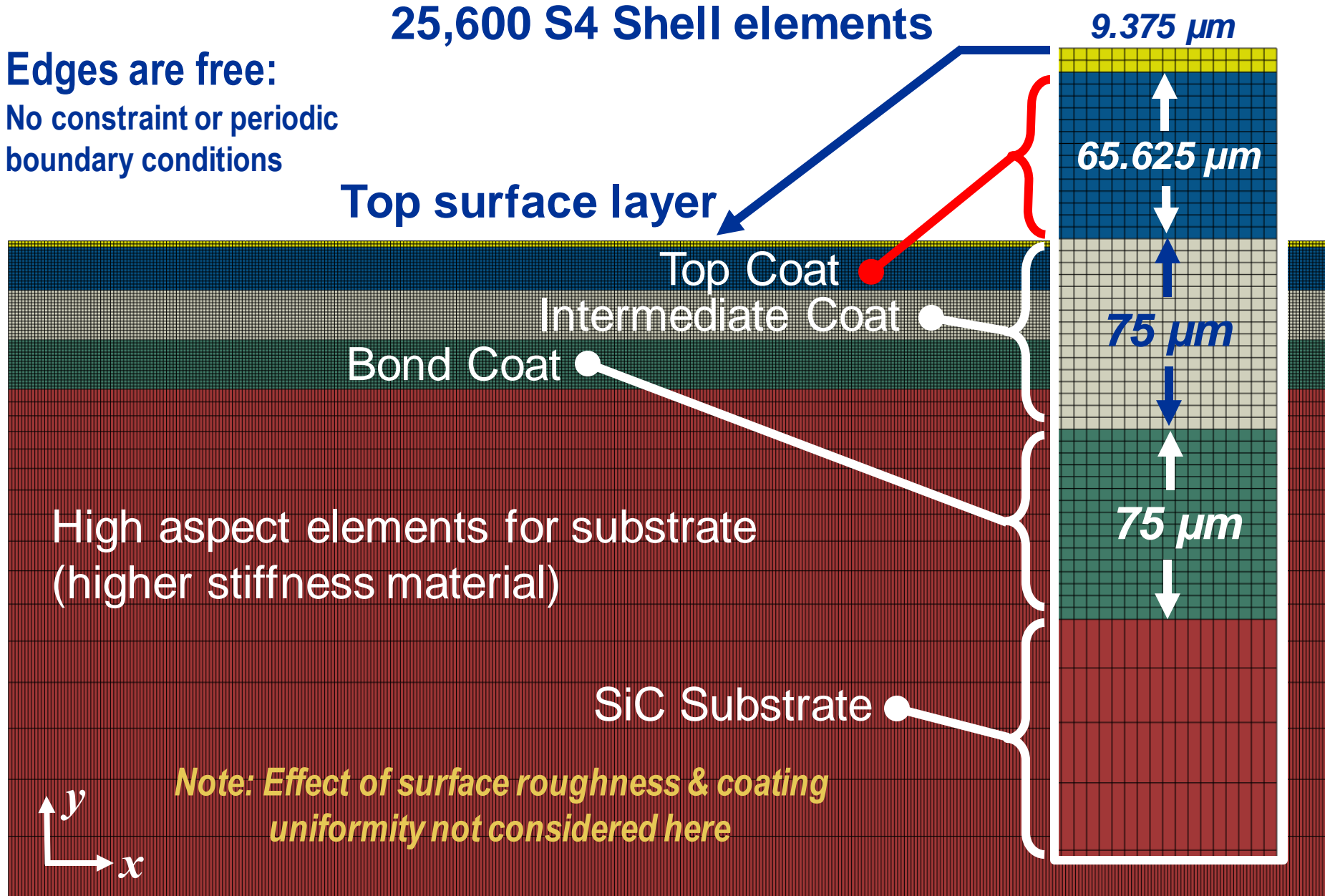
Stresses in undamaged EBC layers : 200° C cool-down from annealing Temp.
Comparison for 75 μm thick top coat

BSAS (Abdul-Aziz et al. 2014)

Top
Intermediate
Bond

| +3.000e+07 |
| +2.482e+07 |
| +1.964e+07 |
| +1.446e+07 |
| +9.287e+06 |
| +4.109e+06 |
| -1.069e+06 |
| -6.247e+06 |
| -1.143e+07 |
| -1.660e+07 |
| -2.178e+07 |
| -2.696e+07 |
| -3.214e+07 |

Substrate

σ_x

Ytterbium Monosilicate (Richards et al. 2016)

| +4.522e+07 |
| +3.977e+07 |
| +3.433e+07 |
| +2.888e+07 |
| +2.344e+07 |
| +1.800e+07 |
| +1.255e+07 |
| +7.111e+06 |
| +1.668e+06 |
| -3.776e+06 |
| -9.219e+06 |
| -1.466e+07 |
| -2.011e+07 |

σ_x

Top
Intermediate
Bond

| +4.835e+07 |
| +4.382e+07 |
| +3.930e+07 |
| +3.477e+07 |
| +3.024e+07 |
| +2.571e+07 |
| +2.119e+07 |
| +1.666e+07 |
| +1.213e+07 |
| +7.607e+06 |
| +3.080e+06 |
| -1.447e+06 |
| -5.974e+06 |

Substrate

σ_y

| +3.579e+07 |
| +3.239e+07 |
| +2.900e+07 |
| +2.560e+07 |
| +2.221e+07 |
| +1.881e+07 |
| +1.542e+07 |
| +1.202e+07 |
| +8.626e+06 |
| +5.232e+06 |
| +1.837e+06 |
| -1.558e+06 |
| -4.953e+06 |

σ_y

Top
Intermediate
Bond

| +1.191e+07 |
| +9.923e+06 |
| +7.938e+06 |
| +5.954e+06 |
| +3.969e+06 |
| +1.985e+06 |
| +0.000e+00 |
| -1.985e+06 |
| -3.969e+06 |
| -5.954e+06 |
| -7.938e+06 |
| -9.923e+06 |
| -1.191e+07 |

Substrate

σ_{xy}

| +1.073e+07 |
| +8.938e+06 |
| +7.151e+06 |
| +5.363e+06 |
| +3.575e+06 |
| +1.788e+06 |
| -1.750e+00 |
| -1.788e+06 |
| -3.575e+06 |
| -5.363e+06 |
| -7.151e+06 |
| -8.938e+06 |
| -1.073e+07 |

σ_{xy}

Unit Sphere Multiaxial (Batdorf) Model:
Puts linear elastic fracture mechanics into Weibull weakest-link theory

➤ **Incremental failure probability is the product of two probabilities:**

$$\Delta P_f = P_1 \cdot P_2$$

P_1 = Probability of the existence of a crack having a critical strength between σ_c and σ_c + $\Delta\sigma_c$ in the incremental volume ΔV

P_2 = Probability a crack having a critical strength of σ_c will be oriented in a direction such that it will fail under the applied multiaxial stress state

➤ **Component failure probability:**

$$P_f = 1 - \exp\left\{ -\int_V \left[\int_0^{\sigma_e} P_1(\sigma_c)\, P_2(\sigma_c)\, d\sigma_c \right] dV \right\}$$

P_2 involves Integration of an equivalent stress σ_e, where $\sigma_e \geq \sigma_c$, over the surface of a unit radius sphere (all possible flaw orientations) divided by the total surface area of the unit radius sphere
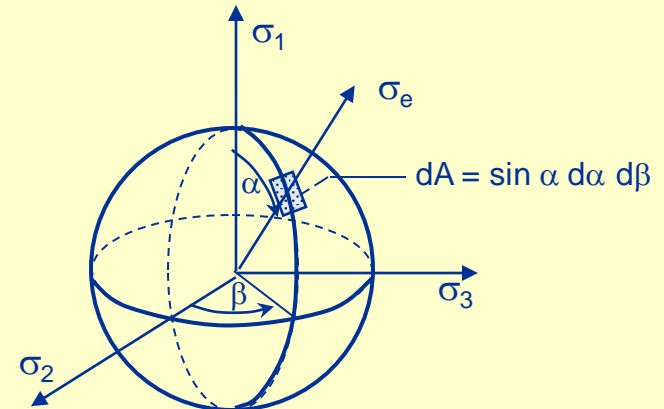
σ_e is a function of an assumed crack shape and multiaxial fracture criterion

Mixed-Mode Fracture Criteria:
• Normal stress (shear-insensitive cracks)
• Maximum tensile stress
• Total coplanar strain energy release rate
• Noncoplanar (Shetty)

Flaw Shapes:
• Griffith crack
• Penny-shaped crack

# Modeling flaw planes/texture anisotropy

Coating layers have complex microstructures with defect/flaw populations that can have orientation bias of pores, voids, and cracks

This can be accounted with the Unit Sphere stochastic-strength multiaxial criterion model

➤ **Two models for transverse isotropy**

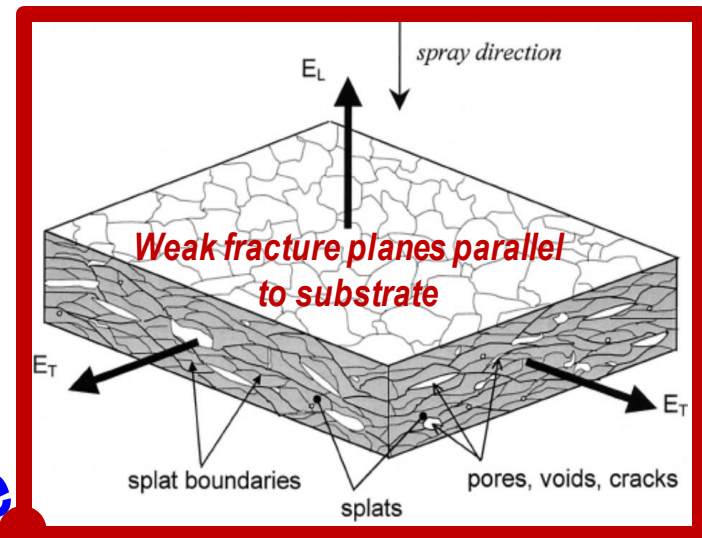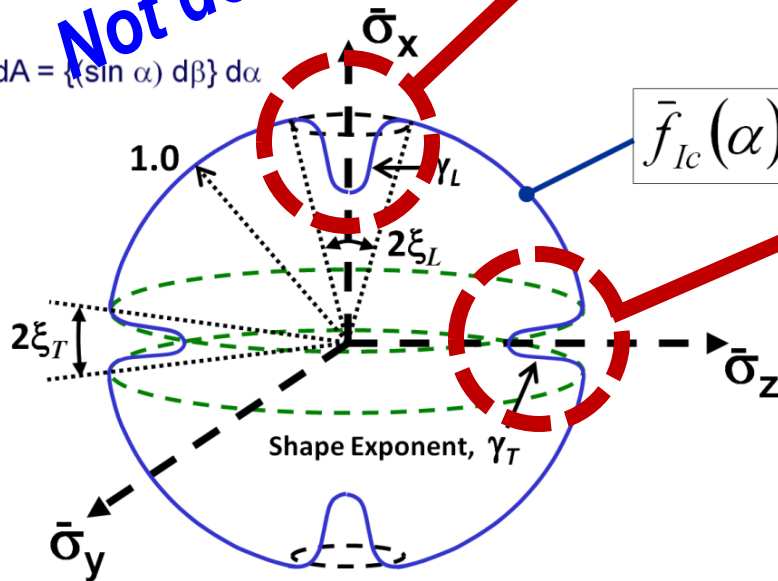**(1) Flaw / Fracture-Plane Orientation Anisotropy**

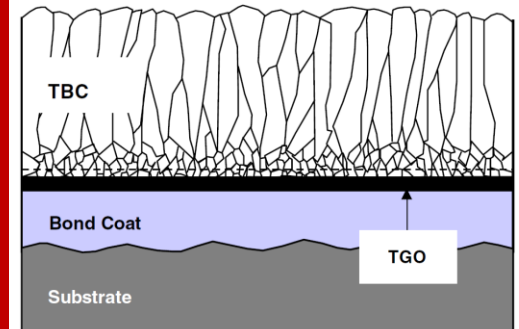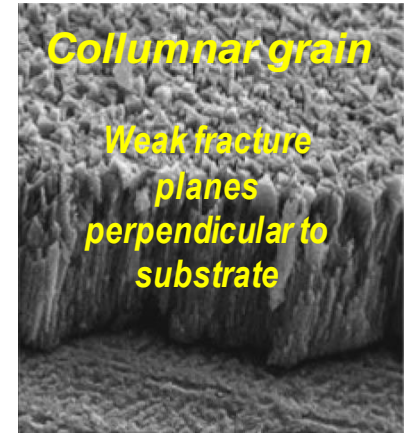**(2) Strength Orientation Anisotropy**

Orientation of intrinsic flaw

$[dA = (\sin \alpha) \, d\beta] \, d\alpha$

Not demonstrated here
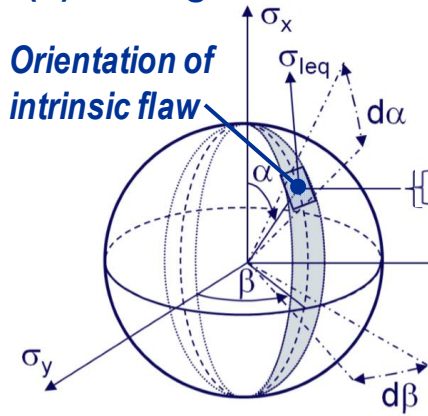
Unit Sphere

- crack shape
- mixed-mode fracture criterion

$\bar{f}_{Ic}(\alpha)$

1.0

γ_L

$2\xi_L$

$2\xi_T$

$\bar{\sigma}_x$

$\bar{\sigma}_z$

$\bar{\sigma}_y$

Shape Exponent, γ_T

spray direction

E_L

E_T

Weak fracture planes parallel to substrate

splat boundaries

pores, voids, cracks

splats

Collumnar grain

Weak fracture planes perpendicular to substrate
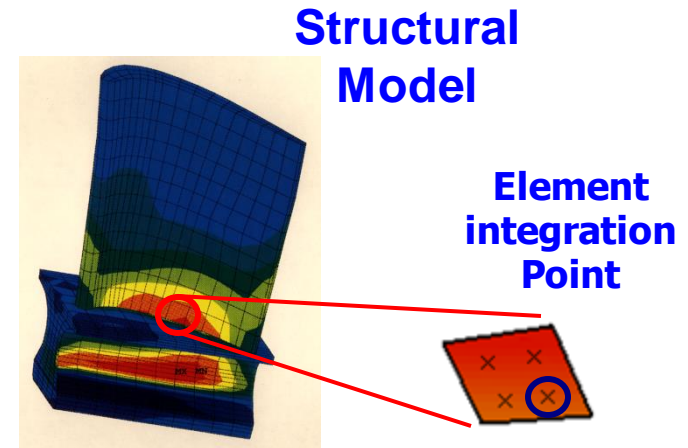
TBC

Bond Coat

TGO

Substrate

CARES: *Ceramics Analysis and Reliability Evaluation of Structures*

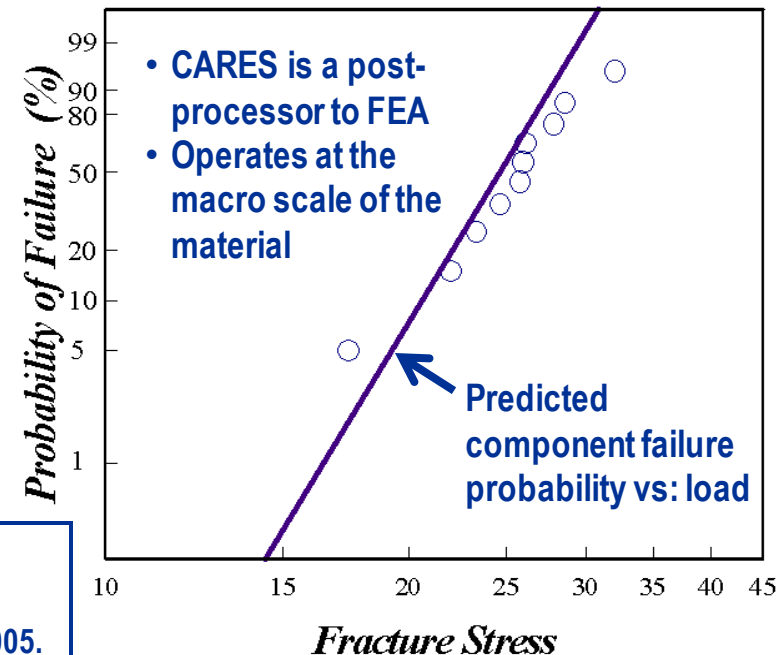## Life Prediction & Component Design Code For Advanced Ceramics

- Developed to predict the probability of failure of ceramic components under complex thermomechanical loading

- Combines Weibull & Weakest Link theory with concepts of linear elastic fracture mechanics (the *Batdorf Unit Sphere* model)

Component Reliability Analysis Capability:

➤ Transient loads and temperatures
➤ Fast-Fracture Rupture
➤ Time-dependent (da/dt) crack growth
➤ Cycle-dependent (da/dn) crack growth
➤ Multiaxial stress failure models
  (PIA & Unit Sphere & Tsai-Wu & Tsai-Hill)
➤ Proof test

Structural Model

Element integration Point

(CARES)
reliability analysis

- CARES is a post-processor to FEA
- Operates at the macro scale of the material

Predicted component failure probability vs: load

*Probability of Failure (%)* vs *Fracture Stress*

Nemeth, Jadaan, Gyekenyesi.: "Lifetime Reliability Prediction of Ceramic Structures Under Transient Thermomechanical Loads." NASA/TP-2005-212505, 2005.

MAC/GMC Micromechanics Analysis Code

Repeating Unit Cell (RUC) of composite material

❖ RUC made of material subcells

❖ Multiscale capability

RUC

Material 1

Material 2

Subcell
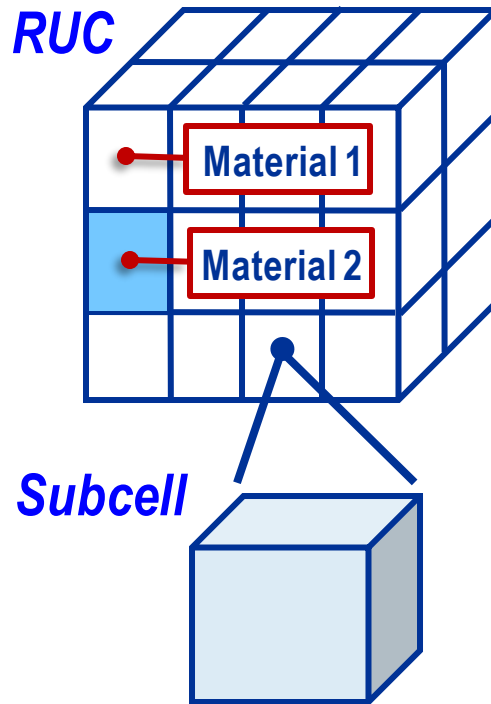
▪ *FEAMAC: MAC/GMC embedded in FEA as constitutive material*

CARES/Life: Life Prediction Code For Advanced Ceramics

• **Predicts the probability of failure of ceramic components under thermomechanical loading**

• **Combines Weibull & Weakest Link theory with concepts of linear elastic fracture mechanics (the *Batdorf Unit Sphere* model)**

• **CARES is a post-processor to FEA**

Component Reliability Analysis Capability:

➢ **Transient loads and temperatures**

➢ **Fast-Fracture Rupture**

➢ **Time-dependent (da/dt) crack growth**

➢ **Cycle-dependent (da/dn) crack growth**

➢ **Multiaxial stress failure models (PIA & *Unit Sphere* & Tsai-Wu & Tsai-Hill)**

➢ **Proof test**

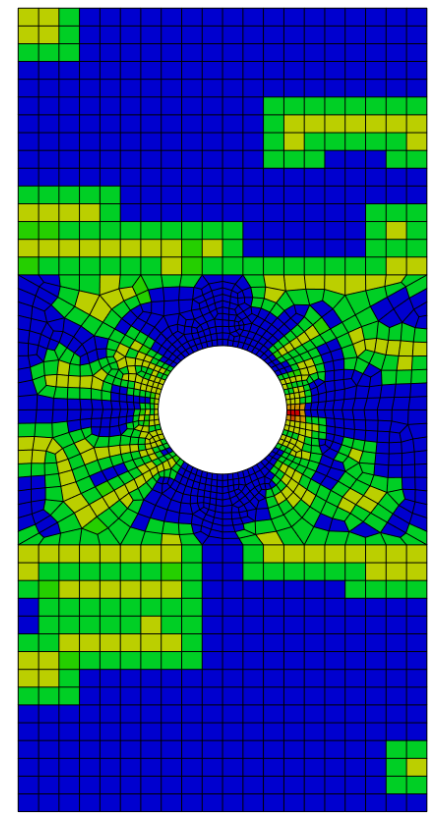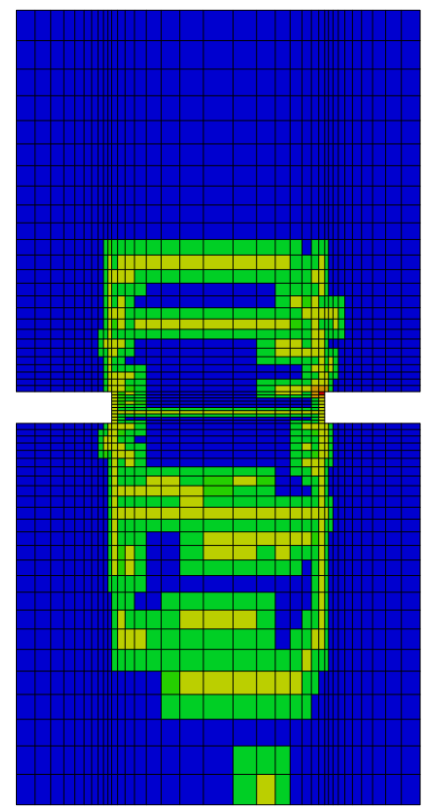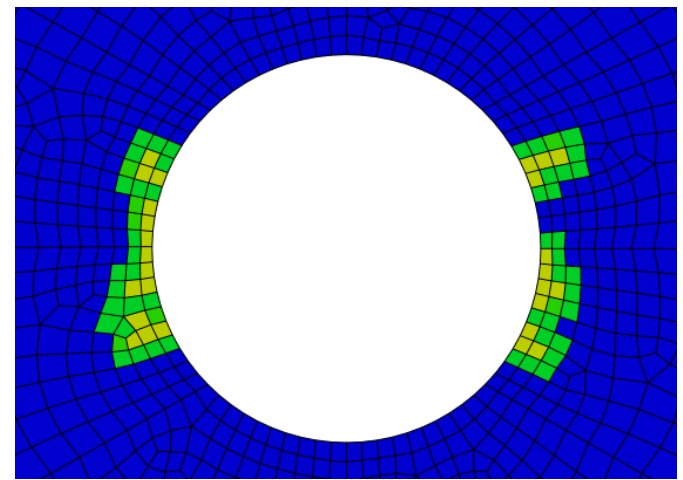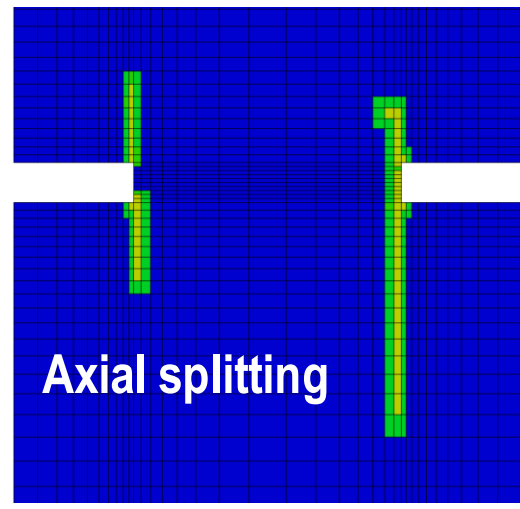0° CMC Double-Notched vs: Central-Hole Tensile Specimen

FEAMAC/CARES damage simulation:

Loading and fiber Direction

Axial splitting

Early matrix damage

Matrix damage progression

Matrix failure
Adjacent to failed matrix

Fiber failure
Adjacent to failed fiber

No failure