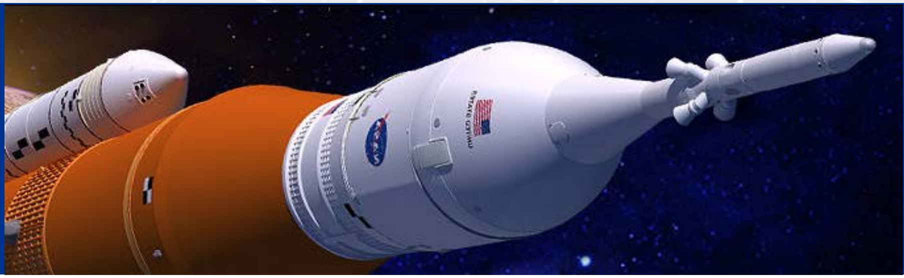
The Implementation of Maximum Likelihood Estimation in Space Launch System Vehicle Design



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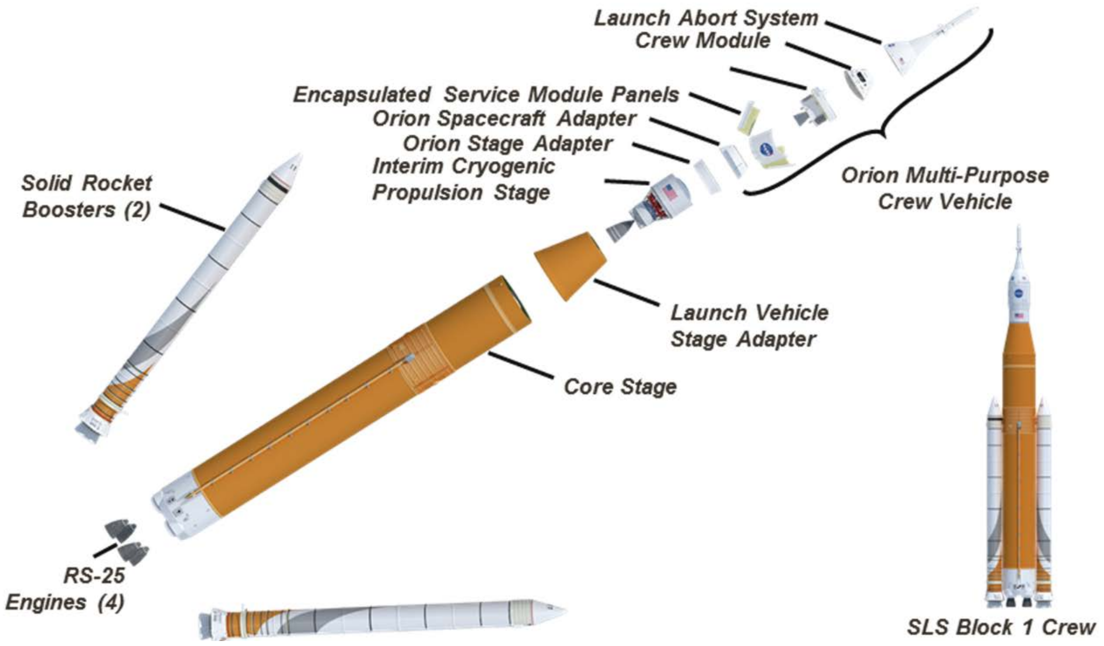
Agenda

1. Background and problem definition
2. The response surface and maximum likelihood solution
3. How well it worked

Background

- SLS design is a complex optimization problem
  - Many different groups with requirements and constraints
- Initial design had many sources of uncertainty
  - Large parameter space to search over
  - Manufacturing uncertainties
  - Day of flight uncertainties
  - Time and computational resources were limited
- How did we work through these problems?
  - Developed a response surface methodology coupled with a maximum likelihood estimation (MLE) process
  - Divide the uncertainty space into two groups
    - Manufacturing uncertainties
    - Flight day uncertainties



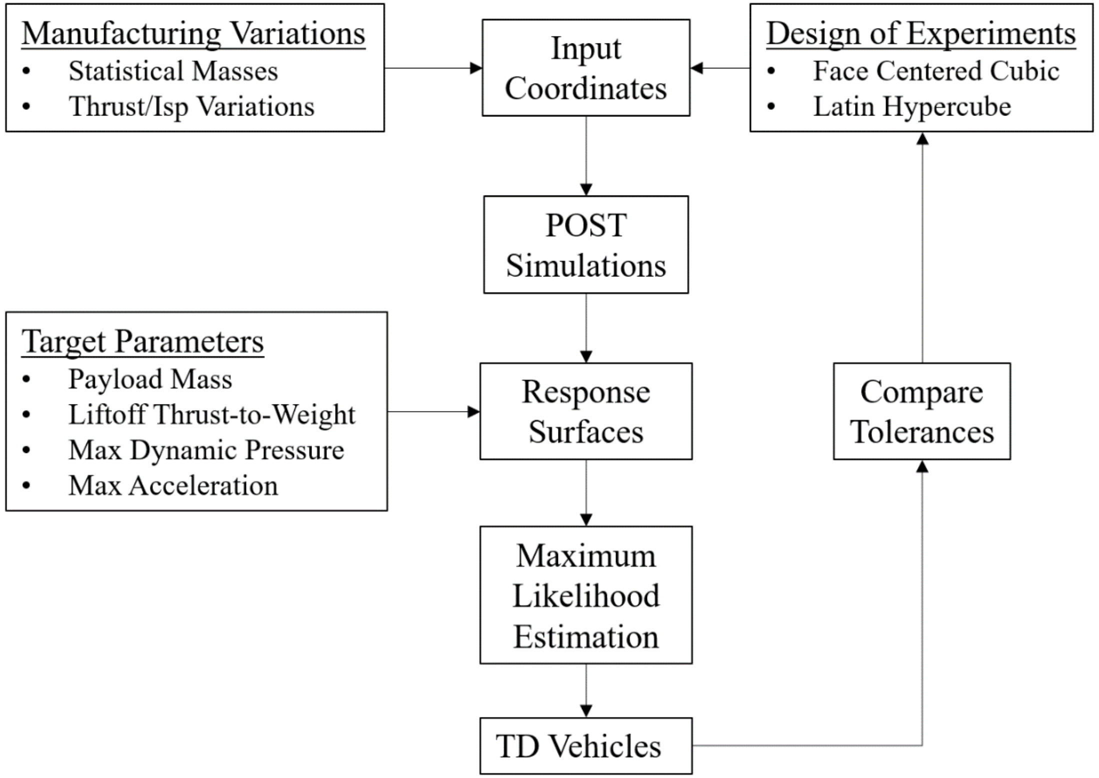
Trajectory Dispersed (TD) Vehicle Design

- The purpose of TD vehicles is to separate the manufacturing uncertainties from the day of flight uncertainties
- We do this because it helps qualify/quantify these effects from the flight day effects
- Allows for specifically constructed vehicles that stress critical parameters
  - Payload
  - Max Dynamic Pressure
  - Acceleration
  - Booster loading
- Explore different interactions without doing a worst on worst case coupling
  - This can lead to over design or lost performance

Month	Response Parameter	Applications
	Heavy Slow	
February	Thrust-to-Weight (10th Percentile) Payload (10th Percentile)	Payload Performance Flight Performance Reserve Calculation Lift-Off Clearances
	Light Fast	
July	Thrust-to-Weight (90th Percentile) Max Dynamic Pressure (90th Percentile) Max Heat Rate (90th Percentile) Max 1st Stage Acceleration (90th Percentile)	Vehicle Loads
	Hybrid	
July	Max Dynamic Pressure (90th Percentile) Max Heat Rate (90th Percentile) Max 1st Stage Acceleration (90th Percentile) Payload (10th Percentile)	Payload Performance Inlet Pressure Clearances

Maximum Likelihood Design Process

1. Qualify/Quantify the manufacturing uncertainties
2. Choose a DOE method to produce a set of test cases from the design parameters
3. Fit response surfaces to the outcomes of the test cases
4. Use the response surfaces and the MLE optimization process to develop targets for the desired response offsets
5. Run a final simulation that compares its outcome to the response surface, to ensure the system closes



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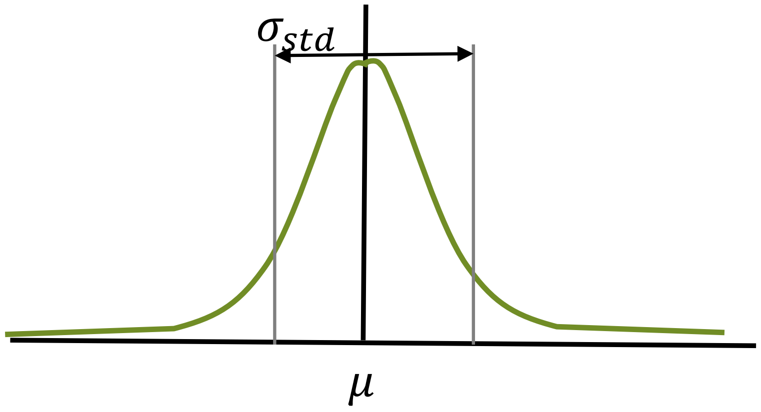
MLE Process: Step 1

- Qualify/Quantify the manufacturing uncertainties



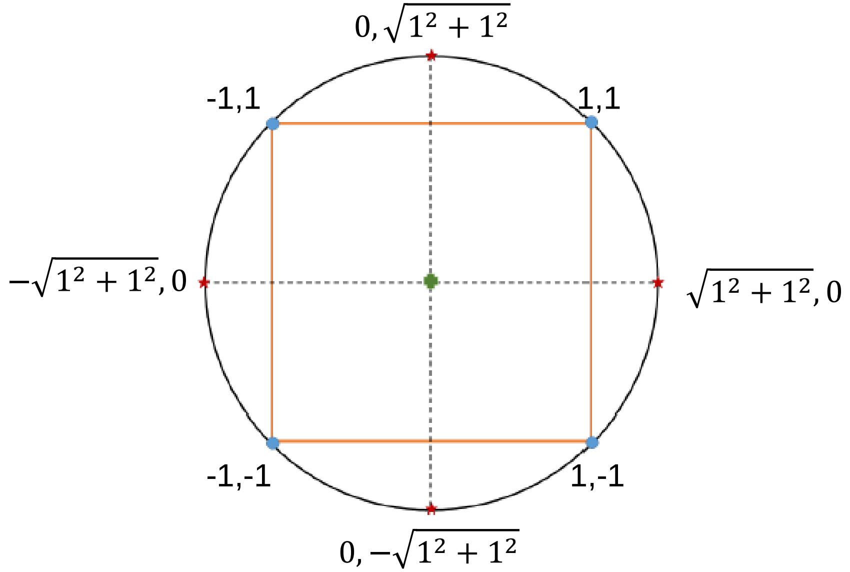
Design Parameters	**Uncertainty mean, σ_{std}**	**Distribution shape**
SRB Propellant Mass	0, 1	Normal
SRB Burn Rate	0, 1	Normal
SRB Burn out mass	0, 1	Normal
RS25 Thrust	0, 1	Normal
RS25 Isp	0, 1	Normal
Core Stage Dry Mass	0, 1	Normal
LAS Mass	0, 1	Uniform

Typically we encode the values to represent the width of the distribution in terms -1 to 1

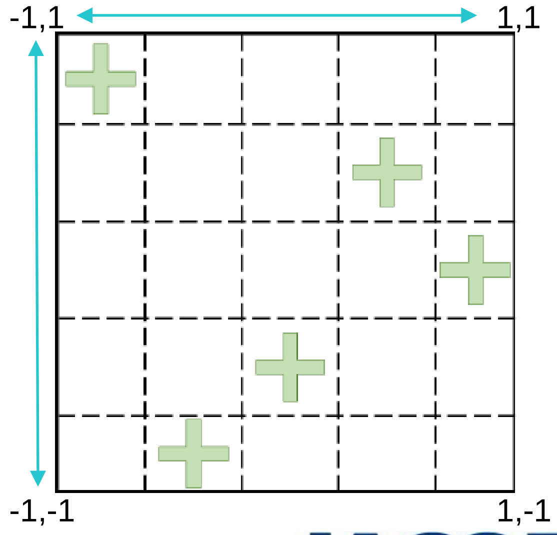
MLE Process: Step 2

- Choose a DOE method to produce a set of test cases from the design parameters
- Parameters are coded from -1 to 1 to represent minimum and maximum values
- The Central Composite method looks at coupled interactions at the vertices of the square
- The Latin Square looks at interior points that are well sampled
- After the design parameters are chosen and a search space produced we run a series of Program to Optimize Simulated Trajectories (POST) trajectories at these grid points

Two design parameters Central Composite



Two design parameters Latin Square



MLE Process: Step 3

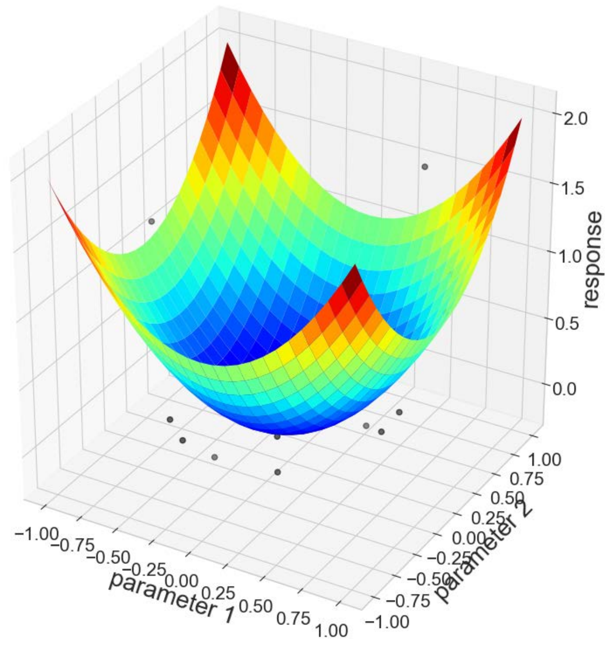
- Fit response surfaces to the outcomes of the test cases
- A standard least squares polynomial is fit

$$R(x_i) = \sum_{i=0}^{N} \beta_i x_i + \sum_{i=0}^{N} \sum_{j=0}^{N} \beta_{ij} x_i x_j + \beta_0$$

- Minimizing the sum of the errors

$$e = \sum_{i=1}^{N} (y_i - R(x_i))^2$$

- Response surfaces allow for prediction of outcomes



Dependent Variable	TD Surface Coefficient	Monte Carlo Surface Coefficient	Difference
SRB Propellant Mass	0.1562	0.1552	0.64%
SRB Burn Rate	0.6685	0.6720	-0.52%
RS-25 Specific Impulse	0.1835	0.1829	0.33%
RS-25 Thrust	1.0703	1.0714	-0.10%
Core Dry Mass	-0.2556	-0.2562	-0.23%
SRB Jettison Mass	-0.4999	-0.5030	-0.62%
LAS Mass	-0.0224	-0.0231	-3.03%

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MLE Process: Step 4

- Use the response surfaces and the MLE optimization process to develop targets for the desired response offsets
- Our goal is to optimize across a surface or surfaces to keep the chance of an offset occurring high while stressing key parameters for a TD vehicle
  - The objective function keeps the probability of occurrence high
  - The constraints become the response surfaces and the target of either 10th or 90th percentile

Notional Values

Design Parameters	**Offset , σ_{std}**
SRB Propellant Mass	0.25
SRB Burn Rate	1.38
SRB Burn out mass	0.36
RS25 Thrust	2.41
RS25 Isp	1.28
Core Stage Dry Mass	0.015
LAS Mass	0.003

Objective Function

$$J(p) = max\left(\sum_{i}^{N} \ln P_i\right)$$

Constraints

$$R_{PAYLOAD} = -1.28\sigma_{PAYLOAD}$$
$$R_{LOTW} = -1.28\sigma_{LOTW}$$

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MLE Process: Step 5

- Run a final simulation that compares its outcome to the response surface, to ensure the system closes
- These offsets go into a final POST run for the TD vehicle
- Typically compare the final output back to the response surface to verify agreement

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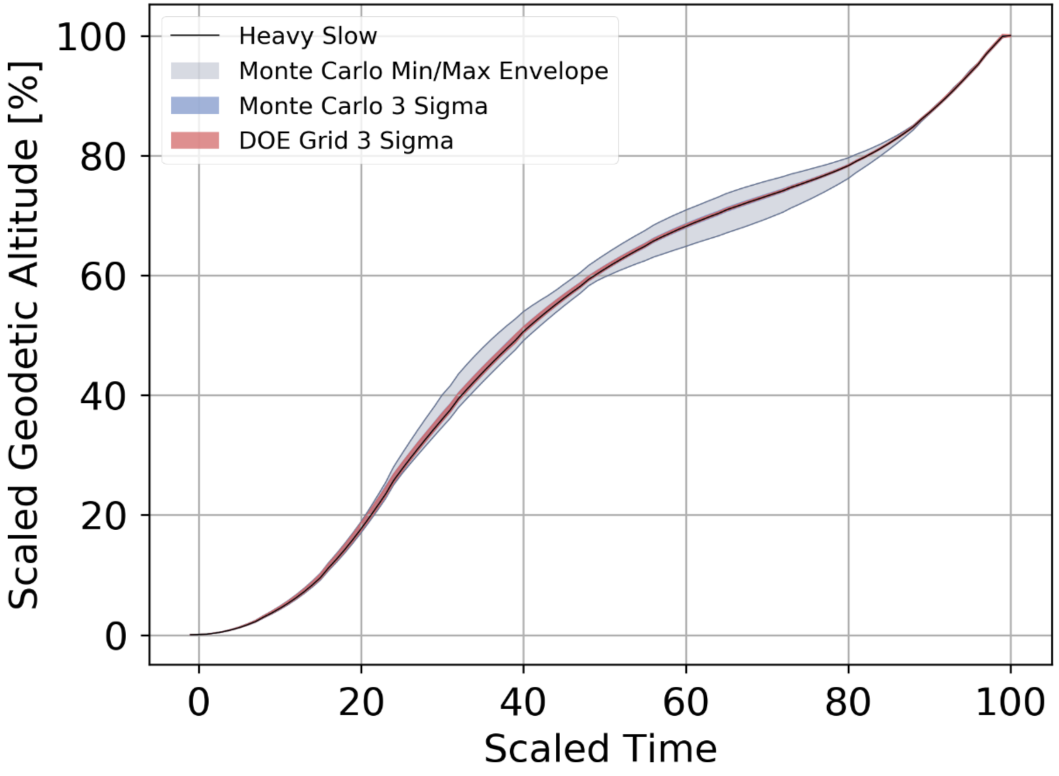
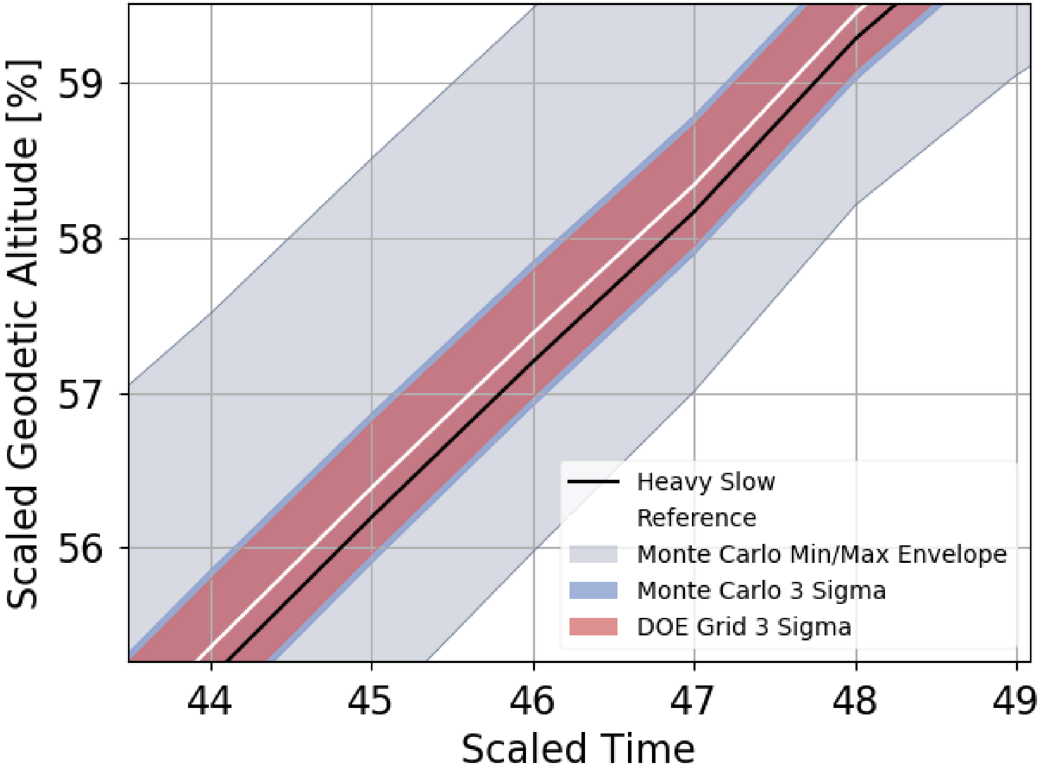
Resulting Vehicle Altitude Time History





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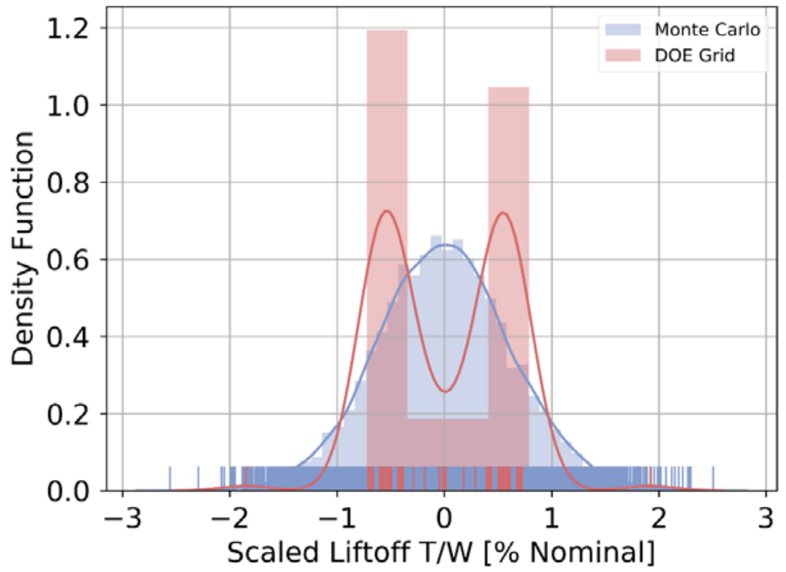
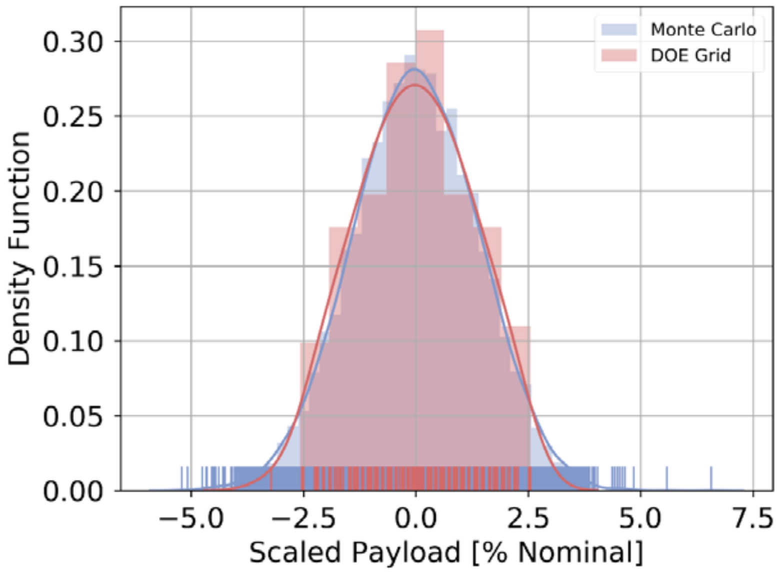
Grid Choice Considerations

- The Central Composite Design had a drawback
  - A dominant parameter would skew results
  - Use a Latin Square that samples the interior, which led to better response surface fits





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Conclusions

- Reduce time and computational requirements by using statistically representative vehicles
- Response surfaces with a constrained MLE process can produce excursion vehicles for analysis
  - Provide a functional representation of a vehicle’s outcome without further need of computational resources
  - Show relative sensitivity of design parameters
  - Process is applicable to other uncertain analyses besides launch vehicle design

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