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# SPACE LAUNCH SYSTEM

## Efficient On-Orbit Singularity-Free Geopotential Estimation

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# Overview

- The complexity of the geopotential model can heavily impact the navigation error in satellites and spacecraft
- Geopotential models of the accuracy needed for spaceflight are too complicated for flight computers to run at the rate needed by the navigation system
- There are methods to make the geopotential model more efficient while maintaining the needed accuracy, which include:
  - Using an efficient method for the full model
  - Propagating to avoid singularities
  - Running the full model at a low rate and propagating to the needed rate
- These methods can decrease the computational requirement enough to be run by the flight computer at the rate required of the navigation system

# Geopotential Model

- The simplest gravity model assumes that the body is a point mass located at its center of gravity or a perfect sphere
- Acceleration in this model is given by the equation

$$\ddot{\boldsymbol{r}} = -\mu \frac{\boldsymbol{r}}{r^3}$$

- Where:
  - r is the vector from the spacecraft to the body
  - μ is the gravitational parameter
- This is the first order term in higher order models, and is the dominant term of any gravity model, becoming more dominant for higher derivatives

# Higher Order Geopotential Models

- **Higher order gravity models have the form**

$$V = \frac{\mu}{r}\left[\sum_{n=0}^{n_{max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\,\bar{P}_{nm}(\sin\phi)\right]$$

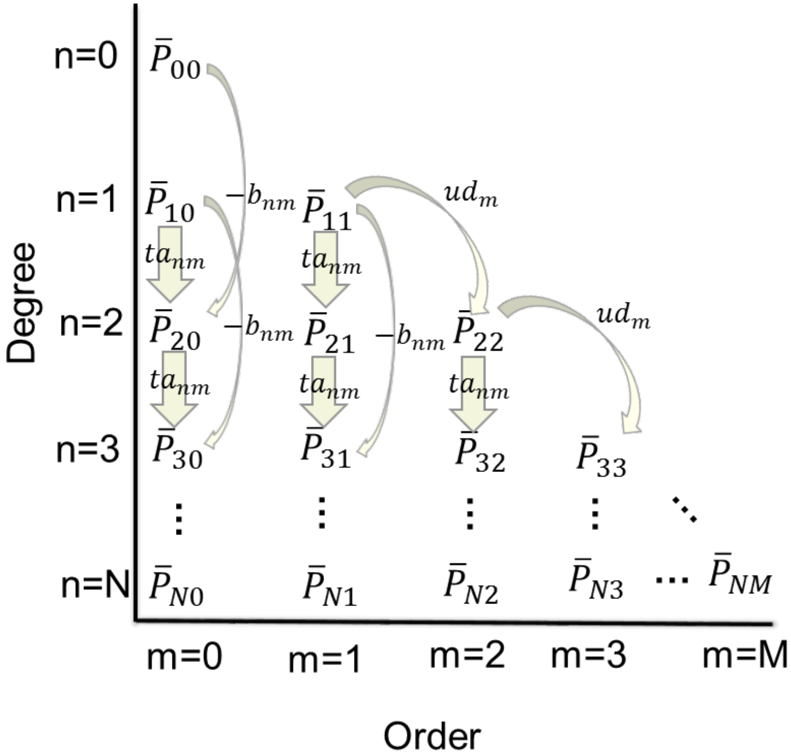
- **Where:**
  - $V$ is the gravitational potential
  - $\phi$ is the geocentric latitude
  - $\lambda$ is the geodetic longitude
  - $a$ is the semi-major axis
  - $\mu$ is the gravitational parameter
  - $\bar{C}_{nm}$, $\bar{S}_{nm}$ are the fully normalized gravitational coefficients given by the model
  - $\bar{P}_{nm}$ is the fully normalized associated Legendre function
- **Non-Normalized or Quasi-Normalized coefficients and Legendre functions can also be used**
- **The first derivative of the potential is acceleration**
- **The second derivative is the Jacobian of the acceleration, or the partial matrix, which is used in the navigation filter**
- **The third derivatives are the Hessian matrixes of acceleration**

# Geopotential Implementation Methods

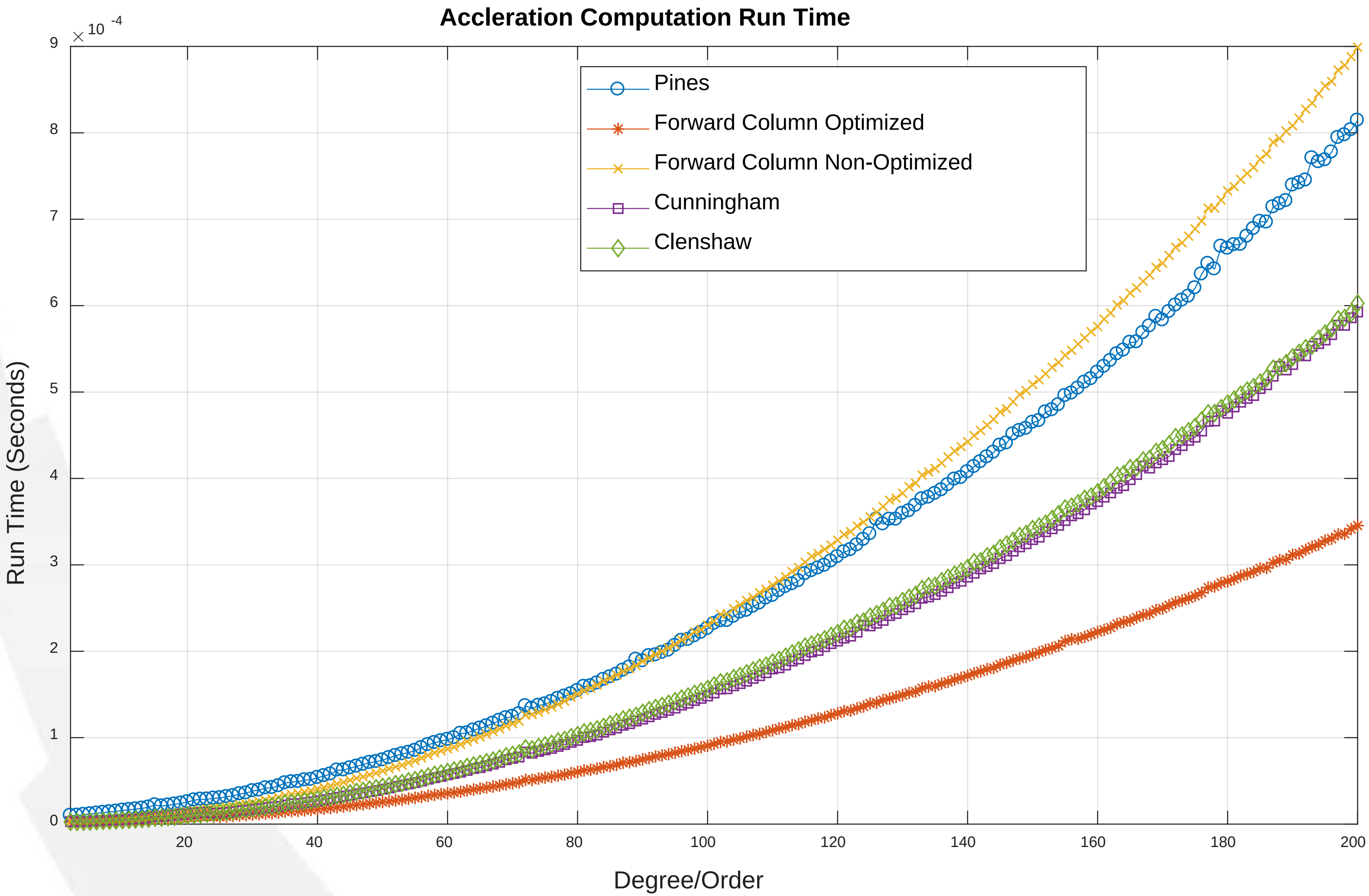
- **There are numerous methods to implement the spherical harmonic gravity equation, four are summarized below:**
- **Forward Column (Legendre)**
  - Uses stable forward column recursion for the Associated Legendre Functions (ALF)
  - Includes singularity at the poles above a latitude of ±89.999999° which can be eliminated through propagation
- **Clenshaw**
  - Revision of Legendre method based on the Clenshaw approach to the summation of products of functions obeying a three-term recurrence relation
  - Includes the same singularity as the Forward Column method
- **Pines**
  - Derived Legendre Polynomials are used instead of ALFs in order to get rid of any singularity, at the cost of using a redundant set of variables
- **Cunningham**
  - Uses forward recursion on a certain differential operator to avoid any singularities, at the cost of using a redundant set of variables

# Forward Column Method

- The forward column method has been shown to be among the most efficient methods of calculating the geopotential
- The associated Legendre Function is calculated using recursion, going forward in the column
- This includes a singularity at the poles, which must be calculated separately
- This method must also be optimized to work efficiently



# Comparison of Geopotential Implementation Methods



# Geopotential Propagation

- Even when using an efficient method of calculating the geopotential, it can have too large of a computational requirement for the flight computer
- One method to lower the computational requirement is to run the full geopotential model at a lower rate, for example 1 Hz, then propagate at a higher rate, for example 100 Hz, between full model runs
- There are multiple options to propagate, including first order, second order, or three-step propagation

# First Order Propagation

- **In order to calculate gravity at a high rate, it can be estimated through a first order expansion, which has the form**

$$g(\mathbf{r}) \approx \mathbf{g}(\mathbf{r}^*) + \mathbf{G}(\mathbf{r}^*)[\mathbf{r} - \mathbf{r}^*]$$

- **Where:**
  - $\mathbf{g}(\mathbf{r})$ is the gravity at location $\mathbf{r}$ (current location)
  - $\mathbf{g}(\mathbf{r}^*)$ is the gravity at location $\mathbf{r}^*$(location where full gravity model was calculated)
  - $\mathbf{G}(\mathbf{r}^*)$ is the partial derivative matrix at location $\mathbf{r}^*$
- **This method does not update the gravity partial matrix, which is only updated during the full gravity call**
- **On Orion EFT1, the full gravity model was called at a 1 Hz rate, resulting in being almost 2 seconds old when used, and propagated at 40 Hz using a first order expansion**
- **If higher accuracy, increased propagation time, or update of the partial matrix is required, a second order approximation can be used**

# Second Order Geopotential Propagation

- **The second order Taylor series can be approximated as**

$$\boldsymbol{g}(\boldsymbol{r}) \approx \mathbf{g}(\mathbf{r}^*) + (\mathbf{G}(\mathbf{r}^*) + \boldsymbol{\Delta}\mathbf{G})[\boldsymbol{r} - \mathbf{r}^*]$$

- **Where: $\Delta\mathbf{G}$ is the change in the G matrix**
- **$\Delta\mathbf{G}$ can be approximated as:**

$$\boldsymbol{\Delta}\mathbf{G} = \frac{\mathbf{G}_{\boldsymbol{PM}}(\boldsymbol{r}) - \mathbf{G}_{\boldsymbol{PM}}(\mathbf{r}^*)}{2}$$

- **Where $\mathbf{G}_{\boldsymbol{PM}}(\boldsymbol{r})$ is the first order (point mass) gravity partial matrix at point r, and $\mathbf{G}_{\boldsymbol{PM}}(\mathbf{r}^*)$ is the first order gravity partial matrix at $\mathbf{r}^*$ which is defined as**

$$\mathbf{G}_{\boldsymbol{PM}} = \frac{\mu}{r^5}\begin{bmatrix} 3x^2 - r^2 & 3xy & 3xz \\ 3xy & 3y^2 - r^2 & 3yz \\ 3xz & 3yz & 3z^2 - r^2 \end{bmatrix}$$

# Second Order Geopotential Propagation

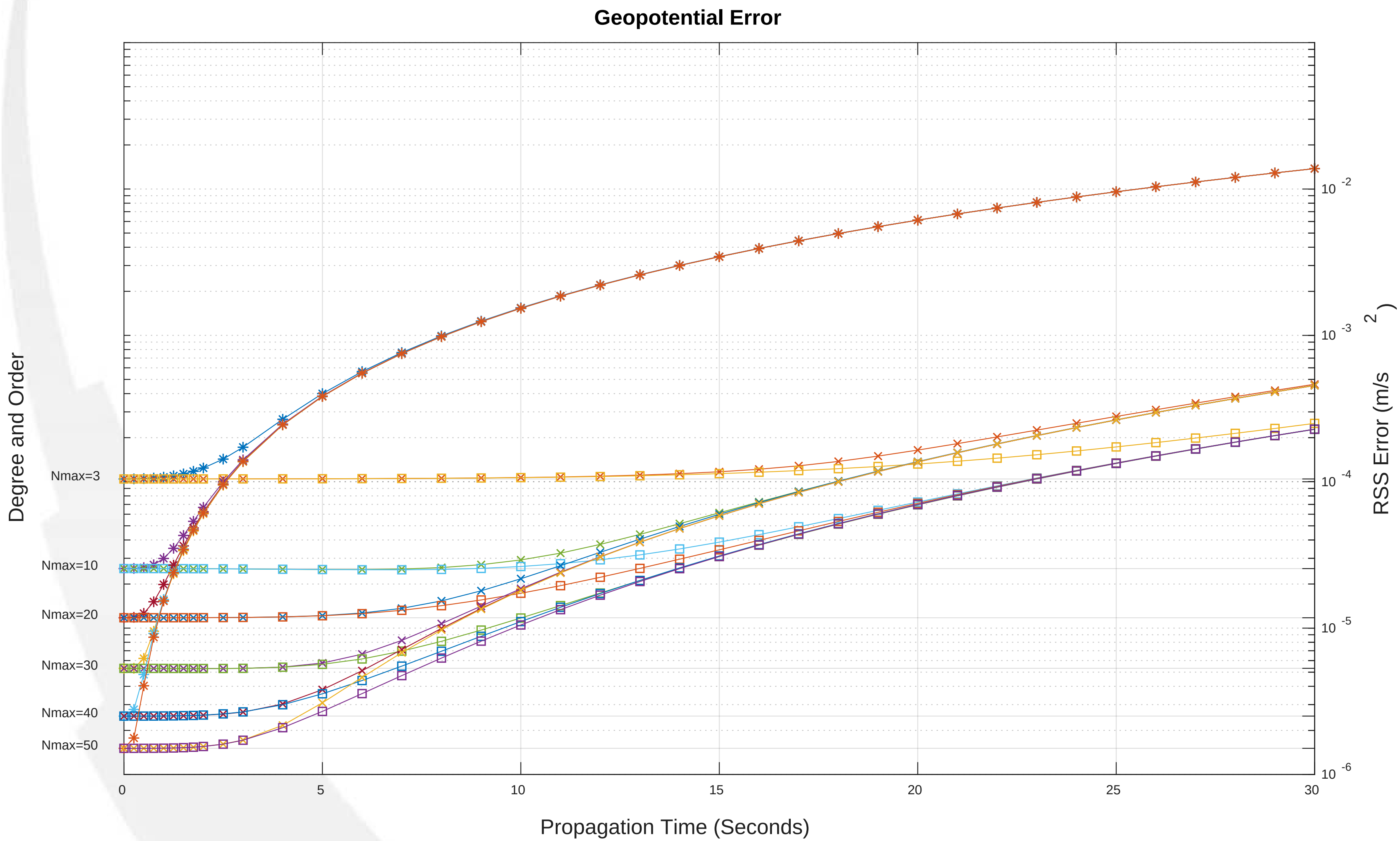
- **Instead of needing to calculate the point mass gravity gradient matrix at each G update, the second order Taylor series expansion of gravity can be calculated by**

$$\boldsymbol{g}(\boldsymbol{r}) \approx \mathbf{g}(\mathbf{r}^*) + \mathbf{G}(\mathbf{r}^*)[\boldsymbol{r}-\mathbf{r}^*] + \frac{1}{2}\begin{bmatrix}[\boldsymbol{r}-\mathbf{r}^*]'\boldsymbol{H}_x(\mathbf{r}^*)[\boldsymbol{r}-\mathbf{r}^*] \\ [\boldsymbol{r}-\mathbf{r}^*]'\boldsymbol{H}_y(\mathbf{r}^*)[\boldsymbol{r}-\mathbf{r}^*] \\ [\boldsymbol{r}-\mathbf{r}^*]'\boldsymbol{H}_z(\mathbf{r}^*)[\boldsymbol{r}-\mathbf{r}^*]\end{bmatrix}$$

- **Where: $\mathrm{H}_i(\mathbf{r}^*)$ are the 3x3 Hessian matrixes of the acceleration components at $\mathbf{r}^*$. The *G* matrix at *r* can be updated as**

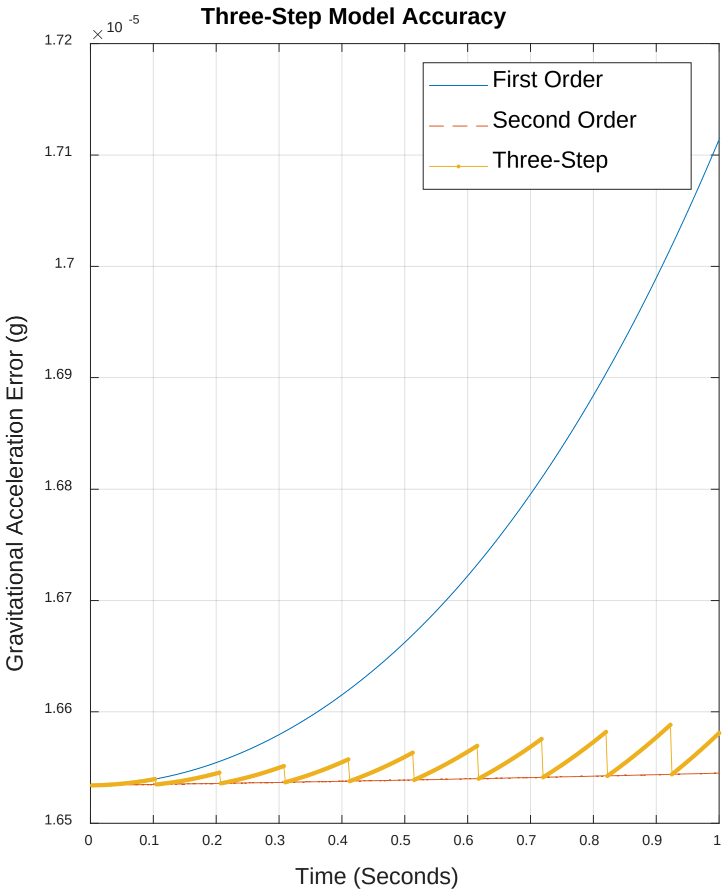
$$\mathbf{G}(\boldsymbol{r}) \approx \mathbf{G}(\mathbf{r}^*) + \frac{1}{2}\begin{bmatrix}[\boldsymbol{r}-\mathbf{r}^*]'\boldsymbol{H}_x(\mathbf{r}^*) \\ [\boldsymbol{r}-\mathbf{r}^*]'\boldsymbol{H}_y(\mathbf{r}^*) \\ [\boldsymbol{r}-\mathbf{r}^*]'\boldsymbol{H}_z(\mathbf{r}^*)\end{bmatrix}$$

# Propagation Accuracy



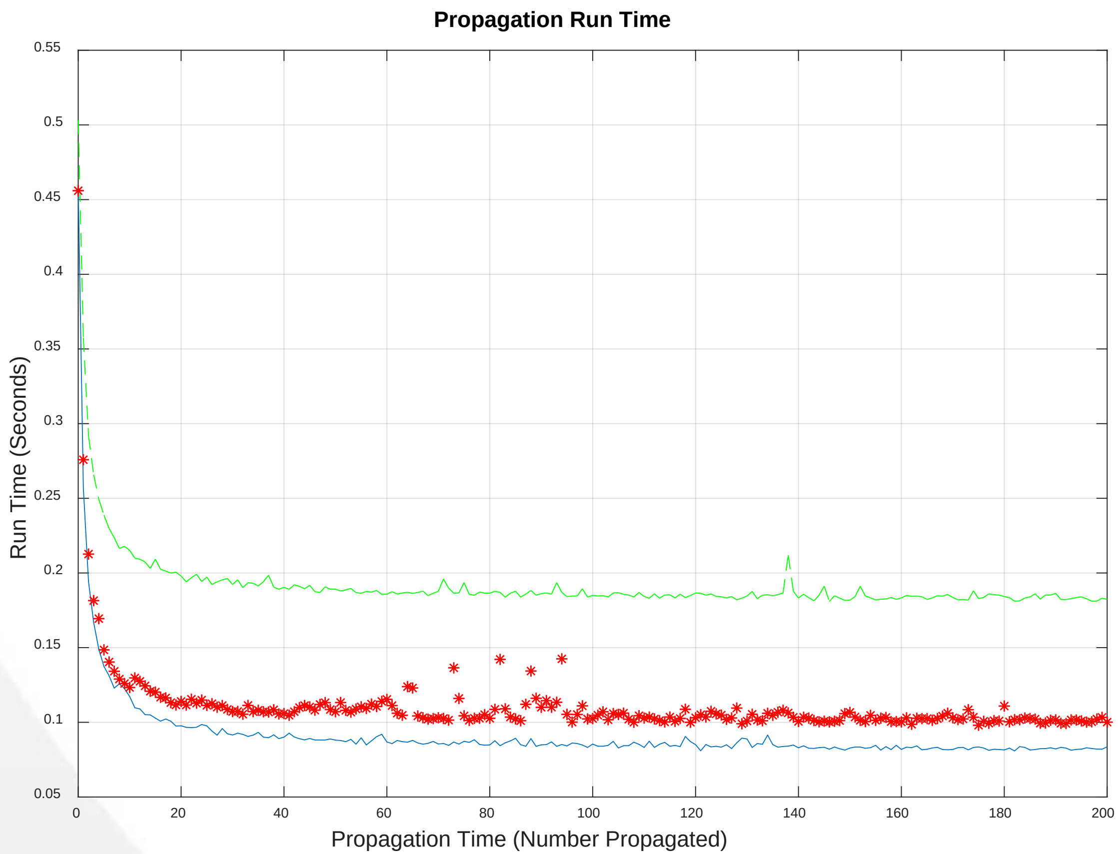
# Three-Step Geopotential Propagation

- The first order and second order methods can be combined to result in a method more computationally efficient than the second order method and more accurate than the first order method
- At a low rate, for example 1 Hz, the full geopotential model is calculated
- At a high rate, for example 100 Hz, a first order model is used to propagate the geopotential to the current location
- At a medium rate, for example 10 Hz, a second order model is used to update the geopotential



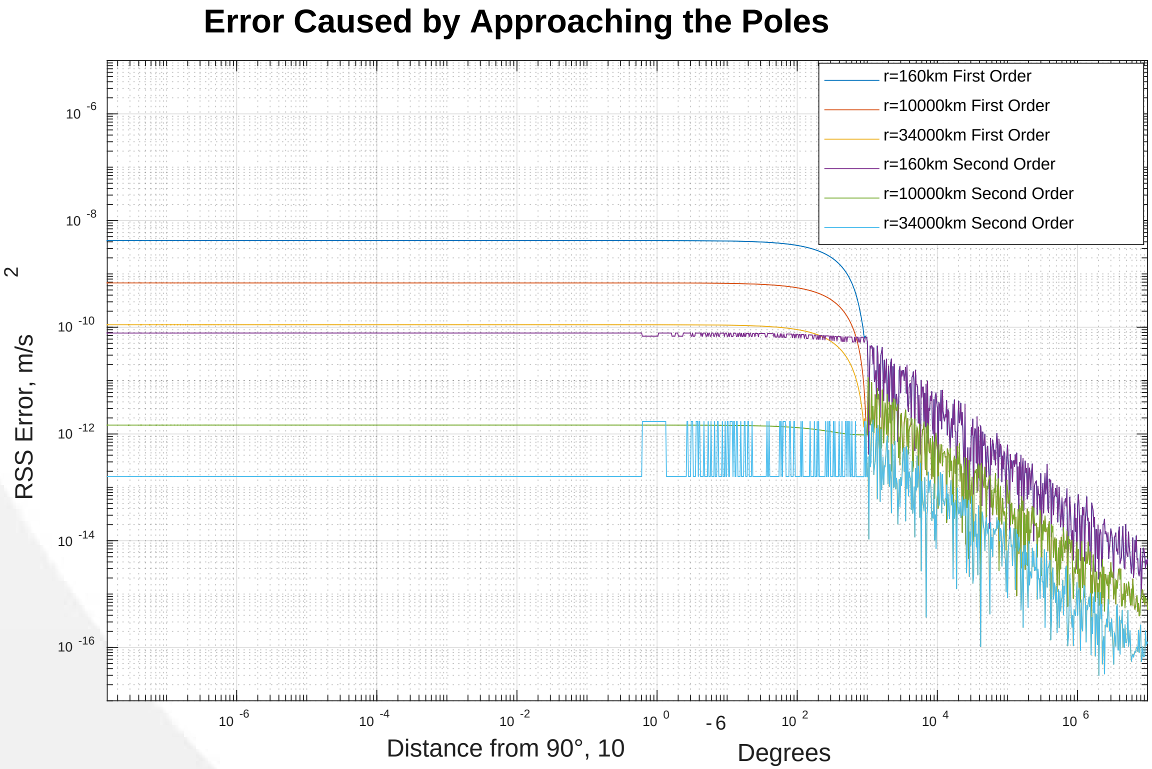
# Propagation Computational Efficiency

- The longer the propagation interval, the lower the effect on the runtime that increasing the propagation interval will have, approaching the computation requirement as the propagation method



# Singularity Avoidance

- Instead of using redundant variables to avoid dividing by zero, a first or second order propagation can be used to avoid any singularities when using the Forward Column or the Clenshaw methods
- This results in maintaining accuracy while avoiding any singularities, while maintaining the increase in computation efficiency from not using redundant variables



# Conclusion

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