THEORY VALIDATION — 2 POINTS OF VIEW

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Introduction

The task of validating any given theory against any given proprotor/propeller data set is of immense importance. Not just to the researchers who are developing a theory, but to the working engineers using the theory to design a proprotor/propeller. However, there is a major difference in how the two groups see validation. This difference is that, most frequently, researchers see the validation in coefficient form as, for example, C_P versus C_T or Figure of Merit versus C_T. In contrast, design engineers much prefer working in the dimensional world as, for example, horsepower required to hover versus aircraft weight. The objective of the design engineer is, of course, to release drawings and specifications to manufacturing so a VTOL aircraft will be built with *reasonably high assurance* that the aircraft will meet the primary specifications.

The magnitude of "reasonably high assurance" was quantified by Charlie Crawford in his 1989 Nikolsky Lecture (Ref. 1) where he concluded – some 25 years ago – that

> "The lack of realistic and detailed rotorcraft analytical tools has resulted in many configuration changes being required after the rotorcraft has entered flight test, in order to have a viable weapon system. These analyses short-falls are most prevalent in the areas of estimates for rotor vibration loads, and flight control system loads, overall vibratory levels, as well as aerodynamic capability. The multi-discipline aspects of rotorcraft dynamics significantly complicates the construction of appropriate analytical tools. The inability of these tools to facilitate a design which would fly adequately "off the drawing board" was illustrated in a development of the Army's competing UTTAS/AAH designs."

Relative to the design engineer's assurance of success, Crawford pointed out with his fig. 22 (reproduced here as Fig. 1) that the power margin deficiency due to missing predicted hover power required was adverse, falling short between + 3 and + 6 percent. *No evidence was found suggesting that rotorcraft were performing better than theory was predicting.* Secondly, his assessment of design takeoff gross weight was that this key design parameter had been under predicted by some 7 to 20 percent.

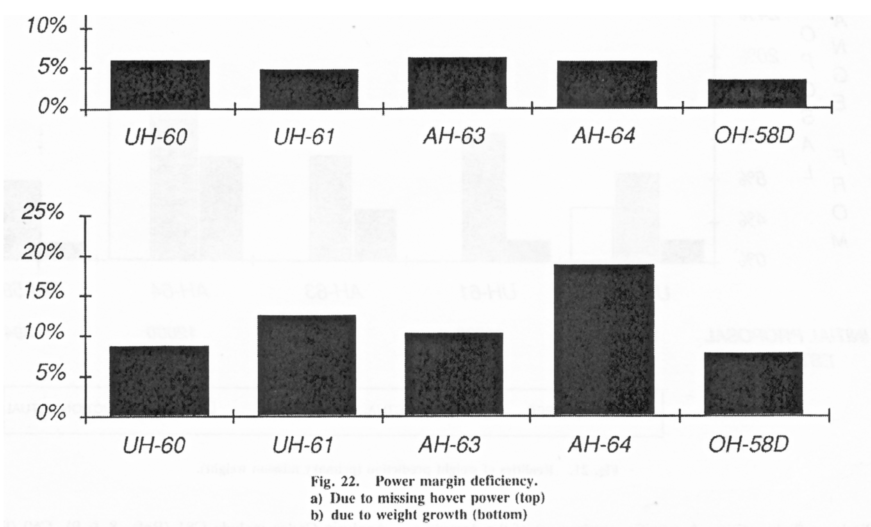

Fig. 1. As of 1990, rotorcraft design engineers still had *very low* assurance that their design would be successful right "off the drawing board". This led to "many configuration changes being required after the aircraft entered flight test."

Crawford's assessment might suggest that the final design should apply performance and weight adverse "correction factors" in the beginning. However, industry experience has shown that this amount of pessimism leads to a very over weight and high cost configuration that is quite unacceptable to the customer. The opposite approach of unbridled optimism has, for many a development program, led to outright cancellation of the activity.[1] The path to "reasonably high assurance" lies in staying well away from these two boundaries.

Now then, consider the researcher's view of validating first; and then the design engineer's view as he navigates along the path to success.

Researchers' Validation Methodology

Quite infrequently, researchers have at hand test results for the same proprotor/propeller tested on two different whirl test rigs. This, in fact, was the interesting case for the XV-15 Metal Blade Proprotor. Reference 2 provides measured hover performance for this proprotor[2] as tested on the NASA Ames' OARF (Fig. 2) and the Air Force's test rig located at the WADC (Fig.3). The test tip Mach number range from the two tests overlapped as indicated by the C_P versus C_T data provided by Fig. 4. It is very clear from Fig. 4 that the two data sets are not in agreement within experimental error. In this particular case, researchers considered that the "gold standard" was the results from the NASA Ames' OARF. Therefore, the validation task under discussion here is directed at the black, open circles data from the XV-15 test on Ames' OARF shown on Fig. 4. Note that this XV-15 OARF data can be quite accurately "curve fitted" with a sixth order polynomial given as

$$\begin{aligned} C_P \;=\; & 9.3378121E+08(C_T)^6 \;-4.2849E+07(C_T)^5 \;+\; 7.91334E+05(C_T)^4 \;-\; 7.4731E+03(C_T)^3 \\ & +4.3010E+01(C_T)^2 \;-\; 7.12964E-02(C_T) \;+\; 2.23367E-04 \end{aligned}$$

While this curve fit equation has been constructed from data taken at tip Mach numbers ranging from 0.60 to 0.73, it should be clear that the difference between test facilities is far more influential than tip Mach number effects – at least within the basically incompressible regime.[3]

Now consider the prediction of the test data given with Fig. 4. Prediction of the XV-15 OARF test by two theories is shown with Fig. 5. This comparison is most certainly very encouraging if for no other reason than its visual appearance of how close the theory lines are to the open, black circles used for the test data. Figure 5 is not, however, a quantified validation. It is just simply a comparison. However, this comparison can be extended to a validation in several ways, one of which is now discussed.

The objective is to see what error there is in predicting power required by the proprotor/propeller to produce any given thrust. It is hardly convenient or necessary to calculate a theory C_P value for each test C_T data point. Rather, the validation methodology only requires that (1) a sweep of collective pitch with the theory be made, (2) the resulting graph of C_P versus C_T be made and then (3) a sixth order polynomial curve fit be found. An example of this intermediate step is shown with Fig. 6. This figure was created in MicroSoft's EXCEL form and the "curve fits" were obtained using EXCEL's trendline tool. The primary advantage to the collective sweep calculation are that theory is not required to do a match thrust subroutine, which can avoid (a) computational problems associated with blade stall at high C_T and (b) inaccurate wake distortion near zero C_T.

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[1] Augustine, N. Augustine's Laws and Major System Development Programs, 2nd Edition, American Institute of Aeronautics and Astronautics, 1983

[2] See Figure A-8, page 48 of Ref. 2.

[3] Readers who wish to examine this point more fully are referred to Fig. A-6, page 46 of Ref. 2.

Fig, 2. XV-15 metal-bladed proprotor tested in hover at the NASA OARF in March of 1984 and 217 thrust–power data points obtained.

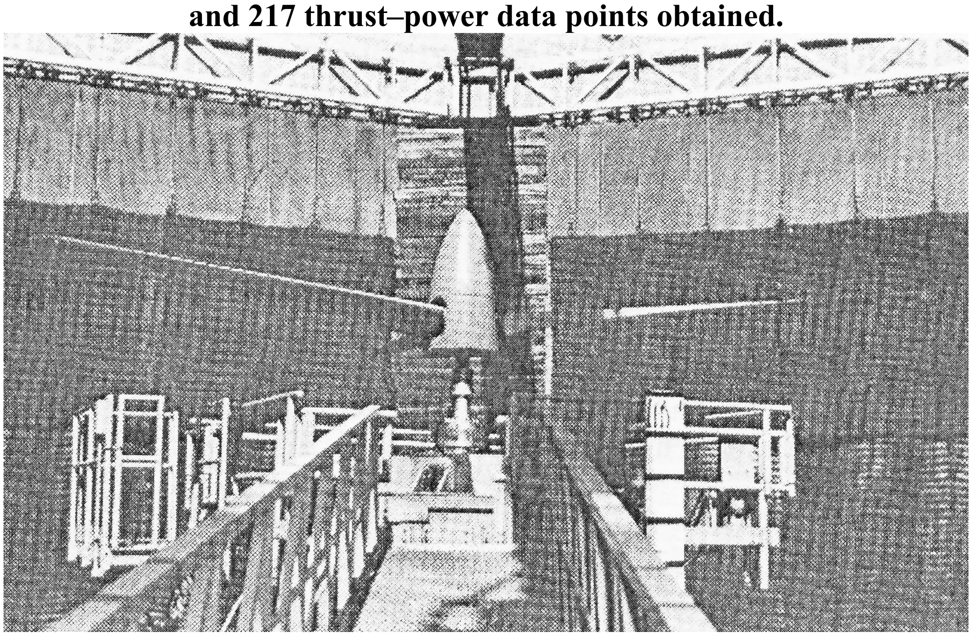

Fig. 3. XV-15 metal-bladed proprotor tested in hover on U.S. Air Force's Wright Air Development Center (WADC) Rig #3 in March of 1973 and 90 thrust –data points obtained.

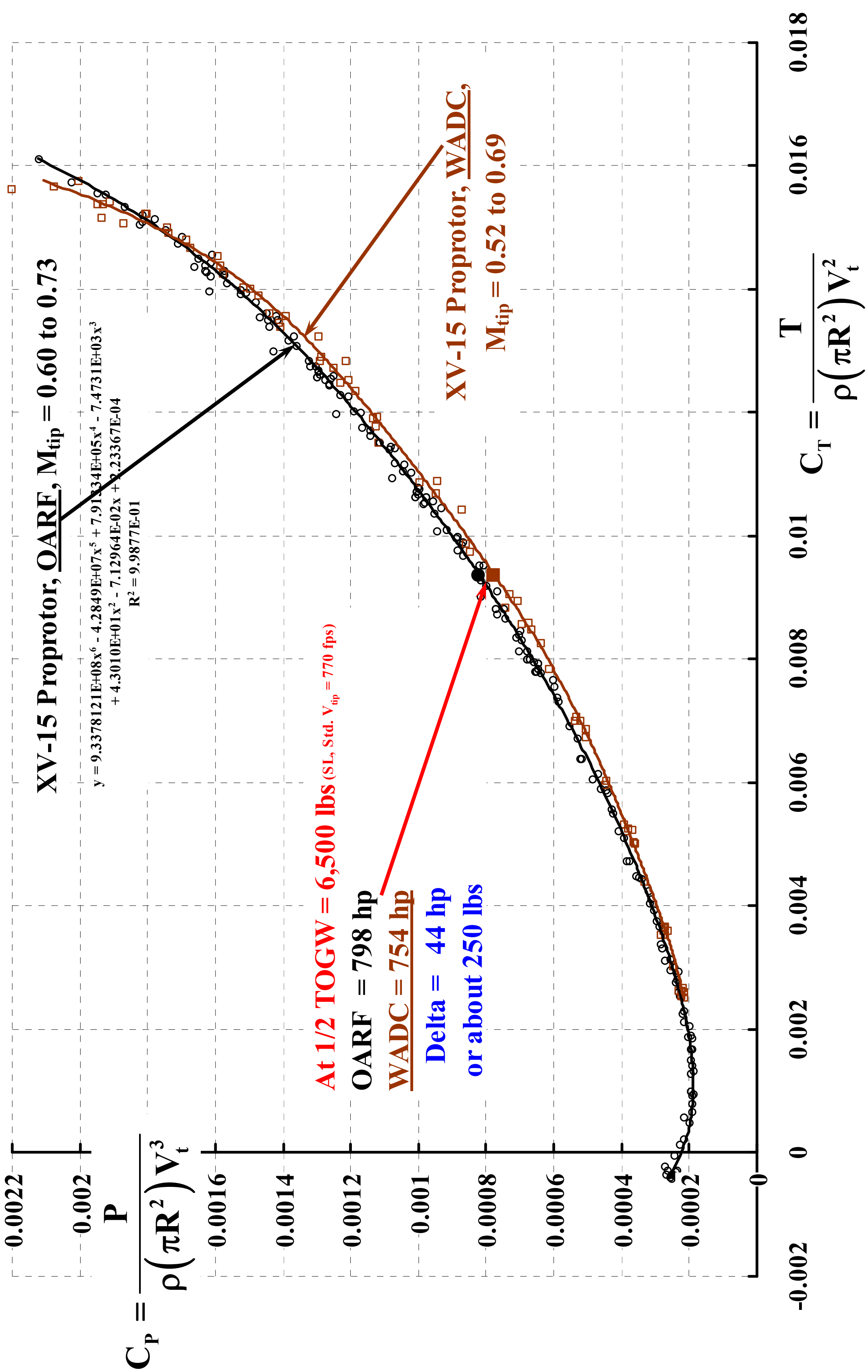

Fig. 4. Do not expect that test data obtained from two different whirl test rigs will be in agreement.

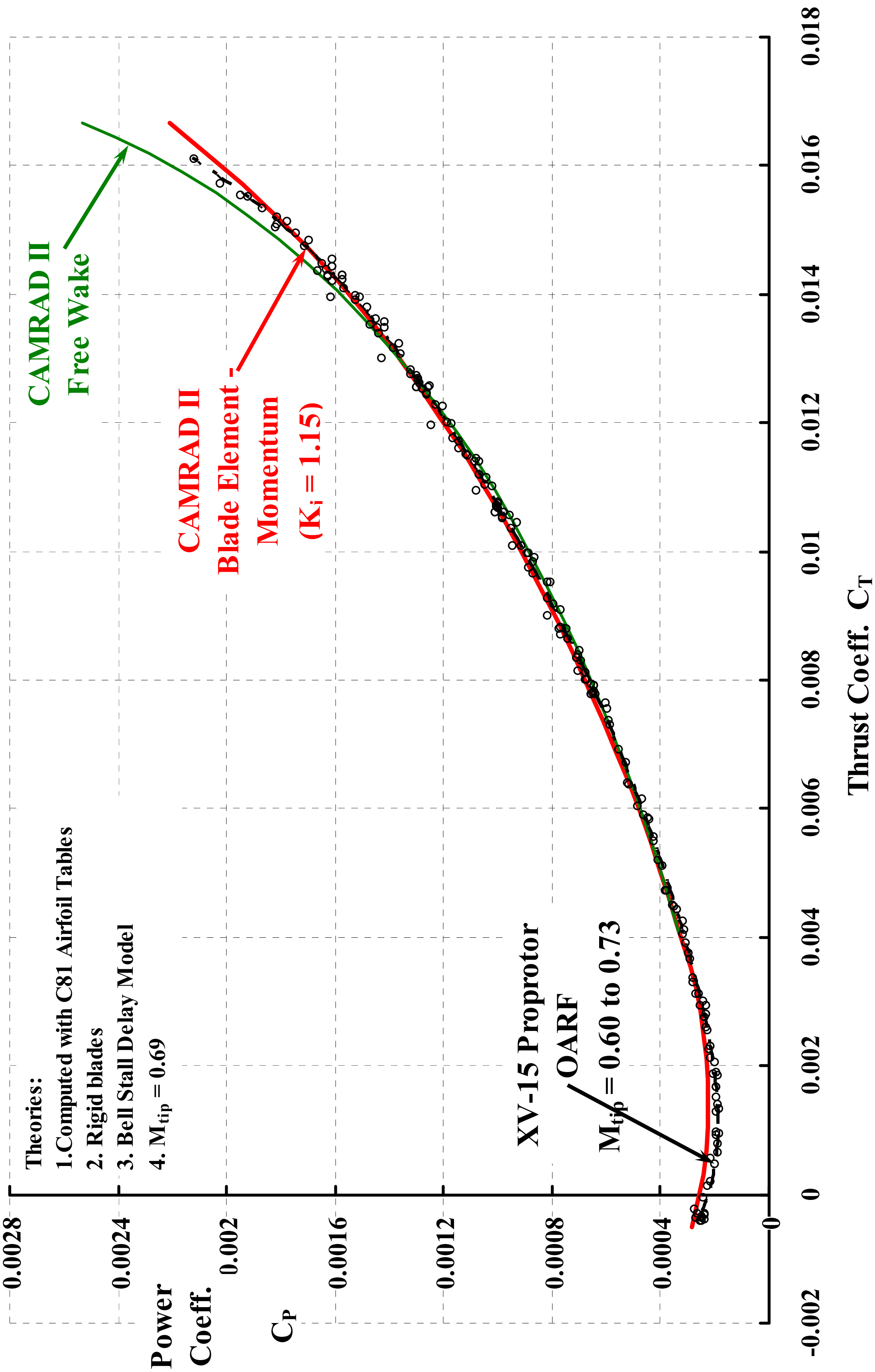

Fig. 5. As of 2017, difference between two different theories is the norm.

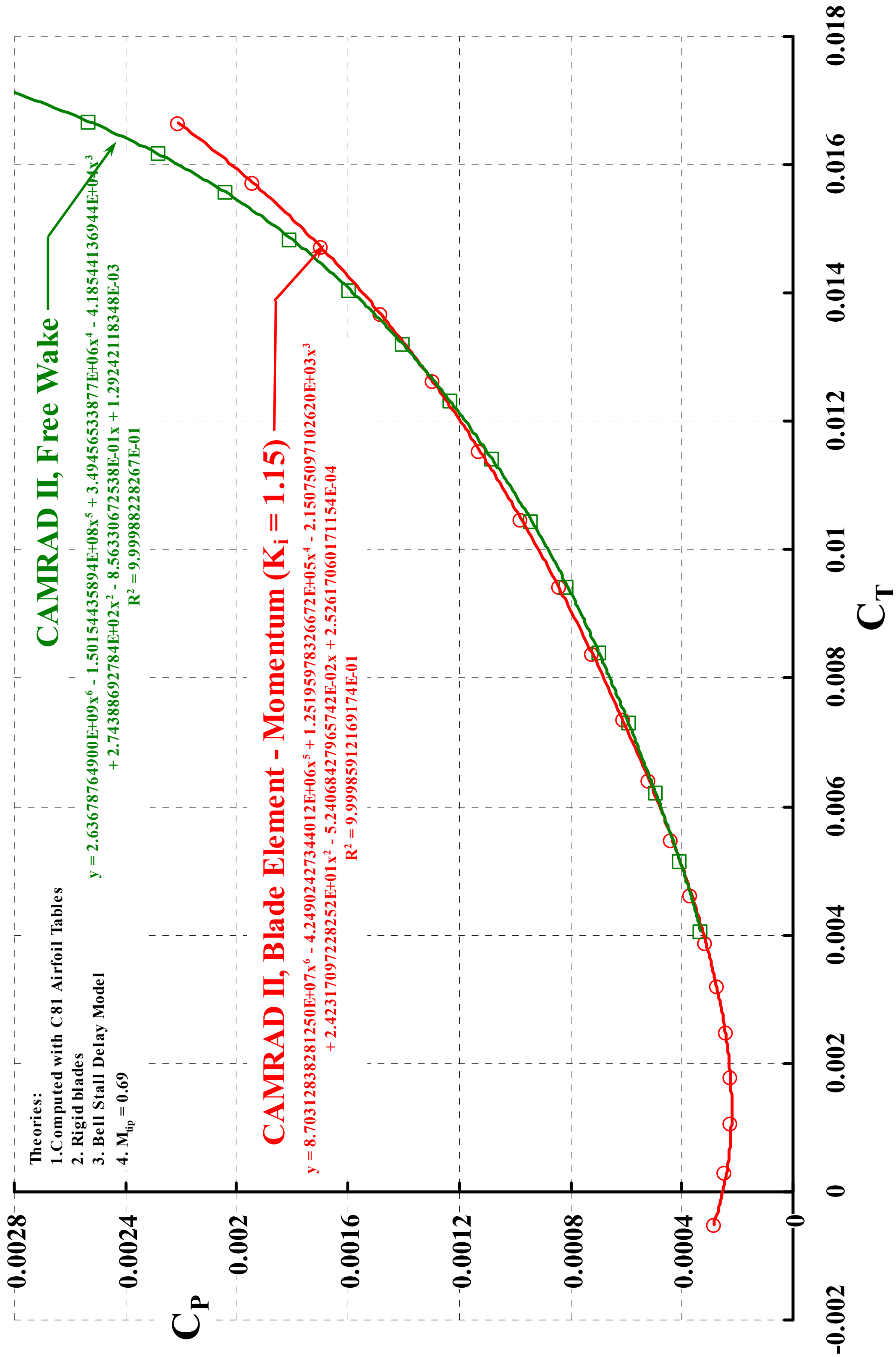

Fig. 6. Curve fitting theory results can be done quite accurately with a sixth order polynomial. This is useful for interpolation at any other C_T.

The next step in the theory validation process is to use the test and theory C_P versus C_T sixth order, interpolation equations to make calculations of C_P at equal C_T. This leads to a table shown for this example as

Table 1. C_P at Equal C_T with 2 Theories and Test Data.

C_T	CAMRAD II Blade Element-Momentum C_P	CAMRAD II Free Wake C_P	XV-15 Test on NASA OARF C_P	CAMRAD II Blade Element-Momentum Test C_P/Theory C_P	CAMRAD II Free Wake Test C_P/Theory C_P
-0.0004	0.0002776		0.0002593	0.934	
0	0.0002526		0.0002234	0.884	
0.001	0.0002224		0.0001884	0.847	
0.002	0.0002294		0.0002044	0.891	
0.003	0.0002646		0.0002492	0.942	
0.004	0.0003211	0.0003290	0.0003106	0.967	0.944
0.005	0.0003939	0.0003971	0.0003833	0.973	0.965
0.006	0.0004792	0.0004756	0.0004657	0.972	0.979
0.007	0.0005748	0.0005622	0.0005582	0.971	0.993
0.008	0.0006794	0.0006575	0.0006614	0.974	1.006
0.009	0.0007926	0.0007632	0.0007756	0.979	1.016
0.010	0.0009151	0.0008824	0.0009005	0.984	1.021
0.011	0.0010484	0.0010183	0.0010359	0.988	1.017
0.012	0.0011952	0.0011755	0.0011829	0.990	1.006
0.013	0.0013595	0.0013600	0.0013457	0.990	0.990
0.014	0.0015463	0.0015813	0.0015346	0.992	0.970
0.015	0.0017626	0.0018535	0.0017686	1.003	0.954
0.016	0.0020175	0.0021985	0.0020800	1.031	0.946

Table 1 illustrates a very important point. The point is that validation can only be quantified in the $C_P – C_T$ range where both theory and test results are available. In this example, the CAMRAD II Blade Element-Momentum theory returned results even down to a C_T of – 0.00326, (which was of analytical interest), but the experimental testing was stopped at a collective pitch where C_T nominally equaled zero. Convergence of the free wake theory at low C_T was less than satisfactory and calculations were stopped at a C_T of + 0.004 as Table 1 shows.

It is, of course, quite valuable to present the tabulated validation results from Table 1 in graphical form. This can be done with two figures. The first, Fig. 7, shows the *test* C_P plotted versus *theory* C_P where each point has been obtained at equal C_T for the test and the theory. When presented as shown with Fig. 7, it is clear both theories used in this example appear to compare well with test data over a fairly wide range in C_T, say, from C_T equal 0.006 up to C_T equal 0.013.

The more informative view of validation is shown with Fig. 8. Here it becomes apparent that both of these theories are valid to only about ± 2 percent over the mid-thrust coefficient range for the XV-15 OARF test data. This suggests that nearly one-half the power margin identified by Crawford on Fig. 1 is due to an inability to accurately predict power required of just an isolated helicopter rotor, an isolated proprotor or an isolated propeller.

Now consider the components of total power, which are induced power and profile power.

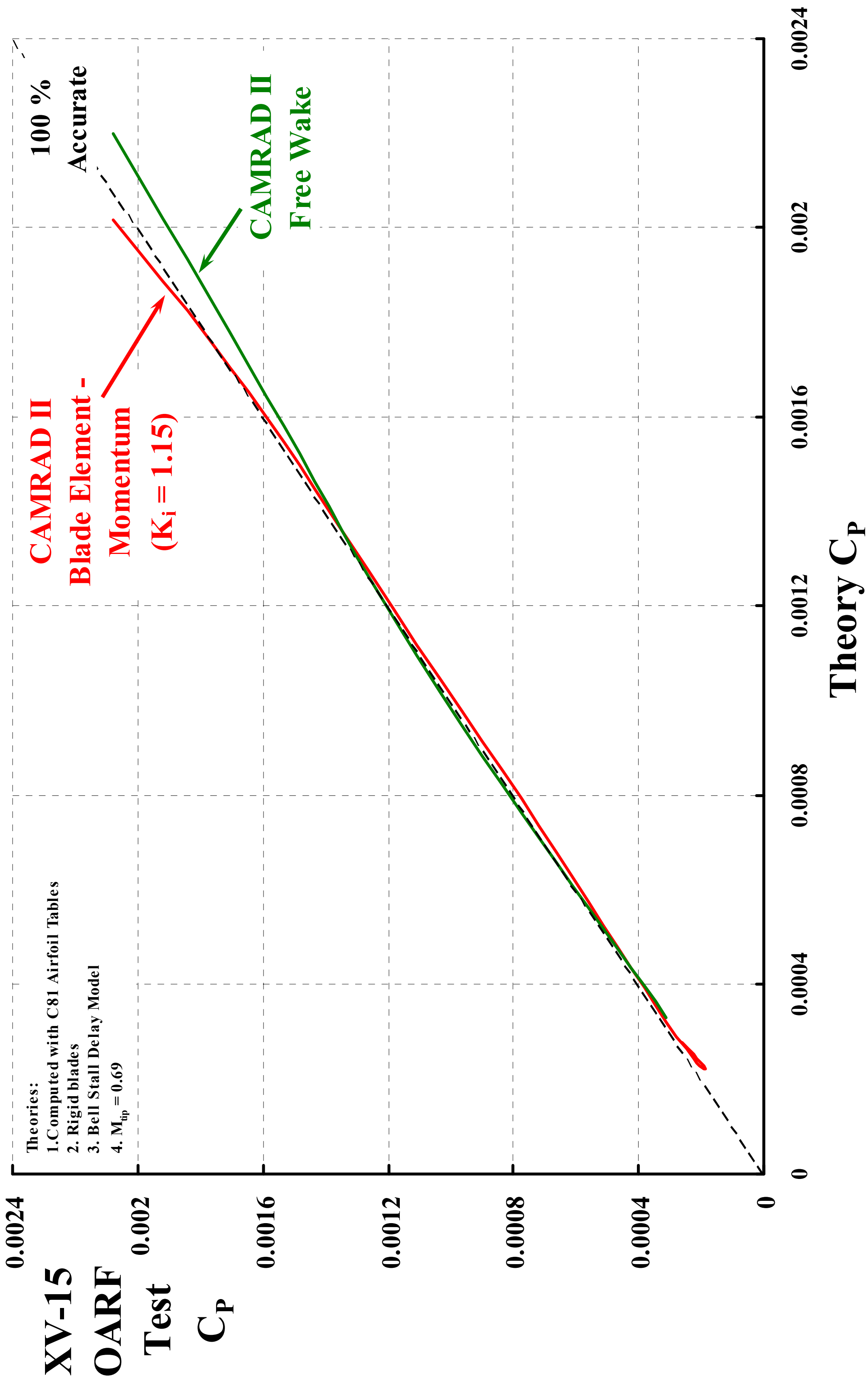

Fig. 7. One presentation form of theory–test validation compares power coefficients at equal thrust coefficients using the interpolation equations. (i.e., If F = ma, then plot F versus ma.)

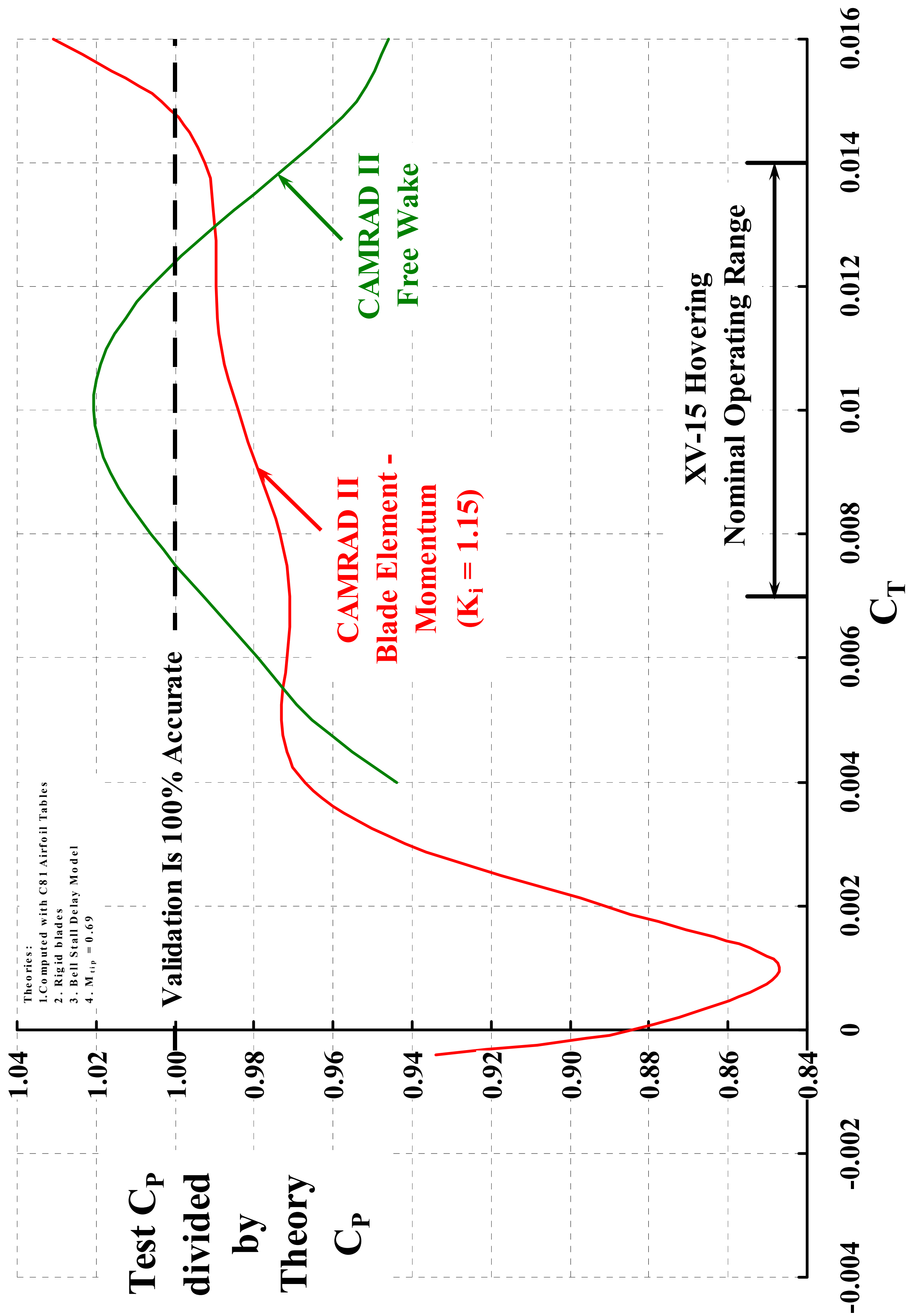

Fig. 8. A better presentation form of theory–test validation quantifies the error between test and theory.

Theories based upon dividing a proprotor's or propeller's blade span in to many blade elements and some induced velocity at each blade element are quite capable of separating the total power coefficient (C_P) into its two components of induced power coefficient (C_{Pi}) and profile power coefficient (C_{Po}). One need only refer to Alfred Gessow and Garry Myers' original classic *Aerodynamics of the Helicopter* first published in 1952 (Ref. 3) to establish a baseline for blade element–momentum theory. When the digital computer became available, the simple representation of a blade element's lift and drag coefficients by equations (as Gessow and Myers did) was replaced with a table lookup for the airfoil's lift, drag and moment coefficients (C_l, C_d and $C_{m1/4c}$) as a function of blade element angle of attack and Mach number. This table became widely known as *the* C81 table. A limitation of a C81 table is that the tabulated airfoil characteristics are for one specific chord and atmosphere. Somewhat later the induced velocity was calculated assuming a free wake controlled by the Biot-Savart law (plus a little semi-empiricism thrown in) rather than by momentum theory.

The contrast between element–momentum theory (with a K_i "correction factor" to induced velocity and thus induced power) and free wake theory apportionment of total C_P into C_{Po} and C_{Pi} is shown with Figs. 9 and 10 respectively. The first thing to notice from Fig. 9 is that up to a thrust coefficient of C_T equal to about 0.011, both theories have calculated virtually identical values of profile power. The differences between the two theories C_{Po} only becomes increasingly evident as the C_T increases and more and more blade elements move into a separated flow regime.

The second point to notice is that it is the difference in calculated induced power (Fig. 10) that is the greater contributor to differences in calculated total power (Fig. 5 and/or 6).

The third point to notice is that in the mid C_T range, induced power (Fig. 10) is about 8 or 9 times profile power (Fig. 9). The importance of this fact is that a one percent error in induced power would need to be corrected by nearly a 10 percent increase in profile power to obtain the same total power.

Based on these three points, it should be clear that further improvements in hover performance theory must come from researchers *first agreeing on induced power*. Then they can deal with profile power issues, which are due to airfoil drag.

Today, computational fluid dynamic (CFD) methods (obeying the first principles of the Navier-Stokes laws) are being applied to the hover performance problem with considerable success. However, one serious shortcoming of this CFD method is that total C_P can not (as yet) be apportioned into induced C_{Pi} and profile C_{Po} components One immediate way around this short coming is to assume the air has no viscosity (i.e., follow Euler's law), which should give a first order approximation to induced power. As to comparing blade element–free wake theories to CFD based theories, it is suggested that both theories compare airfoil normal force (F_N) and chord force (F_C) over the blade span, both theories making calculations at the same thrust. Keep in mind that CFD does not currently define a suitable velocity at the blade element. Therefore, coefficients of the normal and chordwise force are not particularly useful in comparing CFD and blade element-free wake theories.

Now consider the design engineer's view of validation.

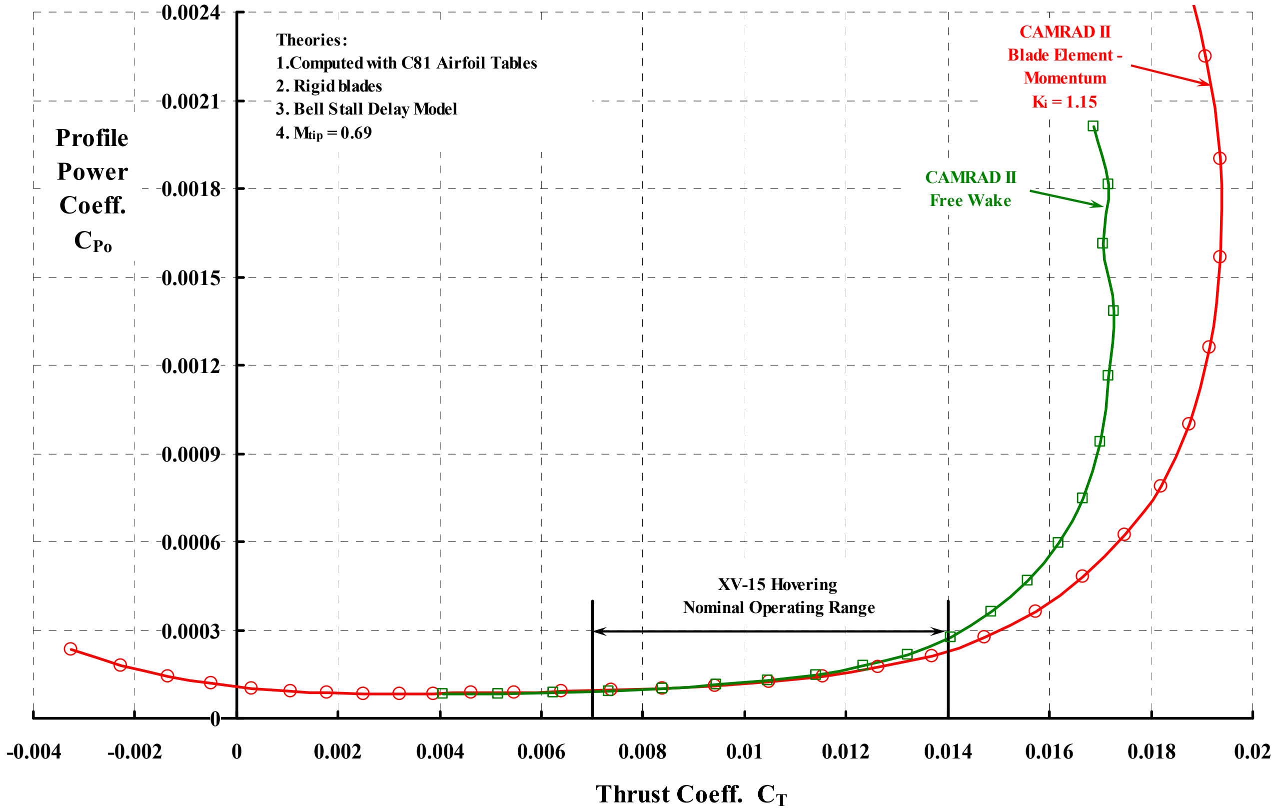

Fig. 9. The integration of profile power over the blade span disguises many errors.

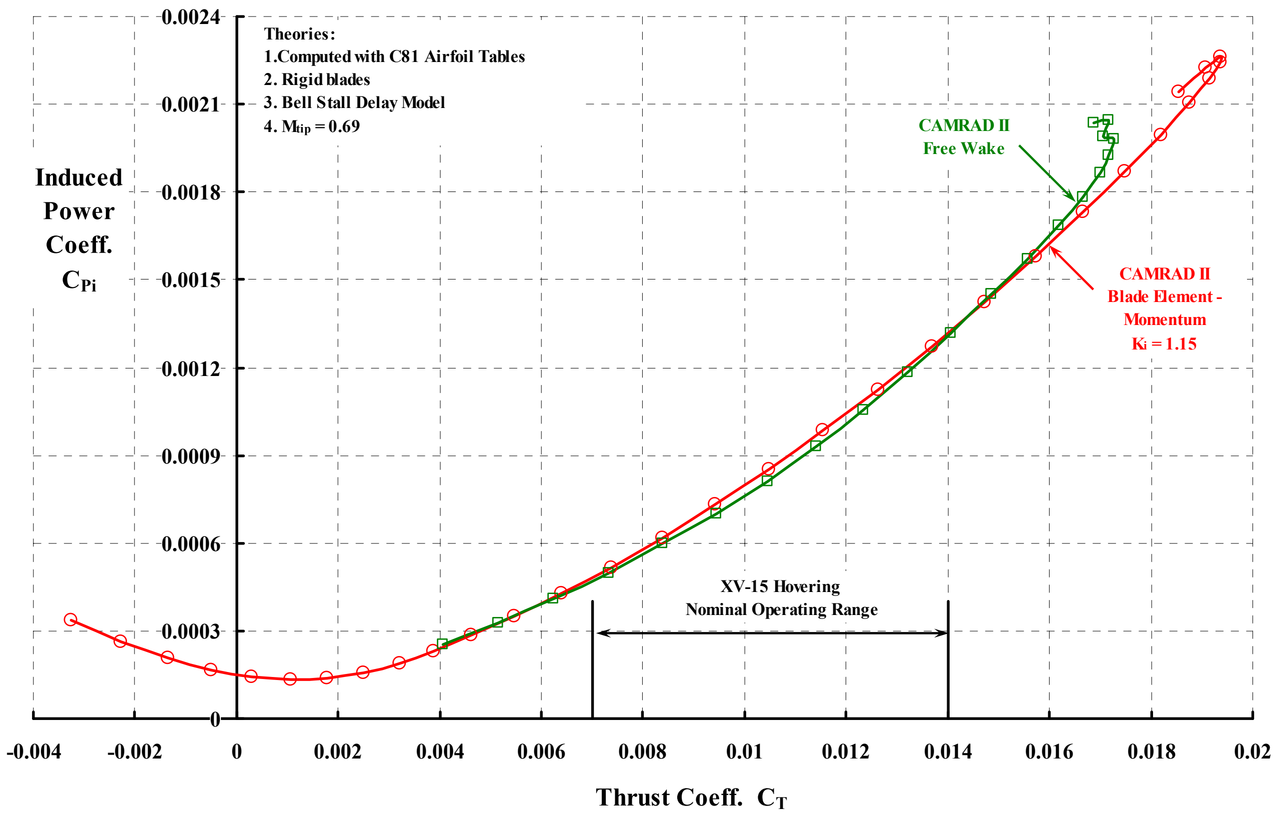

Fig. 10. Induced power is the first component of total power to get right.

The Design Engineer's View

Design engineers work much easier and quicker with hover performance in dimensions. After all, "design to" specifications state, as a typical example, "that the aircraft shall hover out of ground effect at 5,000 feet on a 95°F day at its design gross weight. The aircraft shall be under full control in any wind up to 35 knots regardless of the wind's direction." Furthermore, the customer rarely says anything about deficiencies being acceptable. When the customer does address this issue, it can be conveniently done with a penalty clause in the contract. And this clause might be so severe that the performing company could be forced into bankruptcy should the deficiencies to be repaired require many, many configuration changes.

Given this world that design engineers see daily, consider the researchers C_P versus C_T validation study dimensionalized to fit the XV-15 research aircraft discussed in Ref. 4 and shown in hovering flight here with Fig. 11. This VTOL aircraft was the first *successful* tiltrotor the world had seen and it led directly to the U.S. Marine's V-22B, the first tiltrotor to go into production. The XV-15 research aircraft's key dimensional properties for the following discussion are:

Table 2. Summary of Design Parameters for the XV-15 (Ref. 4)

Parameter	**XV-15**
Installed engine	LTC1K-41K
Takeoff power per engine (shp)	1,550
Gross weight (lb)	13,000
Weight empty (lb)	10,083
Fuel (lbs)	1,436
Proprotor diameter (ft)	25.0
Number of blades	3
Blade material	Metal
Thrust weighted solidity, σ_T	0.0892
Power weighted solidity, σ_P	0.0891
Design tip speed in hover V_t (fps)	771

Fig. 11. The world's first successful tiltrotor VTOL was the XV-15. This experimental aircraft was championed by NASA and the U.S. Army and developed by Bell Helicopter Textron.

Suppose now that the researchers' C_P versus C_T validation study is dimensionalized for design engineers. Assume the view for this user group is presented at sea level on a standard day where the air's density (ρ_o) is 0.002378 slugs per cubic foot. Keep in mind that the proprotor thrusts may exceed the XV-15's weight because of download on the wing (plus several other known and unknown effects) by a factor ranging from 5 to 15 percent. Based on Table 2, the lower bound to thrust coefficient would be

$$\text{(0.1)} \quad \text{Lower Bound } C_T = \frac{\frac{WE}{2\,\text{Props}}(1+DL)}{\rho A V_t^2} = \left(\frac{4}{2\pi\rho_o}\right)\frac{WE(1+DL)}{\sigma D^2 V_t^2} = 268\frac{WE(1+DL)}{\sigma D^2 V_t^2}$$

$$= 0.0007205\left(\frac{WE}{1{,}000}\right)(1+DL) = 0.007205(1+DL) \quad \text{for } \sigma = \rho/\rho_o = 1$$

An upper bound to C_T can be approximated for this example by assuming an overload gross weight of, say, the design weight of 13,000 pounds times 1.15 (for overload) or about 15,000 lbs. Additionally, a density ratio of 0.778 associated with 5,000 feet on a 95°F day might be considered. These assumptions lead to

$$\text{(0.2)} \quad \text{Upper Bound } C_T = \frac{\frac{\text{Design GW}(\text{Overload Factor})}{2\,\text{Props}}(1+DL)}{\rho A V_t^2} = \frac{0.0007205}{0.778}\left(\frac{15{,}000}{1{,}000}\right)(1+DL)$$

$$= 0.01389(1+DL) \quad \text{for } \sigma = \rho/\rho_o = 0.778$$

Thus, the design engineers are most interest in the researchers' validation study in the isolated proprotor C_T range of 0.007 to 0.014 (or even 0.016 if DL =0.15). The sixth order polynomial curve fit equations for both test and theory data cover this C_T range with reasonable accuracy as Figs. 4 and 6 confirm.

Given the test and theory curve fit equations, it is a simple task to calculate and tabulate the shaft horsepower ($SHP_{req,d.}$) each of the two proprotor shafts must have over a relatively narrow range in aircraft weight. For this XV-15 example, the designer's table might look like this:

Table 3. Design Engineer Performance with Table 2 Design Parameters
(Download Factor, DL = 0, Density Ratio = 1.0, M_{tip} = 0.69, V_t = 771 fps)

Parameter	10,000 lbs	11,000	12,000	13,000	14,000	15,000
Proprotor C_T	0.007209	0.007930	0.008651	0.009372	0.010093	0.010814
Test data C_P	0.0005789	0.0006538	0.0007345	0.0008208	0.0009126	0.0010099
Blade element-Mom. C_P	0.0005960	0.0006718	0.0007521	0.0008370	0.0009269	0.0010226
Free Wake C_P	0.0005814	0.0006505	0.0007249	0.0008058	0.0008942	0.0009915
Test $SHP_{req,d.}$	**563**	**636**	**714**	**798**	**887**	**982**
Blade element-Mom. $SHP_{req,d.}$	**579**	**653**	**731**	**814**	**901**	**994**
Free Wake $SHP_{req,d.}$	**565**	**632**	**705**	**783**	**869**	**964**

Table 3's data is graphed in Fig. 12. It is now apparent that the design engineer is faced with a considerable spread in how many pounds of gross weight can be lifted to an out of ground effect hover (HOGE). For example, if each engine were only able to deliver 800 horsepower to each proprotor at takeoff, there are about 363 pounds at doubt of the 1,463 pounds of fuel full capacity the XV-15 had installed. Said in an even graver way to a design engineer, the 363 pounds amounts to about 2 passengers seats out of the 7 passenger seats not being occupied in a production version of the XV-15 if at full fuel load. Alternately, the fuel load must be reduced from 1,463 to 1,100 pounds.

Of course, the correct situation would become evident when flight test data became available. Whereupon, the chief engineer would probably have to change the design, a step he was optimistically hoping to avoid.

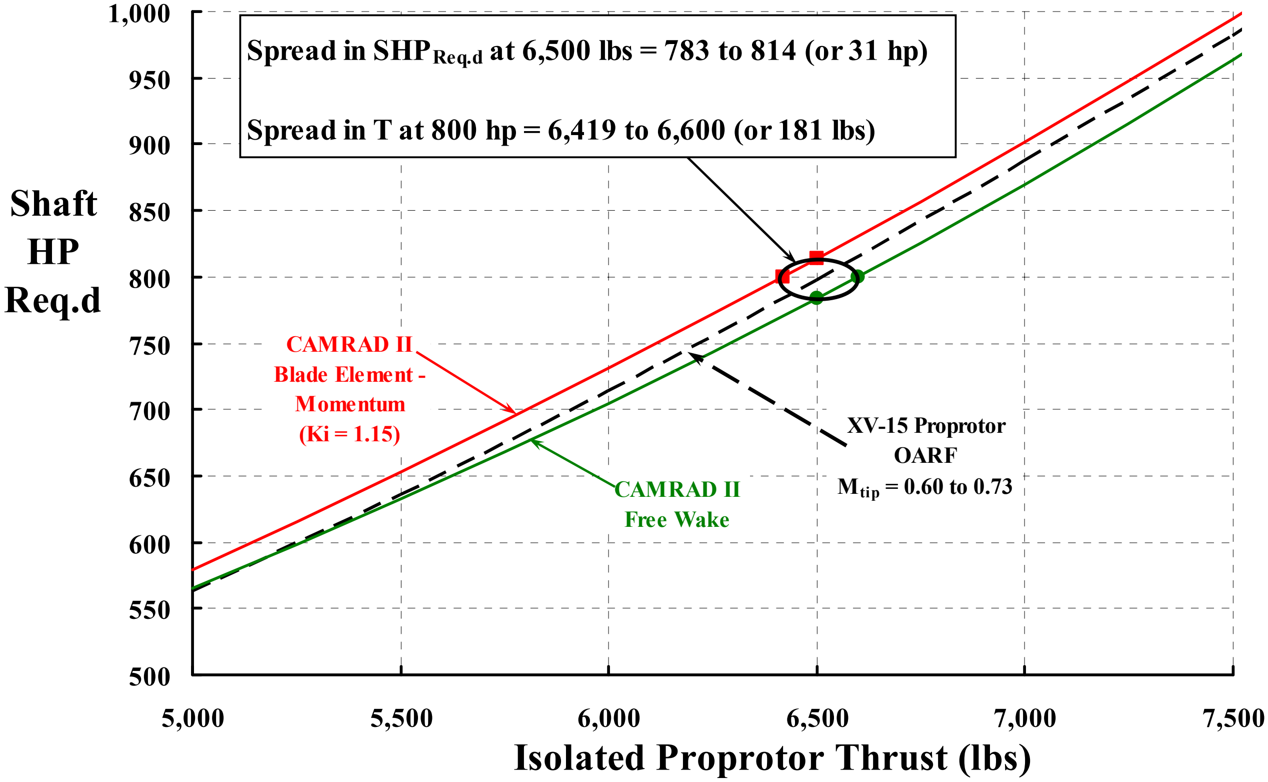

Fig. 12. The chief engineer's view of this XV-15 validation example is most certainly one of disappointment. 181 pounds equals 1 passenger for just one proprotor. That is 2 passengers out of 6 or 7 passenger seats on what could have been a production version of the XV-15.

CLOSING REMARKS

When faced with this 1 passenger issue (per proprotor), what should the chief engineer do about (1) freezing the design, (2) releasing drawing to manufacturing and (3) giving marketing data?

A. Take a conservative approach.

B. Be optimistic.

C. ????

AND, by the way, what about getting from predicting isolated proprotor data to predicting the C_P based on engine power versus C_T based on aircraft weight for the complete machine. This step introduces issues such as:

1. transmission efficiency, which, incidentally, is not a constant,
2. aerodynamic interferences like download and rotor upon rotor,
3. power to drive accessories,
4. engine thrust
5. ETC.

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ACKNOWLEDGEMENT

I owe Wayne Johnson big time for running CAMRAD II for me.