Application of Rapid Distortion Theory to the Prediction of Integrated Propulsion System Noise

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- RANS Flow Solutions:
  - Rick Bozak
- Inlet Distortion, Fan Noise:
  - Ed Envia, Dale Van Zante

Motivation and Objectives

- Proposed aircraft propulsion systems may be tightly integrated into the airframe for performance benefits
- Integrated propulsion systems provide opportunities and challenges for noise
  - Use nearby surfaces for noise shielding
  - New sound sources
- Develop reduced-order prediction methods for flow-surface interaction noise to assess noise impacts of integrated propulsion

Approach: Rapid Distortion Theory (RDT)

- Linear analysis to study interaction of turbulence with solid surfaces
- Valid when turbulence intensity is small and the time scale for the interaction is short compared with eddy decay time
- Versions of RDT
  - Incompressible
  - Uniform mean flow
  - Potential mean flow
- For application to noise predictions in integrated propulsion systems
  - Compressible
  - Sheared Mean Flow
    - Goldstein (1979), Goldstein, Afsar and Leib (2013a,b), Goldstein, Leib and Afsar (2017)

RDT for a Transversely Sheared Mean Flow

$$\bar{\boldsymbol{u}} = U(\boldsymbol{y}_T)\hat{i} \;\; ; \; c^2 = \; c^2(\boldsymbol{y}_T) \qquad\qquad y = \{y_1, y_2, y_3\} = \{y_1, \boldsymbol{y}_T\}$$

General Solution to the Linearized Euler Equations

$$p'(\boldsymbol{x},t) = \int_{-T}^{T}\int_{V} G(\boldsymbol{y},\tau)\widetilde{\omega}_c\left(\tau - \frac{y_1}{U(\boldsymbol{y}_T)}, \boldsymbol{y}_T\right) d\boldsymbol{y}\, d\tau$$

Green’s Function: $L\, G(\boldsymbol{y},\tau|\boldsymbol{x},t) = \dfrac{D_0^3}{Dt^3}\delta(\boldsymbol{y}-\boldsymbol{x})\delta(\tau - t)$; $\dfrac{D_0}{D\tau} = \dfrac{\partial}{\partial t} + U(y_T)\dfrac{\partial}{\partial y_1}$

Rayleigh Operator: $L = \dfrac{D_0}{D\tau}\left(\dfrac{\partial}{\partial y_i}c^2\dfrac{\partial}{\partial y_i} - \dfrac{D_0^2}{D\tau^2}\right) - 2\dfrac{\partial U}{\partial y_i}\dfrac{\partial}{\partial y_1}c^2\dfrac{\partial}{\partial y_i}$

Arbitrary Convected Quantity – “ Gust:” $\widetilde{\omega}_c\left(\tau - \dfrac{y_1}{U(\boldsymbol{y}_T)}, \boldsymbol{y}_T\right)$

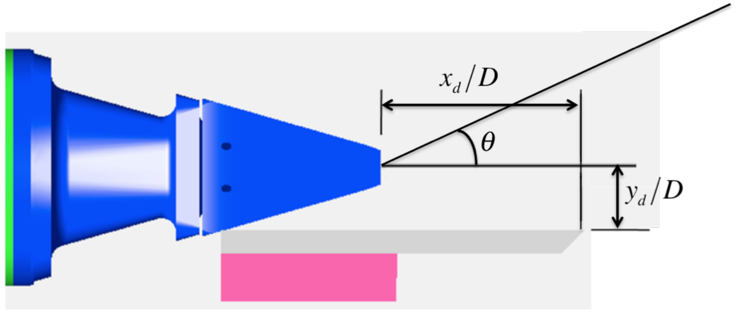
Applications of RDT for a Transversely Sheared Mean Flow to Integrated Propulsion System Noise

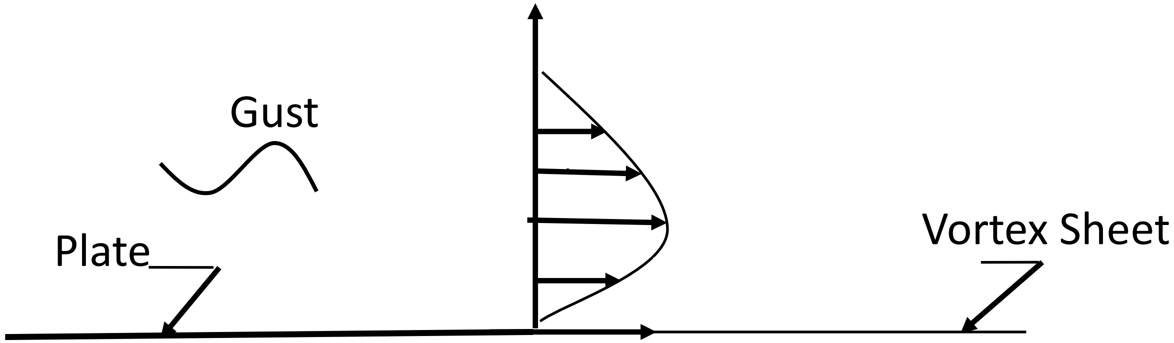
- Trailing-Edge Noise
  - Two-dimensional jet near a flat plate
    - Goldstein, Afsar and Leib (2013), Afsar, Leib and Bozak (2017), Goldstein, Leib and Afsar (2017)
  - Round jet near a flat plate
    - Current work in progress (Goldstein, Leib and Afsar)

- Inlet Turbulence Distortion and Noise
  - Two-dimensional model for turbulence distortion through a boundary-layer ingesting inlet
    - Current work in progress (Leib)

Trailing Edge Noise: Two-Dimensional Jet Near a Flat Plate

- Approximation for large aspect ratio rectangular jet near a flat plate





- Obtain solution for Green's function subject to appropriate boundary conditions
  - Low- / high-frequency solutions
- Derive formula for the acoustic spectrum in terms of spectrum of the "gust"
- Derive relation between "gust" and physically measurable quantities far upstream from the edge

Trailing Edge Noise: Two-Dimensional Jet Near a Flat Plate

- Explicit formula for the trailing-edge noise:

$$I_\omega(\boldsymbol{x}) = \left(\frac{k_\infty}{4\pi[\boldsymbol{x}]}\right)^2 (\beta - \cos\theta) \int_0^\infty \int_0^\infty \frac{[M(y_2)M(\tilde{y}_2)]^{3/2} Q(y_2|\theta,\varphi) Q^*(\tilde{y}_2|\theta,\varphi)}{[1 - M(y_2)\cos\theta][1 - M(\tilde{y}_2)\cos\theta]} \frac{S\left(y_2, \tilde{y}_2;\ k_3^{(s)}, \omega\right)}{\sqrt{1 - \beta M(y_2)}\sqrt{1 - \beta M(\tilde{y}_2)}} dy_2\, d\tilde{y}_2$$

- Source term:

$$S(y_2, \tilde{y}_2;\ k_3, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\omega\tau - k_3\eta_3)} \left\langle \widetilde{\omega}_c(t, \boldsymbol{y}_T) \widetilde{\omega}_c\big(t + \tau, \tilde{y}_{2,} y_3 + \eta_3\big)\right\rangle d\tau\, d\eta_3$$

- Relate arbitrary convected quantity (“gust”) to transverse velocity fluctuations far upstream:

$$S(y_2, \tilde{y}_2;\ k_3, \omega) = \frac{N_2 \widetilde{N}_2}{2\pi\omega^2} \left[\frac{\left({dU}/{dy_2}\right)^2 \left({dU}/{d\tilde{y}_2}\right)^2}{U^3(y_2) U^3(\tilde{y}_2)}\right] \times$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i\omega(\tau - [y_1/U(y_2) - \tilde{y}_1/U(\tilde{y}_2)] - k_3\eta_3)} \frac{\partial^4}{\partial\tau^4} \langle U_\perp(t - y_1/U(y_2), y_T) U_\perp(t + \tau - \tilde{y}_1/U(\tilde{y}_2), \tilde{y}_2, y_3 + \eta_3\,)\rangle\, d\tau\, d\eta_3$$

Two-Dimensional Jet Near a Flat Plate: Results GLA (2017)

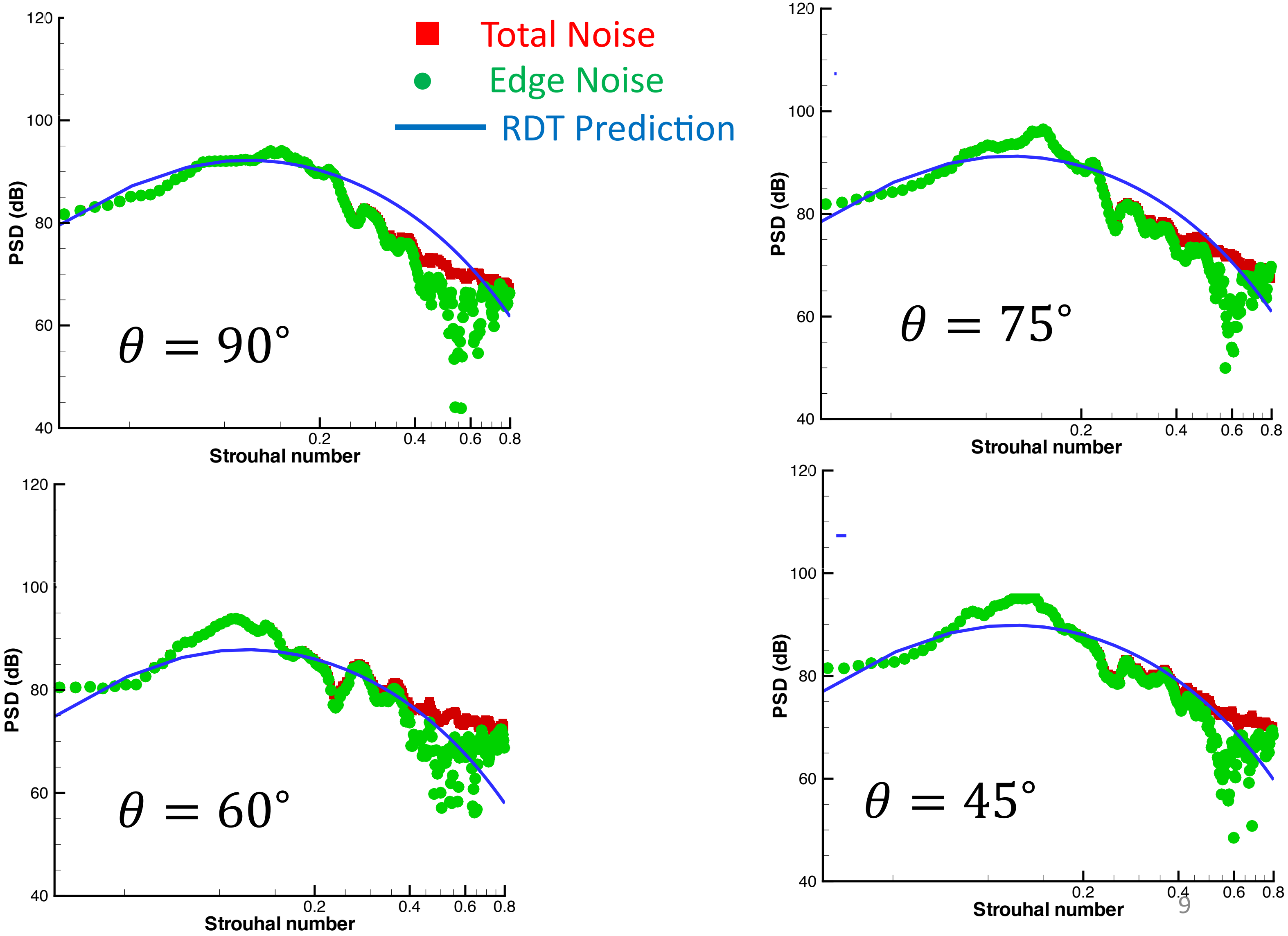
Comparisons of RDT-based predictions with SHJAR data

Aspect Ratio (AR) 8 nozzle

$$Ma = 0.5$$

$$\frac{(x_d, y_d)}{D} = (5.7, 0.98)$$

Observer in shielded location (below the jet)

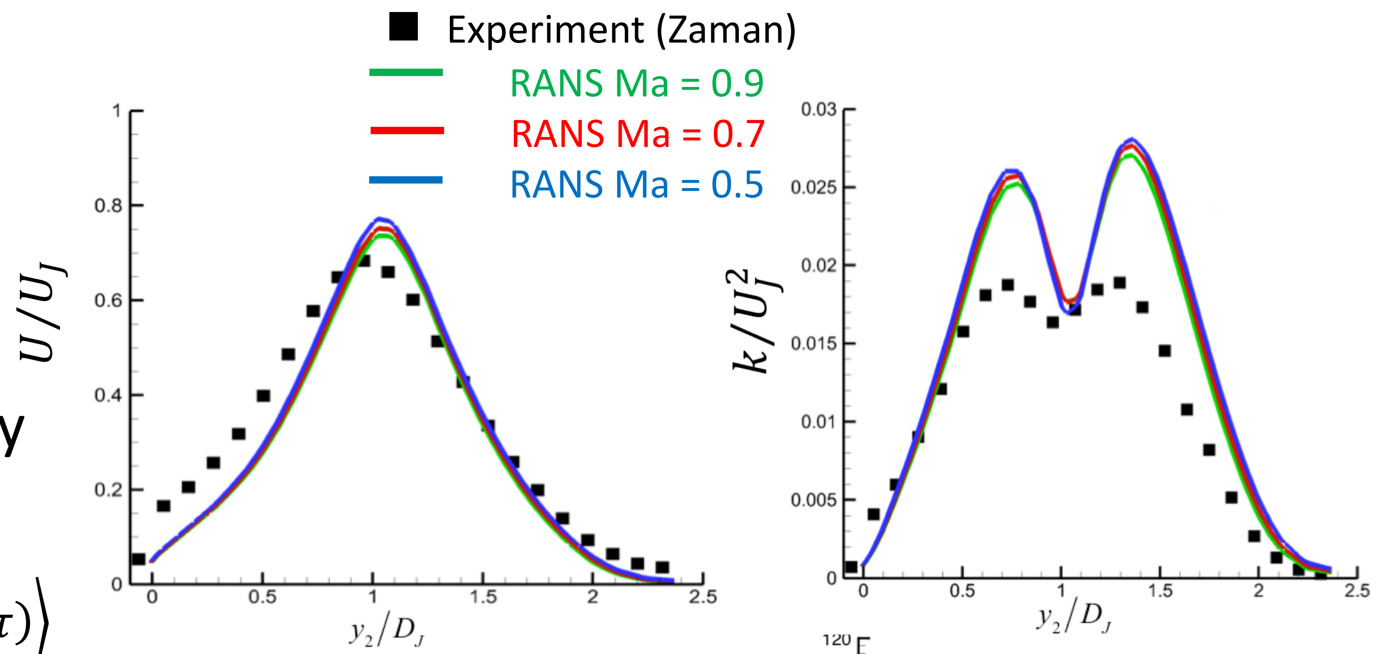


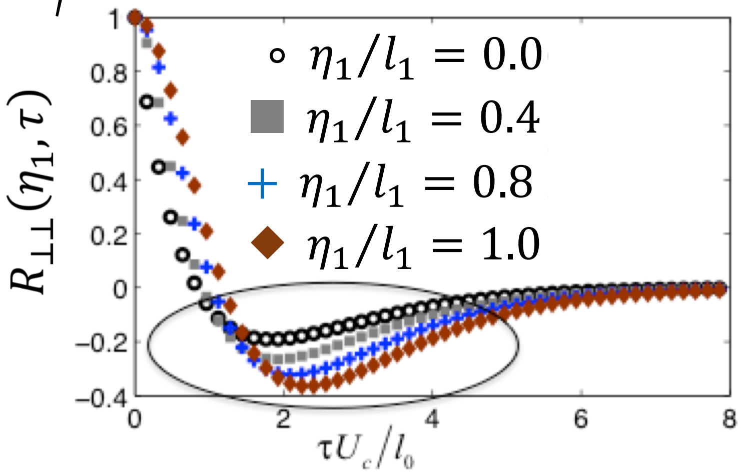
Two-Dimensional Jet Near a Flat Plate Results: RANS-Based Model ALB (2017)

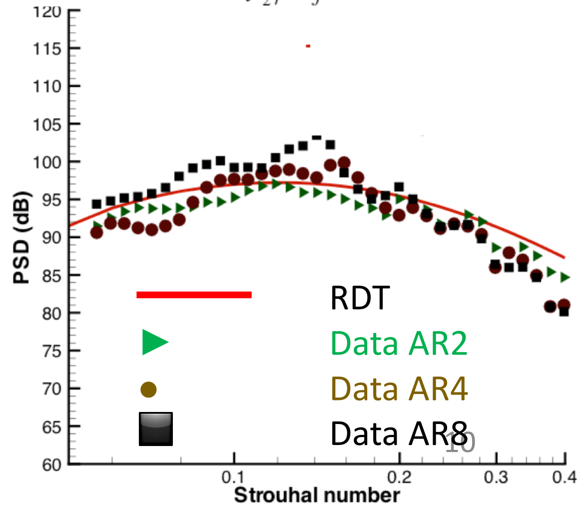
- Mean velocity and turbulence profiles computed from RANS
- Extended source model
  - Negative loops in transverse velocity correlations

$$R_{\perp\perp}(\eta_1,\tau) = \left\langle \rho v'^{(0)}_{\perp}(x,t)\rho v'^{(0)}_{\perp}(x_1+\eta_1 x, x_2, x_3, t+\tau)\right\rangle$$

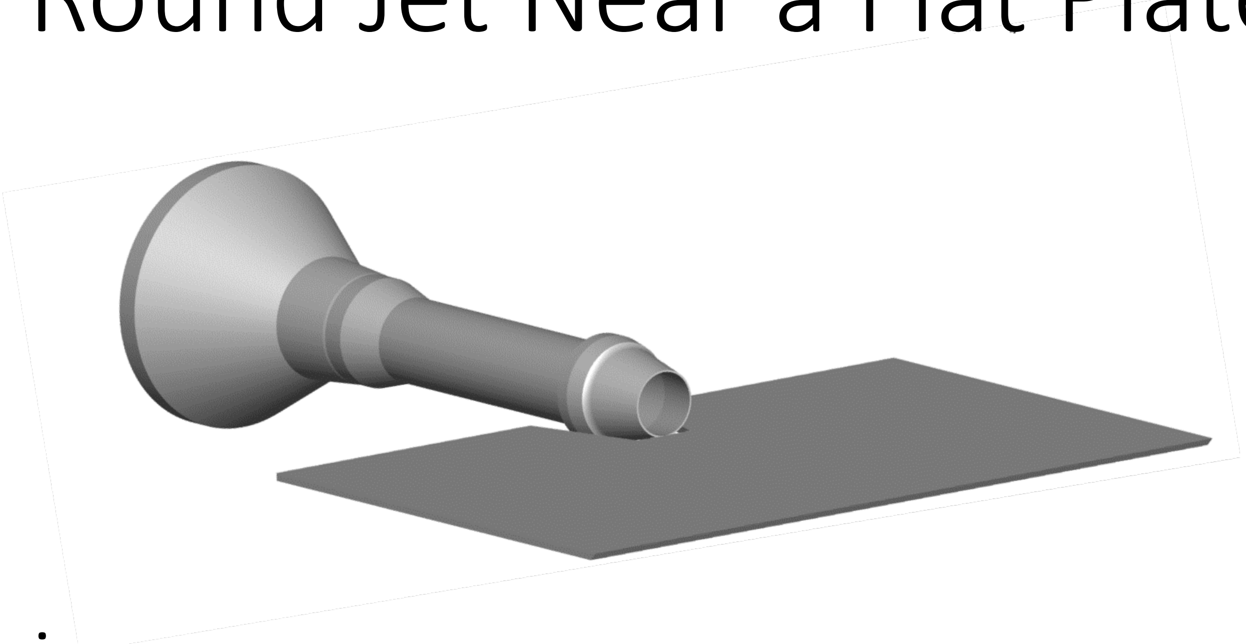
- Apply to smaller aspect ratio nozzles







Trailing Edge Noise: Round Jet Near a Flat Plate

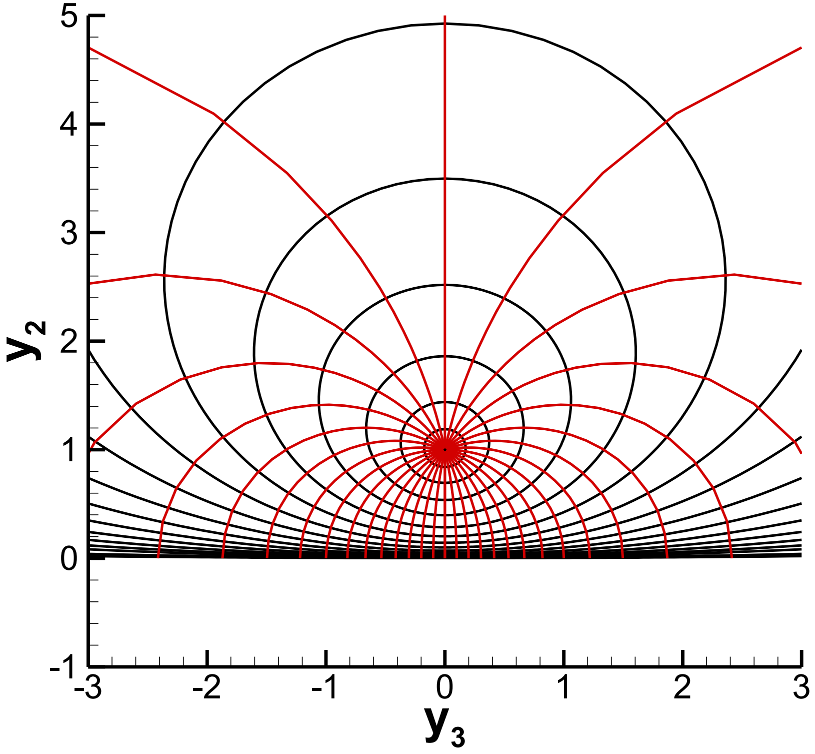


- Conformal mapping
- Low-frequency solution for Green's function
- Relation between "gust" and physical quantities similar to 2D problem
- Model for source similar to 2D problem

Trailing Edge Noise:
Round Jet Near a Flat Plate

Assume: contours of U and c^2coincide

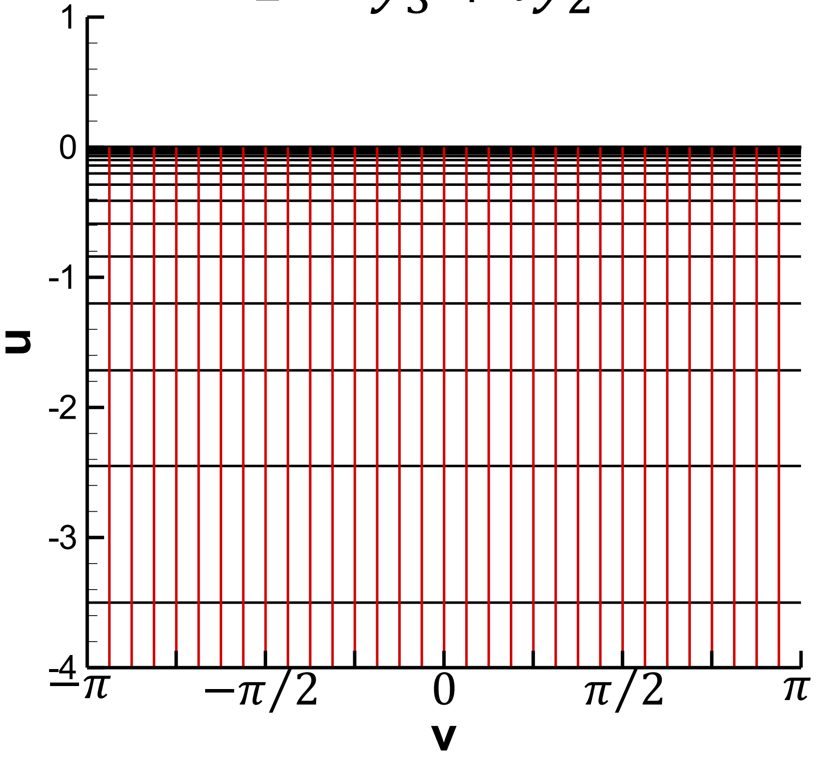
$$U = U(u) \quad ; \quad c^2 = c^2(u)$$



Conformal Mapping:

$$W(z) = u(y_3, y_2) + i\, v(y_3, y_2) = ln\frac{z-i}{z+i}$$

$$z = y_3 + iy_2$$



Trailing Edge Noise: Round Jet Near a Flat Plate

Formula for acoustic spectrum

$$I_\omega(x) = \left(\frac{k_\infty}{4\pi x}\right)^2 \int_{-\infty}^{0} \int_{-\infty}^{0} \frac{(\beta - cos\theta) M^{3/2}(u) M^{3/2}(\tilde{u}) \bar{S}(u, \tilde{u}; \omega)}{\sqrt{1-\beta M(u)}\sqrt{1-\beta M(\tilde{u})}[1 - M(u)cos\theta][1 - M(\tilde{u})cos\theta]} d\tilde{u}\, du$$

$$\bar{S}(u, \tilde{u}; \omega) \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} S(u, \tilde{u}, v, \tilde{v}) \left|\frac{dz}{dW}\right|^2 \left|\frac{d\tilde{z}}{d\widetilde{W}}\right|^2 d\tilde{v}\, dv$$

Spectrum of “gust”

$$S(u, \tilde{u}, v, \tilde{v};\ k_3, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\omega\tau - k_3\eta_3)} \left\langle \widetilde{\omega}_c(t, \boldsymbol{y}_T) \widetilde{\omega}_c\left(t + \tau, \tilde{y}_{2,} y_3 + \eta_3\right)\right\rangle d\tau\, d\eta_3$$

Model $S(u, \tilde{u}, v, \tilde{v};\ k_3, \omega)$ in terms of upstream transverse velocity fluctuation

Trailing Edge Noise: Round Jet Near a Flat Plate

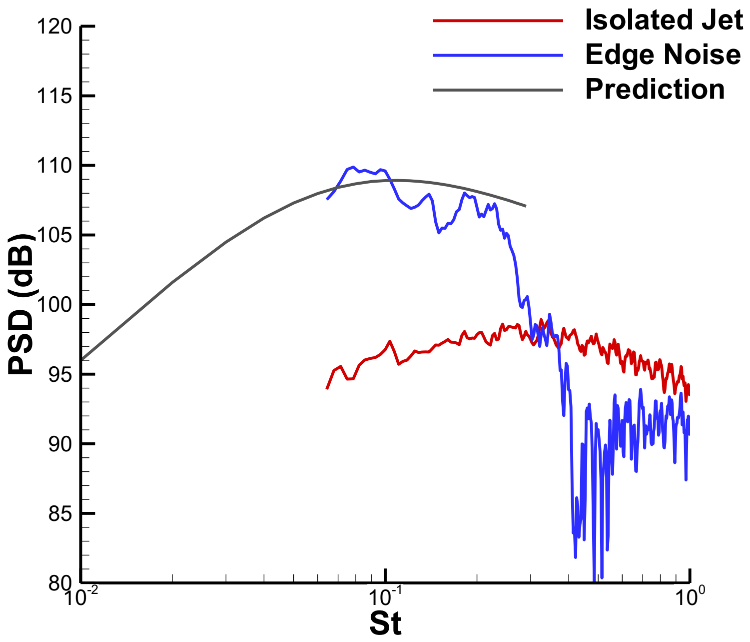
- Status:
  - Computer code for evaluation of edge noise spectrum
  - Models for mean velocity profile and turbulence
    - Obtain from experiment or RANS
  - Preliminary comparisons with SHJAR data (Brown, 2013)

Two-inch round nozzle

$$Ma = 0.5$$

$$\frac{(x_d, h)}{D} = (6.0,1.0)$$

Observer in shielded location (below the jet)



$$\theta = 90°$$

Distortion of Turbulence Through a Boundary-Layer Ingesting Inlet (BLI)

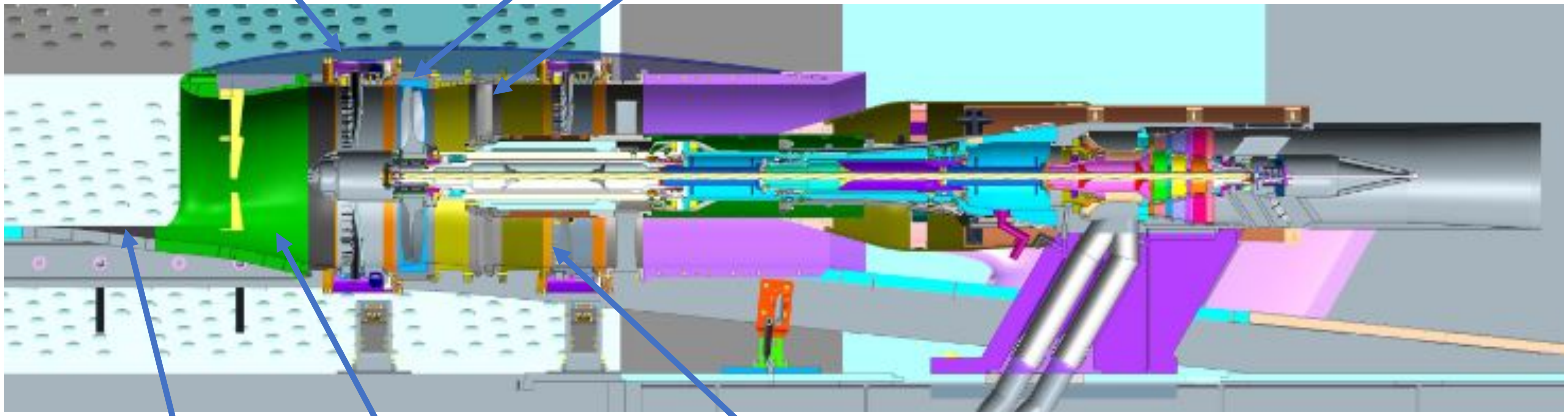
- Boundary-layer ingesting propulsion
  - Performance benefits of ingesting lower momentum fluid (boundary layer)
  - Little work done on noise impacts
- Objective:
  - Apply RDT to evaluate the distortion of an incident turbulent flow though a BLI
    - Apply general solution for fluctuating transverse velocity
    - Derive a relation between the statistics of the turbulence (spectrum) within the inlet (fan face) and those upstream
    - Use the distorted turbulence spectrum to evaluate pressure loading on fan
    - Compute noise

Boundary-Layer Ingesting Inlet-Fan Propulsor

Arend, *et al* AIAA 2017-5041

Aerodynamic Interface
Plane Rotating Rake Array

Distortion-Tolerant Fan
Stage: Rotor and EGV’s



Pre-Entry
Diffusion
Ramp

Boundary Layer
Ingesting Inlet

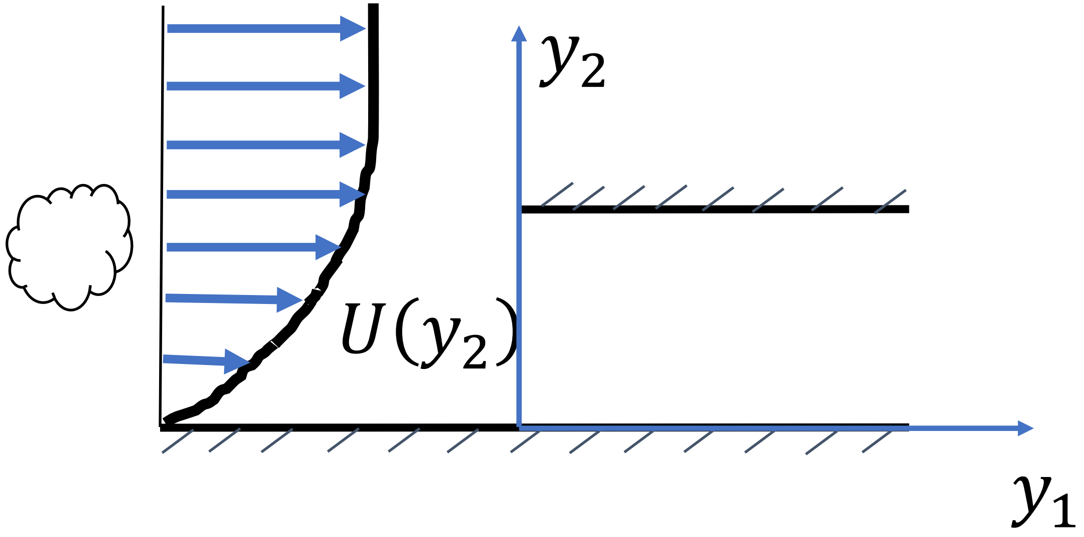
Fan Stage Exit
Rotating Rake
Array

Application of RDT to BLI
Two-Dimensional Model Problem for Analytical Treatment

- Green's function for Rayleigh equation

$$Lg(\boldsymbol{y},\tau|\boldsymbol{x},t) = \delta(\boldsymbol{y}-\boldsymbol{x})\delta(\tau-t)$$

$$L = \frac{D_0}{D\tau}\left(\frac{\partial}{\partial y_i}c^2\frac{\partial}{\partial y_i} - \frac{D_0^2}{D\tau^2}\right) - 2\frac{\partial U}{\partial y_i}\frac{\partial}{\partial y_1}c^2\frac{\partial}{\partial y_i}$$



- Boundary conditions on duct walls and continuity across upstream extension lead to Wiener-Hopf problem
- Further approximations:
    - Low-frequency solution
    - Piece-wise linear mean velocity profile

Transverse velocity spectrum

- Use Green's function in general solution for transverse velocity fluctuation

$$\rho v'_{\perp}(\boldsymbol{x},t) = u_{\perp}(\boldsymbol{x},t) = -\frac{\partial U/\partial x_i}{|\nabla U|}\int_{-T}^{T}\int_V g_i(\boldsymbol{y},\tau|\boldsymbol{x},t)\ \widetilde{\omega_c}\left(\tau - \frac{y_1}{U(\boldsymbol{y_T})}, \boldsymbol{y}_T\right) d\boldsymbol{y} d\tau,$$

$$g_i(\boldsymbol{y},\tau|\boldsymbol{x},t) = \frac{D_0}{Dt}\left(\frac{\partial}{\partial x_i}\frac{D_0}{Dt} + 2\frac{\partial U}{\partial x_i}\frac{\partial}{\partial x_1}\right) g(\boldsymbol{y},\tau|\boldsymbol{x},t).$$

- Relation between transverse velocity spectrum within the duct and that upstream

$$S^*_{\perp\perp}(x_1,x_2,\tilde{x}_2;\omega,k_3) = \int\limits_{l_T}\int\limits_{l_T} e^{i\omega x_1[1/U(y_2) - 1/U(\tilde{y}_2)]}\mathcal{G}\left(y_2|x_1 x_2;\omega, {}^{\omega}/_{U(\tilde{y}_2)}, k_3\right)\mathcal{G}\left(\tilde{y}_2|x_1,\tilde{x}_2;\omega, {}^{\omega}/_{U(\tilde{y}_2)}, k_3\right)\times$$

$$S_{22}(y_2,\tilde{y}_2;\omega,k_3)\, d\tilde{y}_2\, dy_2$$

- Spectra:

Within inlet:

$$S_{\perp\perp}(x_1,x_2,\tilde{x}_2;\omega,k_3) = \frac{1}{(2\pi)^2}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} e^{i(\omega\tau - k_3\eta_3)}\langle\rho v'_{\perp}(\boldsymbol{x},t)\rho v'_{\perp}(x_1,\tilde{x}_2,x_3+\eta_3,\tau,t+\tau)\rangle\ d\tau d\eta_3$$

Upstream of Inlet:

$$S_{22}(y_2,\tilde{y}_2;\omega,k_3) = \frac{1}{(2\pi)^2}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} e^{i(\omega\tau - k_3\eta_3)}\langle\mathcal{U}_{\perp}(t,\boldsymbol{y}_T)\mathcal{U}_{\perp}(t+\tau,\tilde{y}_2,y_3+\eta_3)\rangle\, d\tau\, dy_3$$

Distortion of Turbulence Through a Boundary-Layer Ingesting Inlet: Status and Plans

- Current Status:
  - Formal solution to the Wiener-Hopf problem for the Green's function in the 2D model problem
  - Low-frequency approximation to Green's function
    - Details of WH solution for low-frequency solution – split functions
- Future work – short-term:
  - Incorporate low-frequency solution into formula for transverse velocity spectrum
  - Numerical evaluation of the distorted turbulence spectrum
- Future work – longer-term:
  - Evaluate fan loading due to distorted turbulence
  - Predict noise
  - Use conformal mapping to extend to realistic inlet geometries

THE END