

## THE IMPLEMENTATION OF MAXIMUM LIKELIHOOD ESTIMATION IN SPACE LAUNCH SYSTEM VEHICLE DESIGN

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As NASA's Space Launch System (SLS) approaches first launch, the design has matured to a point where the manufacturing uncertainty has decreased now that many of the components of the launch vehicle have been manufactured and the flight engines have been successfully tested. Prior to this point, a method was required to qualify and capture the impact of the differences between simulation and reality, as well as any uncertainties in the SLS design.

Two primary categories of uncertainty arise during the launch vehicle design process. The first represents flight-day uncertainties including dispersions due to winds and temperatures. These are typically examined by performing a Monte Carlo on 6 Degree of Freedom (6-DOF) simulations [1]. The second category of uncertainties represents any manufacturing variations that are present at the individual component level of the launch vehicle design. These variations are constructed using statistical masses and tend to become better understood and refined as the design cycle matures, finally resulting in the launch vehicle as constructed and tested.

SLS uses a Maximum Likelihood Estimation (MLE) process in conjunction with a Design of Experiments (DOE) grid to develop statistically representative vehicles for the Block-1 and Block-1B configurations. These Trajectory Dispersed (TD) Vehicles are then used to estimate maximum load conditions and accelerations are used as bounding or stressing cases for aerodynamic, structural, and thermal analyses.

Beard and Hanson [2, 3] developed this process during the Constellation program as a way to represent each vehicle as a statistical combination of manufacturing uncertainties. This process utilized DOE to reduce the computational burden on the analyst while capturing the full variability of the manufacturing uncertainties across the design space. This process is outlined as Figure 1.

The TD vehicle process starts with defining the manufacturing uncertainties that are of interest for a particular configuration. These are chosen in a way to properly stress the vehicle or assess performance in a given design dimension. These manufacturing uncertainties are then used as inputs to the TD vehicle process. Examples of manufacturing uncertainties used for SLS design include

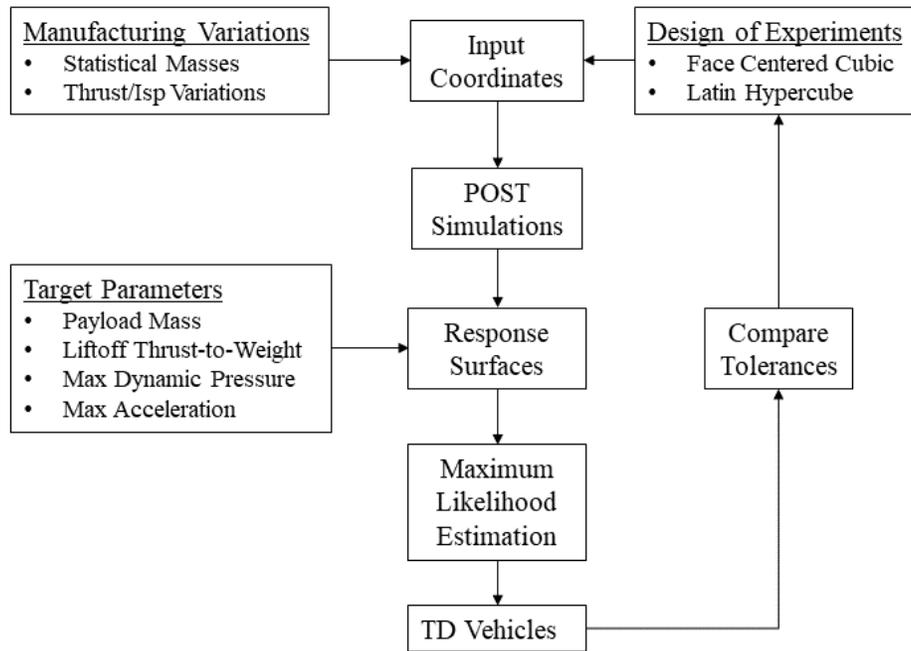
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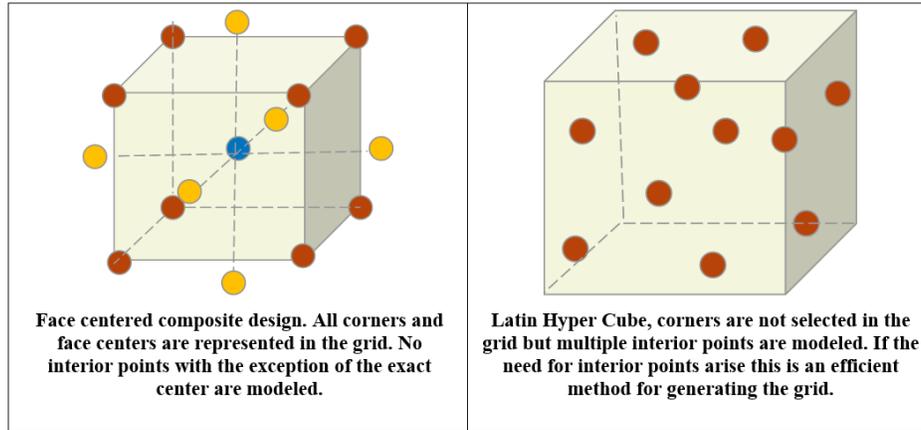


**Figure 1. Flowchart of the Maximum Likelihood Estimation Process**

uncertainties in the RS-25 thrust and specific impulse, RL-10 thrust and specific impulse, inert mass for the boosters, core stage, and Exploration Upper Stage, as well as booster burn rates.

A grid based on design of experiments developed to represent different combinations of the aforementioned manufacturing uncertainties and is illustrated as Figure 2. This multi-dimensional grid is then used to develop POST2 (Program to Optimize Simulated Trajectories II) input decks and are simulated independently at each coordinate combination [2]. This analysis used two types of DOE grids; Block-1 was designed using a face-centered composite design while Block-1B employed a Latin Hypercube design. The face-centered composite design evaluates conditions at the extremes of the an n-dimensional grid, with central points along the dimensions at each face. This method provides performance qualification at the grid extremes, but interior sampling is neglected and thus, under-represents any potential changes interior to the grid. The Latin Hypercube randomly but evenly samples the parameter space. This was employed for Block-1B analysis in order to address poor response surface fits due to interior point underrepresentation.

Targeting parameters are then prescribed for each type of TD vehicle for which response surfaces are generated. These include parameters such as liftoff thrust to weight, payload mass, maximum dynamic pressure, and maximum acceleration. Second order response surfaces are generated for each target parameter and then used as constraints during a statistical process known as maximum likelihood estimation. The MLE process attempts to find the values of the targets which maximize the likelihood function for a given set of observations, or uncertainties in this case.



**Figure 2. Design of Experiment Grid Examples**

$$J(P_i) = \left[ \sum_i \ln(P_i) \right]_{max}$$

This process results in the development of a set of TD vehicles, which are then compared to acceptable tolerances to verify response surfaces have sufficient fidelity to capture the needed variation for a given the target parameter. If the response surface fit is unacceptable, points are added to the original set of coordinates and additional cases are simulated in POST to attempt to capture any missing information in the grid. Upon completion of this process, these TD vehicles are then used as nominal seed cases for further Monte Carlo analyses using 6-DOF simulations.

While this paper focuses on the specific application for the SLS launch vehicle, this statistical process can be applied to any type of vehicle design, whether missile, sounding rocket, launch vehicle, or spacecraft. The MLE process provides a way to conveniently express an uncertainty or variation across a trade space without the need to run Monte Carlo analysis, which can be computationally intensive. As an example, the Block-1 vehicle required only 140 simulations to create a set of TD vehicles, whereas the Block-1B required approximately 350 simulations.

This paper discusses the MLE modeling process that SLS utilizes to develop TD vehicles and how SLS captures manufacturing uncertainty in the launch vehicle design. This paper provides an overview of the MLE process and compares the differences between Block-1 and Block-1B statistical representations. This paper also discusses proper DOE grid choice, as well as which uncertainties drive the vehicle design.

## REFERENCES

- [1] J.M. Hanson and B.B. Beard. Applying Monte Carlo Simulation to Launch Vehicle Design and Requirements Analysis. Technical Report NASA/TP-2010-216447, Marshall Space Flight Center, September 2010.
- [2] J.M. Hanson and B.B. Beard. Applying Monte Carlo Simulation to Launch Vehicle Design and Requirements Verification. *Journal of Spacecraft and Rockets*, 4, January 2012.
- [3] ARES Corporation. Some Notes on the Use of Statistical Methods in the Design of Launch Vehicles. Technical report, Marshall Space Flight Center, January 2009.