

# Shape Sensing for Wings with Spars and Ribs using Simulated Strain



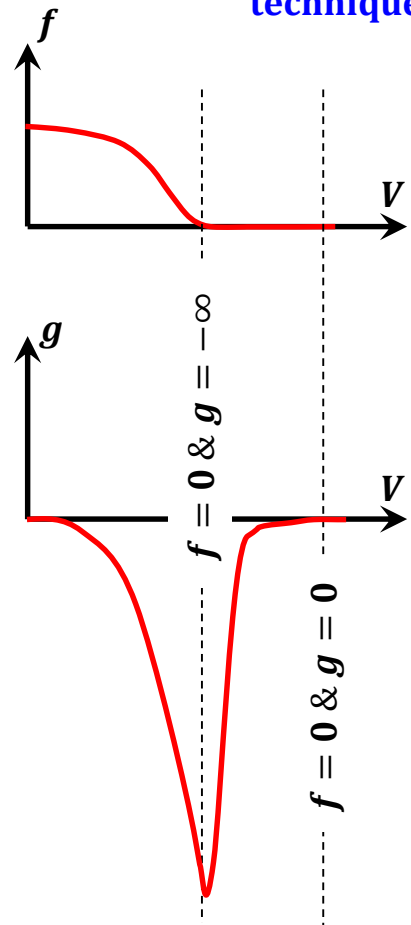
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# Active Shape Control

- ❑ NASA Helios fuel cell aircraft (2004)
  - ❖ Had an over-damped response to external loads before mishap
    - Recommendation 11: develop a method to **measure wing dihedral in real-time** with a visual display available to the test crew.
    - Recommendation 12: **develop manual and/or automatic techniques to control wing dihedral in flight.**



- ❑ NASA Low Boom Flight Demonstration aircraft (2021)
  - ❖ Minimize trim shape error: use “Jig shape optimization”
    - The major issue with this jig shape optimization is that the updated jig shape is optimum **only at the design flight condition.**
    - To overcome this limitation, **an active trim shape control technique can be used to minimize error between the target and the actual trim shapes during flight.**





# Aircraft Shape Sensing from Strain Data

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## ❑ Tessler and Spangler: 2003

- ❖ Inverse finite element method
  - Create simplified **3D structural** model
  - Need a finite element model
  - Use numerical optimization technique to minimize strain error at strain gage locations
  - **Off-line method**
- ❖ Tessler, A., and Spangler, J., “A Variational Principle for Reconstruction of Elastic Deformations in Shear Deformable Plates and Shells,” NASA Langley Research Center TM-212445, Hampton, Virginia, 2003.

## ❑ Ko: 2007

- ❖ Use closed-form equation for deformation computation
  - Deformation along a line is available during flight; **On-line method**
  - Don't need a finite element model
  - **Pitch slope is not available.**
- ❖ Ko, W. L., Richards, W. L., and Tran, V. T., “Displacement Theories for In-Flight Deformed Shape Predictions of Aerospace Structures,” NASA TP-2007-214612, 2007.

## ❑ Pak: 2016

- ❖ Use two-step approach; **On-line method**; **Based on 3D structure**
  - Step 1: deformation along a line (don't need a finite element model)
  - Step 2: expand deformation along the sensor lines to a 3D structure (Need a finite element model)
- ❖ Deformation sensing
  - Pak, C.-g., “Wing Shape Sensing from Measured Strain,” *AIAA Journal*, Vol. 54, No. 3, 2016, pp. 1068–1077.
- ❖ Velocity, Acceleration, and Load sensing
  - Pak, C.-g., “Unsteady Aerodynamic Force Sensing from Strain Data,” *Journal of Aircraft*, Vol. 54, No. 4, 2017, pp. 1476–1485.



# Definition of Curvature $\kappa$

## Upper strain due to pure bending

$$\diamond \epsilon_u - x = \epsilon_u - \frac{\epsilon_u + \epsilon_l}{2} = \frac{\epsilon_u - \epsilon_l}{2}$$

$$\diamond \kappa = -\frac{(\epsilon_u - \epsilon_l)/2}{h/2} = -\frac{\epsilon_u - \epsilon_l}{h}$$

## Lower strain due to pure bending

$$\diamond \epsilon_l - x = \epsilon_l - \frac{\epsilon_u + \epsilon_l}{2} = -\frac{\epsilon_u - \epsilon_l}{2}$$

$$\epsilon_u - x = -(\epsilon_l - x)$$

$$\epsilon_u - x = x - \epsilon_l$$

$$2x = \epsilon_u + \epsilon_l$$

$$x = \frac{\epsilon_u + \epsilon_l}{2} : \text{Strain due to in-plane loading}$$

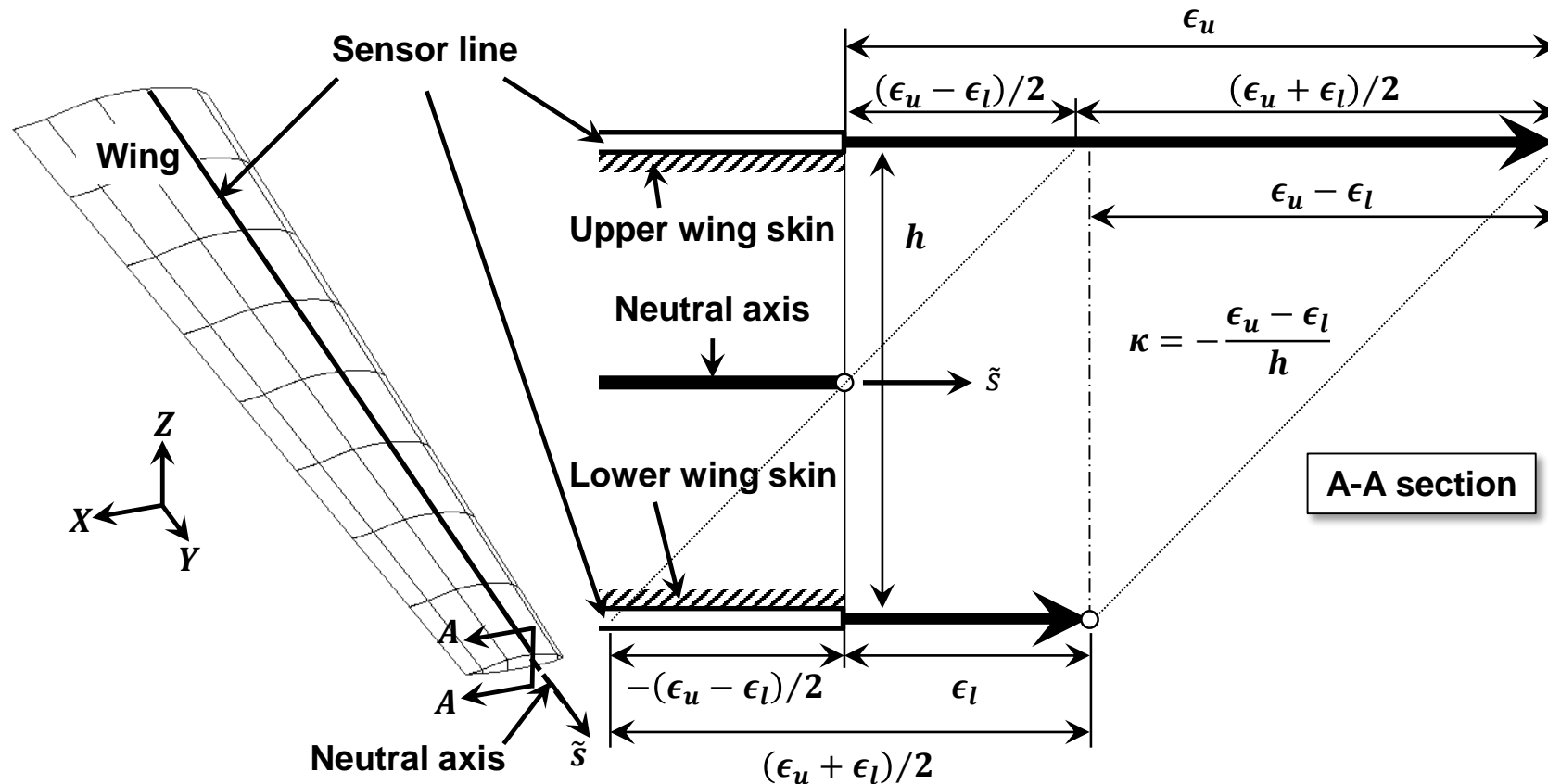


Fig. 1

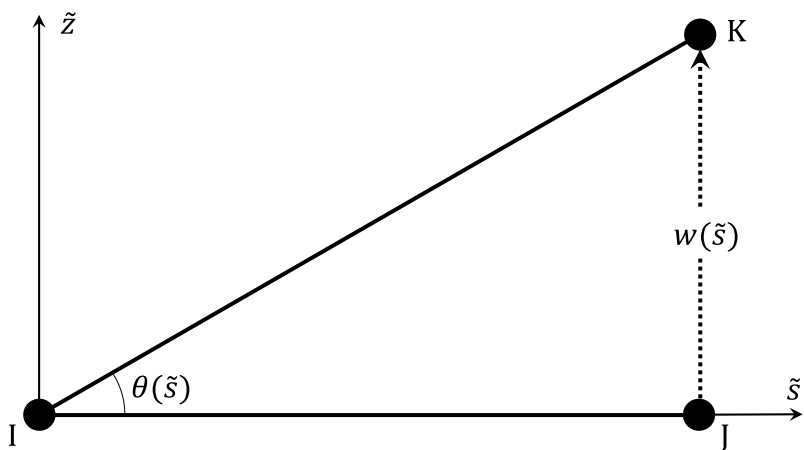
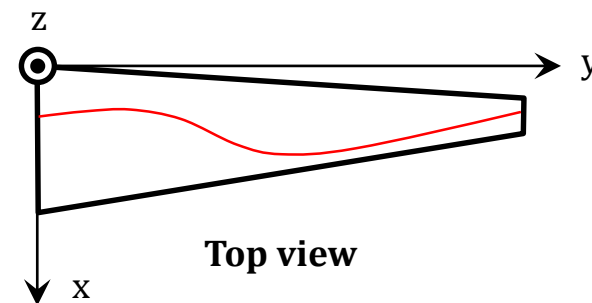
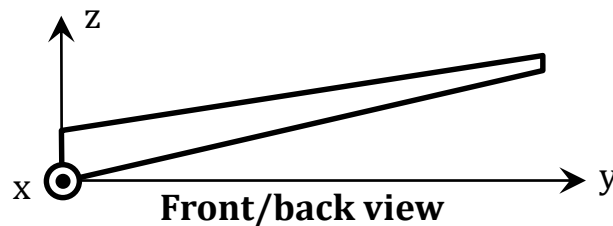


# Definition of coordinate systems for deformation computations

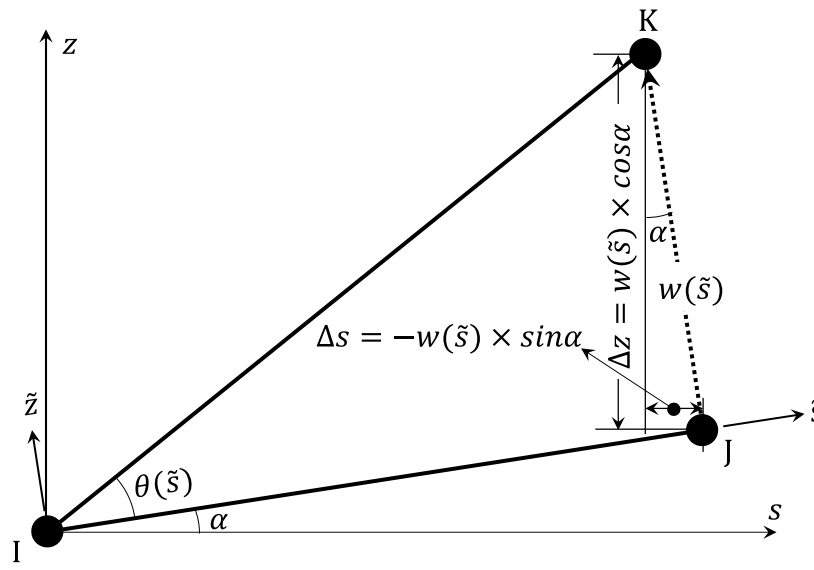
- ❑ Each fiber element
- ❑ Axial strain
- ❑ Rotation  $\theta(\tilde{s})$  and translation  $w(\tilde{s})$  in "Fiber coordinate"
  - ❖ Use linear assumption
  - ❖  $\theta(\tilde{s})$  : integrate curvature with respect to  $\tilde{s}$
  - ❖  $w(\tilde{s})$ : integrate  $\theta(\tilde{s})$  with respect to  $\tilde{s}$

- ❑ Effect of dihedral/anedral and/or taper,  $\alpha$

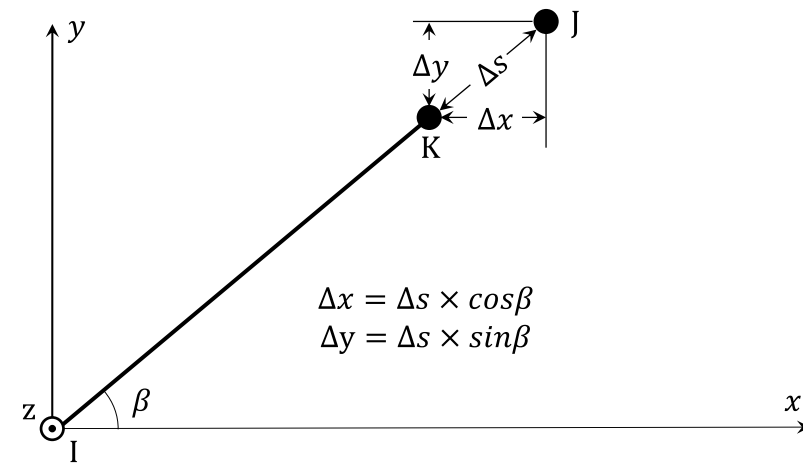
- ❑ Curved fiber element,  $\beta$



(a) Fiber coordinate ( $\tilde{s}, \tilde{z}$ )



(b) Local coordinate ( $s, z$ )



(c) Global coordinate ( $x, y, z$ )

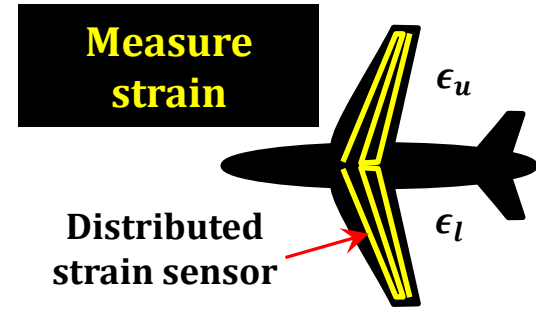
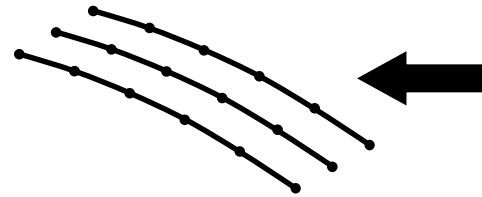
**Fig. 2**



# Mathematical Background of the Two-step Theory

Compute curvature along sensor lines

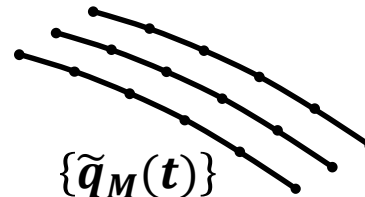
$$\kappa(s) \equiv -\frac{(\epsilon_u - \epsilon_l)/2}{h/2} = -\frac{\epsilon_u(s) - \epsilon_l(s)}{h(s)}$$



Step 1: Compute wing deflection along sensor lines

Curvature → Rotation → Translation

$$\frac{d^2w(s)}{ds^2} = \kappa(s) \quad \frac{dw(s)}{ds} \quad w(s)$$

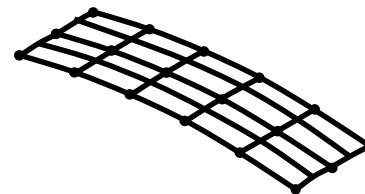


Model independent



Step 2: Expand wing deflection

$$\begin{Bmatrix} q_M(t) \\ q_S(t) \end{Bmatrix} = [T] \{ \tilde{q}_M(t) \}$$



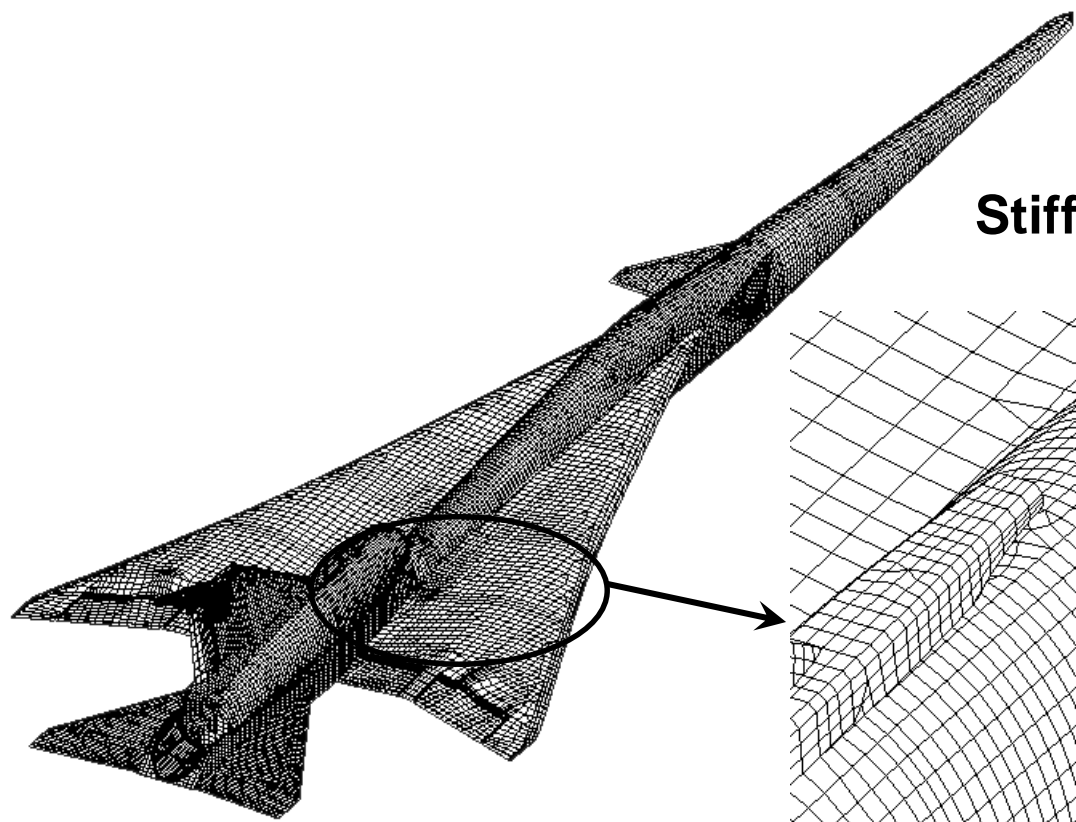
Model dependent

# Low-Boom Flight Demonstration aircraft



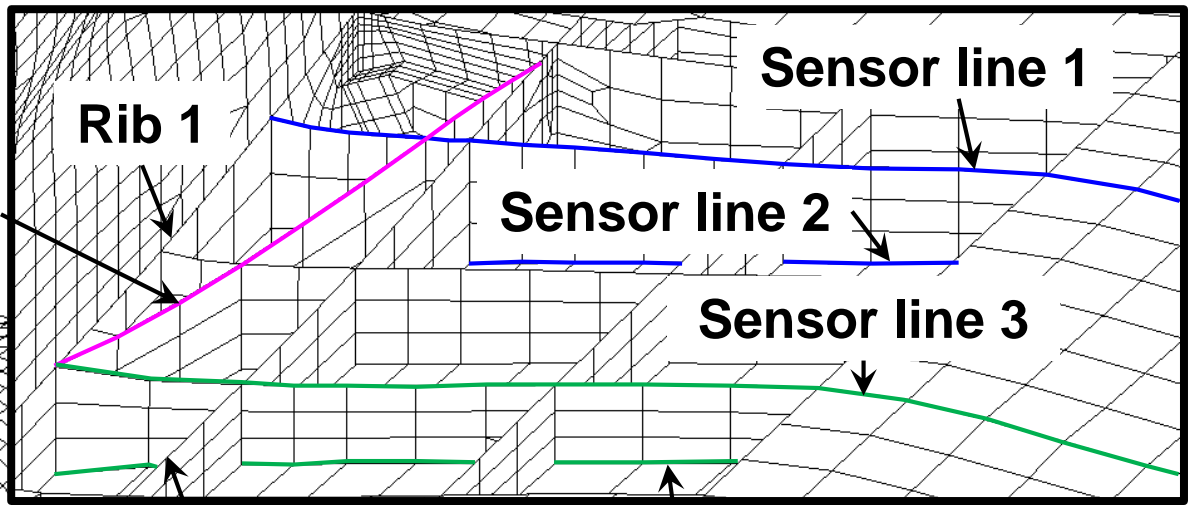


# C607 Wing



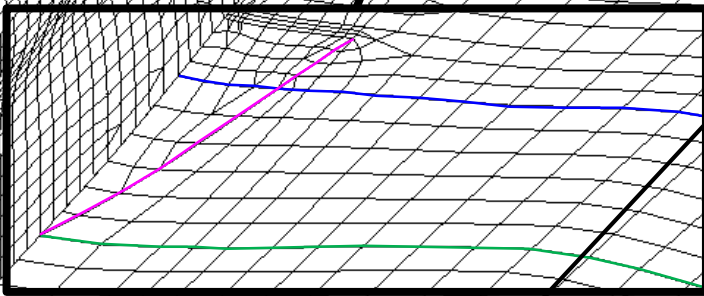
C607 Model

Stiffener



Rib 2

Sensor line 4



Dihedral

Anhedral

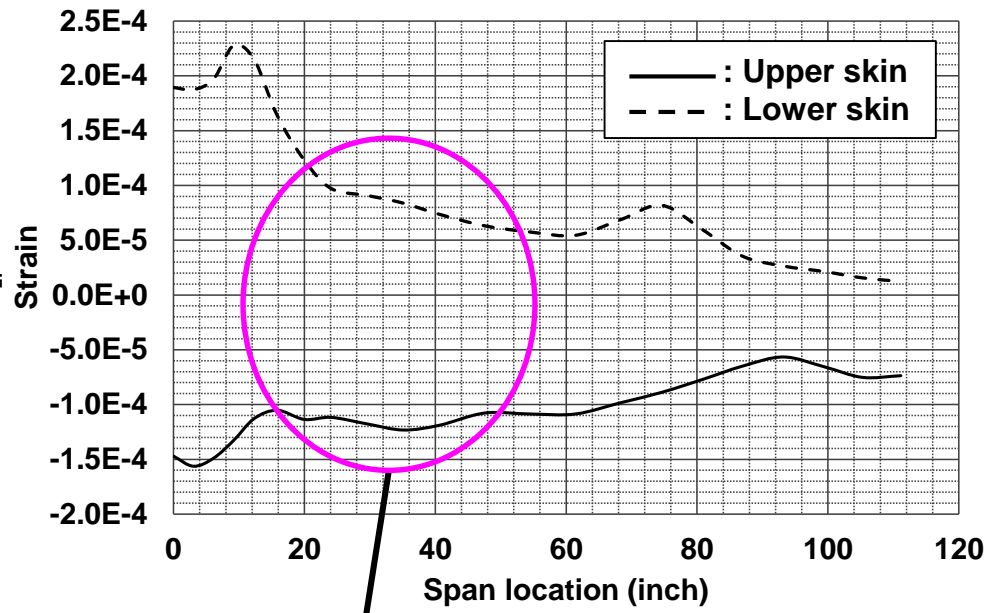




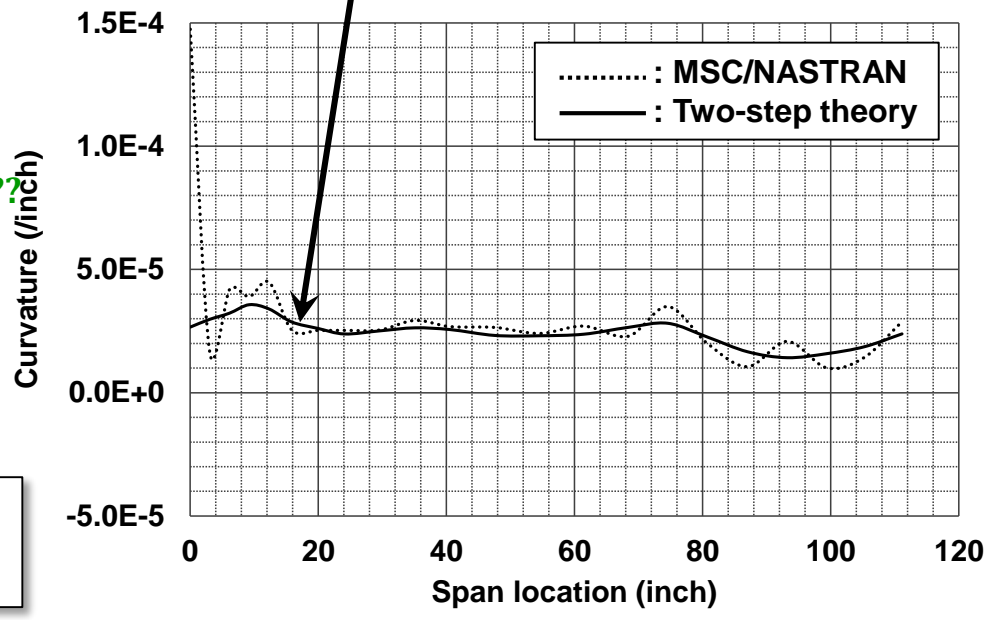
# Strain, curvature, & deformation

- ❑ Trim load under Mach 1.42 flight condition.
  - ❑ Differences at wing tip
    - ❖ Slope: **-11.2%**
    - ❖ Deflection: **-19.8%**
  - ❑ Issue
    - ❖ Curvature definition
      - Looks **fine**
- $$\kappa = -\frac{\epsilon_u - \epsilon_l}{h}$$
- ❖ FE structural model
    - NASTRAN **slope???** near wing root

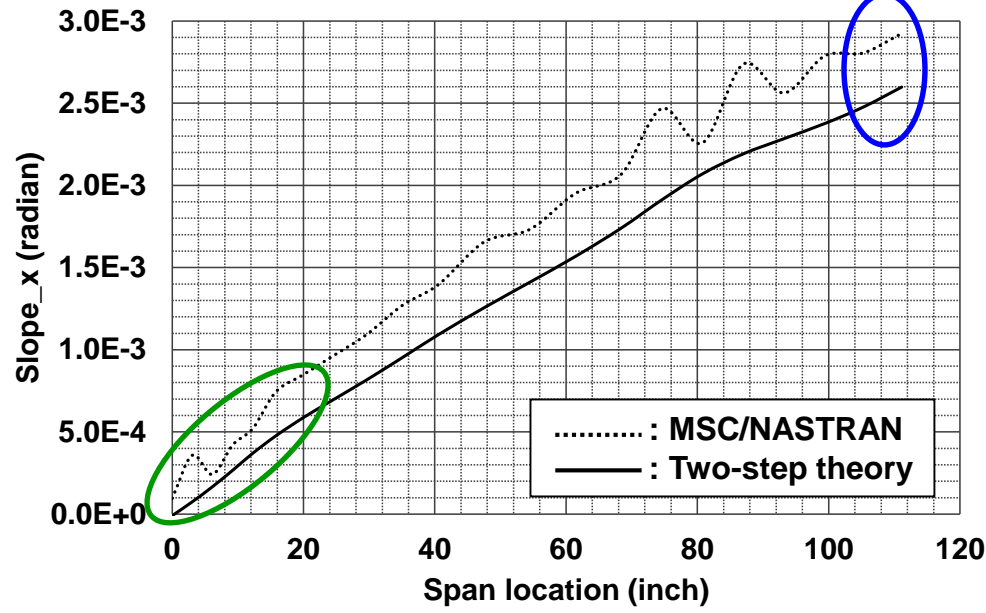
**LBFD aircraft using sensor lines 1 & 2 data**



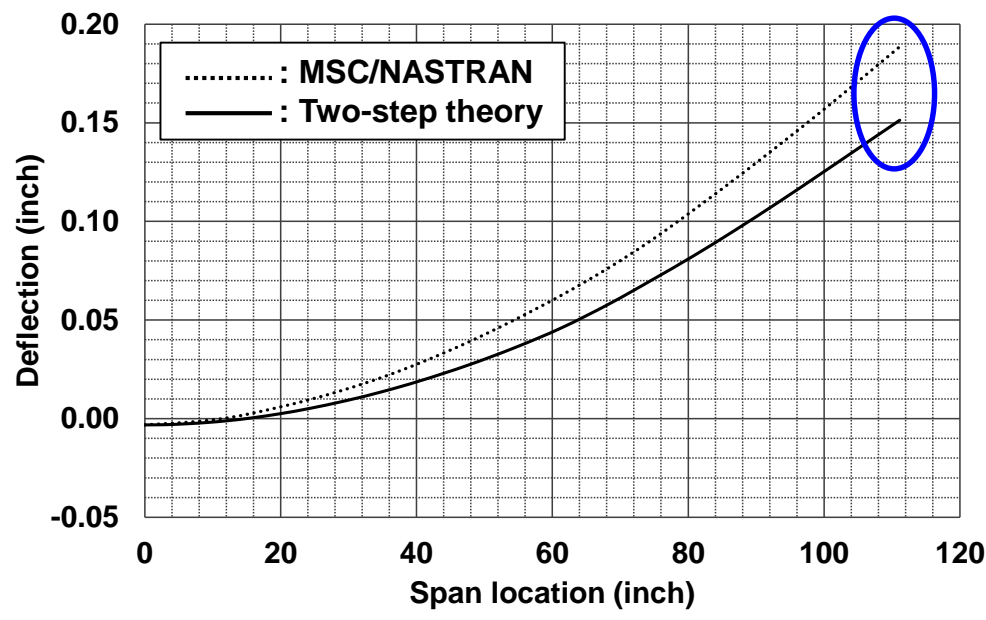
(a) Strain on the upper and lower skin



(b) Curvature



(c) Slope in roll direction



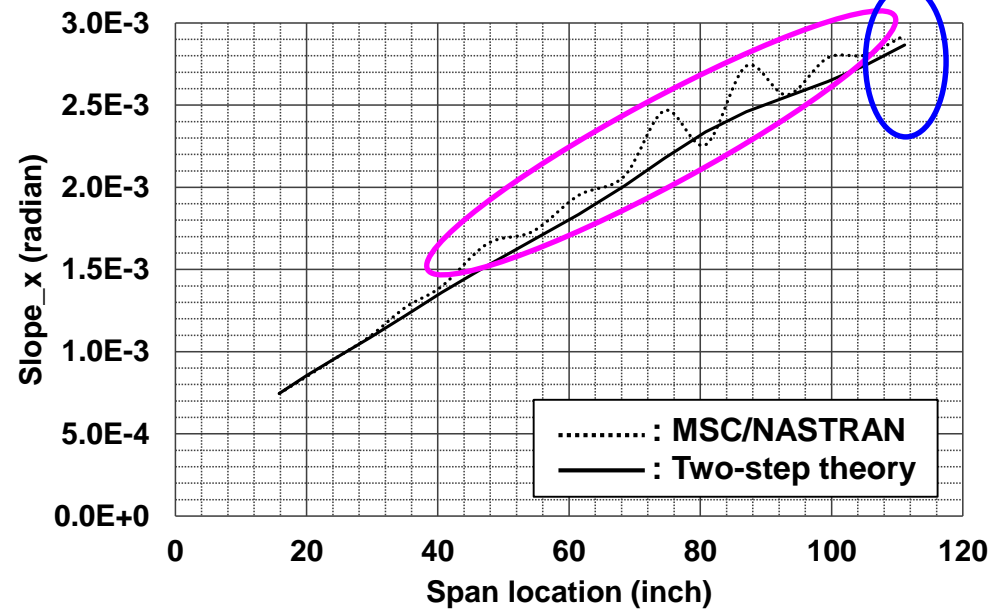
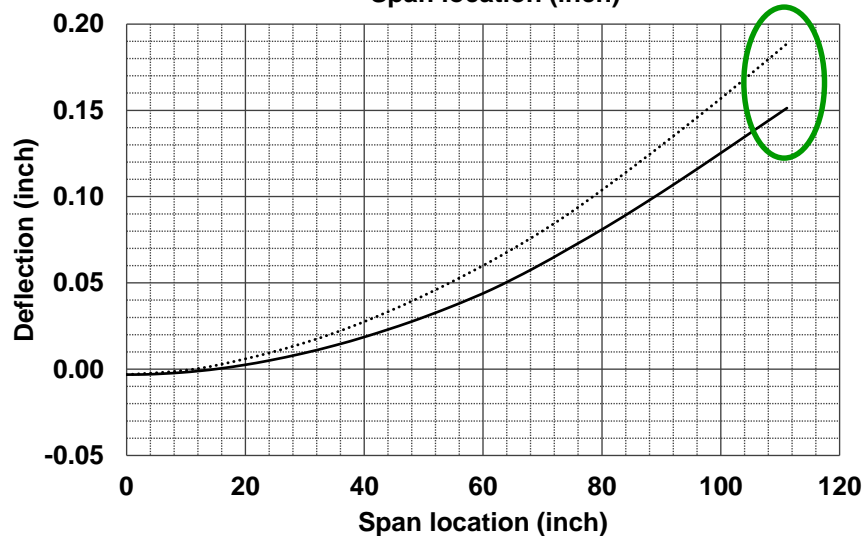
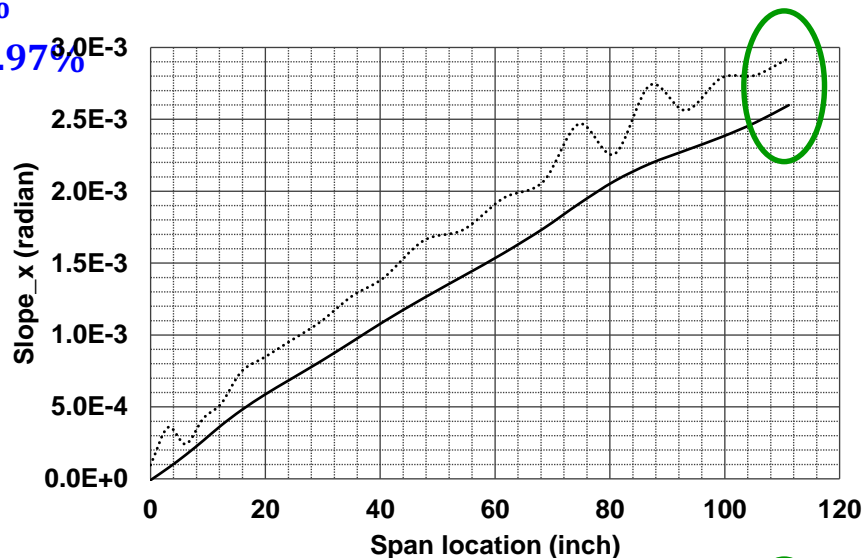
(d) Deflection

**Fig. 4**

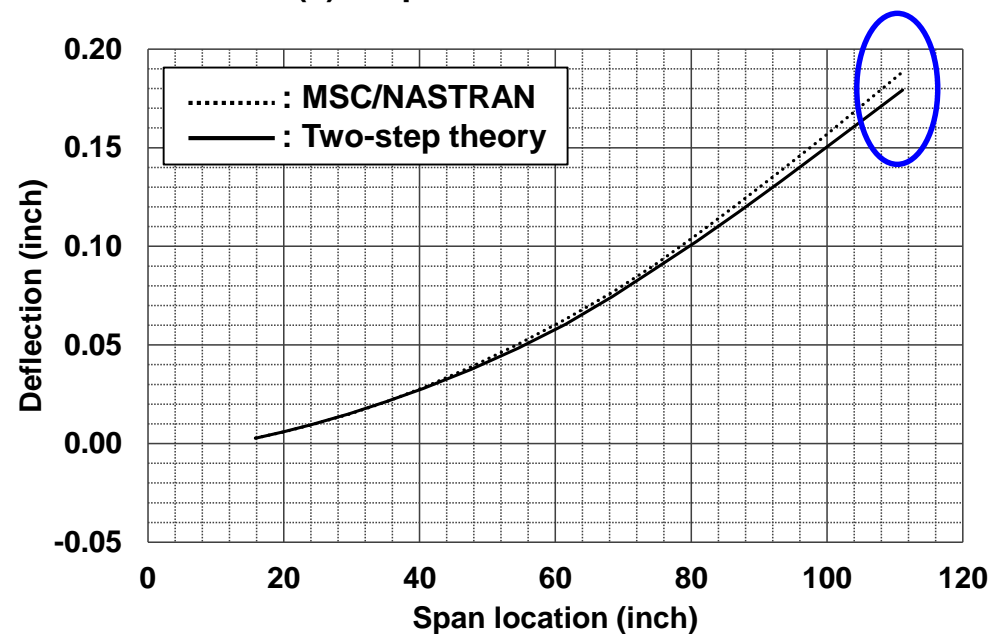


# Deformation of LBFD aircraft integrated from 18 inch using sensor lines 1 & 2 data

- ❑ Don't include wing root to 18 inch
- ❑ Differences at wing tip
  - ❖ Slope: **-11.2%** ---> **-2.02%**
  - ❖ Deflection: **-19.8%** ---> **-4.97%**
- ❑ Issue
  - ❖ Curvature definition
  - ❖ FE structural model
    - NASTRAN **slope**



(a) Slope in roll direction

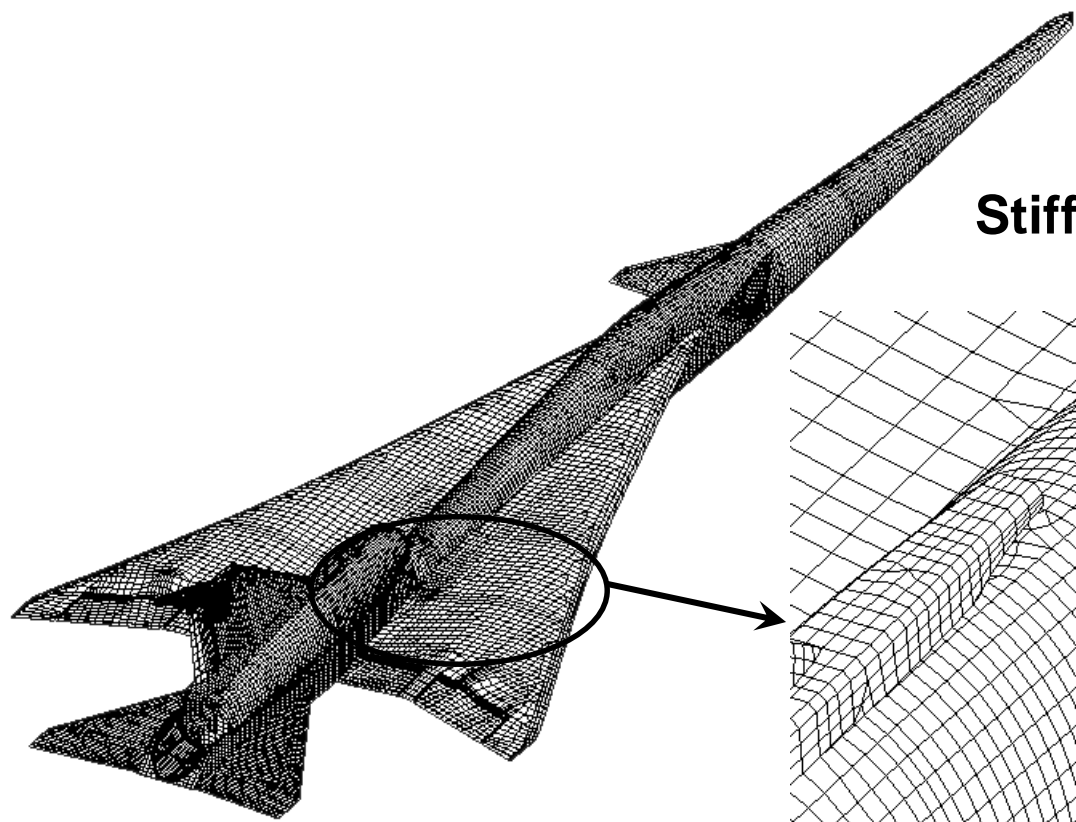


(b) Deflection

**Fig. 5**

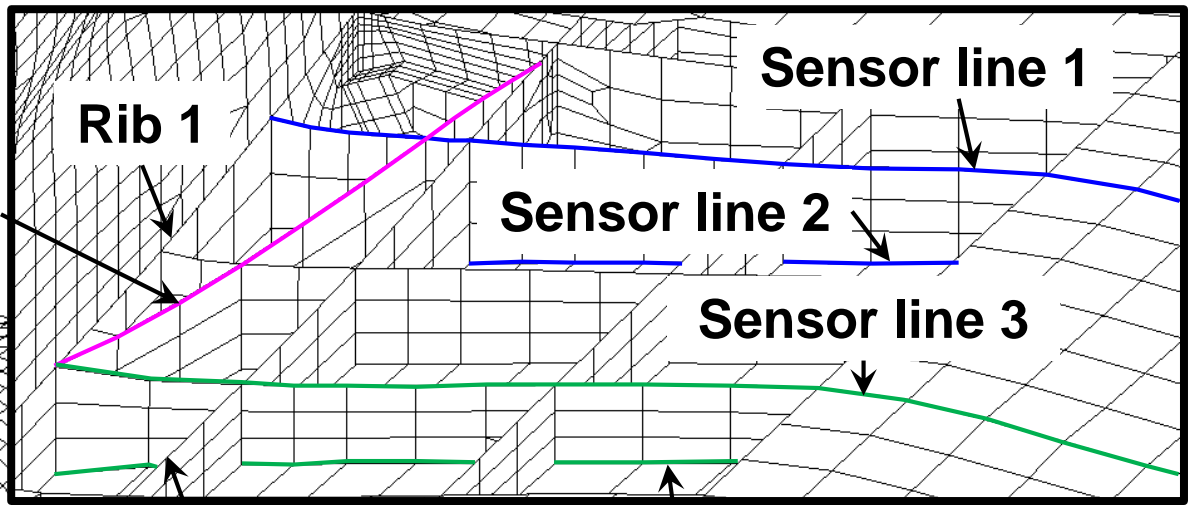


# C607 Wing



C607 Model

Stiffener



Rib 1

Sensor line 1

Sensor line 2

Sensor line 3

Rib 2

Sensor line 4

Dihedral

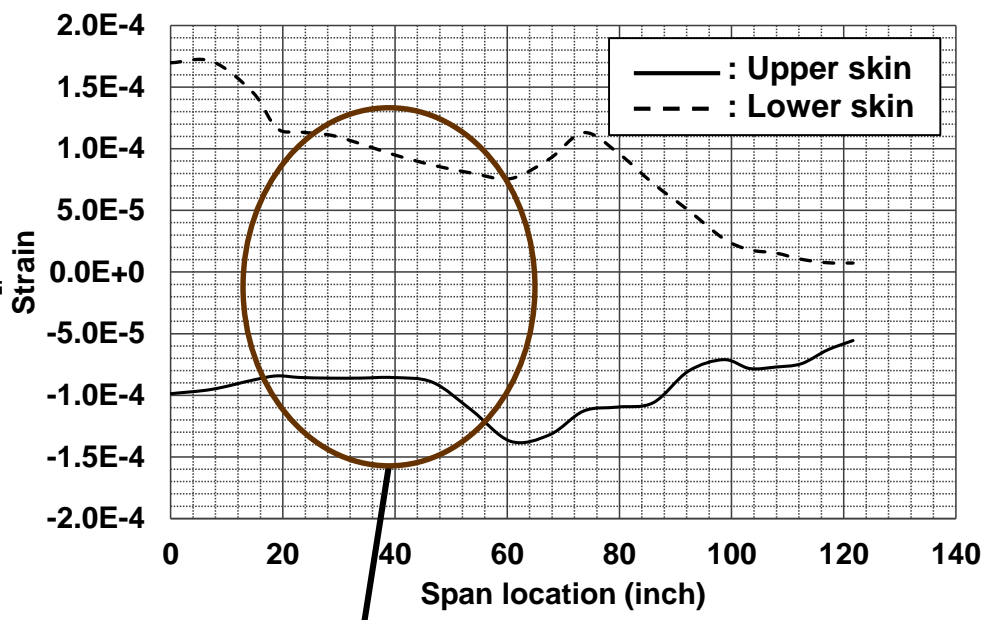
Anhedral



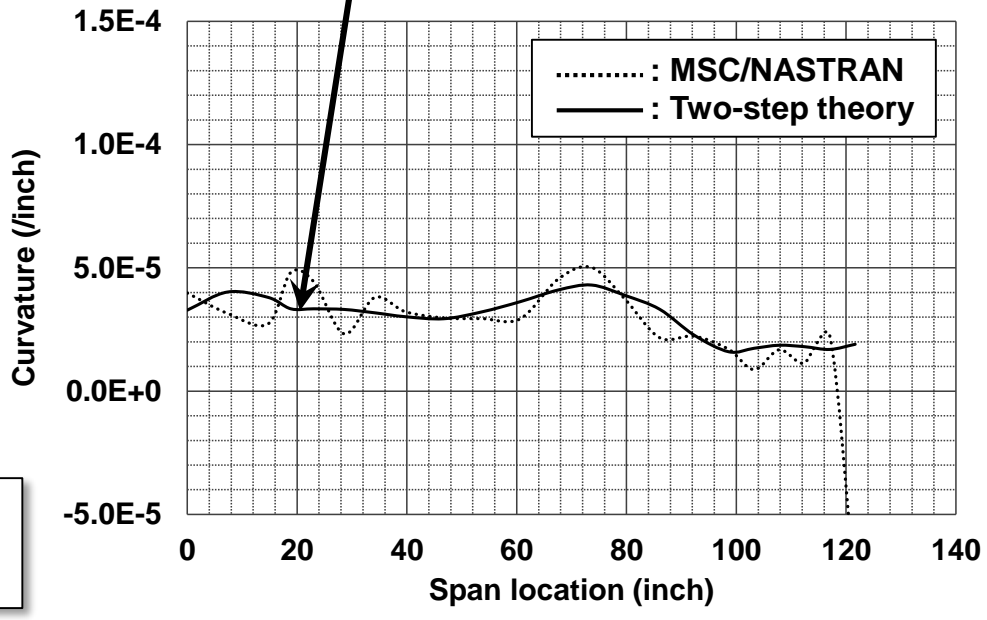
# Strain, curvature, & deformation

- ❑ NASTRAN slope near wing root area becomes better
  - ❖ Stiffening structure effect??
- ❑ Differences at wing tip
  - ❖ Slope: **-1.91%**
  - ❖ Deflection: **-6.63%**
- ❑ Issue
  - ❖ Curvature definition
    - Looks **fine**
  - ❖ FE structural model
    - NASTRAN **slope**

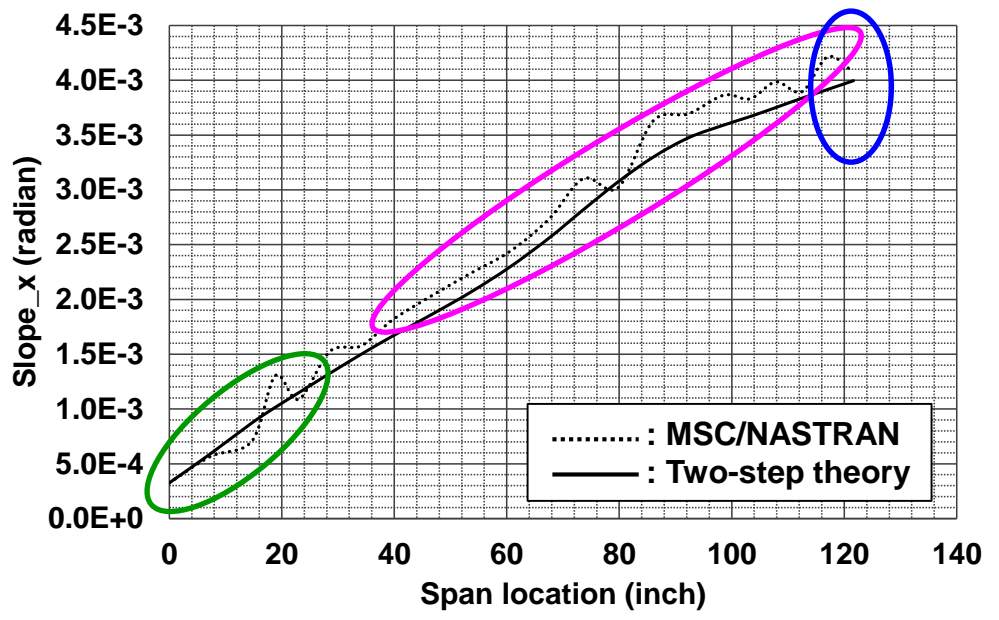
**LBFD aircraft using sensor lines 3 & 4 data**



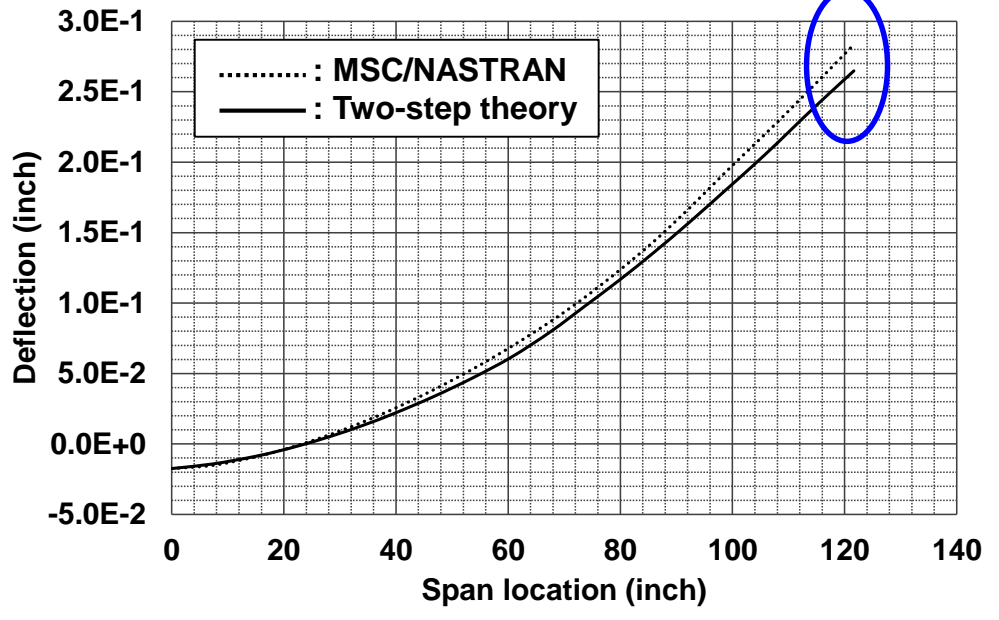
(a) Strain on the upper and lower skin



(b) Curvature



(c) Slope in roll direction



(d) Deflection

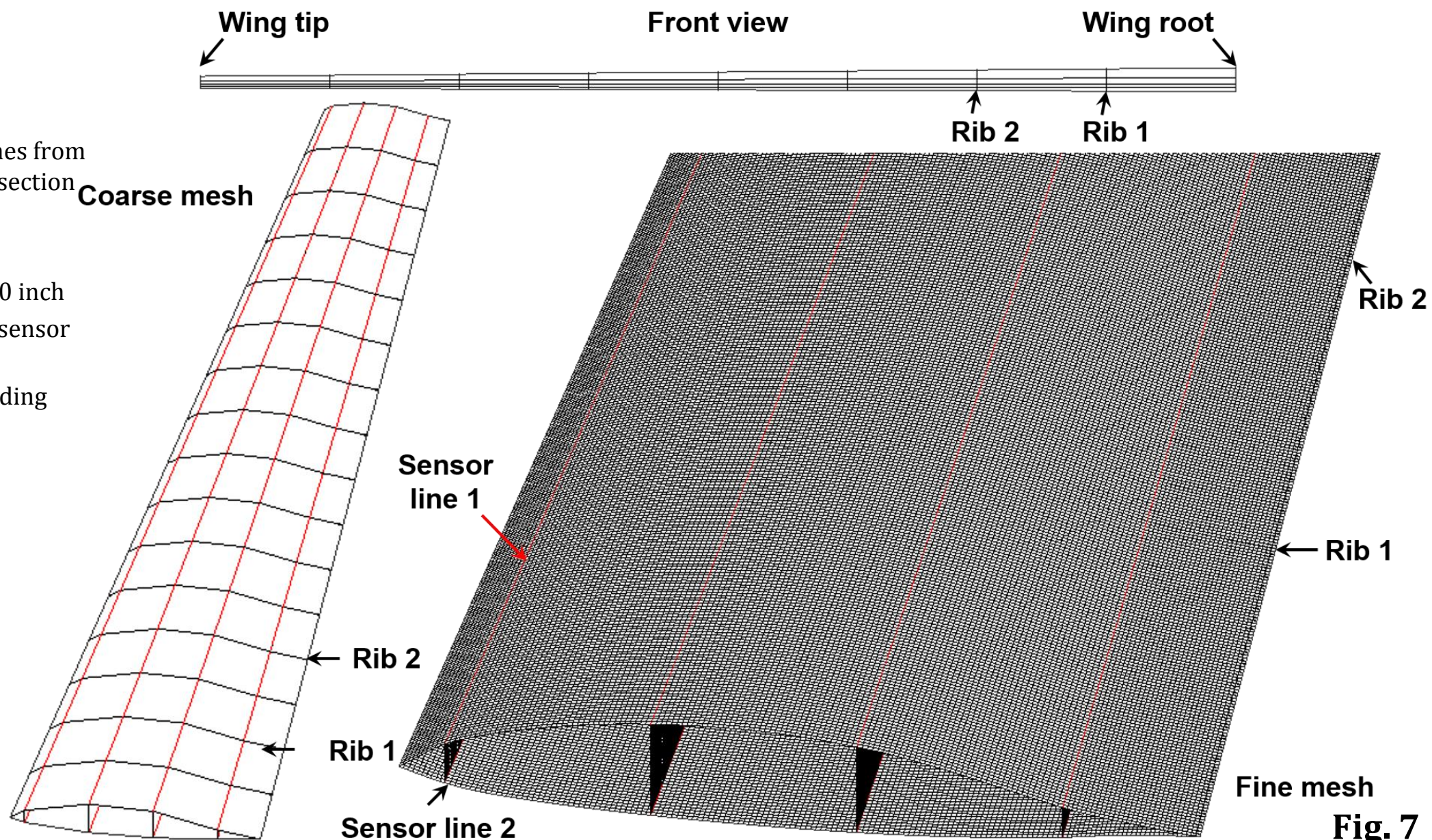
# Tapered Wing





# Tapered wing with coarse and fine meshes

- Aluminum
- Eight ribs
  - ❖ Every 50 inch
- Four spars
  - ❖ 4, 22, 40 58 inches from LE of wing root section
- Root chord: 70 inch
- Tip chord: 35 inch
- Half span length = 400 inch
- Results are based on sensor lines 1 and 2
- External load: 1 G loading



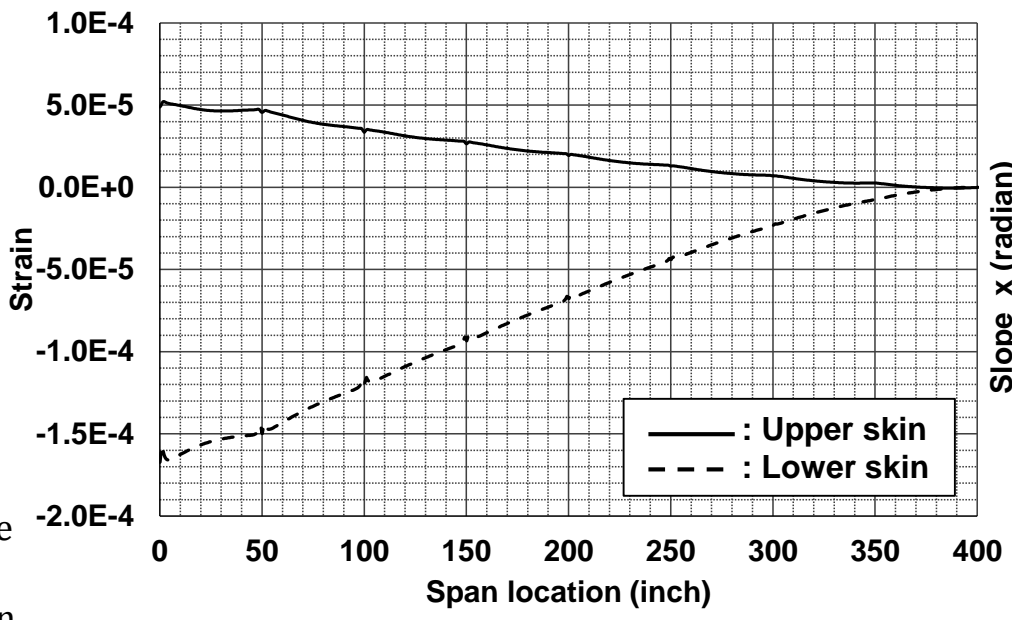
**Fig. 7**



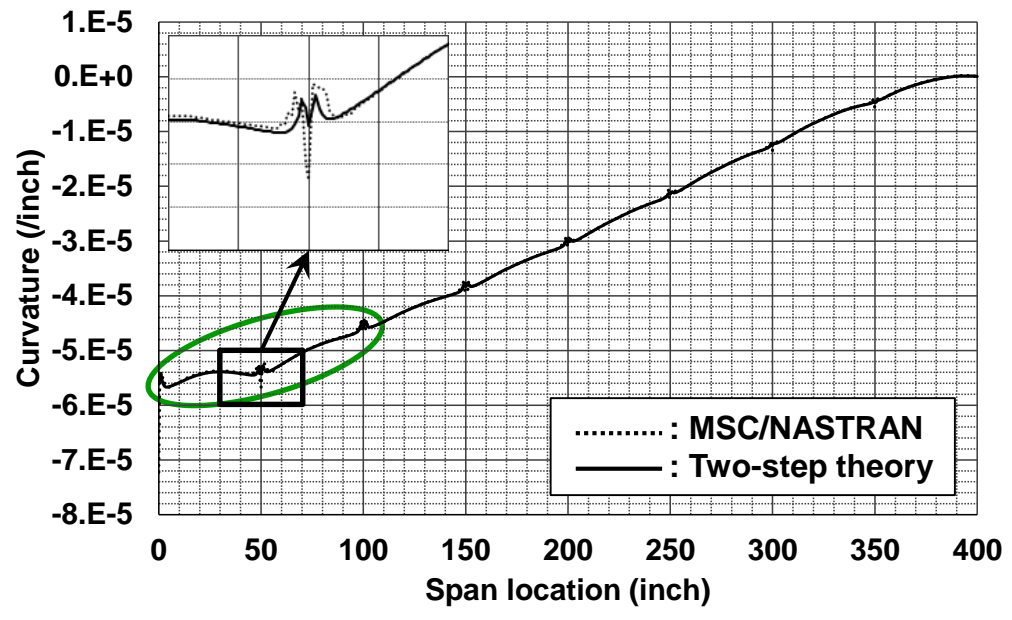
# Strain, curvature, & deformation

- ☐ Differences at wing tip
  - ❖ Slope: **-0.019%**
  - ❖ Deflection: **-0.046%**
- ☐ Curvatures computed from the two-step theory and the MSC/NASTRAN code are excellent matching between root chord and the first two rib.
- ☐ A fine FE mesh gives excellent results

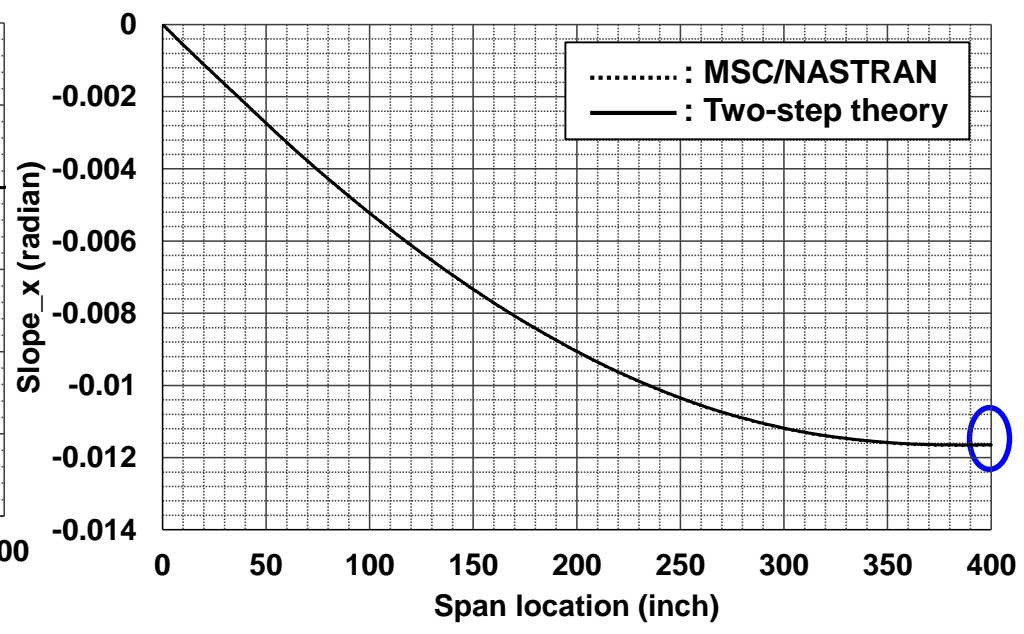
**Tapered wing with fine mesh**



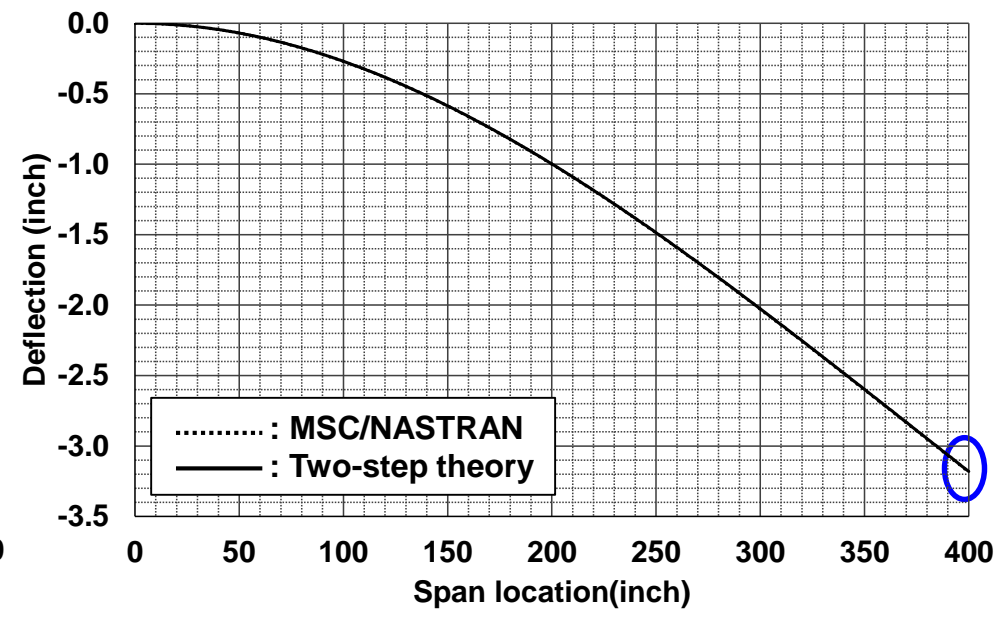
(a) Strain on the upper and lower skin



(b) Curvature



(c) Slope in roll direction



(d) Deflection

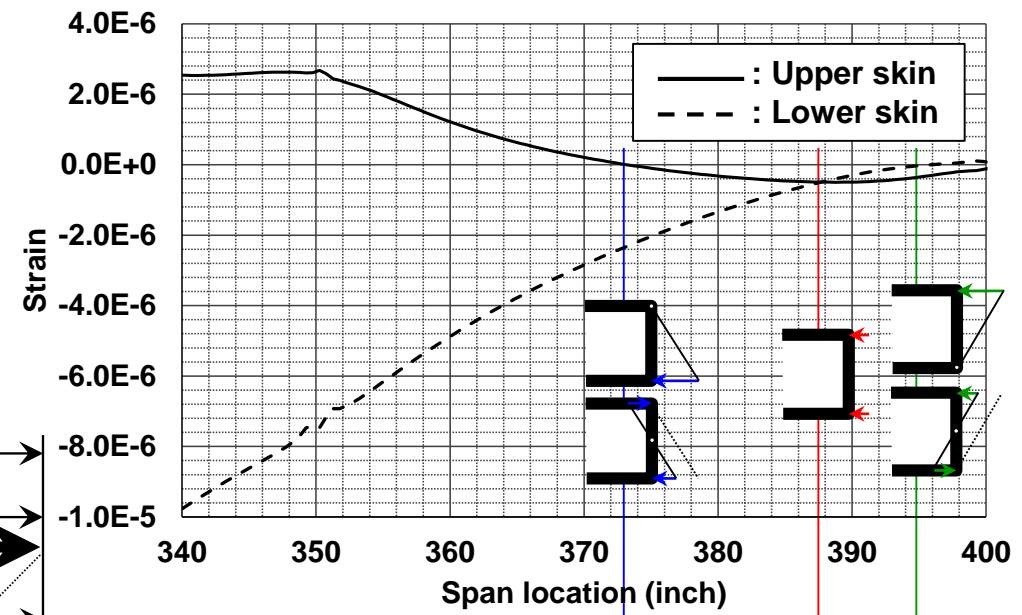
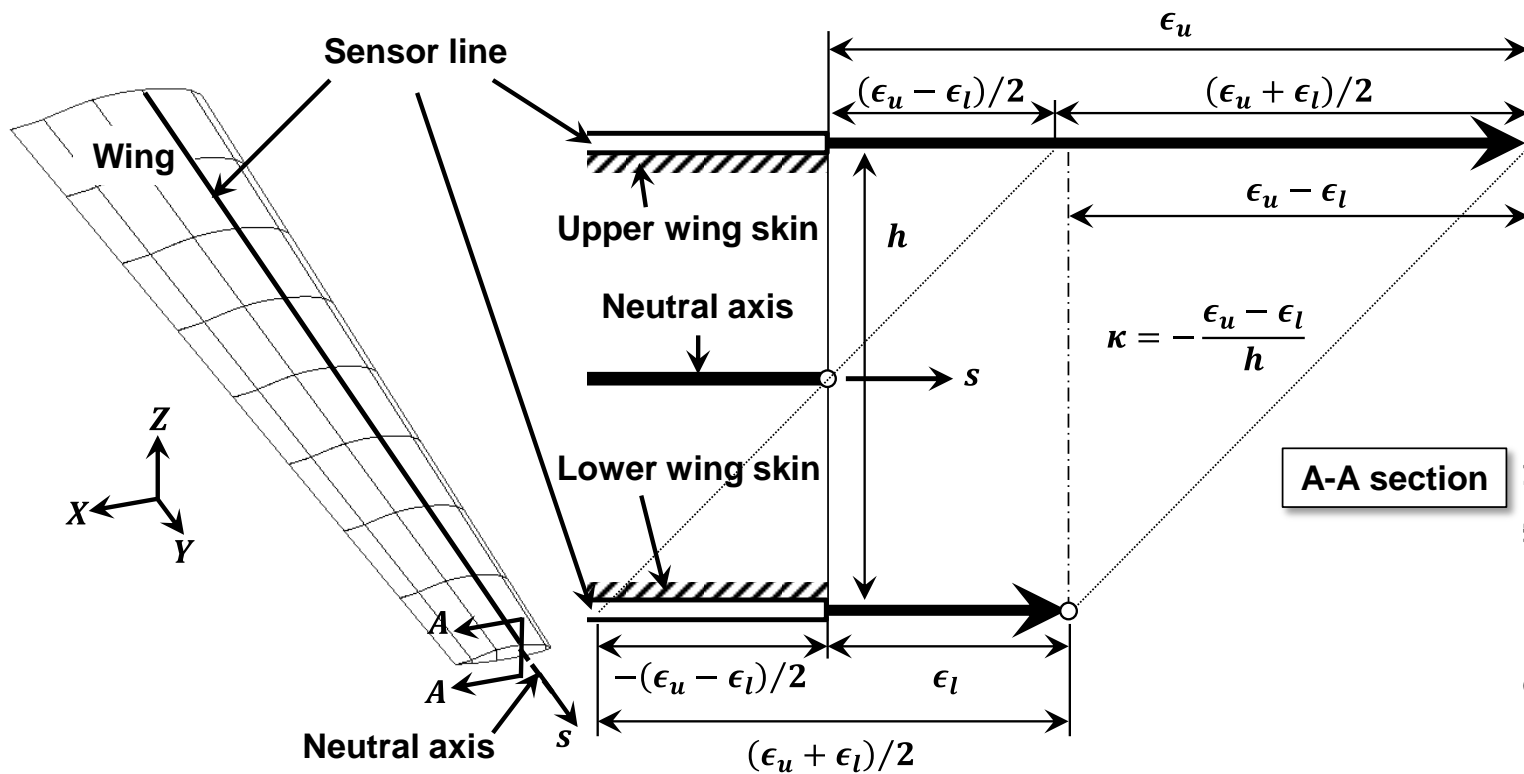
**Fig. 8**



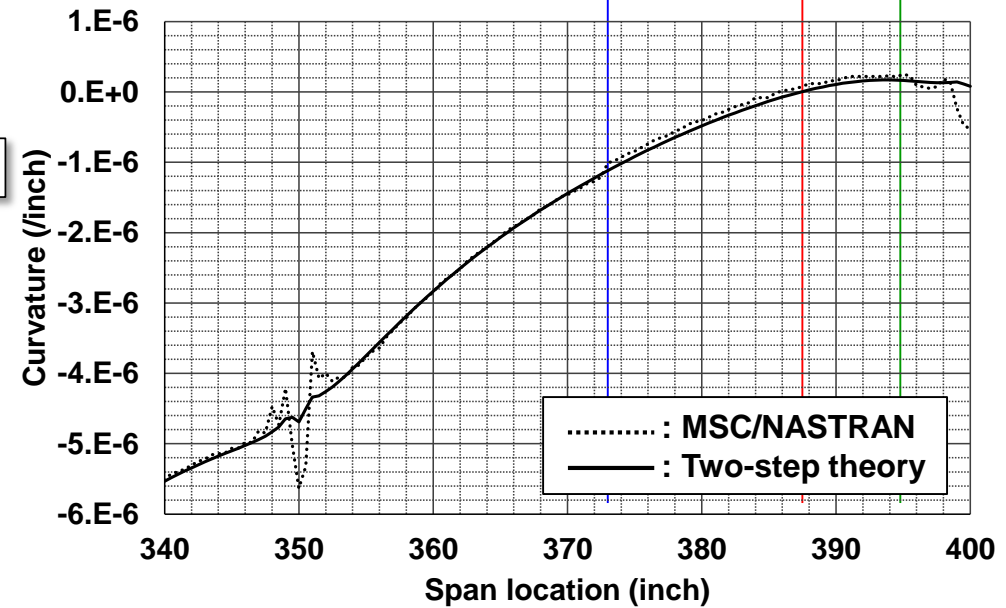
# Tapered wing strain and curvature along reference line (zoom in)

- The upper and lower strains between span stations of 372.5 inch and 395.5 inch are both compressions.

❖ Curvature definition,  $\kappa = -\frac{\epsilon_u - \epsilon_l}{h}$ , is **accurate**.



(a) Strain on the upper and lower skin



(b) Curvature

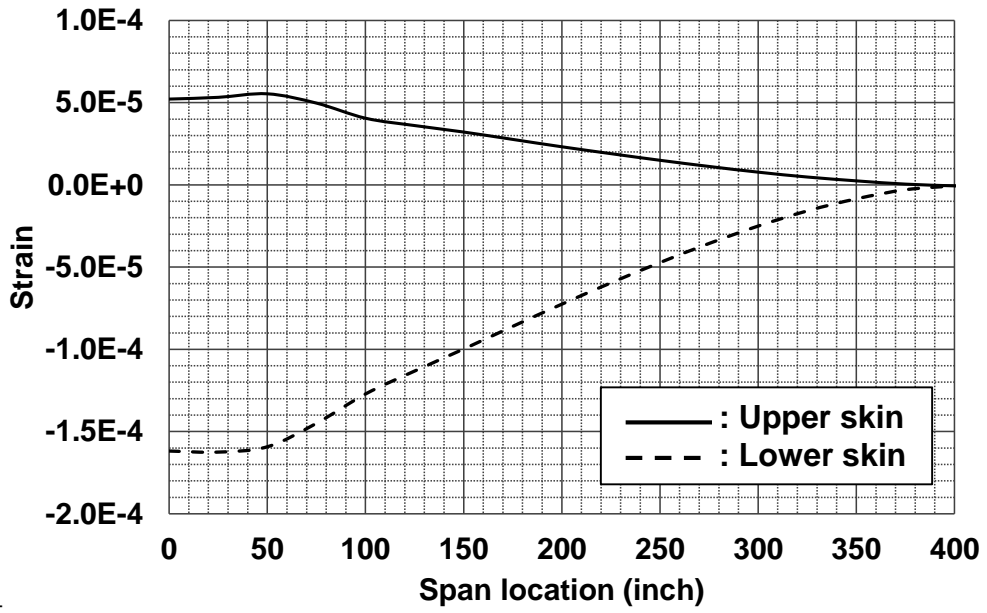




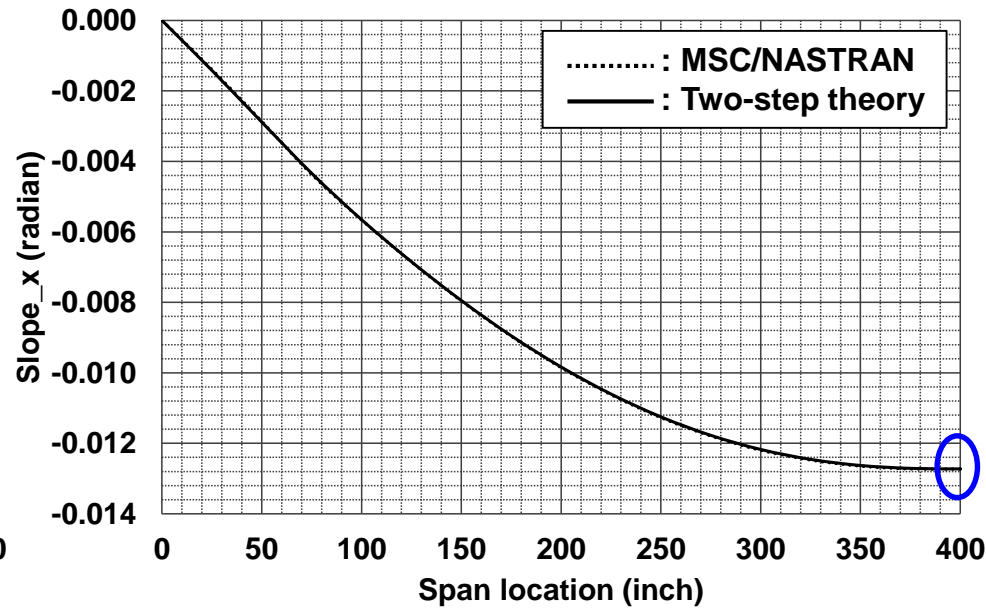
# Strain, curvature, & deformation

- ❑ Differences at wing tip
  - ❖ Slope: **-0.036%**
  - ❖ Deflection: **-0.066%**
- ❑ Curvatures obtained from the two-step theory and the MSC/NASTRAN code are good matching between root chord and the first two rib.
  - ❖ Rib effect??
- ❑ A coarse FE mesh also gives excellent results. (Why??)

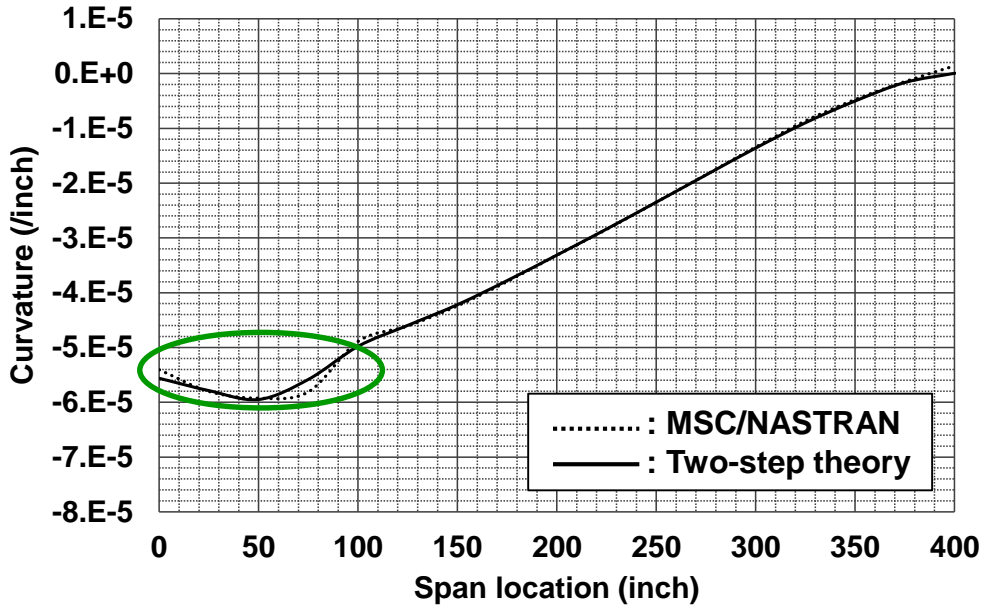
**Tapered wing with coarse mesh**



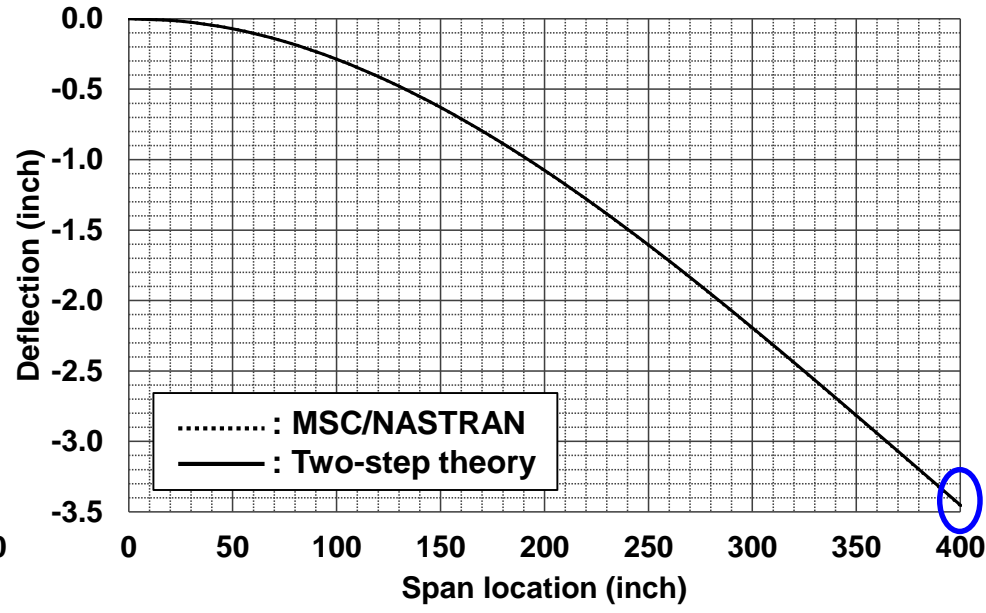
(a) Strain on the upper and lower skin



(c) Slope in roll direction



(b) Curvature



(d) Deflection

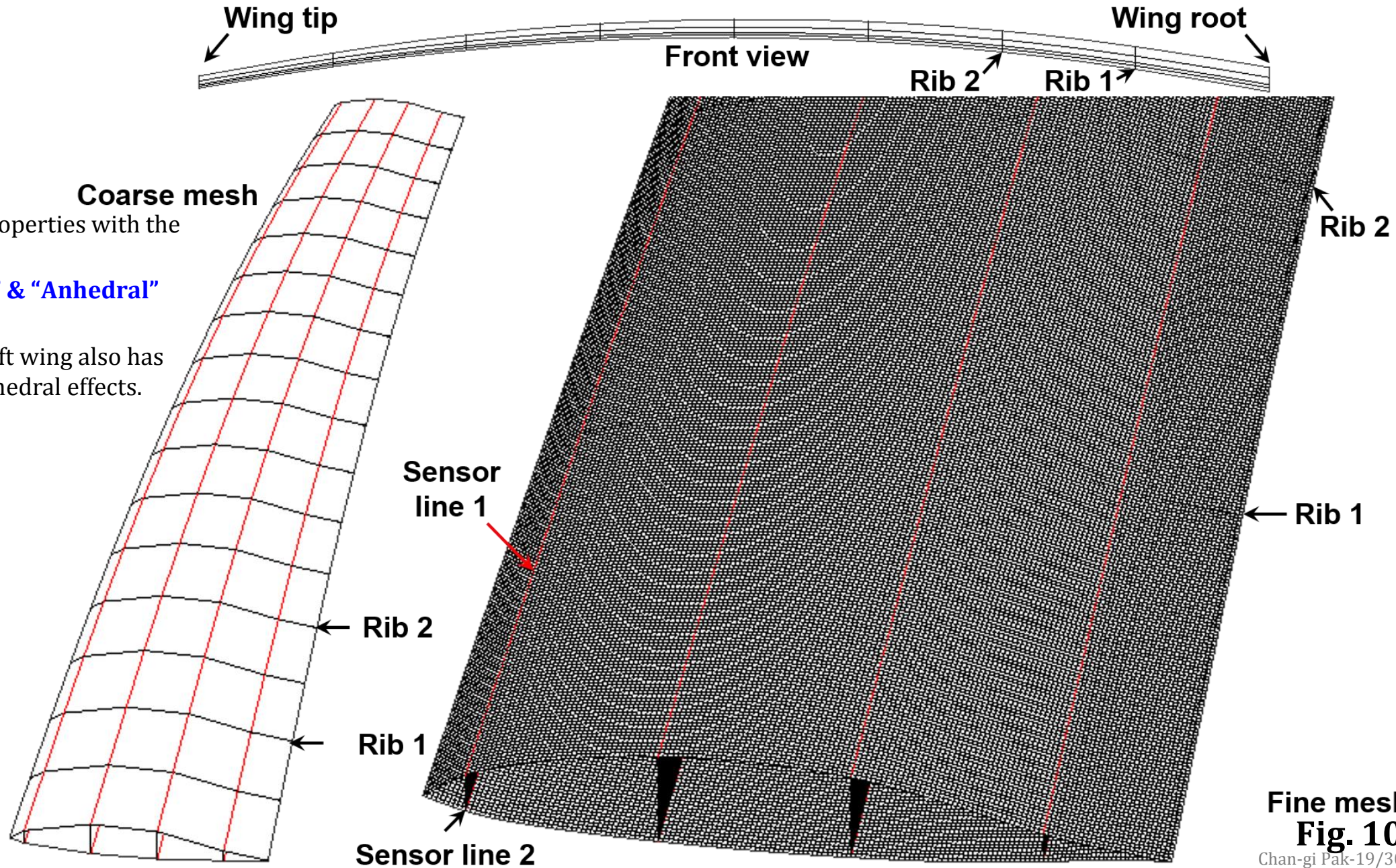
**Fig. 9**

# Tapered Wing with Dihedral/Anhedral





# Dihedral/anhedral wing with coarse and fine meshes



- ❑ Same material and properties with the tapered wing
- ❑ **Wing has “Dihedral” & “Anhedral” effects.**
  - ❖ The LBFD aircraft wing also has dihedral and anhedral effects.

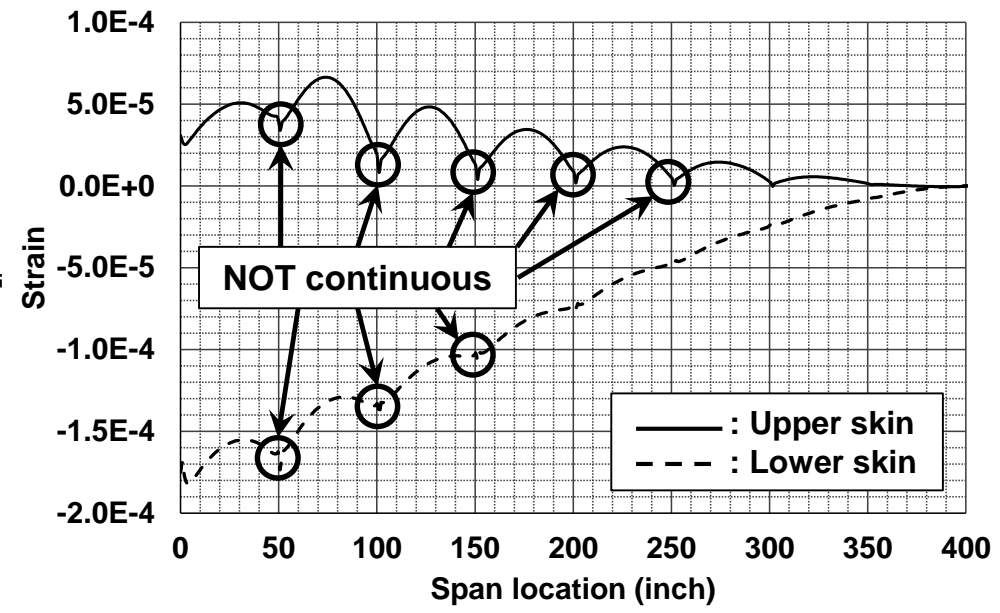
Fine mesh  
**Fig. 10**



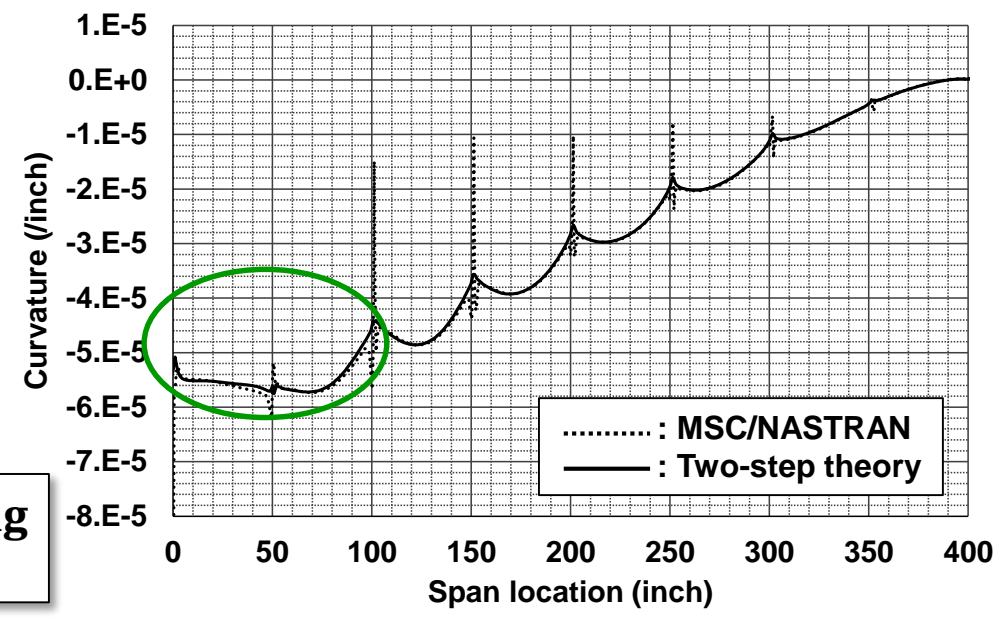
# Strain, curvature, & deformation

- ❑ Differences at wing tip
  - ❖ Slope: **-0.603%**
  - ❖ Deflection: **-0.779%**
- ❑ Bigger difference than tapered wing case.
  - ❖ Strains near the rib location are not continuous.
  - ❖ Needs more fine mesh near rib location
- ❑ Curvatures from two-step theory and MSC/NASTRAN are excellent matching between root chord and the first two rib.
- ❑ A fine FE mesh gives good results

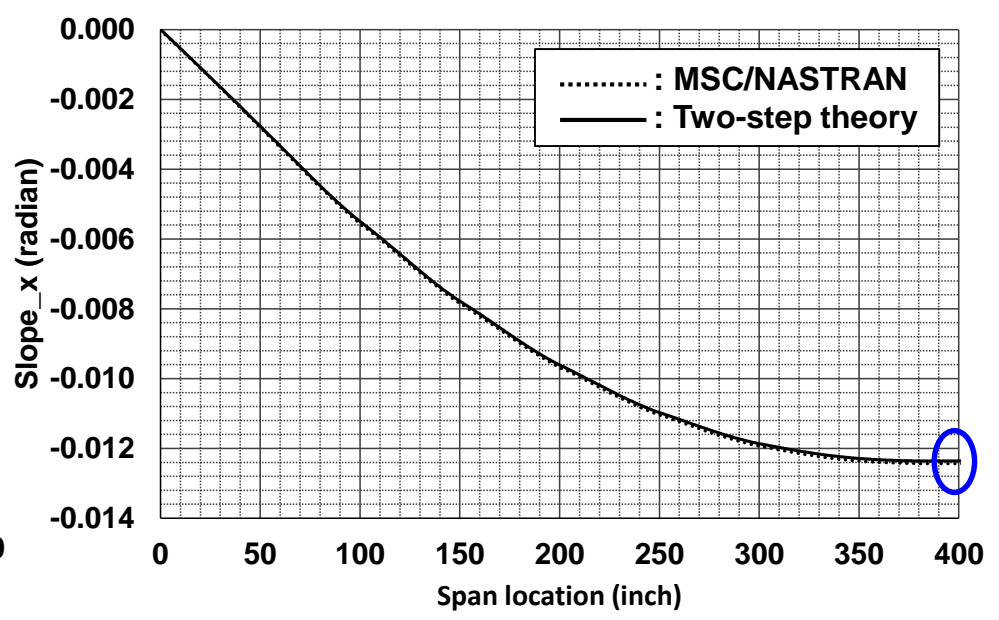
**Dihedral/anedral wing with fine mesh**



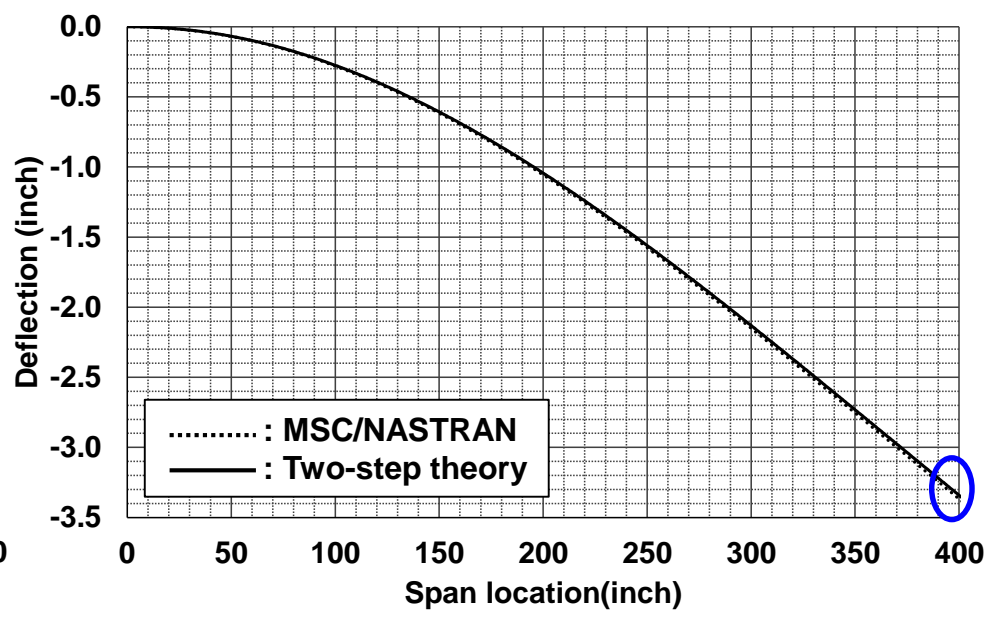
(a) Strain on the upper and lower skin



(b) Curvature



(c) Slope in roll direction



(d) Deflection

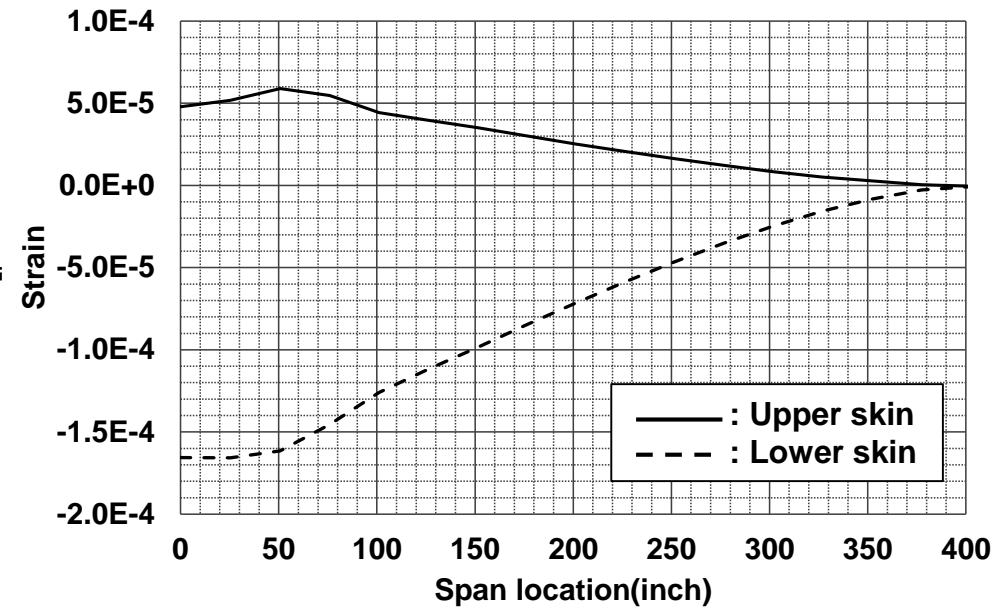
**Fig. 11**



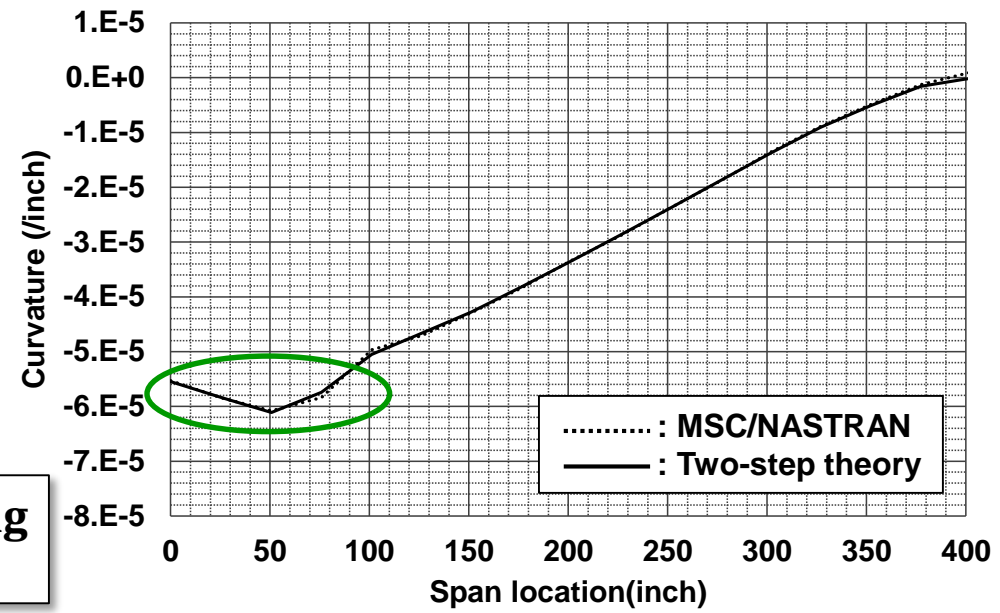
# Strain, curvature, & deformation

- ❑ Differences at wing tip
  - ❖ Slope: **-0.548%**
  - ❖ Deflection: **-0.794%**
- ❑ Deflection difference is bigger than fine mesh.
- ❑ Curvatures from two-step theory and MSC/NASTRAN are good matching between root chord and the first two rib.
  - ❖ Rib effect??
- ❑ A coarse FE mesh also gives good results. (Why??)

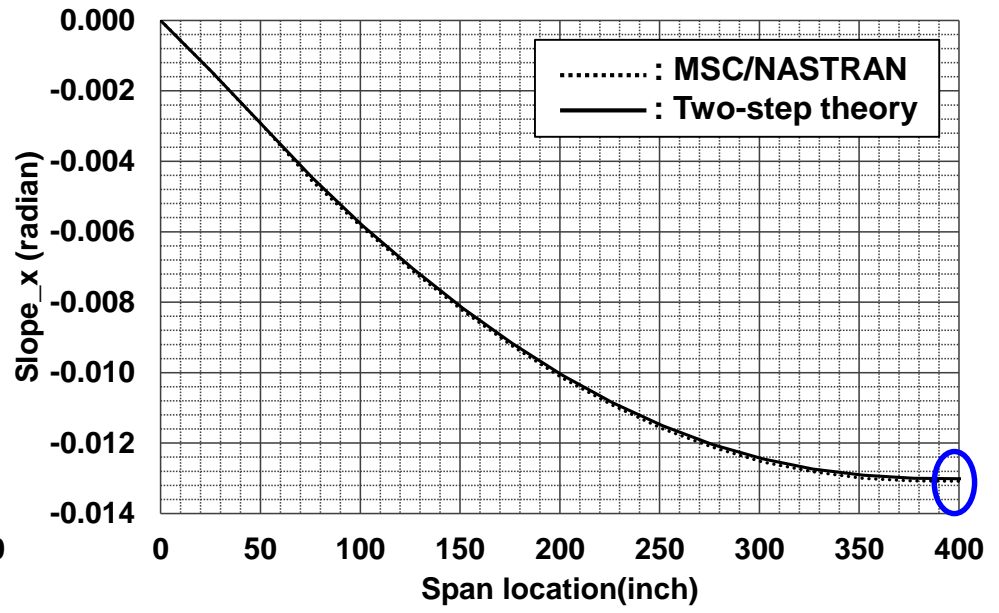
**Dihedral/anedral wing with coarse mesh**



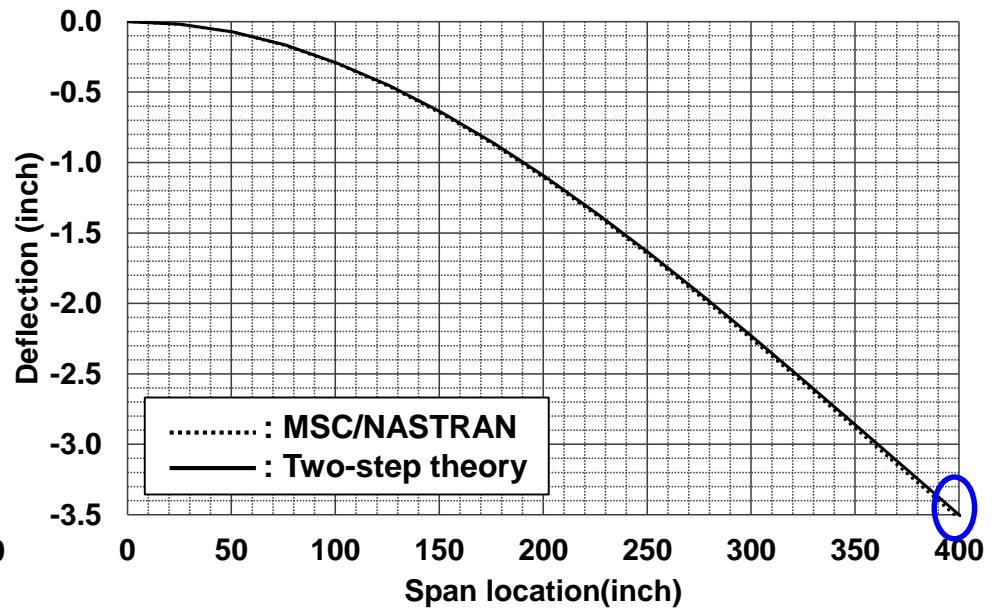
(a) Strain on the upper and lower skin



(b) Curvature



(c) Slope in roll direction



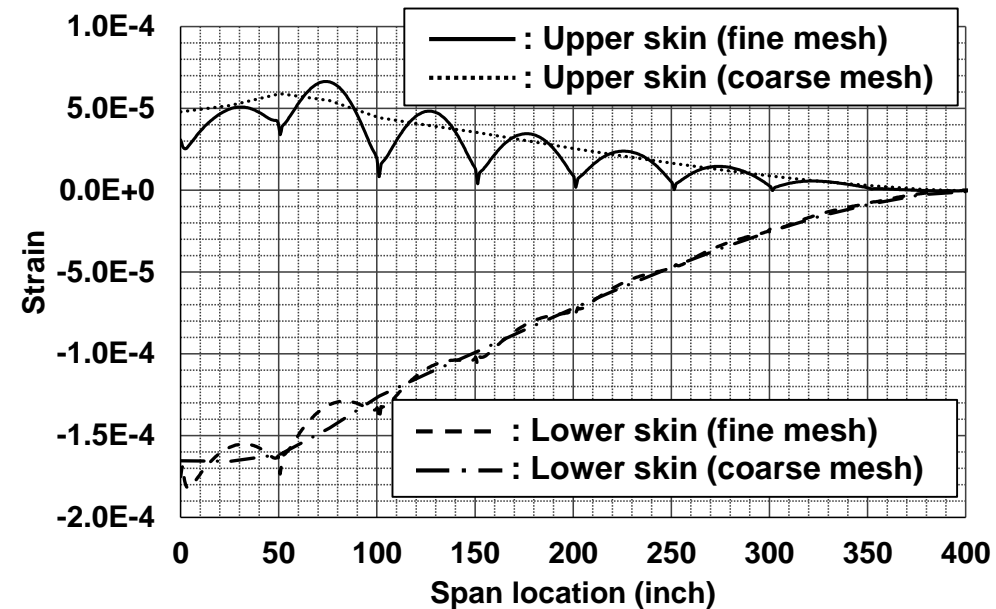
(d) Deflection

**Fig. 12**

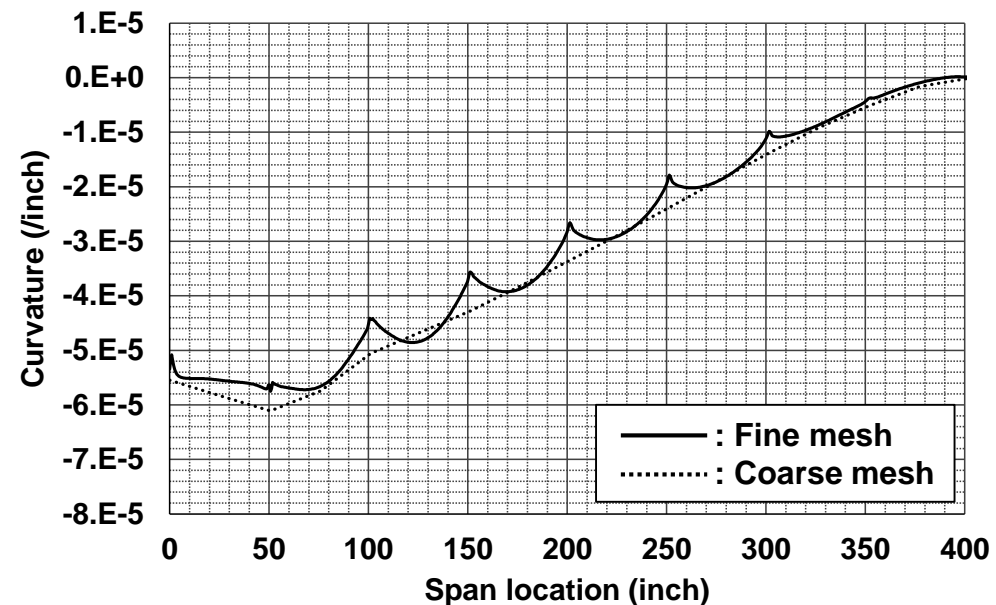


# Comparison of strain and curvature results using coarse and fine meshes

- ❑ Strain values from the coarse mesh are close to the average values of the strain values obtained from the fine mesh.
- ❑ Therefore, curvature values computed from the coarse and fine meshes have similar behavior.
- ❑ However, the fine mesh is needed to have accurate curvature distribution.
- ❑ In general, deformation results obtained from the coarse mesh are good. (why??)



(a) Strain on the upper and lower skin



(b) Curvature from two-step theory

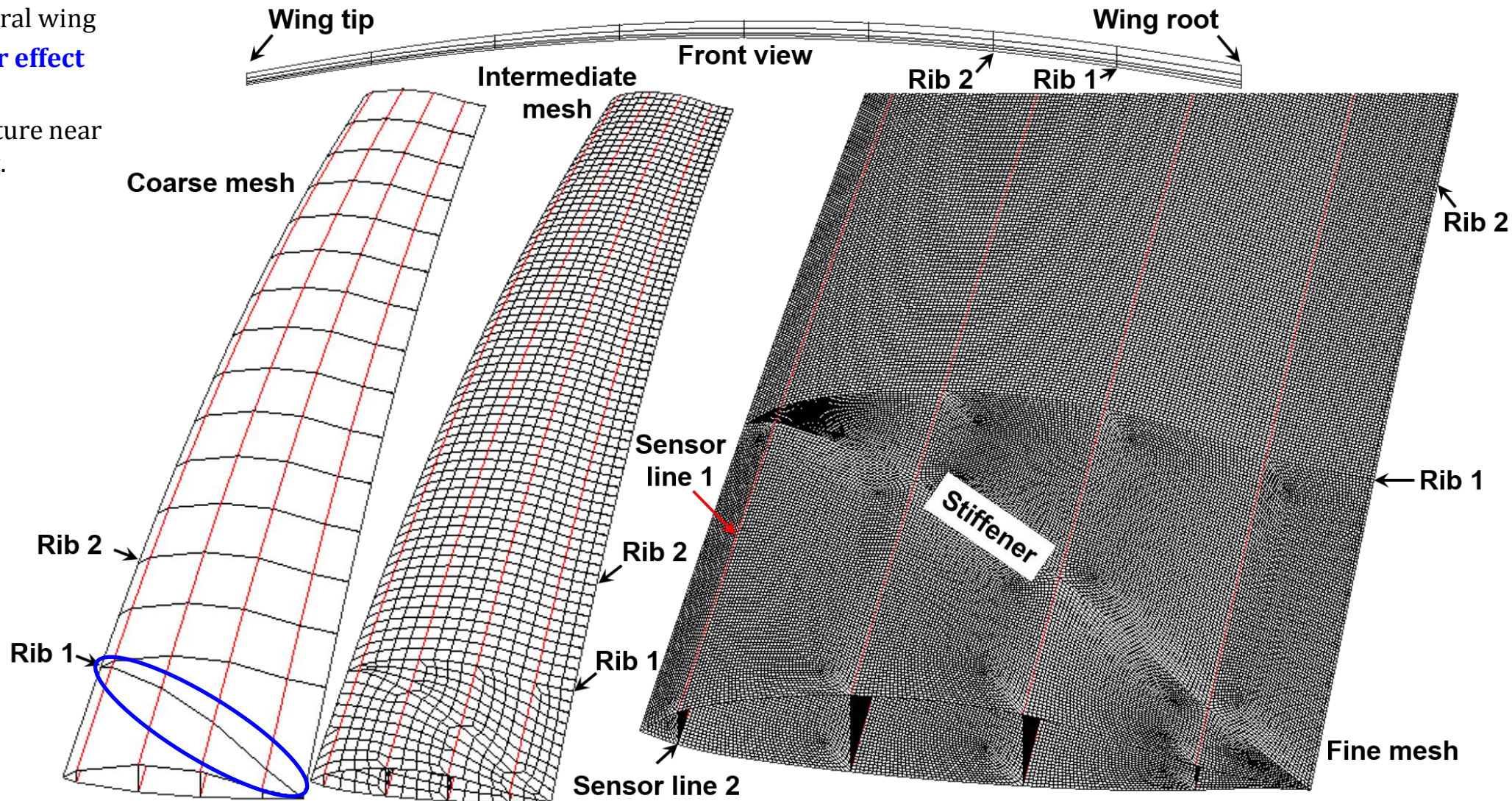
# **Tapered Wing with Dihedral/Anhedral and Wing Root Stiffener**





# Stiffened dihedral/anedral wing with coarse, intermediate, and fine meshes

- ☐ Same material and properties with dihedral/anedral wing
- ☐ **Investigate stiffener effect**
  - ❖ Lbfd also has a stiffening structure near FOSS 1 & 2 root.



**Fig. 13**

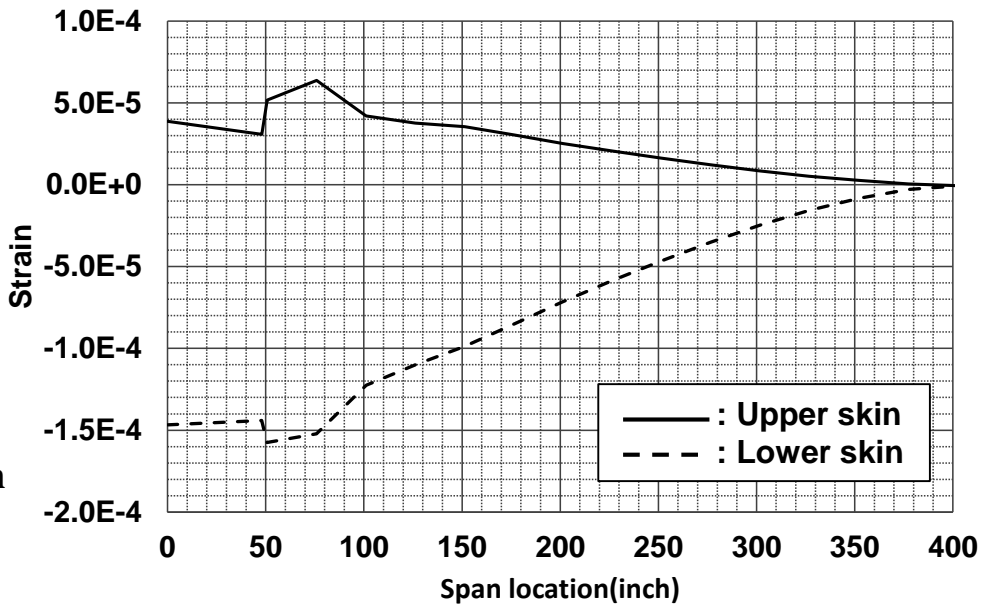




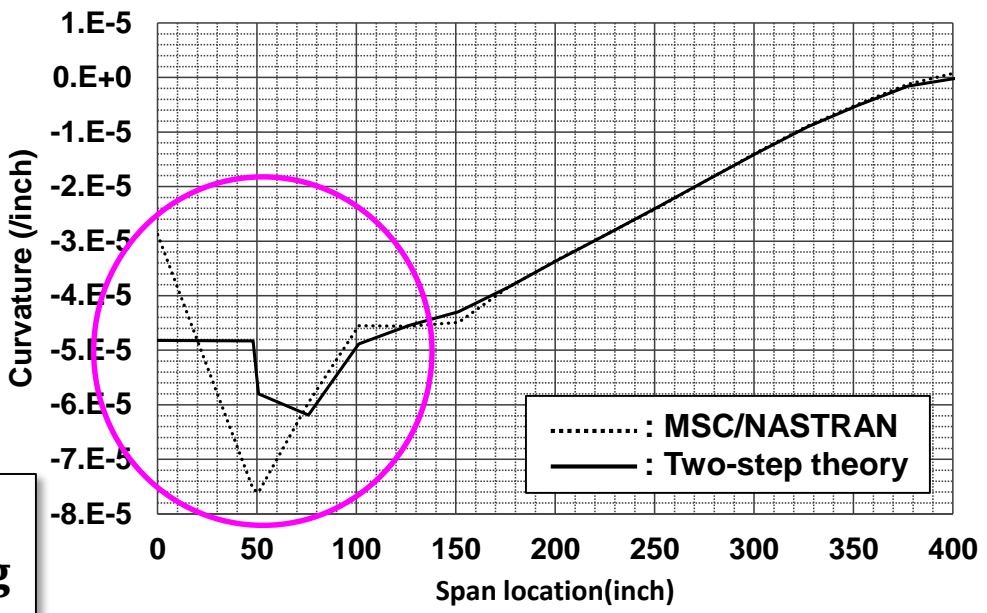
# Strain, curvature, & deformation

- ❑ Differences at wing tip
  - ❖ Slope: **-7.27%**
  - ❖ Deflection: **-10.2%**
- ❑ Similar prediction error with Lbfd case is obtained.
  - ❖ Mainly cause by **curvature error** near wing root area
- ❑ Curvatures from two-step theory and MSC/NASTRAN are not matching between root chord and the first two rib.
  - ❖ Rib effect??

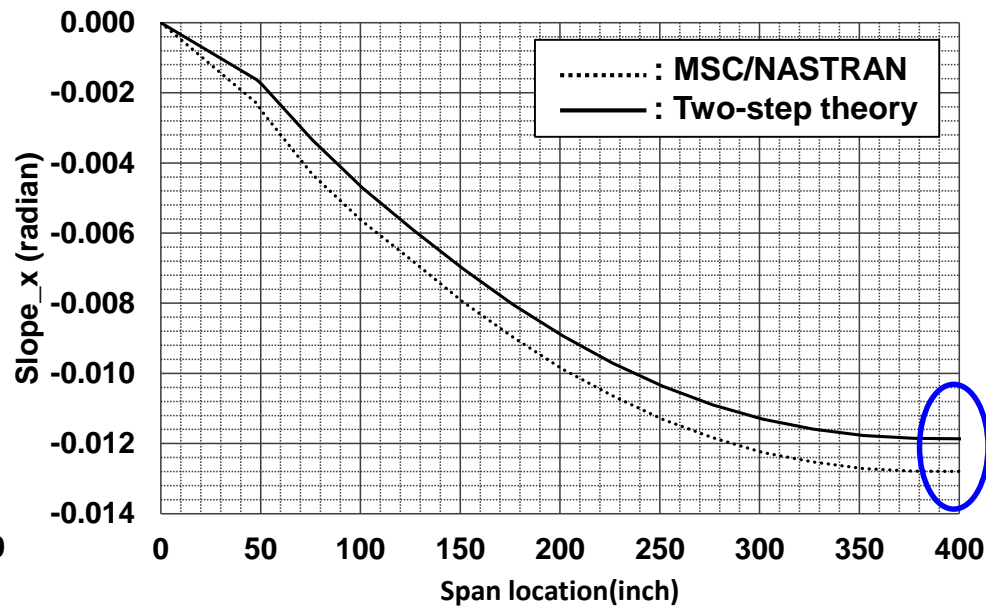
**Stiffened dihedral/anedral wing with coarse mesh**



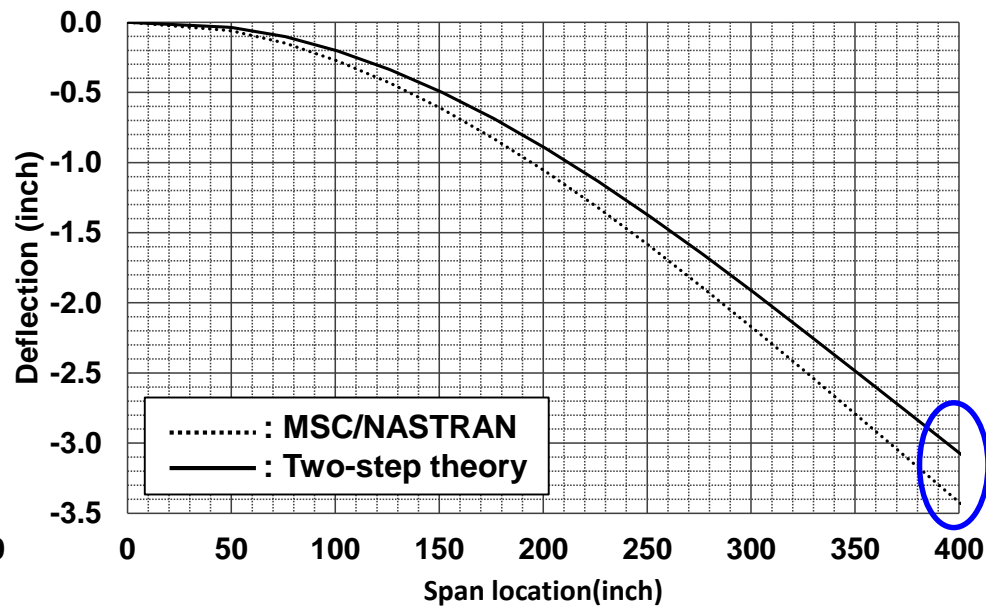
(a) Strain on the upper and lower skin



(b) Curvature



(c) Slope in roll direction



(d) Deflection

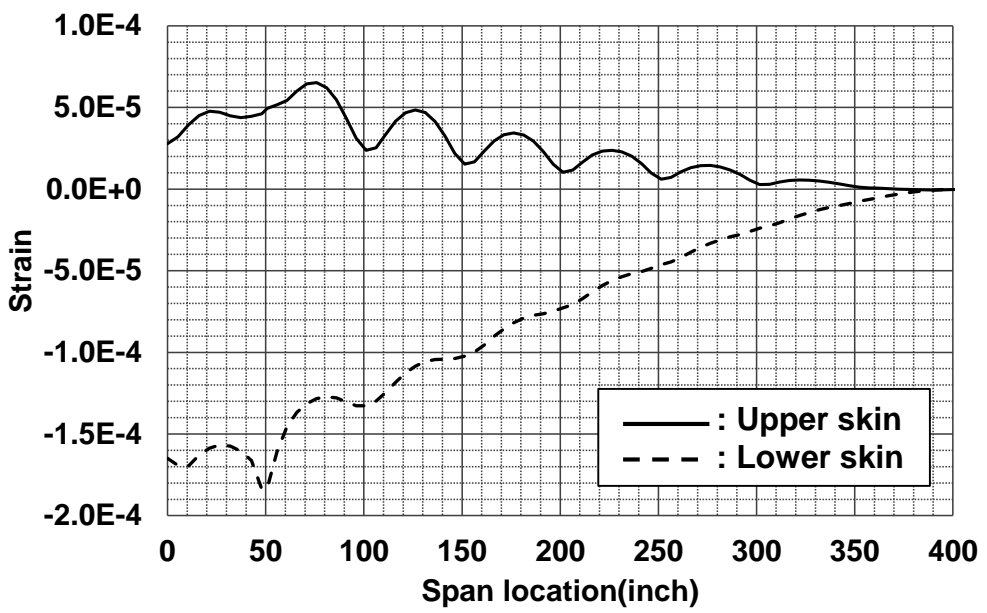
**Fig. 14**



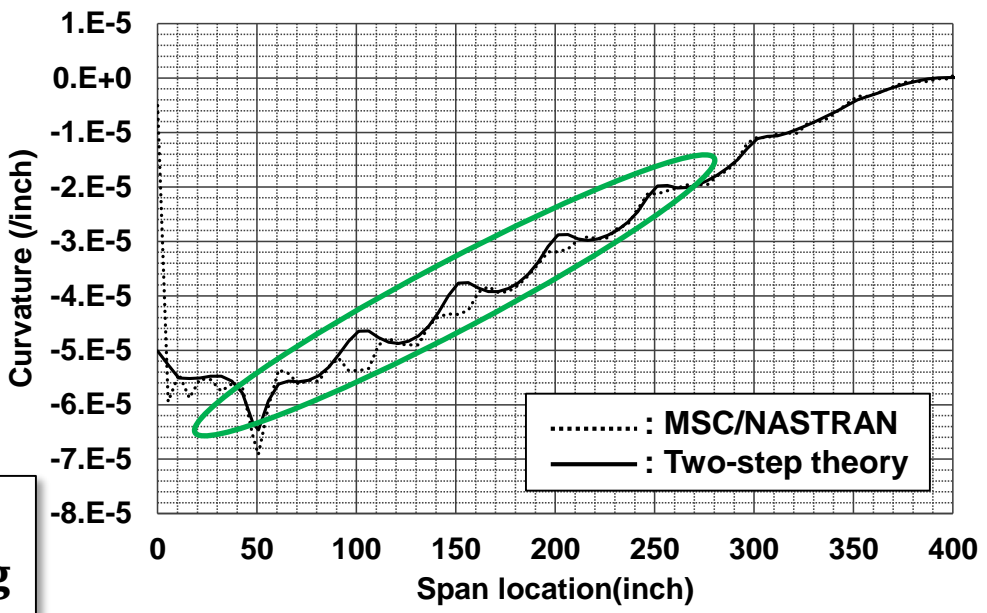
# Strain, curvature, & deformation

- ☐ Differences at wing tip
  - ❖ Slope: **-0.535%**
  - ❖ Deflection: **-0.694%**
- ☐ A medium FE mesh gives good results.
- ☐ Curvature values computed from the medium mesh is **similar** to the NASTRAN results.
  - ❖ Rib effects exist

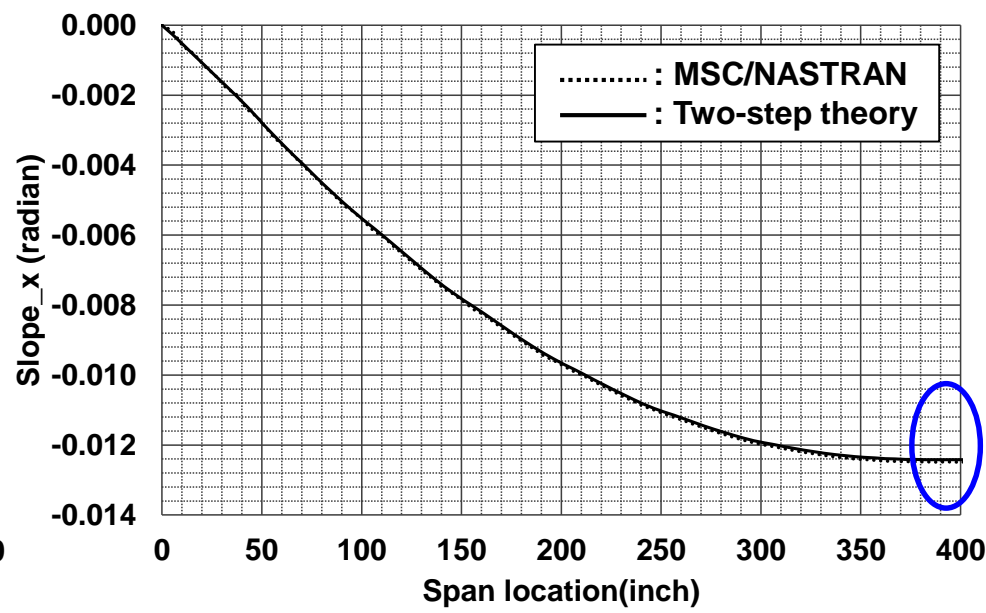
**Stiffened dihedral/anedral wing with intermediate mesh**



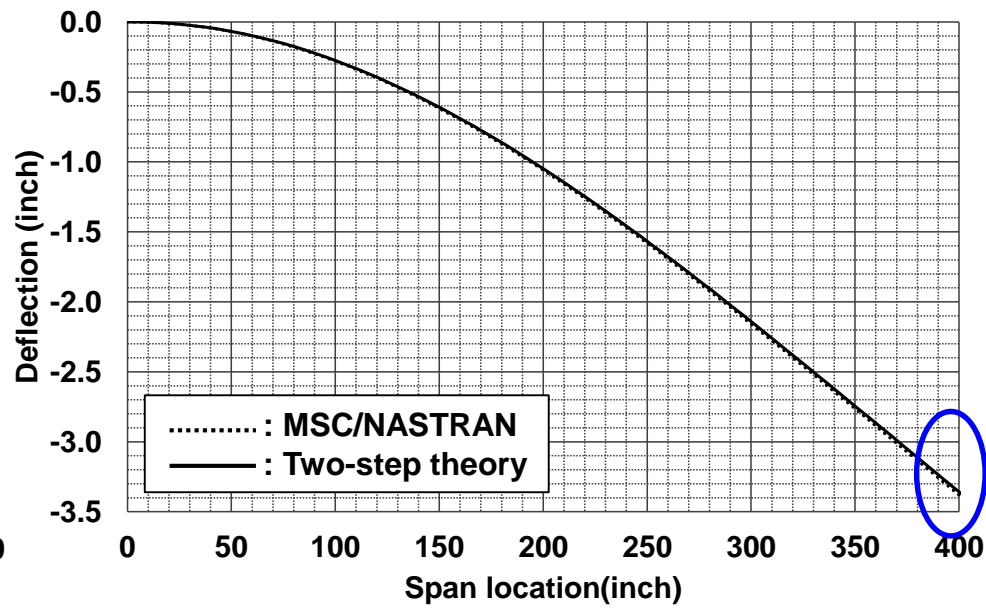
(a) Strain on the upper and lower skin



(b) Curvature



(c) Slope in roll direction



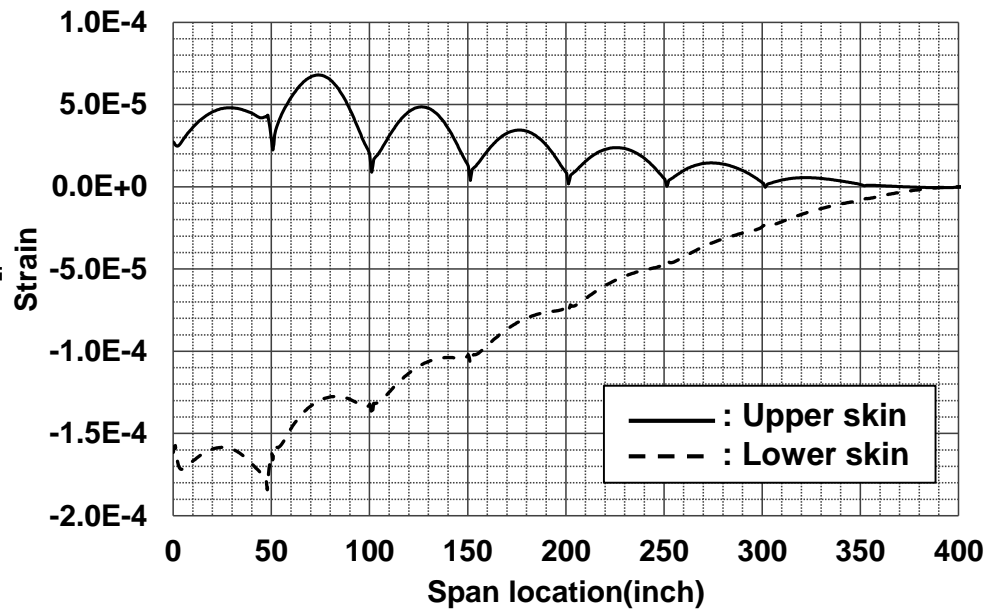
(d) Deflection



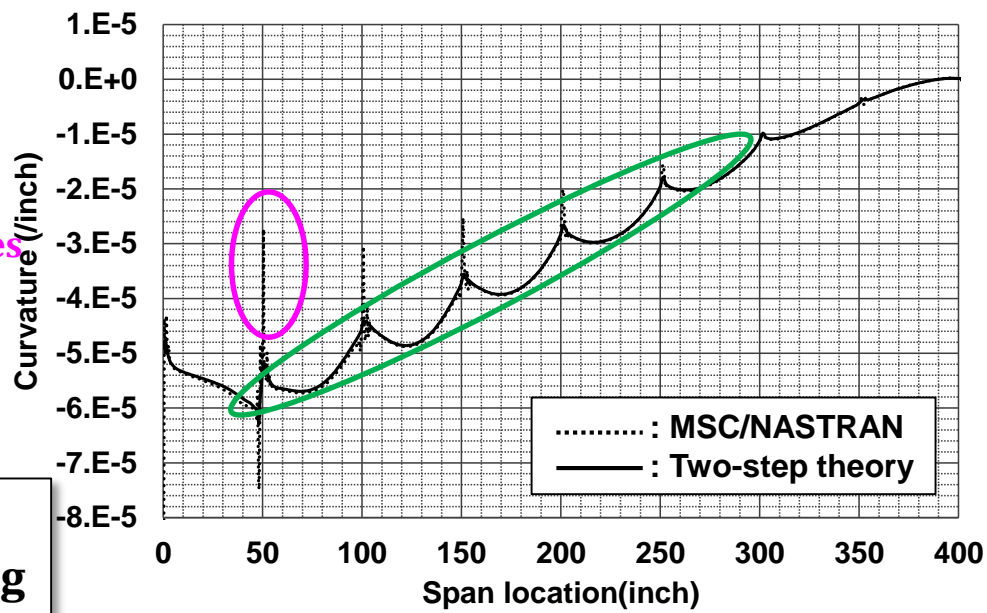
# Strain, curvature, & deformation

- ❑ Differences at wing tip
  - ❖ Slope: **-0.564%**
  - ❖ Deflection: **-0.796%**
- ❑ A fine FE mesh gives good results.
- ❑ Curvatures from two-step theory and MSC/NASTRAN are excellent matching between root chord and the first two rib.
- ❑ The fine mesh is needed to have **accurate** curvature distribution
  - ❖ **Numerical derivatives** are used for the computation of curvatures from MSC/NASTRAN

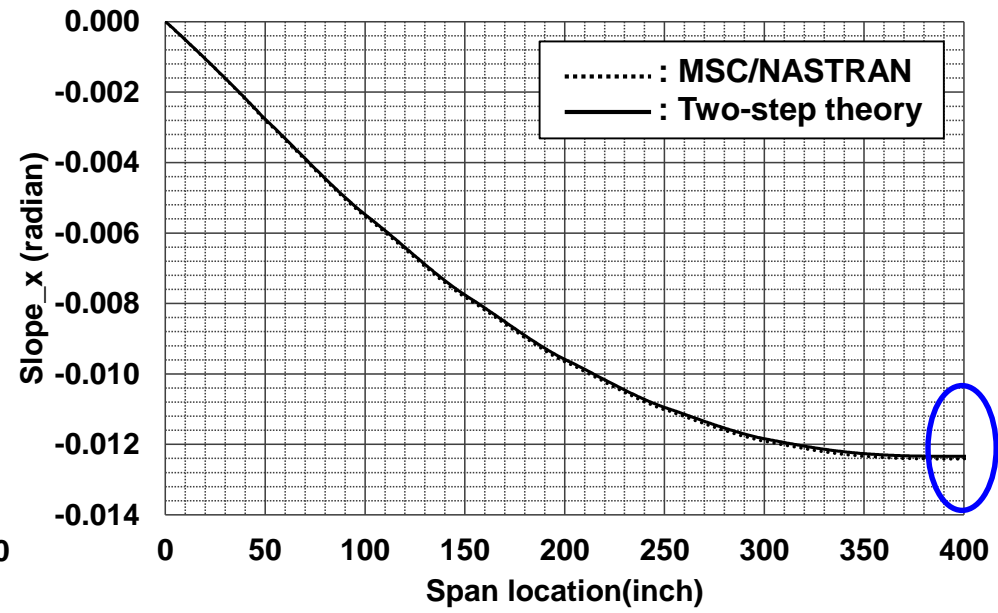
**Stiffened dihedral/anedral wing with fine mesh**



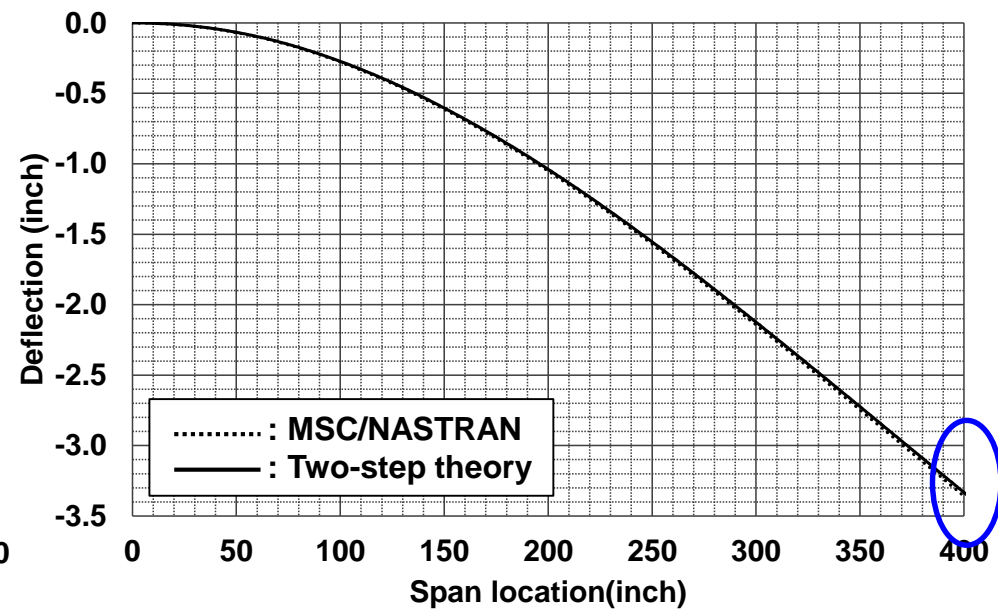
(a) Strain on the upper and lower skin



(b) Curvature



(c) Slope in roll direction



(d) Deflection

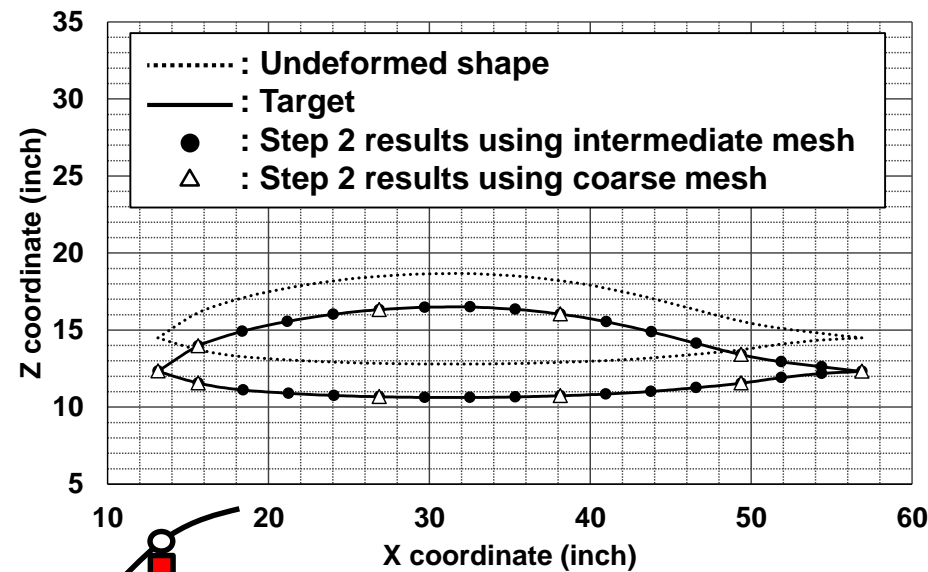


# Chord-wise deformation of Stiffened dihedral/anedral wing

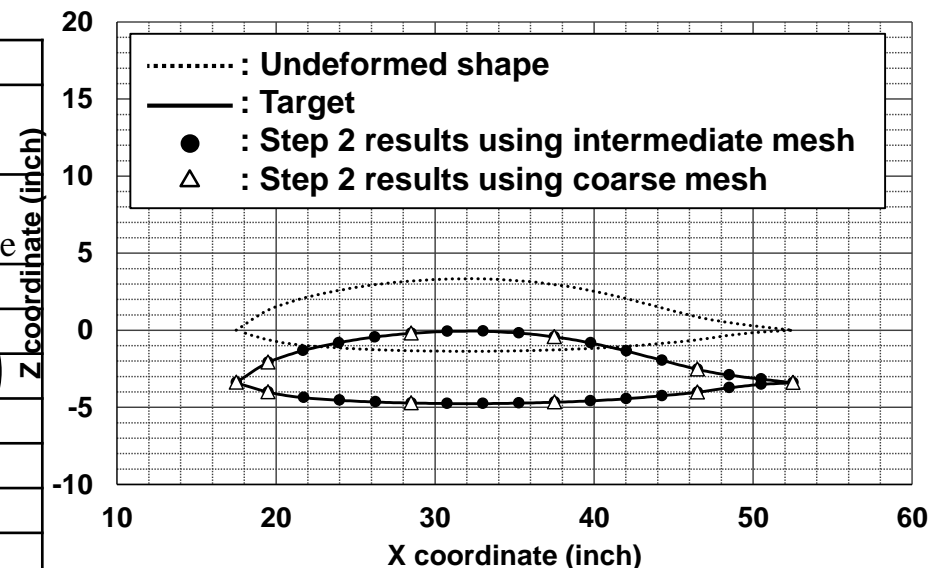
- Step 2 is the FE model dependent procedure.
- Expand measured master DOF to master and slave DOF

$$\{q(t)\} = \begin{Bmatrix} q_M(t) \\ q_S(t) \end{Bmatrix} = \begin{bmatrix} \Phi_M(\Phi_M^T \Phi_M)^{-1} \Phi_M^T \\ \Phi_S(\Phi_M^T \Phi_M)^{-1} \Phi_M^T \end{bmatrix} \{\tilde{q}_M(t)\} \quad \text{Values are based on the fine mesh.}$$

- DOF of  $\{\tilde{q}_M(t)\}$  = coarse mesh (51); intermediate mesh (972); & fine mesh (2403)
- The first six flexible mode shapes are selected as the basis functions.
- Eigen-matrices,  $\Phi_M$  &  $\Phi_S$ , are computed based on the FE model with **coarse or intermediate meshes**. (computer speed and memory issue with fine mesh)
  - DOF of coarse mesh = 1,356       $\Phi_M(51 \times 6)$ ;  $\Phi_M^T \Phi_M(6 \times 6)$ ; &  $\Phi_S(1305 \times 6)$
  - DOF of intermediate mesh = 21,192       $\Phi_M(972 \times 6)$ ;  $\Phi_M^T \Phi_M(6 \times 6)$ ; &  $\Phi_S(20220 \times 6)$
  - DOF of fine mesh = 2,240,442       $\Phi_M(2403 \times 6)$ ;  $\Phi_M^T \Phi_M(6 \times 6)$ ; &  $\Phi_S(2238039 \times 6)$



(a) At typical section



(b) At wing-tip section

Table 3. Deformation of stiffened dihedral/anedral wing at wing-tip section

Deformation	Target	Step 1 (fine mesh)		Step 2 (coarse mesh)		Step 2 (intermediate mesh)	
		Value	% difference	Value	% difference	Value	% difference
X	-0.06057			-0.05113	-15.6	-0.05571	-8.02
Y	-0.1740	-0.1821	4.66	-0.1725	-0.86	-0.1738	-0.11
Z	-3.392	-3.365	-0.80	-3.367	-0.74	-3.368	-0.71
Roll	-0.01241	-0.01234	-0.56	-0.01234	-0.56	-0.01237	-0.32
Pitch	3.205E-4			2.924E-4	-8.77	3.019E-4	-5.80
Yaw	1.981E-4			1.601E-4	-19.2	1.820E-4	-8.13

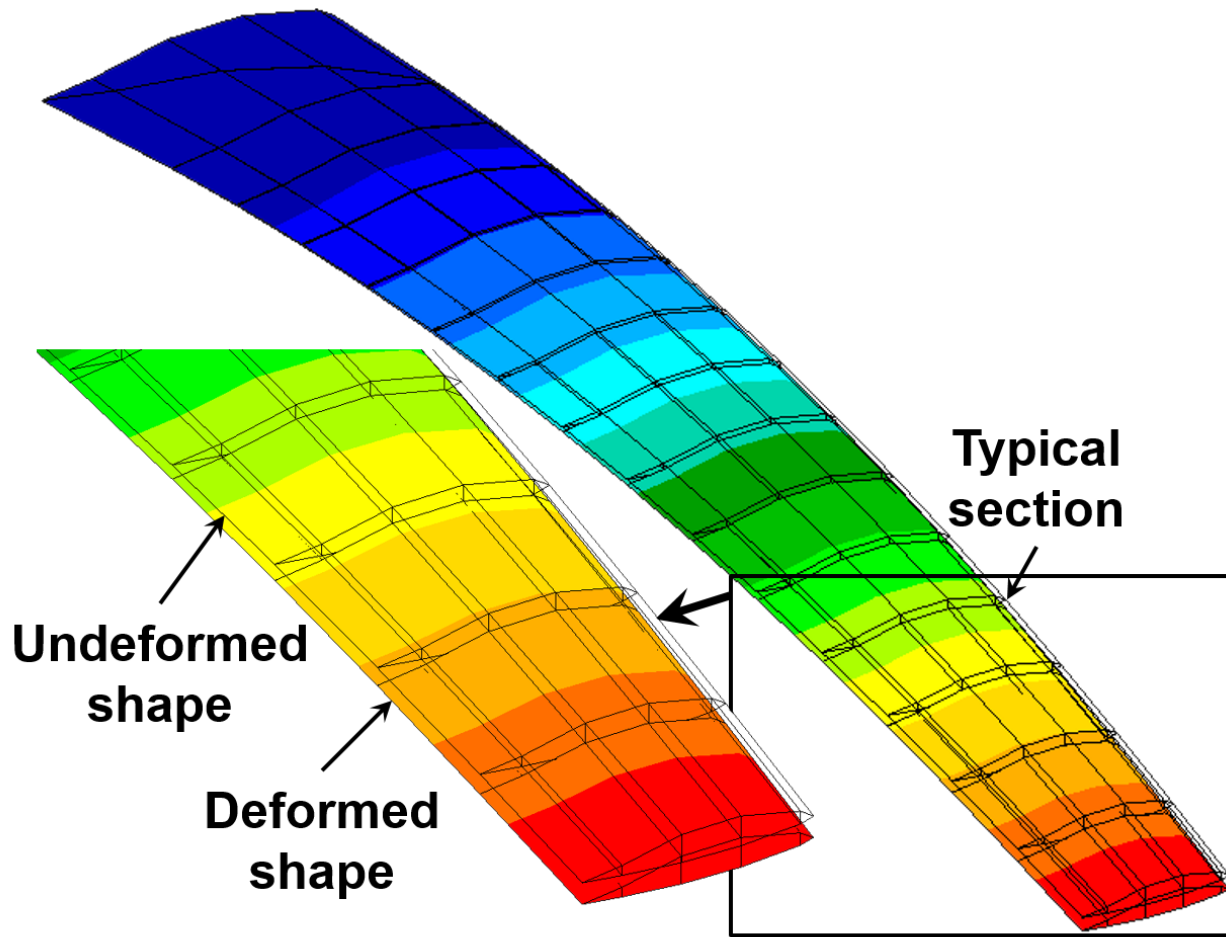
Results are based on strains along the sensor lines 1 and 2

Least squares surface fitting technique (SEREP)

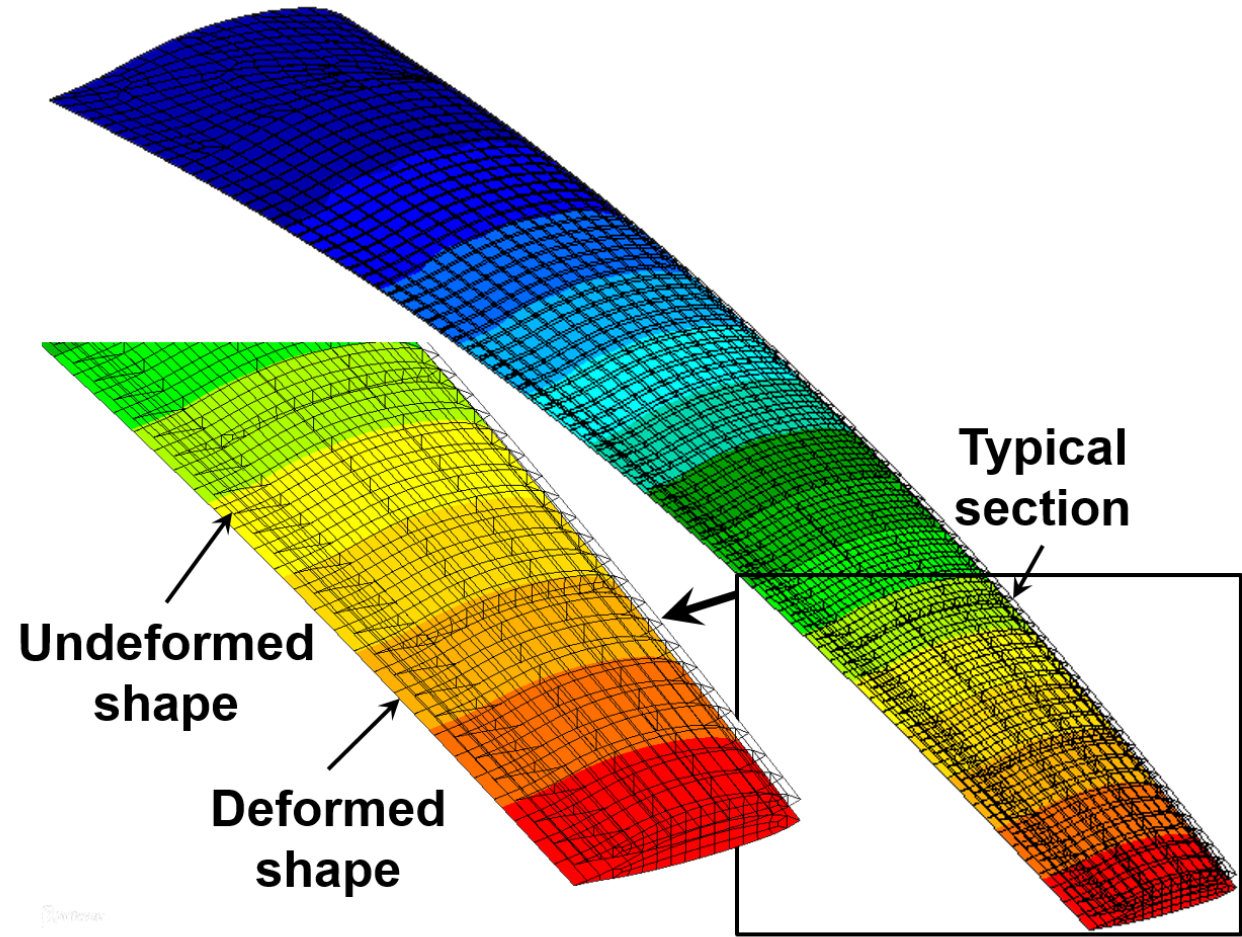
Fig. 17



# Deformed shape of stiffened dihedral/anhedral wing after step 2



(a) Use coarse mesh for step 2



(b) Use intermediate mesh for step 2

**Fig. 18**



# Conclusion

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- ❑ A finite element structural model with **a fine mesh is desired** to have **accurate curvature distributions** during a pre-test analysis for the wing shape sensing of a wing with ribs and spars.
  
- ❑ In case of a finite element (FE) model with **a regular rib configuration**, such as the tapered wing and the dihedral/anedral wing in this study, even the FE models with **coarse mesh give acceptable** strain data and slope and deflection information.
  - ❖ However, there's no guarantee that the strain data obtained from the coarse mesh is acceptable.
  - ❖ A FE model with a fine mesh may be needed to have accurate curvature distribution.
  - ❖ A FE model with **a fine mesh is needed for the pre-test analysis of the LBFD aircraft.**
  
- ❑ It is proved that **the two-step theory** used in this study **works excellent** for the wing shape sensing of the tapered wing, the dihedral/anedral wing, and the stiffened dihedral/anedral wing.
  - ❖ The curvature equation based on the decomposition of the in-plane strain and pure bending strain was successfully applied to the wing with spars and ribs.

# Questions?

