



Learning Model Structural Uncertainty with Gaussian Processes

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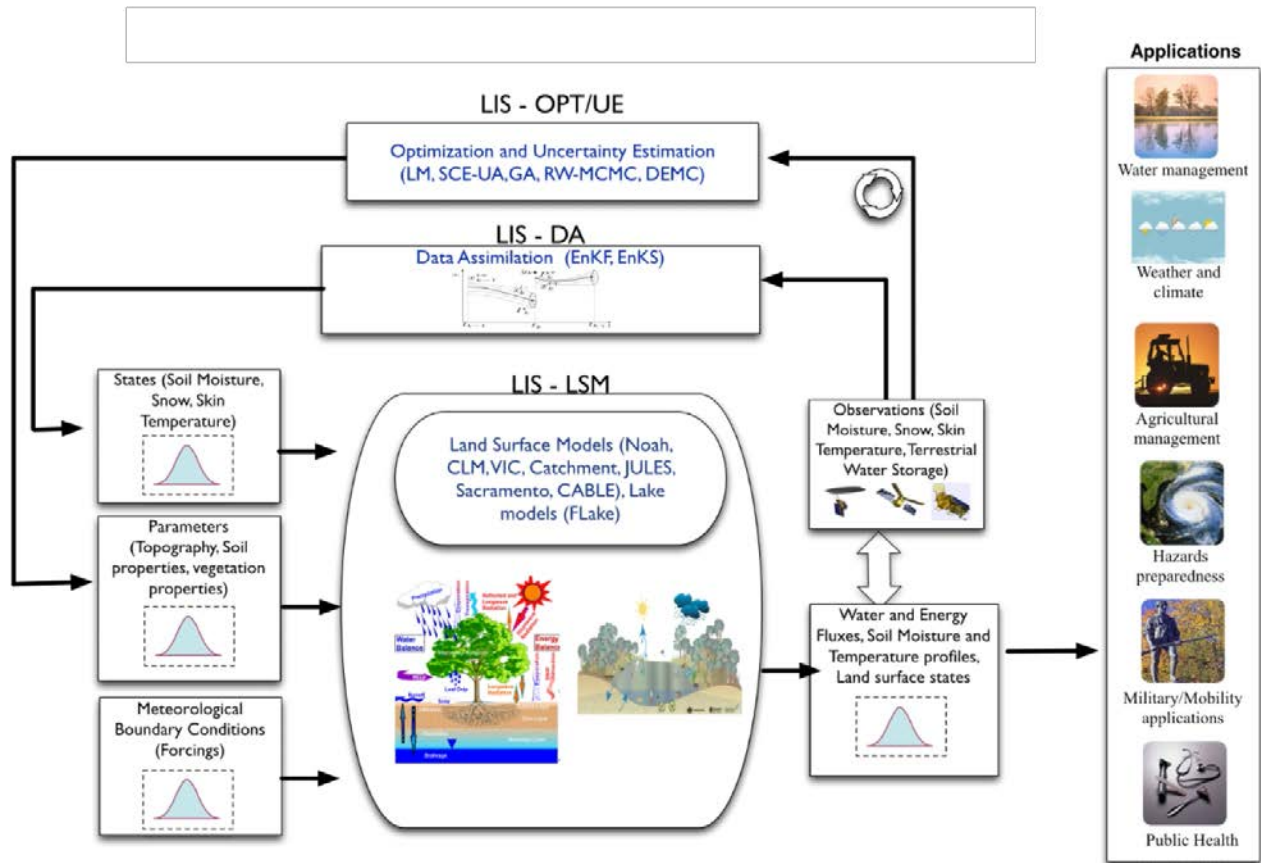
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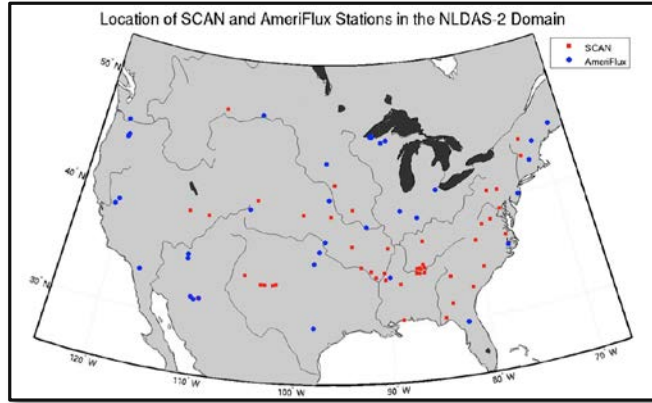
LIS Model



Structural Uncertainty in the Models

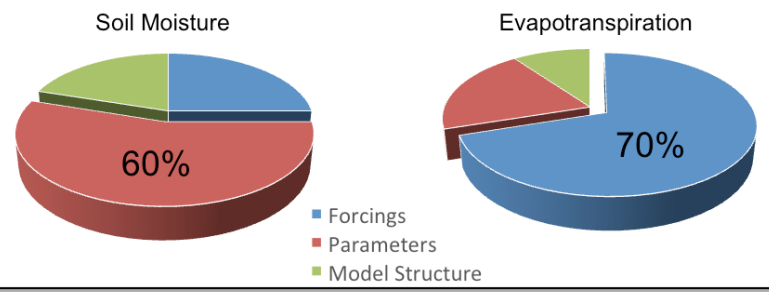


NLDAS-2



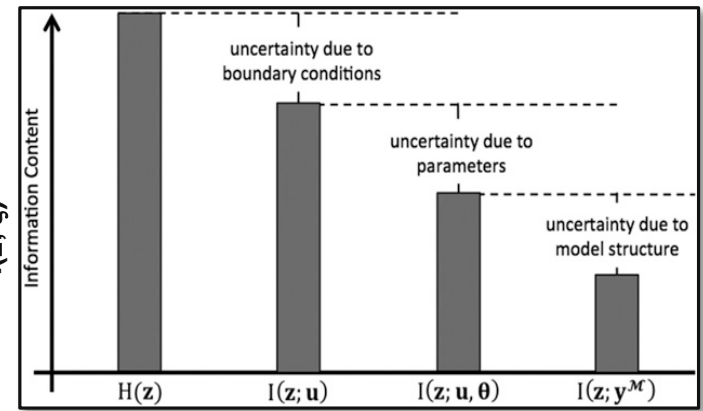
VIC
Mosaic
Noah
SAC-SMA

% of Total Uncertainty



Shannon's mutual information function $I(z; \zeta)$

Nearing, G.S., et al. (2016). Benchmarking NLDAS-2 soil moisture and evapotranspiration to separate uncertainty contributions. Journal of Hydrometeorology



Information (Entropy) in observations z
 Information about observations z in forcings u
 Information about observations z in forcings u plus parameters q
 Information about observations z in model y^M



Dynamic Post-Processing with ML

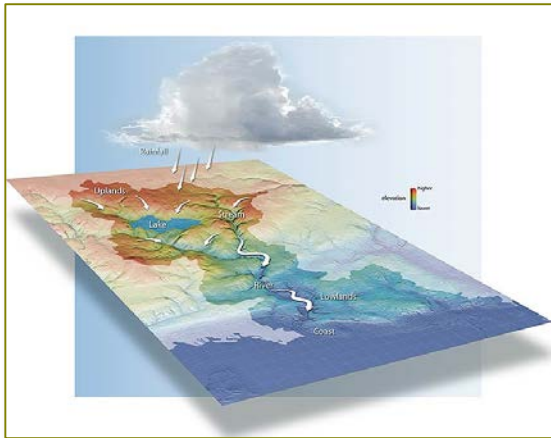


$$dx = f(x, u, \theta_f) dt + \mathcal{GP}_\mu(f, x, u, \theta_{\mathcal{GP}}) + \mathcal{GP}_\sigma(f, x, u, \theta_{\mathcal{GP}})$$

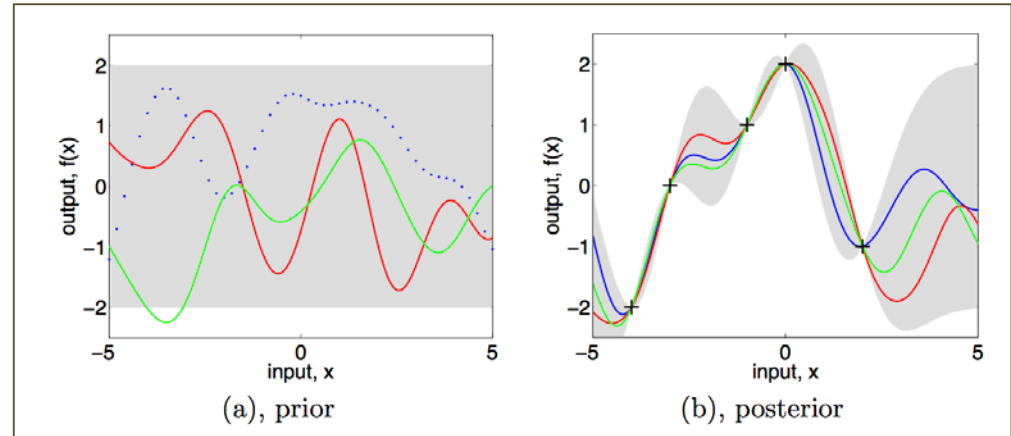
Biogeophysics
lives here

Structured
model error
is here

Uncertainty
lives here



+



Gaussian Processes Regression



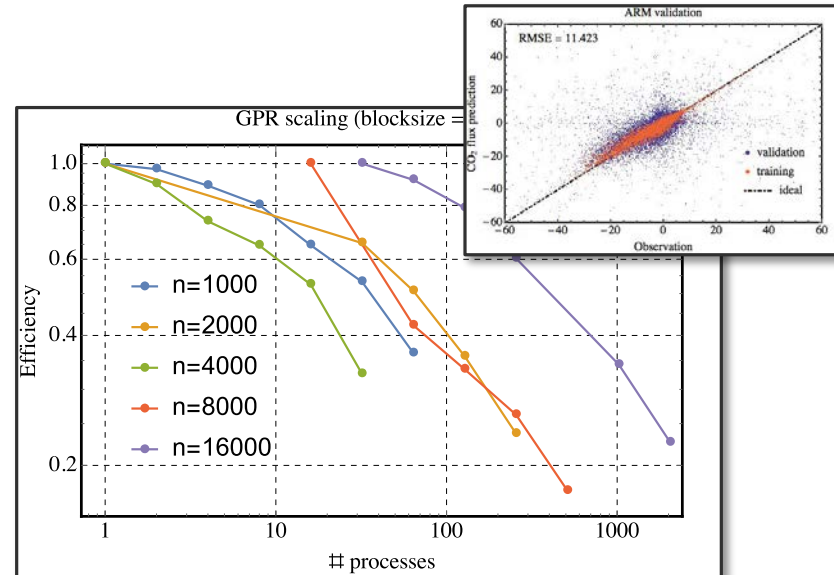
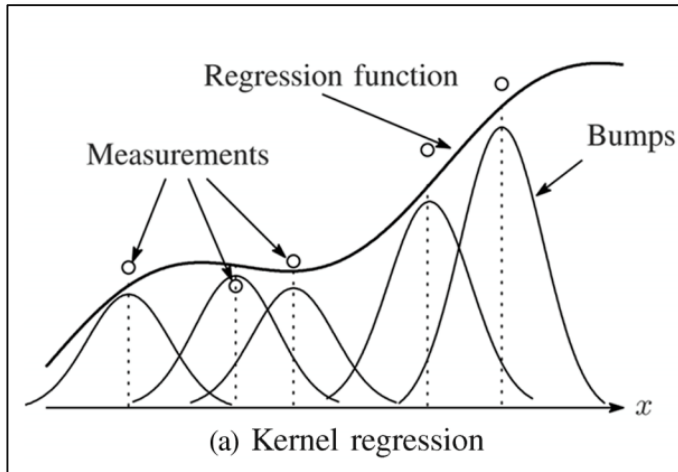
Gaussian Process Regression with ARD Kernel Efficient MPI-enabled c++ GPR

$$K(x, x', \theta) = \sigma_f^2 \exp\left[-\frac{1}{2}(x - x')^T \Sigma^{-1} (x - x')\right]$$

$$\Sigma_{ij} = \sigma_i^2 \delta_{ij}$$

$$p(y|f) = \mathcal{N}(f, \sigma_y^2)$$

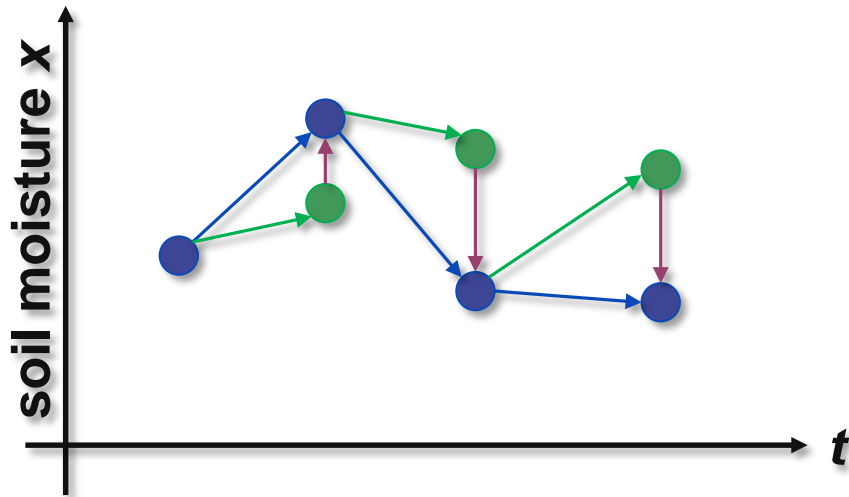
- Uses ScaLapack.
- Can do 50k-100k samples.
- Kernels: ARD w/wo noise and NIGP.



Training a Corrective Model



If everything were perfect ...



$$dy = f(x, u, \theta_f) dt + \Delta$$

Truth

Physical model

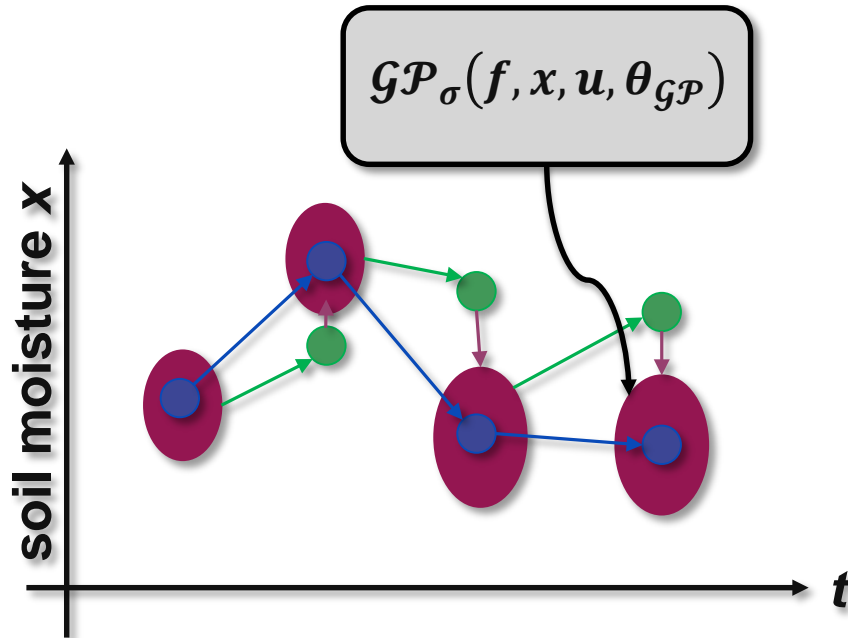
ML correction

Creating a training set

1. Equilibrate Noah.
2. Start from observation "truth" at time t .
3. Make prediction $f(x, u, \theta_f)dt$ (Noah).
4. Record training target = Δ , and training input = $(f dt, x, u)$
5. Repeat steps 2-4 for $t+1$

Noisy Training Sets

Uncertainty in correction needs to be accounted for ...

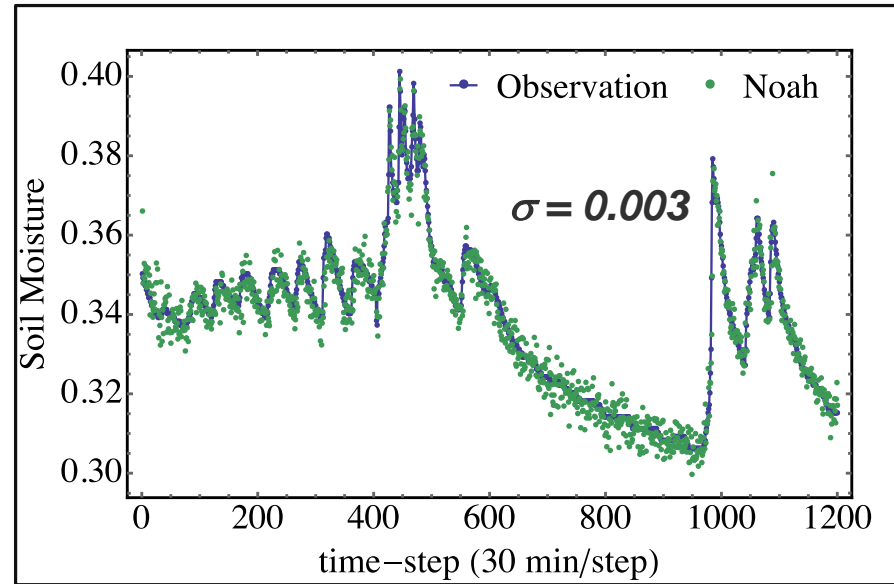
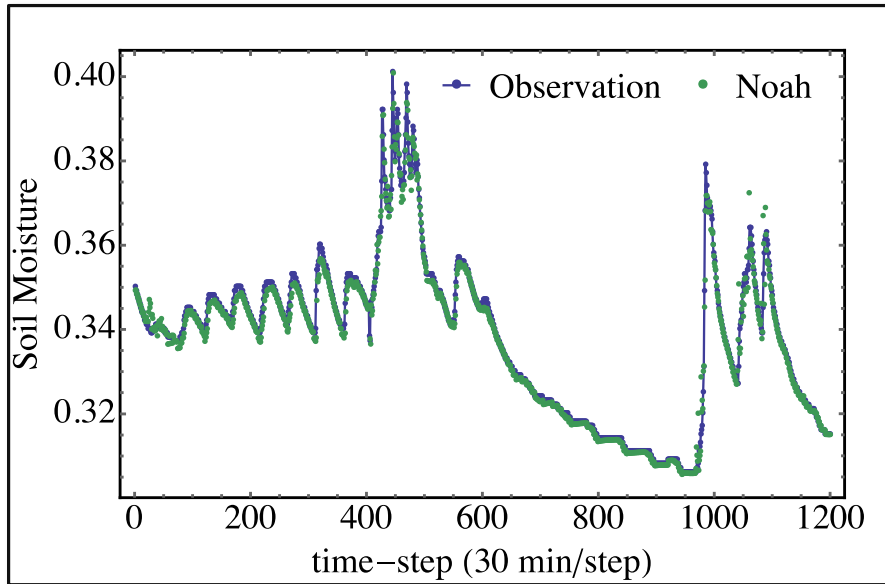


Modify procedure

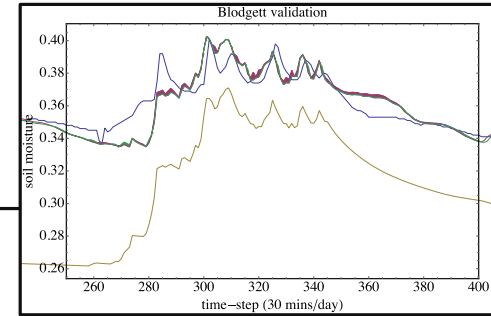
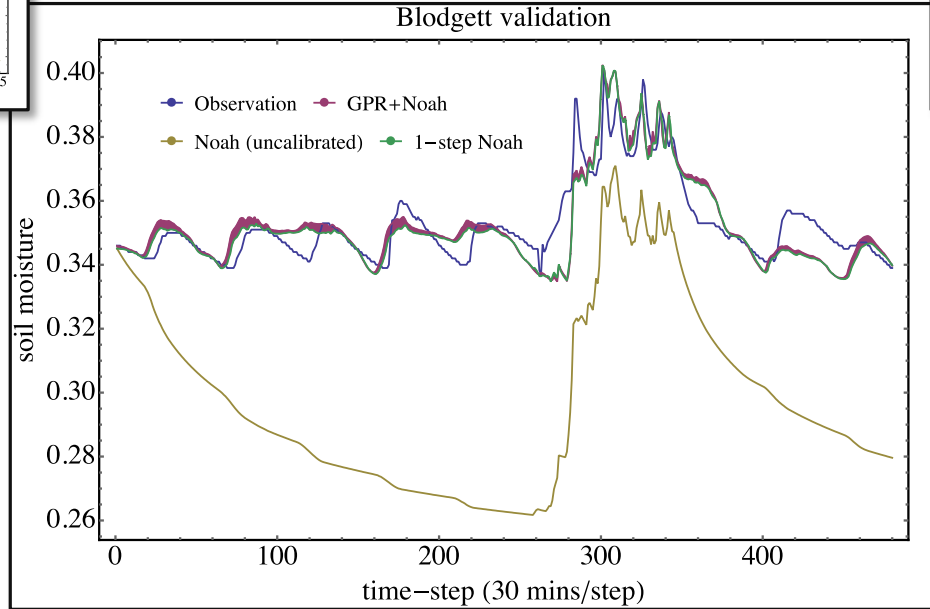
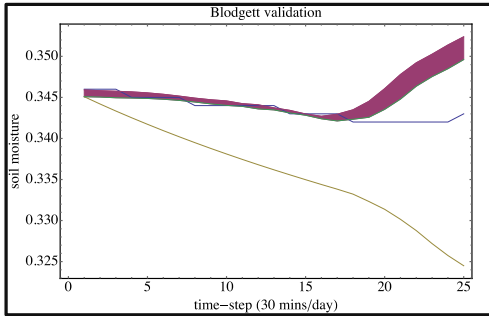
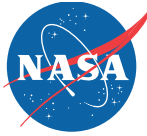
Creating a training set

1. Equilibrate Noah.
2. Start from observation “truth” at time t .
3. Add $N(0, \sigma)$ noise to lagged value to propagate uncertainty to the prediction $f(x, u, \theta_f)dt$ (Noah).
4. Make prediction $f(x, u, \theta_f)dt$ (Noah).
5. Record training target = Δ , and training input = (fdt, x, u)
6. Repeat steps 2-5 for $t+1$

Blodgett Training Sets



Blodgett AmeriFlux Tower



Noisy Input GPs

$$y = f(\mathbf{x} - \boldsymbol{\epsilon}_x) + \epsilon_y \approx f(\mathbf{x}) + \boldsymbol{\epsilon}_x^T \nabla f_{\mu}(\mathbf{x}) + \epsilon_y$$

Noisy Input GPs with ARD Kernel

$$K(\mathbf{x}, \mathbf{x}', \theta) = \sigma_f^2 \exp\left[-\frac{1}{2}(\mathbf{x} - \mathbf{x}')^T \Sigma^{-1}(\mathbf{x} - \mathbf{x}')\right]$$

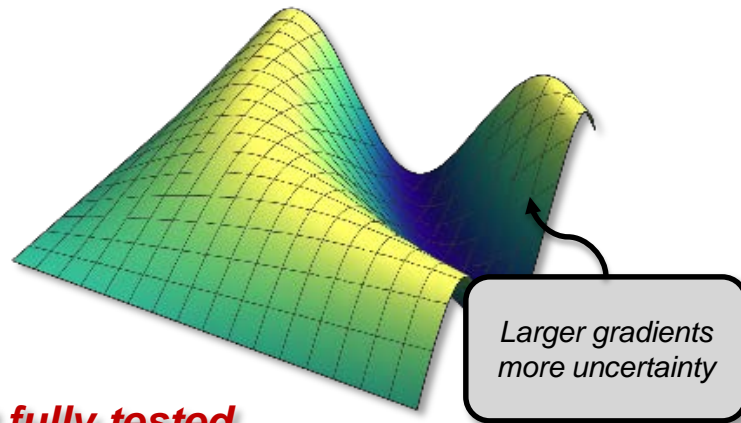
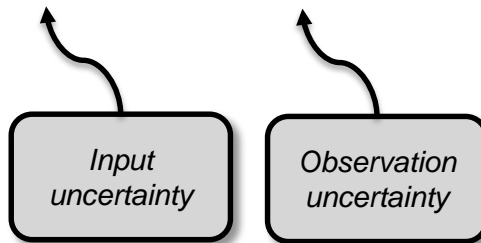
$$\Sigma_{ij} = \sigma_i^2 \delta_{ij}$$

$$p(\mathbf{y}|\mathbf{f}) = \mathcal{N}(\mathbf{f}, \sigma_y^2 + \nabla \mathbf{f}^T \Sigma^{\text{noise}} \nabla \mathbf{f})$$

$$\Sigma_{ij}^{\text{noise}} = \sigma_i^2 \delta_{ij}$$

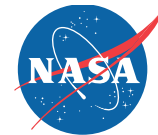
Heteroscedastic
output noise

Tunable
hyperparameters



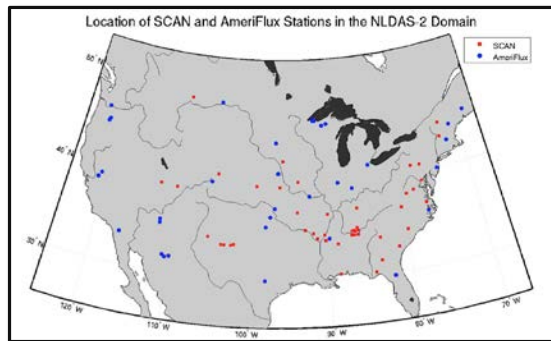
Implemented but not fully tested

Outlook



Coming soon

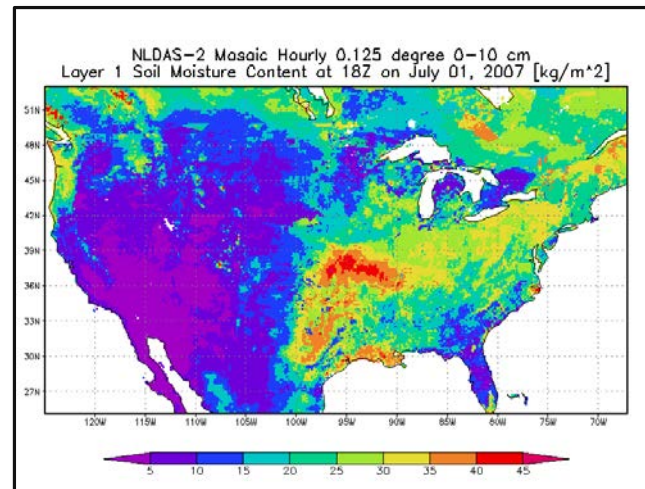
Validate spatially and temporally over 1 year at the flux towers sites.



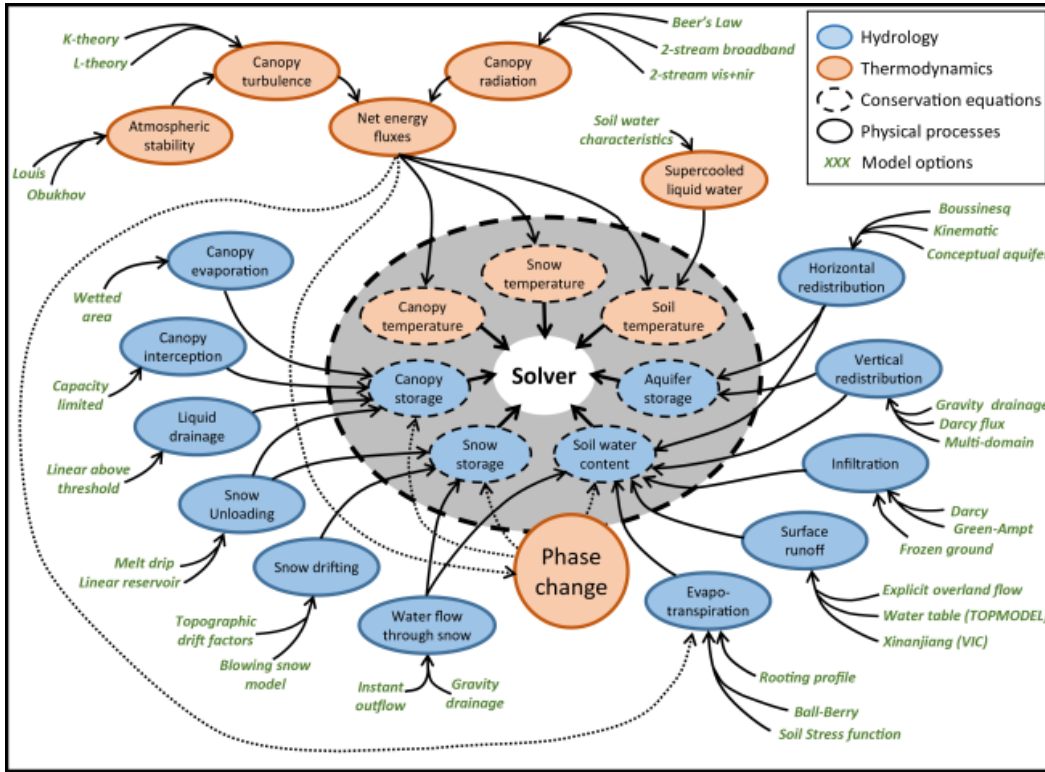
1. Incorporate NIGPs to reduce uncertainty.
2. Validate at each tower individually for 1 year with at least 10 day forecasting (temporal).
3. Perform LOO validation over the towers (spatial).

In the near future

Create NLDAS maps with hourly 0.125 degree resolution



An Emerging Vision



We need radically novel strategies for merging complex process models with the powerful ability of machine learning to extract information from data.

M. Clark et al. (2015) "A unified approach for process-based hydrologic modeling: 1. Modeling concept" *Water Resources Research*



Thanks!

