



- Quantum Approximate Optimization Algorithm: review and status
- The «Quantum Alternating Operator Ansatz»
 - Mixing Operators
 - Examples
- Compiling and Executing
 - The gate synthesis problem
 - Review of compilation methods
 - Compiling framework in nearest-neighbor architectures

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READING LIST

- Optimizing Variational Quantum Algorithms Using Pontryagin's Minimum Principle Z. Yang, A. Rahmani, A. Shabani, H. Neven, and C. Chamon Phys. Rev. X 7, 021027 (2017)
- Quantum Approximate Optimization with Hard and Soft Constraints. Hadfield, S., Wang, Z., Rieffel, E. G., O'Gorman, B., Venturelli, D., & Biswas, R. (2017, November). In Proceedings of the Second International Workshop on Post Moores Era Supercomputing (pp. 15-21). ACM.
- From the quantum approximate optimization algorithm to a quantum alternating operator Ansatz Hadfield, S, Z. Wang, B. O'Gorman, E. G. Rieffel, D. Venturelli, and R. Biswas. arXiv preprint arXiv:1709.03489 (2017).
- Compiling quantum circuits to realistic hardware architectures using temporal planners Venturelli, D., Do, M., Rieffel, E. and Frank, J., 2018. Quantum Science and Technology, 3(2), p.025004.



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1. Review on QAOA Research





- 1. Design a binary optimization classical Hamiltonian ("phase separation")
- 2. Design a unitary operator that can connect and allow jumps between different states ("mixing")
- 3. Prepare a QAOA state for some parameters

$$Q_p(\boldsymbol{\beta}, \boldsymbol{\gamma}) = U_{\mathrm{M}}(\beta_p) U_{\mathrm{P}}(\gamma_p) \cdots U_{\mathrm{M}}(\beta_1) U_{\mathrm{P}}(\gamma_1)$$

4. Measure the state in the computational value and compute the exp. value of C(z)

$$F_{p}(\boldsymbol{\gamma},\boldsymbol{\beta}) = \langle \boldsymbol{\gamma},\boldsymbol{\beta} | C | \boldsymbol{\gamma},\boldsymbol{\beta} \rangle \qquad M_{p} \ge M_{p-1}$$
$$M_{p} = \max_{\boldsymbol{\gamma},\boldsymbol{\beta}} F_{p}(\boldsymbol{\gamma},\boldsymbol{\beta}) \qquad \lim_{p \to \infty} M_{p} = \max_{z} C(z)$$

5. Change the parameters if they are not proven optimal and repeat 3-4





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1. UNDERSTANDING

- a. Power/Supremacy: Fahri, Arrow (2016), Jiang (2017)
- b. Parameter Setting: Yang (2017), Wang (2017)

2. PRACTICAL ASPECTS

- a. Compilation: Venturelli (2017), Guerreschi (2017), Oddi (2018)
- b. Hybridization: Guerreschi (2016)

3. GENERALIZATIONS/APPLICATIONS

- a. Combinatorial Optimization: Hadfield (2016)
- b. Factoring: Anschuetz (2018)
- c. Deep Learning: Verdon (2018)

4. EXPERIMENTS

- a. Superconducting: Otterbach (2017)
- b. Neutral Atoms: Pichler (2018)

mization Algorithm

A Quantum Approximate Optimization Algorithm

Edward Farhi and Jeffrey Goldstone Center for Theoretical Physics Massachusetts Institute of Technology Cambridge, MA 02139

Sam Gutmann

Abstract

We introduce a quantum algorithm that produces approximate solutions for combinatorial optimization problems. The algorithm depends on an integer $p \ge 1$ and the quality of the approximation improves as p is increased. The quantum circuit that implements the algorithm consists of unitary gates whose locality is at most the locality of the objective function whose optimum is sought. The depth of the circuit grows linearly with p times (at worst) the number of constraints. If p is fixed, that is, independent of the input size, the algorithm makes use of efficient classical preprocessing. If p grows with the input size a different strategy is proposed. We study the algorithm as applied to MaxCut on regular graphs and analyze its performance on 2-regular and 3-regular graphs for fixed p. For p = 1, on 3-regular graphs the quantum algorithm always finds a cut that is at least 0.6924 times the size of the optimal cut.



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 Optimizing Variational Quantum Algorithms
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 Zhi-Cheng Yang, Armin Rahmani, Alireza Shabani, Hartmut Neven, and Claudio
 Chamon Phys. Rev. X 7, 021027 Published 18 May 2017
 a. Con
 b. Fac
 QAOA is a Bang-Bang quantum control protocol
 - Same «ansatz» (not params) is robust to noise/decoherence
 It seems that certain aspects of parameter settings are university
 - It seems that certain aspects of parameter settings are universal (to be verified)

4. EXPERIMENTS

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- a. Superconducting: Otterbach (2017)
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This tutorial !

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NS adfield (2016)



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2. QAOA for Constrained Optimization Problems





- Maximum Cut
- Max-SAT, Min-SAT, NAE-SAT
- Set Splitting
- MaxE3LIN2
- Max-ColorableSubgraph
- Graph Partitioning
- Maximum Bisection
- Max Vertex k-Cover
- MaxIndependentSet
- MaxClique
- MinVertexCover
- MaxSetPacking
- MinSetCover
- TSP
- SMS with various metrics and constraints
- ...

Objective Function: Soft Constraints **Feasible States**: Hard Constraints

Quantum Approximate Optimization with Hard and Soft Constraints

Stuart Hadfield*, Zhihui Wang^{+,**}, Eleanor G. Rieffel⁺,

Bryan O'Gorman^{+,†}, Davide Venturelli^{+,**}, Rupak Biswas⁺ * Department of Computer Science, Columbia University, New York, NY ⁺ Quantum Artificial Intelligence Lab., NASA Ames Research Center, Moffett Field, CA ** Universities Space Research Association, Mountain View, CA †Stinger Ghaffarian Technologies, Inc., Greenbelt, MD

ABSTRACT

Challenging computational problems arising in the practical world are frequently tackled by heuristic algorithms. Small universal quantum computers will emerge in the next year or two, enabling a substantial broadening of the types of quantum heuristics that can be investigated beyond quantum annealing. The immediate question is: what experiments should we prioritize that will give us insight into quantum heuristics? One leading candidate is the quantum approximate optimization algorithm (QAOA) metaheuristic. In this work, we provide a framework for designing QAOA circuits for a variety of combinatorial optimization problems with both hard constraints that must be met and soft constraints whose violation we wish to minimize. We work through a number of examples, and discuss design principles.

CCS CONCEPTS

 Mathematics of computing → Approximation algorithms;
 Hardware → Emerging technologies; Quantum computation;
 Theory of computation → Quantum computation theory; Mathematical optimization; advantage, and if so, how to design quantum algorithms that realize such advantages. Today, challenging computational problems arising in the practical world are frequently tackled by heuristic algorithms, which by definition have not been analytically proven to be the best approach, or even proven analytically to outperform the best approach of the previous year. Rather, these algorithms are empirically shown to be effective, by running them on characteristic sets of problems, or demonstrating their effectiveness in practical applications. As prototype quantum hardware emerges, this approach to algorithm design becomes available for the evaluation of quantum heuristic algorithms.

For several years now, special-purpose quantum hardware has been used to explore one quantum heuristic algorithm, quantum annealing. Emerging gate-model processors, which are universal in that, once scaled up, they can run any quantum algorithm, will enable investigation of a much broader array of quantum heuristics beyond quantum annealing. Within the last year, IBM has made available publicly through the cloud a 5-qubit gate-model chip [13], and announced recently an upgrade to a 17-qubit chip. Likewise, Google [3] and Rigetti Computing [22], anticipate providing processors with 40–100 qubits within a year or two [18]. Many academic

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$Q_p(\boldsymbol{\beta}, \boldsymbol{\gamma}) = U_{\mathrm{M}}(\beta_p)U_{\mathrm{P}}(\gamma_p)\cdots U_{\mathrm{M}}(\beta_1)U_{\mathrm{P}}(\gamma_1)$

 $|\boldsymbol{\beta}, \boldsymbol{\gamma}\rangle = Q_p(\boldsymbol{\beta}, \boldsymbol{\gamma}) |s\rangle$

Some initial state respecting:

- It is a superposition of several solutions in the feasible subspace
- It can be prepared efficiently

Some unitary respecting:

 $eta H_{\mathbf{M}}$

- Preserve the feasible subspace
- Provide all-to-all nonzero transitions between all feasible states
- Non-necessarily time evolution of a local Hamiltonian

Some unitary respecting:

- Is diagonal in the computational basis
- The spectrum of HP encodes the objective function
 - $H_f \left| \mathbf{x} \right\rangle = f(\mathbf{x}) \left| \mathbf{x} \right\rangle$

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Node u is colored by c $X_{u,c}=1$



$$H'_{\rm P} = \frac{4-\kappa}{4}mI + \frac{1}{4}\sum_{\{u,v\}\in E}\sum_{a=1}^{\kappa} \left(Z_{u,a} + Z_{v,a} - Z_{u,a}Z_{v,a}\right)$$





Dallas, hpc TX inspires.

Initial state:

$$|W\rangle_v = \frac{1}{\sqrt{k}} (|100\cdots0\rangle + |010\cdots0\rangle + |0\cdots01\rangle)$$

- Babbush (2017)
- Verstraete (2009)
- Wang (2009)
- Childs (2002)

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$$H_{\text{ring}}^{(\text{enc})} = \sum_{a}^{d} \left(X_a X_{a+1} + Y_a Y_{a+1} \right)$$

Respects the Hamming Weight constraint

 $Exp(iH_{ring})$ is difficult to implement





$$\begin{split} H_{\mathrm{ring}}^{(\mathrm{enc})} &= \sum_{a}^{d} \left(X_{a} X_{a+1} + Y_{a} Y_{a+1} \right) & \underset{\mathrm{Constraint}}{\operatorname{Respects the}} \\ \mathrm{Hamming Weight} \\ \mathrm{Exp(iH_{ring}) is difficult to implement} \\ U_{\mathrm{M}} &= \prod_{v=1}^{n} U_{v,\mathrm{parity}}^{(\mathrm{enc})} & \prod_{\mathrm{a \in parity}} \mathrm{Exp(iX_{a} X_{a+1} + Y_{a} Y_{a+1})} \\ U_{\mathrm{M}} &= \left[U_{1} U_{3} U_{5} U_{7} \right] \left[U_{2} U_{4} U_{6} U_{8} \right] \left[U_{1} U_{3} U_{5} U_{7} \right] \left[U_{2} U_{4} U_{6} U_{8} \right]. \\ & \underset{\mathrm{Maxing}}{\operatorname{This couples only distance 2;}} \\ & \underset{\mathrm{has to be repeated k/2 times}}{\operatorname{Respects the}} \end{split}$$

All these 2-qubit $k^2/2$ gates need to be scheduled

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Davide Venturelli – SC18 Tutorial Quantum Computing for Scientific Applications – November 12th 2018 (Dallas, TX)

6



Finding the largest induced subgraph colorable by k colors



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In traveling salesman encoding



$$\sum_{\{u,\upsilon\}\in E} d_{u,\upsilon} \sum_{j=1}^n \left(x_{u,j} x_{\upsilon,j+1} + x_{\upsilon,j} x_{u,j+1} \right)$$

$$H_{\text{PS},\{i,j\},\{u,v\}}^{(\text{enc})} = S_{u,i}^+ S_{v,j}^+ S_{u,j}^- S_{v,i}^- + S_{u,i}^- S_{v,j}^- S_{u,j}^+ S_{v,i}^+,$$

(partitioned using edge coloring and parity $\approx (n-1)n^2/4$ mixers) (needs to be repeated n(n-1)/2 times for all-to-all)

In single machine scheduling



$$X_{jt} = 1 \text{ if job j starts at time t}$$
$$C = \sum_{j} w_j \sum_{(d_j - p_j) < t < h} x_{j,t}(t + p_j - d_j)$$

$$H_{\mathrm{TS},t,\{i,j\}}^{(\mathrm{enc})} = S_{i,t+p_j}^+ S_{j,t}^+ S_{i,t}^- S_{j,t+p_i}^- + S_{i,t}^+ S_{j,t+p_i}^+ S_{i,t+p_j}^- S_{j,t}^-$$

(But if we add release dates then we need controls on the no-overlap constraint)

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Bitflip mixers

- Maximum Cut
- Max-SAT, Min-SAT, NAE-SAT
- Set Splitting
- MaxE3LIN2

Controlled Bitflip mixers

- MaxIndependentSet
- MaxClique
- MinVertexCover
- MaxSetPacking
- MinSetCover
- •••

...

XY mixers

- Max-ColorableSubgraph
- Graph Partitioning
- Maximum Bisection
- Max Vertex k-Cover

Controlled XY mixers

- Max-k-ColorableInducedSubgraph
- MinGraphColoring
- MinCliqueCover

Permutation mixers

. . .

...

• TSP

. . .

• SMS with various metrics and constraints

From the Quantum Approximate Optimization Algorithm to a Quantum Alternating Operator Ansatz

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September 12, 2017

The next few years will be exciting as prototype universal quantum processors emerge, enabling implementation of a wider variety of algorithms. Of particular interest are quantum heuristics, which require experimentation on quantum hardware for their evaluation, and which have the potential to significantly expand the breadth of applications for which quantum computers have an established advantage. A leading candidate is Farhi et al.'s Quantum Approximate Optimization Algorithm, which alternates between applying a cost-function-based Hamiltonian and a mixing Hamiltonian. Here, we extend this framework to allow alternation between more general families of operators. The essence of this extension, the Quantum Alternating Operator Ansatz, is the consideration of general parameterized families of unitaries rather than only those corresponding to the time-evolution under a fixed local Hamiltonian for a time specified by the parameter. This ansatz supports the representation of a larger, and potentially more useful, set of states than the original formulation, with potential long-term impact on a broad array of application areas.





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3. QAOA in the "Real World"

Quantum Circuits can be composed by single and two-qubit gates of universal set* CNOT, $R_v(\theta)$ and $R_z(\alpha)$

Each single qubit gate can be decomposed by single qubit rotations. U1= $R_z(\alpha) R_y(\beta) R_z(\gamma) e^{i\phi}$

Each two qubit gate is reversible and it is representable by a Unitary Matrix.

R_z gates can be «virtually» compiled. (McKay 2017 and refs)

* active research to natively support multi-qubit gates

Maximum number of elementary 1-qubit gates: **15** Maximum number of CNOTs: **3** Maximum depth assuming R_w, R₇ and simplifications: **11**

Barenco et al. (1995) Kraus, Cirac (2001) Vatan, Williams (2003)

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SWAP-Compilation (review)



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Performance of algorithms in NISQ will depend on aspects such as gate fidelities, parallelization, idle time, crosstalks..

Different Metrics to optimize correlate to final performance:

- Total Quantum Factor
- Quantum Volume
- Number of Two-Qubit Gates
- Makespan

Guerreschi and Park (2018). Two-step approach to scheduling quantum circuits. *arXiv preprint arXiv:1708.00023*.

Khatri, Sumeet, et al. "Quantum assisted quantum compiling." *arXiv preprint arXiv:1807.00800* (2018).

Li, G., Ding, Y., & Xie, Y. (2018). Tackling the Qubit Mapping Problem for NISQ-Era Quantum Devices. *arXiv preprint arXiv:1809.02573 (2018)*.

Oddi, Angelo, and Riccardo Rasconi. "Greedy Randomized Search for Scalable Compilation of Quantum Circuits." *International Conference on the Integration of Constraint Programming, Artificial Intelligence, and Operations Research.* Springer, Cham, (2018.

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Example: MaxCut



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$$\mathsf{S_i=\pm 1} \\ U = \frac{1}{2} \sum_{(i,j) \in E} (1 - s_i s_j)$$

Defines the cut

) Counts the edges in the cut

 $\sum_{i} X_{i}$ Mixes the two partitions

$$U_{PS} = \prod_{\langle jk \rangle} Exp(ibZ_jZ_k)$$

 $U_{M} = \prod_{j} Exp(igX_{j})$

Interaction graph obtained from quadratic objective function (MAXCUT)



PS1	MX	PS2
$P-S(q_1,q_4)$	$MIX(q_1)$	$P-S(q_1,q_4)$
$ ext{P-S}(q_1,q_3)$	$MIX(q_3)$	$\operatorname{P-S}(q_1,q_3)$
$ t P-S(q_3,q_4)$	$MIX(q_A)$	$ t P-S(q_3,q_4)$
$ ext{P-S}(q_3,q_7)$	$MIX(q_2)$	$ t P-S(q_3,q_7)$
$ extsf{P-S}(q_4,q_7)$	$MIX(q_5)$	$ t P-S(q_4,q_7)$
$P-S(q_6,q_7)$	$MIX(q_6)$	$P-S(q_6,q_7)$
$ t P-S(q_5,q_6)$	$MIX(q_7)$	$P-S(q_5,q_6)$
$P-S(q_1, q_5)$		$ extsf{P-S}(q_1,q_5)$

- Every edge is a gate that needs to be executed (in arbitrary order)
- The same graph has to be executed multiple times (*p* rounds).
- Every qubit has to complete all the gates of round p before being involved in p+1

Interaction graph obtained from Initial assignment quadratic objective

 n_1

Circuit Execution Schedule

 n_2 n_3

 n_7

 $\boldsymbol{q_i} \rightarrow \boldsymbol{n_i}$

n₅

n₈

- Every edge is a gate that needs to be ٠ executed (in arbitrary order)
- The same graph has to be executed multiple times (*p* rounds).
- Every qubit has to complete all the ٠ gates of round *p* before being involved in *p*+1

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function (MAXCUT)













SWAPS can also be inserted as part of the UZZ interaction without the need to be sequential.





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Objective: finding the makespan-minimizing Gantt Schedule for p=1, p=2, N=8, N=21





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All actions of round 1 are completed – qubit can be mixed. Qubit 1 can start participating to round 2.







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Planning Domain Specification Language (PDDL)



Planning & Compilation



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LPG (Gerevini 2003): Local Search SGPlan (Wah 2004): Subproblem planning decomposition CPT (Vidal 2006): Partial-Order-Causal Link (POCL) TFD (Eyerich 2009): Fast-Forward Search (FFS) + postprocessing POPF (Coles 2010): POCL+FFS+Simple Temporal Network

Plan quality relative comparison

	p=1, N8		p=1, N21		p=2, N8	
Utilization u	0.9	1.0	0.9	1.0	0.9	1.0
LPG	0.88	0.89	0.91	0.87	0.50	0.52
TFD	0.87	0.87	0.95	0.90	0.99	0.99
SGPlan	0.67	0.68	0.64	0.67	0.75	0.79
POPF	0.81	0.81	0.90	0.94	0.87	0.91

(CPT is provably optimal but timing scales badly)





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https://ai.googleblog.com/2018/07/announcing-cirq-open-source-framework.html







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